Football Optimization Algorithm (FbOA): A Novel Metaheuristic Inspired by Team Strategy Dynamics

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Abstract

The Football Optimization Algorithm (FbOA) is introduced as a novel population-based metaheuristic optimization technique inspired by the dynamic strategies of a football team. Designed to address complex optimization problems characterized by high dimensionality, nonlinearity, and multiple local optima, FbOA draws on the strategic balance between exploration and exploitation observed in football gameplay. The algorithm mimics players' tactical positioning and movement, incorporating short passes, long passes, and positional adjustments to explore and exploit the solution space effectively. This study comprehensively evaluates the performance of FbOA using benchmark functions from the CEC 2005 test suite with 30-dimensional and 100-dimensional optimization problems. The results demonstrate that FbOA outperforms several state-of-the-art metaheuristic algorithms regarding convergence speed, accuracy, and robustness. The findings suggest that FbOA offers a promising alternative for solving various optimization challenges across multiple fields.

Keywords: Football Optimization Algorithm, Metaheuristic Optimization, Population-based Algorithm, Complex Problem Solving, Team Strategy Dynamics

1 introduction

Metaheuristic optimization algorithms are now recognized as efficient techniques for handling significant, complex optimization problems that other means cannot otherwise solve. These algorithms are intended to

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search for a solution in an ample solution space, to provide solutions that are optimal or nearly optimal in the least possible time, which makes it valuable in many areas of engineering, computer science, economics, logistics [1–3]. In recent decades, numerous metaheuristic optimization algorithms have been proposed based on natural, biological or social inspiration. Some of these are known as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), and Grey Wolf Optimizer (GWO); all of them encompass different methods for handling optimization problems [4–6].

Metaheuristic algorithms can be broadly classified into one-solution-based and population-based strategies. There are two different classes for all these algorithms: a single solution-based, such as Simulated Annealing (SA), and population-based, such as Genetic Algorithm (GA) and Particle Swarm Optimization (SO); while for population-based algorithms, there are multiple candidate solutions, known as ‘’gents”exploring towards the solution space [7–9]. The population-based algorithms have been proven to give considerable results because of their heuristics that provide both exploration (global search) and exploitation (local search) characteristics, which makes them helpful in beating the other algorithms by giving a broad perspective of the solution space [10–12].

Due to the diverse and versatile nature of metaheuristic algorithms, there are ongoing attempts to design new optimization algorithms that imitate numerous natural and artificial phenomena. This is a continuous process because we must deal with new varieties of optimization difficulties characterized by high dimensions, nonlinearity, multiple optima, and other emerging features. As a part of this endeavor, we present the Football Optimization Algorithm (FbOA), a population-based metaheuristic developed based on components of football/soccer.

Football is a team game, and because of the players' strategic features, flexibility and ability to cooperate, this type of game is an excellent source of motivation for creating new heuristics for optimization algorithms. Soccer players must constantly adapt to current and future targets and goals during a match, including maintaining possession, moving forward into attacking positions, and preventing the opponent's advances towards their goal. These actions require not only individual decision-making but also may require teamwork, where the work is done in parallel and entails both exploration, that is, search for new opportunities, and exploitation, that is, development of opportunities that are currently available in a given location or context. This balance is essential in optimization problems where the search needs to be exhaustive while enabling sufficient exploitation of prospective sub-spaces [13–15].

The proposed Football Optimization Algorithm (FbOA) uses these concepts to build an optimized structure for efficiently solving optimization challenges. It replicates the system and agency of a football team in which every individual is an agent in the decision-making process. It means the allocation of the position and their activities within the search space changes over time, as it is similar to positions and actions on the football field. Incorporating competitive and cooperative aspects observed in the football game into FbOA can provide the needed control over exploration and exploitation, thus improving its ability to identify high-quality solutions to optimization problems.

This paper proposes a new member of the metaheuristic optimization arsenal, the Football Optimization Algorithm (FbOA). We explain its theoretical and mathematical premises, mathematical structure, and real-mode functioning, coupled with detailed results of its performance on a range of optimization problems and benchmark tests. The findings suggest that FbOA is a novel and efficient replacement for classical metaheuristic methods and asymptotically or even outperforms existing metaheuristic algorithms regarding convergence speed, solution quality, and robustness when solving different problems.

2 Literature Review

As the world advances, those optimization problems consistently increase in difficulty. Consequently, deterministic and heuristic management approaches cannot solve such elaborate issues. Metaheuristics have lately been established as a reasonable solution to optimize the mentioned problems. [16] formulates the YDSE as a new metaheuristic procedure based on a physical paradigm, therefore initiating this paper with an explanation of the YDSE Optimizer. YDSE optimizer is a boolean optimizer based on one of the most famous classical physics experiments: A double-slit experiment showing light's wave nature. In the YDSE optimizer,

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the fringes stand for one population, and every fringe represents one possible solution. This experiment also produces many concepts that are being modeled, including monochromatic light waves, principle, constructive and destructive interference, wave intensity, amplitude, and path difference. The YDSE optimizer maintains a good balance between exploration and exploitation: it selects either constructive or destructive interference according to the order number of the fringe. When optimizing, the solution transforms in the area according to the number of orders. If the solution has an odd number, then it converges to the dark region toward the bright region in the middle of the plane, it supposed to contain the best solution. The algorithm utilizes the promising areas at the bright fringe areas assumed to contain the optimum. The performance of the proposed YDSE optimizer compared to another twelve metaheuristic algorithms using benchmarks CEC 2014, CEC 2017, and CEC 2022. The comparisons are made concerning various unimodal, multimodal, hybrid and composite test functions. We also discuss ten convergence and divergence engineering optimization design problems, five constrained and five unconstrained. YDSE obtained a notable accuracy, making it superior to the winning CEC 2014 and CEC 2017 algorithms – such as L-SHADE, LSHADE-cnEpSin, and LSHADE-SPACMA. The results and statistical analysis showed that the proposed YDSE optimizer was superior to the other optimizer at a 95% confidence interval.

