On the Problem of Inverting Discrete Self-Regression Models to Continuous Models

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Abstract

In this paper, we discuss the problem of converting auto-regression models at a discrete time into auto-regression models at continuous time, based on the idea of converting auto-regression models from first to second order. We study the general formula of AR (p) and its ability to convert from discrete to continuous time. Also, we use our model to study some real-life problems as a direct application of our approach.

Keywords: Discrete-time; Continuous time; Self-regression; Mathematical model

1. Introduction

A time series is a set of observed values of a certain phenomenon at successive times, and time series analysis is used to determine the reality of changes in the values of observations of the phenomenon, and to predict future values for it. This is done by building a mathematical model of the series, such as linear or non-linear depending on the nature of the studied time series. Such models are usually expressed as differential equations with constant coefficients. When these models are represented in continuous time, they can be expressed as differential equations with constant coefficients.

Time series are one of the important mathematical concepts adopted for scientific research and studies and are the main pillar of development plans, the development of planning methods and a key input for resolutely addressing some of the existing problems and gaps in the medical, economic, and military and service aspects [3].

The importance of time series in scientific research and studies is highlighted in their dependence on the observed values of the phenomenon and the study of how these values change over equal time periods to determine the reality of changes in the observed observations of the phenomenon and to know their causes and factors influencing them to build a suitable mathematical model to express the time series of the phenomenon.

In this research, the process of reversing auto- regression models at discrete time into auto-regression models at continuous time was studied based on the basic idea of the scientist Priestley.in the applied aspect, we relied on a time series study in the medical aspect, where the research dealt with the phenomenon of viral hepatitis Type-B, which is a health and social problem in many countries of the world and threatens many people with death or disability [1] [5].

The available statistics on the number of infected people were obtained from the main blood bank in Nineveh Governorate from 1997 until 2006. Was divided at the level of months, into 120 views, which represent the numbers of people with this disease. There are many studies in the field of embedding and vice versa (Converting), among which we mention:

Researchers Tong and Chan (1987) published their joint research on immersing auto- regression models at discrete time into auto-regression models at continuous time, and explained that it is not always possible to immerse the real intermittent parameters values of the Kaos model of First-Order auto- regression AR (1) in the real continuous parameters values of the Kaos model of First-Order auto- regression AR (1) [9].

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

He and Wang (1989) dealt with the topic of immersion of intermittent ARMA (p,q) models into models Continuous ARMA(p', q'),the researchers explained that the real intermittent parameters. The values for the caus of autoregressive and moving circles model ARMA (p,q) and for each q < p can be immersed in real continuous parameters the values for the kaos model of autoregressive and moving models ARMA (p',q') and for each q'< p' [7]. **Brockwell and Brockwell (1999)** showed that autoregressive and integrated moving circles model when one of the roots of the moving circles model is on the unit circle cannot be immersed in any auto- regression model and time-continuous moving circles [6].

In the field of inverting (Converting) discrete-time auto- regression models to continuous-time auto- regression models, the scientist Priestley (1981) demonstrated the mechanism of converting intermittent auto-regression models of the first rank AR (1) and intermittent self-regression models of the second rank AR (2) to continuous auto- regression models of the first and second ranks, respectively [8].

1-The theoretical aspect (the reverse of autoregressive models from discontinuous to continuous time).

Regression models in intermittent time can be expressed as differential equations with constant coefficients, and when these models are represented in continuous time, they are written as differential equations with constant coefficients [8]. Because most studies are carried out by collecting readings in intermittent time, knowing that they are constantly happening, so we will try in this paper to reverse the models of auto- regression in intermittent time to models of auto- regression in continuous time.
Let's have the following form:

$$\sum_{i=0}^{n} a_i X_{t-i} = Y_t$$

It can be written in the following form:

$$X_t = a_1 X_{t-1} - \ldots - a_n X_{t-n} + Y_t$$

It is an auto- regression model of rank (n), where $\{Y_t\}$white noise and{t} represents the time for each *t = 0,±1,±2,…*and that a_i constants represent the parameters of the model where *i = 1,2,…,n* ، $a_0 = 1$.
The researcher M.B.Priestley reverses the first-and second-order auto- regression model in discontinuous time to the first-and second-order auto- regression model in continuous time.