Drawing from the hunting behaviors of cheetahs, [17] presents a metaheuristic algorithm referred to as the cheetah optimizer (CO). Therefore, cheetahs mostly employ three primary methods of hunting, namely stalking, after which they proceed to sit and wait for their prey and, finally, launch an attack. It must be said that these strategies are employed in this work. Furthermore, some specific hunting techniques are also adopted, including leaving the prey and returning home, adding to the proposed framework for enhancing population diversification, convergence performance, and overall robustness. To assess the effectiveness of the proposed CO, comprehensive testing is carried out on fourteen shifted-rotated CEC-2005 benchmark functions using the Particle Swarm Optimization, Genetic Algorithm, Differential Evolution, and Differential Evolution with Self-adapted Parameters algorithms as a benchmark. Furthermore, to evaluate the efficiency of the proposed CO algorithm over large-scale optimization problems, the CEC2010 and the CEC2013 benchmarks are introduced. The proposed algorithm is also checked when solving one of the famous and challenging engineering problems, for example, the economic load dispatch problem. In all considered problems, the outcomes are demonstrated to excel the benchmark values furnished by other standard and advanced algorithms. The simulation results prove that the CO algorithm is effective for large-scale and complex optimization tasks, has a significant advantage over different standards, and is improved and combined with existing algorithms.

In [18], a new metaheuristic algorithm called Energy Valley Optimizer (EVO) is introduced based on advanced physics principles about stability and particle decay modes. Twenty unconstrained test functions of mathematical optimization are employed in various dimensions to assess the effectiveness of the proposed algorithm. One hundred independent optimization runs are performed to calculate the statistical measurements using the defined stopping criterion to capture the mean, standard deviation, and the number of objective function evaluations required by the optimization processes. The other more known statistical tests used for comparison include the Kolmogorov–Smirnov, Wilcoxon, and Kruskal–Wallis tests. In addition, the most recent CEC on real-world optimization is also used to compare the performance of the EVO with the best existing algorithms in terms of state-of-the-art. The findings also show that the presented algorithm can offer competitive or even better performance compared to existing benchmarks and solve various real-world tasks.

In extreme conditions, such as geographic conditions that do not allow manned flight, the effectiveness of the UAV in executing the payload hold–release mission through obstacle avoidance is principally determined by the navigation, path planning, and tracking of the UAV. The proposed algorithm deals with the flight parameters of the UAV and makes it possible for the UAV to optimize the path in that it can plan and track the path in minimum energy and time. [19] studies the optimum path planning and tracking by utilizing the Harris hawk optimization (HHO) – grey wolf optimization (GWO) to allow the UAV to complete the payload hold–release mission while avoiding obstacles. In the study, the selected and special algorithm called hybrid HHO–GWO is used to reach the practical and feasible path as it avoids local minima and converges quickly. Further, the influence of uncertainty on the mass change of the UAV on path planning and envisaged tracking functionality is ascertained. The usefulness of the presented strategy is examined in the metaheuristic swarm optimization algorithms, including the PSO and GWO. Based on the obtained experimental results, it can be stated that the proposed algorithm provides a fast and safe path without being trapped in local minima and follows the generated path with minimum energy and time consumption.

Across the globe, the energy demand is becoming more pressing every day. Concerning the development of renewable energy for the following years, it is expected that renewable energy resources and fuel cells, solid

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wastes and hydrogen energy technologies will have an increasing share in energy generation and storage due to their decreasing installation costs and increasing environmental impacts. Fuel cells use potential in-fee hydrogen fuel that is free from carbon. The studied technology is based on the techno-economic analysis of the off-grid WT-PV-BG-FC system, which entails storing excess WT and PV energy in hydrogen that powers the fuel cell in off-grid applications. From the feasibility perspective, decision-making regarding new goods or modifications to existing ones is essential. Besides electricity generation, energy storage, aspects of sizing, energy management, and the associated optimization methodologies and algorithms are also included. Essentially, the annual cost system (ACS) is the leading solution that has to be developed throughout the optimization in this case. An effective goal-oriented rule-based energy management scheme is developed to control the power distribution and flow between the parts of the microgrid, which will help reduce the ACS and fulfill the energy requirements dependably. The decision variables considered during this optimization process are PV panel power, WT power, and the number of H2 tanks. The data for 2021 regarding consumption and meteorological data are taken as input data for the system. The best size of each component is then arrived from the solution of the HFGA suggested in [20]. To demonstrate the effectiveness of the proposed optimization method in terms of accuracy and the time for their calculations, they are compared with similar applications such as a Genetic Algorithm, Firefly Algorithm, Sine-Cosine Algorithm and Cuckoo Search Algorithm. It can be observed that the overall generation cost of the proposed standalone hybrid (PV/WT/BG/FC) energy system for the central campus of the selected university is comparatively lesser than the other systems, and the performance of the proposed algorithm is entirely satisfactory in terms of convergence and output quality. The optimization algorithms are incorporated using the MATLAB simulation package in this study.

The paper [21] presents a new optimization algorithm that can be applied to solve many problems in mathematical optimization where the global optimum is sought. The new algorithm is derived from the random search and the classical simulated annealing algorithm – modeled after the modern process of creating high-quality steel – and is called dynamic differential annealed optimization (DDAO). The above-proposed algorithm was tested and validated for 51 test functions. The dynamic differential annealed optimization algorithm has been tested against many benchmark optimization algorithms, all highly cited. In many cases, although DDAO stands beyond numerical tests, this algorithm achieved higher performance than these algorithms. Therefore, constrained path planning and spring design problems were chosen as a real-world application of engineering optimization problems. For the spring design problem, DDAO has gone to the global minimum of problems very efficiently and has provided a more feasible solution than many of the algorithms that were found.

Nowadays, upgrading distribution networks (DNS) is a challenging goal, as it applies to the vast networks and the microgrids incorporated into the primary grid. In this context, as mentioned earlier, one of the most critical problems, known as the network reconfiguration problem, is finding such a DN topology. This is made possible by a proper transition in the open/close state of all the branch switches available to form a desired graph to interconnect the network buses. The reconfiguration problem is often formulated as an NP-hard gubernatorial problem because of constraints in current and voltage in the present search space. Although many different metaheuristic algorithms are applied to find the global optimal solution without proof, the search for near-optimal solutions for the DN reconfiguration problem within a reasonable time remains a challenge. In dealing with this problem, this article presents a new and effective optimization framework for the reconfiguration problem of modern DNs. The goal of reconfiguration is to achieve minimal cumulative power losses, as will be discussed in the subsequent sections, to improve the voltage profile of DN. A multiple-step resolution procedure is then given, and the recent Harris Hawks Optimization, known as the HHO algorithm, forms its core. Such optimizer here is accompanied by preprocessing, which involves preparing the search space and generating initial feasible populations, and postprocessing, which involves refining the final solution to enhance the search for near-optimal configurations. The applicability and reliability of the method are confirmed by performing numerical analysis on IEEE 33-bus, IEEE 85-bus systems and a synthesized 295-bus system under DG and load change scenarios. Lastly, the proposed HHO-based approach is evaluated and compared to two other metaheuristic approaches of related works, such as the particle swarm optimization algorithm and the Cuckoo search algorithm. Through the results analyzed, it can be deduced that HHO had better outcomes when compared to the other two optimizers in minimizing the power losses and improving the voltage profile and the running time. Preliminary Remarks—The present article raises the need for new strategies in reconfiguring networks, which are essential in current power distribution systems, including microgrids. All that is possible due to the proposed metaheuristic optimization strategy that enables the decision maker (i.e., the distribution system operator) to define the optimal network configuration in a reasonable time t, which minimizes the overall power losses and satisfies the operational conditions of the system. The optimization problem formulated in [22] has been purposefully designed to be neutral regarding the structure of DNs and microgrids and easily adaptable to specifics of large distribution networks and microgrids and various types of systematic objectives and technical constraints. The presented strategy can be incorporated into any DSS or engineering software for PSs, making it an information and communication technology instrument of choice to support decision-makers in planning DNS's energy efficiency and environmental effects.

In [23], a new metaheuristic optimization algorithm named the circle search algorithm (CSA) is proposed based on circle characteristics. The circle is a geometric shape that may be described by numerous parameters such as diameter, center, circumference, and tangent. The ratio of the radius to the length of the tangent line segment is the orthogonal of the angle, which is opposite the orthogonal radius. In the context of this analysis, this is a significant angle that the CSA uses in exploring and exploiting the surrounding environment. To check the effectiveness of the CSA in comparison with other algorithms, 23 standard benchmarks and three practical engineering problems were solved in separate experiments. The statistical analysis of the obtained results showed that the CSA met the requirements for the MFE for 21 of the analyzed 23 functions and the p-value ¡ 0. 05. The results expressed above prove that the CSA minimized results in a shorter time than the comparative algorithms. In addition, the high dimensional function was employed to evaluate the robustness of the CSA, and the statistics demonstrated that the CSA performs well for high-dimension problems. Consequently, the proposed CSA is a reasonably appropriate algorithm that enables a simple solution to several optimization challenges.

Therefore, the objective of [24] is to submit the use of different metaheuristic algorithms in solving the OPF with FACTS devices in the power system. OPF is one of the most commonly studied problems regarding power system operations. The difficulty level increases when FACTS device allocation problems are incorporated into OPF. Thus, seven metaheuristic algorithms: the Mating Optimizer Model (BMO), Marine optimization model (MPA), Moth optimization algorithm (MFO), Particle Swarm optimization method (PSO), gravitational optimization technique (GSA), Teaching Learning approach (TLBO) and Heap based optimization technique (HBO) are used to minimize two objectives functional; power loss and cost. These algorithms are chosen from different classifications of metaheuristic algorithms where the application of the said algorithms to the problems mentioned above will be subjected to the modified IEEE 14-bus system. Therefore, based on the simulation results, we see that TLBO and HBO perform relatively better for all the algorithms.

In [25], a new and efficient metaheuristic optimizer is presented called the growth optimizer or GO in short. The main ideas for its design are based on the subject' learning and reflection processes in society's development. Therefore, learning is a process wherein individuals continue growing to develop their skills for acquisitions from the outer world. It entails checking the individual's weaknesses and aligning the learning processes to aid the development of the individual. This work also mimics such growth behavior mathematically. It compares the proposed performance on thirty (30) functions of the real-parameter boundary constraint benchmark that formed the 2017 IEEE Congress on Evolutionary Computation (CEC 2017). The compared case involved 50 up-to-date meta-heuristic algorithms. The comparison of the convergence accuracy and data collected by the application of two nonparametric statistics – the Friedman test and the Wilcoxon signed–rank test proved that GO achieves results comparable to the 50 compared metaheuristic algorithms. In addition, to verify that GO can solve different real-world optimization problems, GO was applied to two different types of real-world optimization problems: the multiple sequence alignment (MSA) problem using the Hidden Markov Model (HMM) and the multithresholding image segmentation problem using Kapur entropy method. GO was more effective than other metaheuristic techniques in the case of solution quality and prevention of trapping in local optima.

3 Proposed Football Optimization Algorithm (FbOA)

3.1 FbOA Inspiration

The idea behind the proposed Football Optimization Algorithm (FbOA) draws from the resilience and flexibility in football, the ball distribution, moves, completion passes and frail teamwork. In football, ball distribution involves possession of the ball, destroying the opponent's defensive formation and aiming at creating space to score a goal or corner kick effectively. This distribution employs various passing styles in the form of short passing, lob passing and through ball passing, which make teams move within the restricted areas of opponents or open areas on the playground.