In this research, we will try to take advantage of the idea used by the researcher (Priestley)in reversing the auto-regression models of the first and second rank in intermittent time to auto- regression models of the first and second rank in continuous time to reverse the regression models of the rank p and p=1,2,…,k.

1-1 Inversion of the third-order auto- regression model: AR (3)

Let the third-order self-regression model be in the following form:

$$X_n + a_1 X_{n-1} + a_2 X_{n-2} + a_3 X_{n-3} = Y_n$$

Where a_i are constants for each *i=1,2,3* and $\{Y_n\}$ white noise, for each *n = o,±1,±2,...* and based on a similar formula in the inverse of AR (p) where p=1,2 that was used by the scientist M.B.Priestley We will get the following:

$$-a_3\Delta^3 X_n + (a_2 + 3a_3)\Delta^2 X_n - (a_1 + 2a_2 + 3a_3)\Delta X_n + (1 + a_1 + a_2 + a_3)X_n = Y_n$$
….(1)

Where Δ denotes the differential coefficient and includes the back-displacement factor B, as follows:

$$\Delta = (1 - B)$$

Let $t = n.\Delta t$ by dividing the sides of Equation (1) by the amount $(-a_3(\Delta t)^3)$, we get:

$$\frac{\Delta^3 X_n}{(\Delta t)^3} - \frac{(a_2 + 3a_3)\Delta^2}{a_3(\Delta t)^3} X_n + \frac{(a_1 + 2a_2 + 3a_3)\Delta}{a_3(\Delta t)^3} X_n - \frac{(1 + a_1 + a_2 + a_3)}{a_3(\Delta t)^3} X_n = \frac{Y_n}{-a_3(\Delta t)^3}$$

$$\frac{\Delta^3 X_n}{(\Delta t)^3} - \frac{(a_2 + 3a_3)}{a_3(\Delta t)} . \frac{\Delta^2 X_n}{(\Delta t)^2} + \frac{a_1 + 2a_2 + 3a_3}{a_3(\Delta t)^2} . \frac{\Delta X_n}{\Delta t} - \frac{(1 + a_1 + a_2 + a_3)}{a_3(\Delta t)^3} X_n = \frac{-Y_n}{a_3(\Delta t)^3}$$

We assume that:

$$\in(t) = -Y_n/[a_3(\Delta t)^3]$$
$$\alpha_1 = -(a_2 + 3a_3)/[a_3(\Delta t)]$$

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

$$\alpha_2 = (a_1 + 2a_2 + 3a_3)/[a_3(\Delta t)^2]$$

$$\alpha_3 = -(1 + a_1 + a_2 + a_3)/[a_3(\Delta t)^3]$$

Taking the approach from zero to Δt, we obtain the formal differential equation of the third rank:

$$\dddot{X}(t) + \alpha_1\ddot{X}(t) + \alpha_2\dot{X}(t) + \alpha_3 X(t) = +(t)$$

Which represents a continuous auto- regression model of the third order.

1-2 Inversion of the fourth-order auto- regression model AR(4):

Let us have the following fourth-order auto- regression model:

$$X_n + a_1X_{n-1} + \dots + a_4X_{n-4} = Y_n$$

Where a_i are constants for each $i=1,2,3,4$ and $\{Y_n\}$ white noise, for each $n = o,\pm1,\pm2,\dots$ and depending on the relations in the inverse of AR (p) where p=1,2,3 above we will get the following:

$$a_4\Delta^4X_n - (a_3 + 4a_4)\Delta^3X_n + (a_2 + 3a_3 + 6a_4)\Delta^2X_n - (a_1 + 2a_2 + 3a_3 + 4a_4)\Delta X_n + (1 + a_1 + a_2 + a_3 + a_4)X_n = Y_n \quad \dots(2)$$

Let $t = n.\Delta t$ dividing the sides of Equation (2) by the amount $(a_4(\Delta t)^4)$we get:

$$\frac{\Delta^4X_n}{(\Delta t)^4} - \frac{(a_3 + 4a_4)\Delta^3}{a_4(\Delta t)^4}X_n + \frac{(a_2 + 3a_3 + 6a_4)\Delta^2}{a_4(\Delta t)^4}X_n - \frac{(a_1 + 2a_2 + 3a_3 + 4a_4)\Delta}{a_4(\Delta t)^4}X_n + \frac{(1 + a_1 + a_2 + a_3 + a_4)}{a_4(\Delta t)^4}X_n = \frac{Y_n}{a_4(\Delta t)^4}$$

Or in another form we get:

$$\frac{\Delta^4X_n}{(\Delta t)^4} - \frac{(a_3 + 4a_4)}{a_4\Delta t}.\frac{\Delta^3X_n}{(\Delta t)^3} + \frac{(a_2 + 3a_3 + 6a_4)}{a_4(\Delta t)^2}.\frac{\Delta^2X_n}{(\Delta t)^2} - \frac{(a_1 + 2a_2 + 3a_3 + 4a_4)}{a_4(\Delta t)^3}.\frac{\Delta X_n}{(\Delta t)} + \frac{(1 + a_1 + a_2 + a_3 + a_4)X_n}{a_4(\Delta t)^4} = \frac{Y_n}{a_4(\Delta t)^4}$$

then:

$$\in(t) = Y_n/a_4(\Delta t)^4$$

$$\alpha_1 = -(a_3 + 4a_4)/a_4(\Delta t)$$

$$\alpha_2 = (a_2 + 3a_3 + 6a_4)/a_4(\Delta t)^2$$

$$\alpha_3 = -(a_1 + 2a_2 + 3a_3 + 4a_4)/a_4(\Delta t)^3$$

$$\alpha_4 = (1 + a_1 + a_2 + a_3 + a_4)/a_4(\Delta t)^4$$

We take Δt approaching zero to obtain the formal differential equation of the fourth rank:

$$X^{(4)}(t) + \alpha_1X^{(3)}(t) + \alpha_2X^{(2)}(t) + \alpha_3X^{(1)}(t) + \alpha_4X(t) = \in(t)$$

Which represents a continuous Fourth-order auto- regression model.

1-3 Inversion of the fifth-order auto- regression model AR(5):

Let us have the following fifth-order auto- regression model:

$$X_n + a_1X_{n-1} + a_2X_{n-2} + \dots + a_5X_{n-5} = Y_n$$

Where a_i are constants for each $i=1,2,\dots,5$ and $\{Y_n\}$ white noise, for each $n = o,\pm1,\pm2,\dots$

We will rely on the relations in the inverse of AR(p)where p=1,2,3,4 above to get the following:

$$-a_5\Delta^5X_n + (a_4 + 5a_5)\Delta^4X_n - (a_3 + 4a_4 + 10a_5)\Delta^3X_n + (a_2 + 3a_3 + 6a_4 + 10a_5)\Delta^2X_n - (a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5)\Delta X_n + (1 + a_1 + a_2 + a_3 + a_4 + a_5)X_n = Y_n \quad \dots(3)$$

Let: $t = n.\Delta t$, and dividing the sides of Equation (3) by the amount $(-a_5(\Delta t)^5)$ we get:

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

$$\frac{\Delta^5 X_n}{(\Delta t)^5} - \frac{(a_4+5a_5)\Delta^4}{a_5(\Delta t)^5}X_n + \frac{(a_3+4a_4+10a_5)\Delta^3}{a_5(\Delta t)^5}X_n - \frac{(a_2+3a_3+6a_4+10a_5)\Delta^2}{a_5(\Delta t)^5}X_n + \frac{(a_1+2a_2+3a_3+4a_4+5a_5)\Delta}{a_5(\Delta t)^5}X_n - \frac{(1+a_1+a_2+a_3+a_4+a_5)}{a_5(\Delta t)^5}X_n = -\frac{Y_n}{a_5(\Delta t)^5}$$

Or it can be written in the following form:

$$\frac{\Delta^5 X_n}{(\Delta t)^5} - \frac{(a_4+5a_5)}{a_5\Delta t}.\frac{\Delta^4 X_n}{(\Delta t)^4} + \frac{(a_3+4a_4+10a_5)}{a_5(\Delta t)^2}.\frac{\Delta^3 X_n}{(\Delta t)^3} - \frac{(a_2+3a_3+6a_4+10a_5)}{a_5(\Delta t)^3}.\frac{\Delta^2 X_n}{(\Delta t)^2} + \frac{(a_1+2a_2+3a_3+4a_4+5a_5)}{a_5(\Delta t)^4}.\frac{\Delta X_n}{(\Delta t)} - \frac{(1+a_1+a_2+a_3+a_4+a_5)}{a_5(\Delta t)^5}X_n = -\frac{Y_n}{a_5(\Delta t)^5}$$