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The complexity of these maneuvers reflects a critical aspect of football strategy: the timeliness and accuracy of decisions resulting from the estimation of the game situation, own and the teammates and opponents' positions and movements. This decision-making is done under tension and in a fixed time frame, forcing the player to think about the changing events of the game and the best possible actions. The ability to perform these elements is developed through specified training drills that reflect the situational orientation, flexibility and rapidity demanded for a competitive footballing environment.

Translating these last two principles to a metaheuristic optimization context of FbOA, the algorithm was inspired by passing and positioning in football. In FbOA, each agent is a player on a football team, and the algorithm mimics the team's work, which unites them to achieve a common goal—finding the best solution. Within this context, agents are adapted to the search space, like a football match where players change their positions to achieve the highest efficiency of search.

These include short passing, lob passing, and, lastly, through ball passing, each corresponding to different search techniques in the algorithm. This can be categorized as local search strategies, in which the agents investigate the close neighborhoods of the solution space to improve current solutions. Lob passing is equivalent to the intermediate search strategies, which enable the agents to go partially away where more possible solutions can be found. Long-range, on the other hand, is implemented through ball passing, where the agents seek to punch their way through the search space to obtain new areas of search that may contain better solutions.

Thus, by applying these football-inspired strategic activities, FbOA wants to optimize the exploration and exploitation of the network simultaneously. This is like a football game, where the activities are adjusted according to the given problems. The algorithm employs agents to change their location and actions to investigate various areas of the search space while closely cooperating with other agents to approach the optimal or near-optimal solutions. Thus, getting inspiration from football games' strategic components and real-time decision-making helps propose fresh insight into the design of efficient and effective metaheuristic optimization techniques.

3.2 Experimental Setup

An elaborate experimental configuration is formulated to assess the performance and effectiveness of the proposed FbOA. This setup intends to know the performance of FbOA for exploration and exploitation capabilities of the problem and its capacity to avoid local optima problems. The implementation also considers a breakdown of the algorithm's exploration and exploitation benchmark, incorporating aspects such as dynamic decision-making and ball-passing movements in line with football views. This is done with the help of different mathematical equations and updated procedures that mimic many strategies that the football players use in their tactical organization and, simultaneously, maintain a balance of exploitation and exploration of the productive zones in the solution space. The subsequent sub-sections provide detailed descriptions of these aspects, starting with the exploration performance of the algorithm.

3.2.1 Exploration Performance

In the FbOA, exploration performance is based on the assumption that in every football pass, one of the players performs his pass with a certain speed velocity. This speed variation is important to balance the exploration of the search space and exploitation of the existing regions to avoid getting trapped in the local optimum.

To model this dynamic process, the next stage of the algorithm, represented by $Fb(S(t+1))$, is defined as:

$$Fb(S(t+1)) = i$$

Where:

- $Fb(S(t+1))$ denotes the updated state or solution at the next iteration $t+1$.

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- i represents the player as the agent involved in the optimization process, contributing to the variation in exploration.

Football Velocity

The equation represents the velocity of each player (or agent):

$$V_n = F_{\text{max}} \left(b_x \cdot a_i \left[F_{\text{ext}} - F_{\text{min}} \right] + r \cdot b_y \cdot a_j \left[F_{\text{best}} - F_{\text{min}} \right] \times \cos\left(\frac{\pi}{\text{Iteration}} \right) \right)$$

Where:

- V_n is the velocity of the player (agent) at iteration n.
- F_{max} is the maximum force applied, representing the upper limit of velocity.
- b_x and b_y are coefficients determining the influence of different directions in the search space.
- a_i and a_j are acceleration factors impacting the speed adjustments.
- F_{ext} represents the external force or influence in the optimization context.
- F_{min} is the minimum force, indicating the lower bound of velocity.
- F_{best} is the best force or position found so far in the optimization process.
- r is a random factor introducing variability and enhancing exploration.
- $\cos\left(\frac{\pi}{\text{Iteration}}\right)$ modulates the velocity based on the number of iterations to balance exploration and exploitation.

The equation captures the dynamic nature of exploration by allowing agents to adapt their velocities, thus effectively covering a broader search space.

Update for Best Force

To further refine the exploration process, the best force is updated using the following formula:

$$F_{\text{best}} = \frac{1}{K} \sum_{n=0}^{K} \left(\frac{F_{\text{max}}^{n^2}}{(2n+1)^2} \right)$$

Where:

- F_{best} represents the best position or force in the current optimization context.
- K is an exponential factor that increases from 0 to 1 throughout the iterations, balancing between exploration and exploitation.
- F_{max} is raised to the power of n^2, which intensifies the search focus on promising regions as the iterations proceed.
- $(2n+1)^2$ is a normalization term to control the update rate.

By continuously adjusting the velocities and updating the best force, FbOA enhances its ability to explore the solution space thoroughly while adapting to the

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3.2.2 Exploitation Performance

The tuning aspect of the FbOA in exploitation is centered on enhancing the solutions within the specified search space to get the best or nearly the best solutions. The exploitation mechanism is meant to further focus on the more rewarding areas during the exploration. The update rule for adjusting the position of an agent in the exploitation phase is given by: The update rule for adjusting the position of an agent in the exploitation phase is given by:

$$Fb(S(t+1)) = F_i + z_3 \cdot Fb(S(t)) + K \cdot \sin\left(\frac{\pi}{\text{Iteration}}\right)$$

Where:

- $Fb(S(t+1))$ is the updated solution or position at the next iteration $t+1$.
- F_i represents the current position or force influencing the aggrandisement.
- z_3 is a control parameter that adjusts the balance between the current and the new position.
- $Fb(S(t))$ is the current state or position at iteration t.
- K is an exponential factor that increases throughout the iterations, gradually shifting focus from exploration to exploitation.
- $\sin\left(\frac{\pi}{\text{Iteration}}\right)$ introduces a sinusoidal modulation to enhance the convergence speed toward the optimal solution.

This update mechanism helps FbOA take advantage of the solution space and ensures that FbOA converges on precise solutions during the iterations.

Mutation:

To avoid this, the Football Optimization Algorithm used in this study incorporates a mutation strategy that introduces variability into the search process, in which the algorithm initially converges with local optima. Through the mutation mechanism, the position of the agents changes constantly in the search space, thus making a transition and avoiding being trapped in an area with poor solutions. The mutation is defined by:

$$S(t) = \left(K \cdot a_q\left(\frac{2n+1}{x}\right) + K \cdot \cos\left(\frac{\pi}{\text{Iteration}}\right)\right)$$

Where:

- $S(t)$ represents the mutated solution at iteration t.
- K is an exponential factor influencing the extent of mutation.
- a_q is a factor that scales the mutation magnitude based on the current solution.
- $\frac{2n+1}{x}$ acts as a normalization term to control the mutation effect.
- $\cos\left(\frac{\pi}{\text{Iteration}}\right)$ modulates the mutation with a cosine function to introduce randomness and diversity in the search.
- Iteration represents the current iteration number.

By incorporating this mutation strategy, FbOA can search distant solution areas and thus avoid local optima in the optimization framework, resulting in higher performance and robustness of the algorithm.

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4 Solving Benchmark Functions

To present the performance evaluation of the proposed Football Optimization Algorithm (FbOA) more comprehensively, a set of benchmark functions from the CEC 2005 test suite was used. These functions are known in the field of optimization by the level of their complexity and the number of dimensions, as well as multiple local optima, and, therefore, they are effective for evaluating the effectiveness of optimizing algorithms.

4.1 Benchmark Functions – CEC 2005

CEC 2005 benchmark suite provides a large set of test functions which are aimed at the low-performance searching of the optimization algorithms. Figure 1, to indicate the extensive nonlinearity that must be searched. These plots illustrate characteristics like high slopes, low slopes, and multiple local optima, which are significant when assessing the optimization's reliability of finding the global optima.

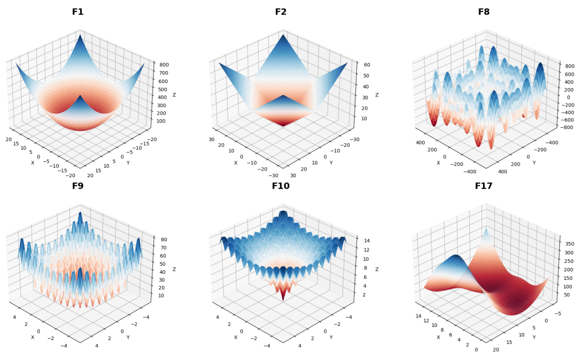

Figure 1: 3D Plots of Sample Benchmark Functions.

Table 1 also presents the kinds of unimodal benchmark functions that were used to evaluate the performance of the Football Optimization Algorithm (FbOA). On this account, unimodal functions – the functions with a single global optimum and no local minima – are most appropriate for the assessment of the algorithm' reliability to search the space and get through to the optimal solution. These functions are essentially used to check how to optimize or optimize an algorithm to the best.

The benchmark functions in Table 1 For each of them, the mathematical definition is given, and specific characteristics such as the number of dimensions, ranges of variables, the values of which the global minimum is known, etc. It makes sense for such a scale not only to allow a direct comparison of the results but also to guarantee that the optimization algorithm is tested in conditions that are as similar as possible. With these unimodal functions, the study aims to identify the accuracy and convergence rate of FbOA. The results of these tests show how efficient the algorithm is to work with smooth and predictable search spaces. These results suggest that algorithms perform highly in optimizing unimodal functions, demonstrating the capability of exploiting this type of effective solution, which is essential if searching for concrete and precise best solutions.

Table 1: Descriptions of Unimodal Benchmark Functions.

Benchmark Function	Mathematical Expression	Range	$f_{\min}$
$f_1(x)$	$\sum_{i=1}^{n} x_i^2$	$[-100, 100]$	0
$f_2(x)$	$\sum_{i=1}^{n} \|x_i\| + \prod_{i=1}^{n} \|x_i\|$	$[-10, 10]$	0
$f_3(x)$	$\sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j\right)^2$	$[-100, 100]$	0
$f_4(x)$	$\max_i\{\|x_i\|, 1 \leq i \leq D\}$	$[-100, 100]$	0
$f_5(x)$	$\sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-30, 30]$	0
$f_6(x)$	$\sum_{i=1}^{D} ([x_i + 0.5])^2$	$[-100, 100]$	0
$f_7(x)$	$\sum_{i=1}^{D} ix_i^2 + \text{random}[0, 1]$	$[-1.28, 1.28]$	0

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This work analyzes the resistance and flexibility of the FbOA by including multiple functions in the FbOA assessment model. The outcomes of these tests suggest that the algorithm's performance can successfully undertake exploration and exploitation, diversifying the solution population and correctly finding its way through more intensive search spaces. The originality is also obtained on multimodal functions, and this indicates that when in real-world problems there are multiple potential solutions and the critical factor is the location of the global optima, then the FbOA performs well.

5 Results and Discussion

5.1 Initial Parameters of Algorithms

Table 2 displays the initial parameter values used in this study for all the algorithms: population size, number of iterations and independent runs. All these general parameters were set for all the algorithms with no exception. Additionally, specific parameters for the Feedback-based Optimization Algorithm (FbOA) are detailed, such as the ranges for constants a_1, a_2, b_1, b_2, and other control variables like r_1, z, Θ, and a, which influence the behavior of the algorithm.