Let:

$$\in(t) = -Y_n/a_5(\Delta t)^5$$

$$\alpha_1 = -(a_4+5a_5)/a_5(\Delta t)$$

$$\alpha_2 = (3a_3+4a_4+10a_5)/a_5(\Delta t)^2$$

$$\alpha_3 = -(a_2+3a_3+6a_4+10a_5)/a_5(\Delta t)^3$$

$$\alpha_4 = (a_1+2a_2+3a_3+4a_4+5a_5)/a_5(\Delta t)^4$$

$$\alpha_5 = -(1+a_1+a_2+a_3+a_4+a_5)/a_4(\Delta t)^5$$

When Δt approaches zero, that is, $\Delta t \to 0$, we obtain the formula of the differential equation of the fifth rank in the following form:

$$X^{(5)}(t) + \alpha_1 X^{(4)}(t) + \alpha_2 X^{(3)}(t) + \alpha_3 X^{(2)}(t) + \alpha_4 X^{(1)}(t) + \alpha_5 X(t) = \in(t)$$

Where $X^{(5)}(t)$ is the rank of the equation, which represents a continuous auto- regression model of the fifth rank.

1-4 inversion of the auto- regression model of rank p: AR (p)

By continuing to find the limits of the displacement factor Δ^i, for each *i = 1,2,3,4,5*, it was possible to design a table that performs a quick calculation of the decoder Δ^i, for each *i = 1,2,…, p* and arrive at the numerical values $Z_{i,j}$ for the coefficients a_i associated with the displacement factor Δ^i and for each *i = 1,2,…, p*, as shown in the following table (1) :

For each *i = 1,2,…, p* and for each *i = 1,2,…,i*

Table 1: Calculation of the coefficients of the decoder of the differential coefficient $\Delta(\Delta^0, \Delta^1, \Delta^2, \dots)\ X_{n-j}$

	X_n	X_{n-1}	X_{n-2}	X_{n-3}	X_{n-4}	X_{n-5}	….	X_{n-p}
$a_0\Delta^0$	$Z_{0,0}=1$	0	0	0	0	0	….	0
$a_1\Delta^1$	$Z_{1,0}=1$	-1	0	0	0	0	….	0
$a_2\Delta^2$	$Z_{2,0}=1$	-2	1	0	0	0	….	0
$a_3\Delta^3$	$Z_{3,0}=1$	-3	3	-1	0	0	….	0
$a_4\Delta^4$	$Z_{4,0}=1$	-4	6	-4	1	0	….	0
$a_5\Delta^5$	$Z_{5,0}=1$	-5	10	-10	5	-1	….	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Where the columns represent coefficients $X_n, X_{n-1}, \dots$ and the differential coefficient rows $\Delta^0, \Delta^1, \Delta^2, \dots$ and that $\Delta = (1-B)$and B is the backward Shift operator previously defined $B^r X_n = X_{n-r}$. And that $Z_{i,j}$ is represented by the following equation:

$$Z_{i,j} = \sum_{f=i-1}^{0} |Z_{f,j-1}|$$

Continuing in a similar way, AR(p) is obtained, where the inverse operation can be represented by the following table (2), through which the inverse operation can be generalized to include all p.

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

For each i=1,2,…,p and for each j=1,2,…,i.

Table 2: Representation and generalization of the inverse process X_{n-j}

AR(p)	$\Delta^0 x_n$	$\Delta^1 x_n$	$\Delta^2 x_n$		$\Delta^3 x_n$	$\Delta^4 x_n$	$\Delta^5 x_n$	**….**
a_0	$Z_{0,0}$	0	0		0	0	0	….
a_1	$Z_{1,0}$	$Z_{1,1}$	0		0	0	0	….
a_2	$Z_{2,0}$	$Z_{2,1}$	$Z_{2,2}$		0	0	0	….
a_3	$Z_{3,0}$	$Z_{3,1}$	$Z_{3,2}$		$Z_{3,3}$	0	0	….
a_4	$Z_{4,0}$	$Z_{4,1}$	$Z_{4,2}$		$Z_{4,3}$	$Z_{4,4}$	0	….
a_5	$Z_{5,0}$	$Z_{5,1}$	$Z_{5,2}$		$Z_{5,3}$	$Z_{5,4}$	$Z_{5,5}$	….
⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮

By finding time-continuous models using the inverse process we deduced from Tables (1) and (2) the following:

$$\left.\begin{aligned} &1)\ Z_{i,j} = -i \quad \forall j = 1 \\ &2)\ Z_{i,j} = (-1)^i \quad \forall j = j \\ &3)\ Z_{i,j} = 1 \quad \forall j = 0 \\ &4)\ Z_{i,j} = |Z_{i-1,j}| + |Z_{i-1,j-1}| \quad \forall i > 1 \& j \geq 2 \\ &5)\ Z_{i,j} = 0 \quad \forall i < j \& j \geq 1 \\ &6)\ a_0 = 1 \end{aligned}\right\} \qquad (4)$$

Based on Table (1) and the conclusions in (4), the following general formula of the inversion process from discontinuous models to continuous models can be obtained:

The general formula for finding AR(p) in continuous time is:

$$AR(p): \sum_{i=0}^{P} S_i = y_n \qquad \dots(5)$$

Where:

$$S_i = a_i \sum_{j=0}^{i} Z_{i,j} \Delta^j X_n \qquad \dots(6)$$

And:

$$Z_{i,j} = \sum_{f=i-1}^{0} |Z_{f,j-1}| \qquad \dots(7)$$

By dividing the sides of Equation (5) by the amount $((\Delta t)^p$ *coefficient $\Delta^P x_n)$, we arrive at the general formula AR(p). In order to verify that the general formula for finding AR(p)is correct, we will apply it to the models of auto- regression in discontinuous time of the first and second orders.

First: continuous-time auto- regression model of the first order: AR(1)

Let be a discrete-time auto- regression model of the first order (the differential equation) in the following form:

$$X_n - a_1 X_{n-1} = Y_n \qquad \dots(8)$$

Applying equation (5) when p=1 we get:

$$AR(1): \sum_{i=0}^{1} S_i = Y_n$$

$$S_0 + S_1 = Y_n \qquad \dots(9)$$

Applying equations (4) and (6) to equation (9), we obtain:

$$S_0 = a_0 \sum_{j=0}^{0} Z_{0,j} \Delta^j X_n = a_0 Z_{0,0} \Delta^0 X_n$$

$$\therefore S_0 = X_n$$

We now find S_1 as follows:

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

$$S_1 = a_1(Z_{1,0}\Delta^0 X_n + Z_{1,1}\Delta^1 X_n)$$

Applying equations (4) and (6) to find S_1 from equation (9), we obtain:

$$S_1 = a_1 X_n - a_1 \Delta X_n$$

Substituting the values of S_i, for each $i=0,1$ in equation (9) we obtain the following:

$$AR(1): X_n + a_1 X_n - a_1 \Delta X_n = Y_n$$

$$AR(1): (1 + a_1)X_n - a_1 \Delta X_n = Y_n \qquad \ldots(10)$$

By dividing the sides of equation (10) by the magnitude $(-a_1\Delta t)$, we arrive at the following general formula AR(1):

$$\frac{\Delta X_n}{\Delta t} - \frac{(1 + a_1)}{a_1 \Delta t}.X_n = -\frac{Y_n}{a_1 \Delta t}$$

Which represents the First-Order auto- regression equation in continuous time.

Second: the equation of auto- regression in continuous time of the second order: AR(2)

Let be the second-order differential equation:

$$X_n + a_1 X_{n-1} + a_2 X_{n-2} = Y_n \qquad \ldots(11)$$

Substituting p=2 into equation (5), we obtain:

$$AR(2): \sum_{i=0}^{2} S_i = Y_n$$

$$S_0 + S_1 + S_2 = Y_n \qquad \ldots(12)$$

In AR(1) we found both S_0 and S_1 and applying equations (4), (6) and (7) to equation (12) we get S_2:

$$S_2 = a_2((-1)^0 Z_{2,0}\Delta^0 X_n + (-1)^1 Z_{2,1}\Delta^1 X_n + (-1)^2 Z_{2,2}\Delta^2 X_n)$$