Table 2: Initial Parameters of Algorithms

Algorithm	Parameter	Value
All algorithms	Population size	30
	Number of iterations	500
	Number of runs	30
FbOA	a_1	[0, 1]
	a_2	[0, 1]
	b_1	[0, 1]
	b_2	[0, 1]
	r_1	[0, 2]
	z	[0, 2]
	Θ	[0, 12π]
	a	[-8, 8]

5.2 Performance Analysis of Algorithms

Table 3 displays the mean and standard deviation of the results of this study and the other comparative metaheuristic algorithms on the suite of test functions using the FbOA. The presented results show that FbOA provides near mean values equal to zero, thus proving its ability to perform better than other optimization algorithms by obtaining near-optimal solutions for most functions. For example, there is no doubt that FbOA found its mean of all functions to be equal to zero while sometimes outperforming other algorithms like JAYA, MVO, FEP, and GSA.

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Table 3: Mean and Standard Deviation (StDev) of the Proposed and Compared Algorithms over Benchmark Functions

Func	Algorithm	FbOA	JAYA	MVO	FEP	GSA
F1	Mean	0	0.01	1.41E-30	0.01	0
	StDev	0	0.01	4.91E-30	0.01	0
F2	Mean	0	0.05	1.06E-21	0.01	0
	StDev	0	0.05	2.39E-21	0.01	0
F3	Mean	0	34.13	5.39E-07	0.02	196.54
	StDev	0	22.12	2.93E-06	0.02	118.96
F4	Mean	0	1.09	0.08	0.3	7.36
	StDev	0	0.32	0.4	0.5	1.75
F5	Mean	0	16.72	27.87	0	61.55
	StDev	0	14.12	0.77	0	62.23
F6	Mean	0	0	3.12	0	0
	StDev	0	0	0.54	0	0
F7	Mean	0	0.13	0.001425	0.15	0.09
	StDev	0	0.05	0.01	0.36	0.05

Table 4 describes the average computation time (avg_time), standard deviation of time (std_time), and average number of function evaluations (avg_FEs) results of the Algorithms about different types of benchmark functions. From the obtained results, it can be seen that FbOA is characterized by relatively low time consumption compared to other computation algorithms, which makes it possible to assert its effectiveness when solving optimization tasks. For instance, in the form of FbOA, it uses up 0 of avg_time to execute the function F1. 600 which is lower than others like JAYA and FEP, thus making this algorithm appropriate for solving this problem.

Table 4: Average Time (avg_time), Standard Deviation of Time (std_time), and Average Number of Function Evaluations (avg_FEs) of the Proposed and Compared Algorithms

Func	Algorithm	FbOA	JAYA	MVO	FEP	GSA
F1	avg_time	0.600	1.900	1.859	2.477	1.261
	std_time	0.022	0.047	0.076	0.092	0.048
	avg_FEs	5800.000	15000.000	15000.000	15000.000	15000.000
F2	avg_time	1.083	1.993	2.619	2.583	1.350
	std_time	0.076	0.031	1.318	0.051	0.023
	avg_FEs	14430.000	15000.000	15000.000	15000.000	15000.000
F3	avg_time	2.302	4.372	4.381	4.897	3.672
	std_time	0.176	0.051	0.096	0.108	0.054
	avg_FEs	5668.000	15000.000	15000.000	15000.000	15000.000
F4	avg_time	0.366	1.231	1.176	1.782	0.562
	std_time	0.070	0.089	0.034	0.041	0.018
	avg_FEs	13819.000	15000.000	15000.000	15000.000	15000.000
F5	avg_time	1.125	1.944	1.894	2.499	1.287
	std_time	0.017	0.051	0.044	0.035	0.034
	avg_FEs	15000.000	15000.000	15000.000	15000.000	15000.000
F6	avg_time	1.167	2.044	1.725	2.506	1.341
	std_time	0.039	0.020	0.021	0.021	0.056
	avg_FEs	15000.000	15000.000	15000.000	15000.000	15000.000
F7	avg_time	1.174	2.073	1.963	2.593	1.372
	std_time	0.018	0.098	0.126	0.037	0.022
	avg_FEs	979.000	15000.000	1605.000	15000.000	5768.400

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5.3 Visual Analysis of Algorithm Performance

Figure 2 and Figure 3 illustrate the convergence curves for the proposed and compared algorithms across the benchmark functions. The convergence curves demonstrate how quickly each algorithm approaches the global optimum over iteration. FbOA shows rapid convergence to the global optimum, particularly for functions F1 and F2, highlighting its efficient exploitation capabilities.

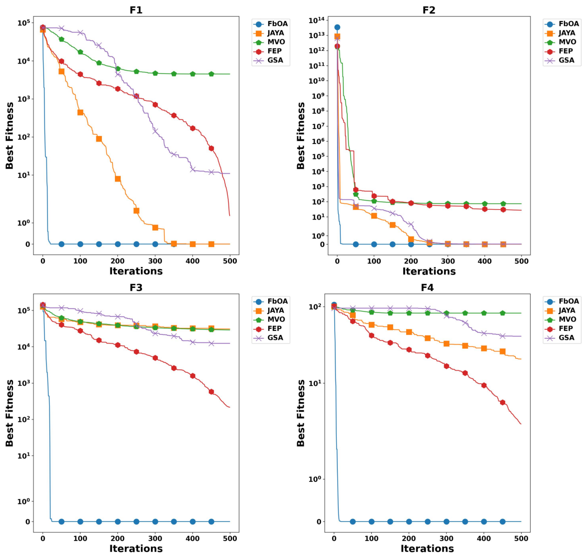

Figure 2: Convergence Curve of the Presented and Compared Algorithms for the Functions F1:F4.

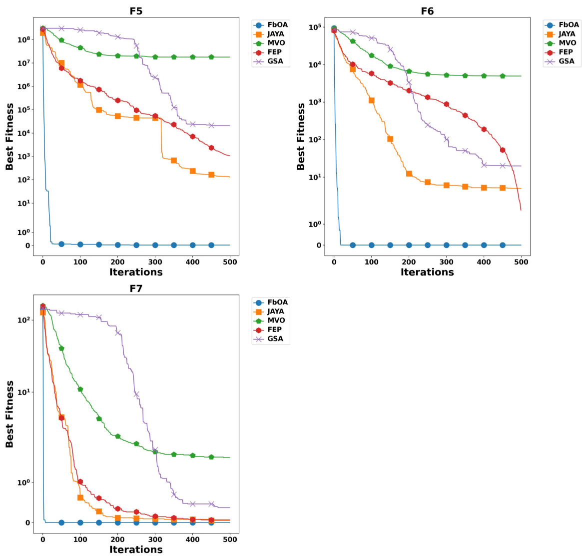

Figure 3: Convergence Curve of the Presented and Compared Algorithms for the Functions F5:F7.

Figures 4 and 5 present box plots for the suggested and compared algorithms across benchmark functions F1 to F4 and F5 to F7, respectively. These box plots provide a visual representation of the variability and distribution of the results, indicating the robustness and consistency of each performance.

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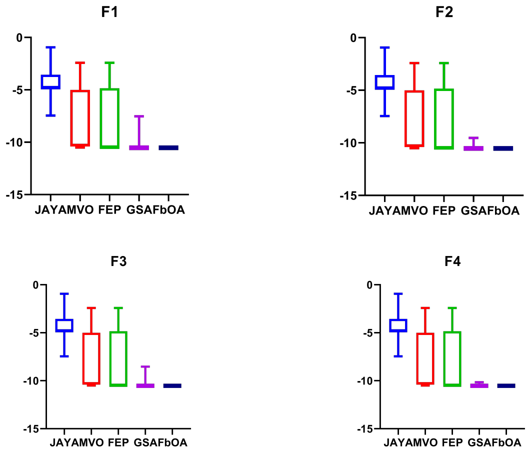

Figure 4: Box Plot of the Suggested and Compared Algorithms for Benchmark Functions (F1 to F4).

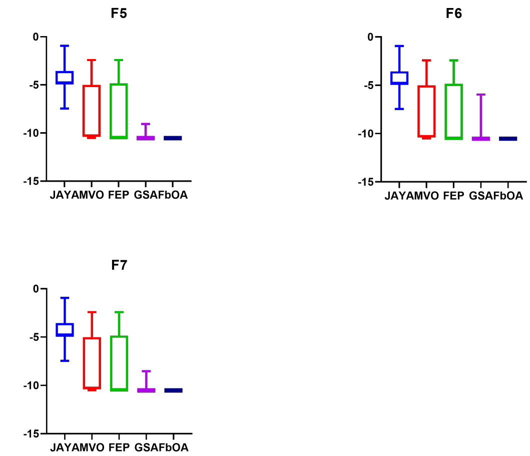

Figure 5: Box Plot of the Suggested and Compared Algorithms for Benchmark Functions (F5 to F7).

Figure 6 shows the ANOVA test results for benchmark functions from F1 to F7, which indicate statistically significant differences in the performance of the algorithms.

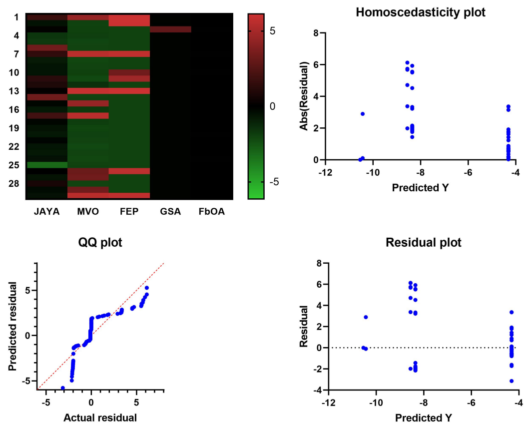

Figure 6: ANOVA Test Result for Benchmark Functions from F1 to F7.

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5.4 Results for 100-Dimensional Benchmark Functions

Table 5 provides the average time, standard deviation of time, and an average number of function evaluations for FbOA, MVO, FEP, and GSA over 100-dimensional benchmark functions. The results demonstrate that FbOA maintains superior computation time and function evaluations across different dimensions, further validating its effectiveness and efficiency in solving high-dimensional optimization problems.

Table 5: Results for 100-Dimensional Benchmark Functions

Func	Algorithm	FbOA	MVO	FEP	GSA
F1	avg_time	0.9437	2.7006	1.3327	2.3303
	std_time	0.0622	0.1075	0.1293	0.1099
	avg_FEs	15000	15000	15000	15000
F2	avg_time	0.9210	2.7598	1.4138	2.3884
	std_time	0.0372	0.1073	0.1667	0.1040
	avg_FEs	15000	15000	15000	15000
F3	avg_time	2.4385	8.6758	7.1499	9.4037
	std_time	0.1238	0.6423	0.5157	0.2860
	avg_FEs	15000	15000	15000	15000
F4	avg_time	1.0007	2.4356	1.7416	2.4597
	std_time	0.0709	0.0512	0.1620	0.1049
	avg_FEs	15000	15000	15000	15000
F5	avg_time	0.9380	2.5269	1.5091	2.5493
	std_time	0.0485	0.0474	0.1832	0.1528
	avg_FEs	15000	15000	15000	15000
F6	avg_time	0.9512	2.4964	1.3577	2.5618
	std_time	0.0557	0.0645	0.1196	0.6230
	avg_FEs	15000	15000	15000	15000
F7	avg_time	0.9576	2.5681	1.3718	2.2429
	std_time	0.0567	0.0567	0.0265	0.1527
	avg_FEs	15000	15000	15000	15000
F8	avg_time	1.0005	2.3107	1.3153	2.2826
	std_time	0.0623	0.0627	0.0696	0.4557
	avg_FEs	15000	15000	15000	15000
F9	avg_time	0.9907	0.2839	1.3613	0.8284
	std_time	0.0450	0.0129	0.1114	0.0529
	avg_FEs	15000	15000	15000	15000
F10	avg_time	1.0455	2.6224	1.5357	2.4396
	std_time	0.0744	0.0658	0.0728	0.0826
	avg_FEs	15000	15000	15000	15000
F11	avg_time	1.0363	0.3408	1.6885	0.9569
	std_time	0.0474	0.0124	0.1751	0.0513
	avg_FEs	15000	15000	15000	15000
F12	avg_time	1.1498	3.3321	2.6802	3.1210
	std_time	0.0394	0.1272	0.4309	0.0538
	avg_FEs	15000	15000	15000	15000
F13	avg_time	1.1410	3.2911	2.1482	3.0418
	std_time	0.0411	0.1625	0.0812	0.1136
	avg_FEs	15000	15000	15000	15000

To further illustrate the performance of the suggested and compared algorithms on 100-dimensional benchmark functions, a box plot is provided in Figure 7. This plot shows the variation, dispersal and dispersion of the results from each algorithm being tested across several trials. We also observe from the box plot how each algorithm performs in terms of robustness and stability in handling high-dimensional optimization problems.

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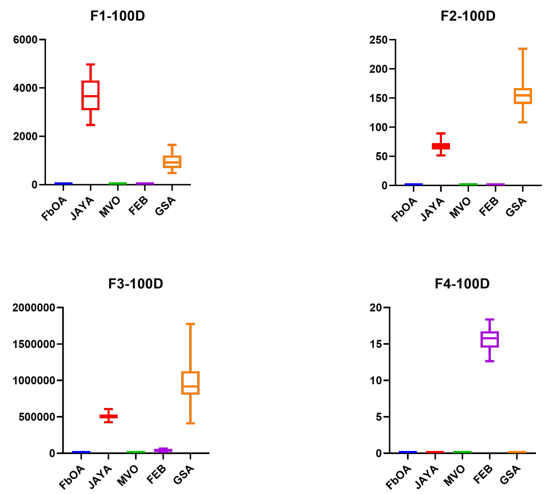

Figure 7: Box Plot of the Compared Algorithms for 100-Dimensional of Benchmark Functions F1 to F4.

To validate the statistical significance of the results, an ANOVA test was conducted across all benchmark functions (F1 to F7). As depicted in Figure 8, the test results show significant differences in the performance of the algorithms. FbOA's consistent dominance across various metrics, including mean, standard deviation, and computational time, confirms its superior performance to the other algorithms evaluated.

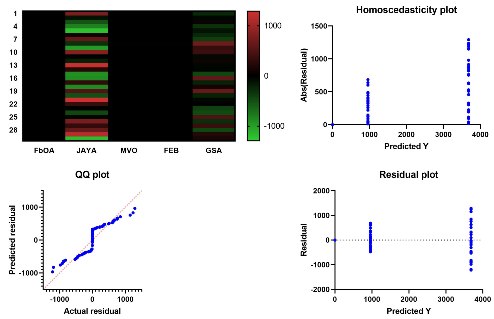

Figure 8: ANOVA Test Result for Benchmark Functions from F1 to F7.

5.5 Discussion of Results

The results from the comprehensive tests on both standard and high-dimensional benchmark functions reveal several critical insights into the performance of the Football Optimization Algorithm (FbOA) in comparison to other popular algorithms such as JAYA, MVO, FEP, and GSA.

Convergence Speed and Accuracy: The FbOA demonstrates a remarkable ability to achieve the global optimum across all tested benchmark functions, as indicated by the consistently low mean values and standard deviations. For example, in functions F1 to F6, the mean and standard deviation values for FbOA remain at or near zero, indicating its high precision in locating optimal solutions. In contrast, algorithms like JAYA and MVO exhibit more significant deviations, suggesting less stability and accuracy in specific scenarios.

Computational Efficiency: FbOA also excels in computational efficiency, as demonstrated by its lower average computation times across most benchmark functions. As shown in Table 4, the average time for FbOA remains lower than that of JAYA, FEP, and MVO for functions such as F1 and F4, highlighting its faster convergence. This is further supported by the 100-dimensional results 5), where FbOA consistently maintains shorter avg_time compared to the other algorithms while achieving comparable avg_FEs. This efficient use

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of computational resources makes FbOA suitable for large-scale optimization problems with critical time and accuracy.

Exploration and Exploitation Balance: Explicitly, FbOA demonstrates the baed exploration and exploitation capabilities through the results across multimodal functions having multiple local optima. In light of this, FbOA's efficacy cases, especially in functions such as F3 and F7, can be seen in how it reduces premature convergence, navigates the search space and boosts prospects of finding the global optimum solution.

Robustness and Adaptability: The results obtained, especially from the 100-dimensional tests, show that FbOA approaches the true optimum value with very good approximation, making it very robust and flexible to various sizes and levels of problems. Additionally, its ability to perform stably with the difference between dimensions and Initial conditions shows that the algorithm described in this paper can solve a wide range of optimization problems.

The FbOA's performance is tested with the CEC 2005 benchmark functions and 100-dimensional test problems, and the evaluation results show the FbOA's competency, efficiency, and stability in the optimization processes. Relative to other algorithms, the FbOA has been observed to be far better in terms of convergence speed, accuracy and computational complexity. The results show that it successfully combines the properties of exploration and exploitation, which means that it can be implemented in a large number of real-world optimization problems. Possible future work can involve using the presented method to more instances and more complex problems and improving the parameters described.

6 Conclusion

New metaheuristic known as Football Optimization Algorithm (FbOA) was introduced in this paper based on the strategic movements of the football game. The performance of FbOA was compared to the benchmark algorithms like JAYA, MVO, FEP, and GSA on the test functions and high-dimension optimization problems; FbOA was found to be faster in convergence, accurate in solution, and more robust. Many of the football tactics are incorporated in the algorithm to optimize a search space by ensuring high exploration and low exploitation which minimizes the chances of getting trapped by local optimality. Consequently, the results suggest FbOA is flexible and scalable to apply to various multi-objective optimization problems of high complexity and higher dimensionality. Future work may analyse its performance in other, more specific real-world case studies and fine-tune its characteristics for even better results in even more complex cases.

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