$$S_2 = a_2 X_n - 2a_2 \Delta X_n + a_2 \Delta^2 X_n$$

$$\therefore \sum_{i=0}^{2} S_i = X_n + a_1 X_n - a_1 \Delta X_n + a_2 X_n - 2a_2 \Delta X_n + a_2 \Delta^2 X_n$$

$$\Rightarrow a_2 \Delta^2 X_n - (a_1 + 2a_2)\Delta X_n + (1 + a_1 + a_2)X_n$$

Substituting the values of S_i, for each in equation (12) we obtain the following:

$$AR(2): a_2 \Delta^2 X_n - (a_1 + 2a_2)\Delta X_n + (1 + a_1 + a_2)X_n = Y_n \qquad ...(13)$$

Dividing the sides of equation (13) by the amount $(a_2 * (\Delta t)^2)$, we arrive at the following general formula AR(2) :

$$\frac{\Delta^2 X_n}{(\Delta t)^2} - \frac{(a_1 + 2a_2)}{a_2 \Delta t}.\frac{\Delta X_2}{\Delta t} + \frac{(1 + a_1 + a_2)}{a_2 (\Delta t)^2}.X_n = \frac{Y_n}{a_2 (\Delta t)^2}$$

Which represents the equation of auto- regression of the second order in continuous time and is identical to the inverse equation of the researcher (Priestley).

2- The Practical Part

In this research, the time series of viral hepatitis Type-B was studied, which represents the numbers of people infected with this disease, where data were obtained from the main blood bank/ virus division, and for the years 2006-1997 and at the level of months.

And figure number (1) represents the autocorrelation function of the studied series.

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

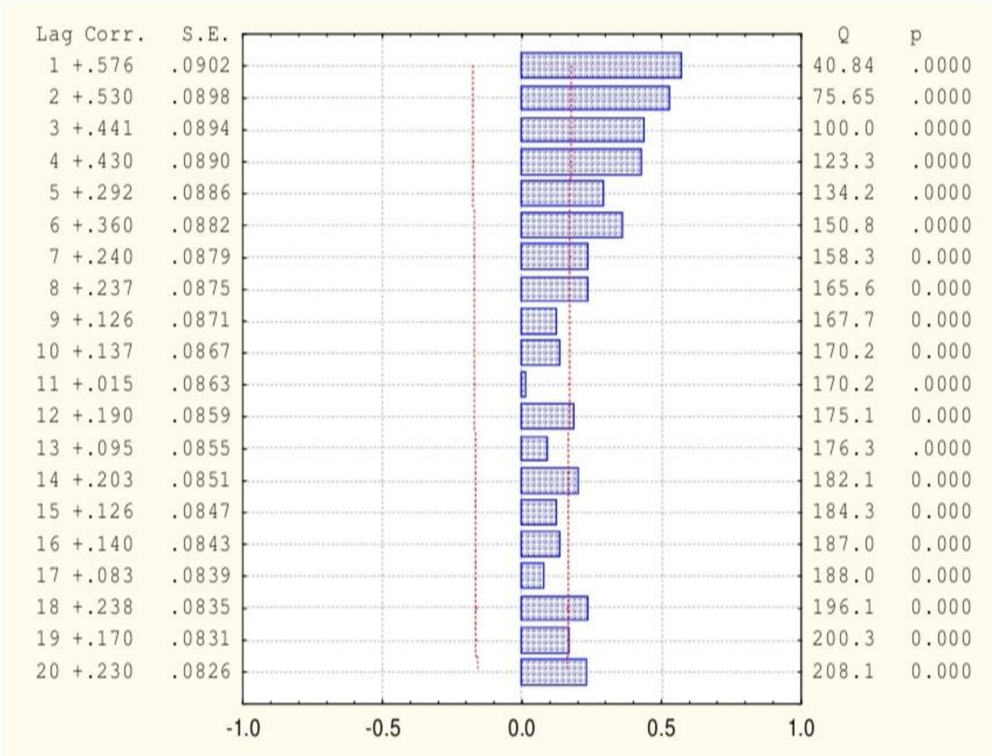

Figure 1. The autocorrelation function of the time series of the number of cases of viral hepatitis disease pattern-B-for the period (2006-1997)

From the observation of the drawing of the autocorrelation function [figure No.(1)] we find that most of the autocorrelation coefficients fall outside the constraint $\mp\frac{1.96}{\sqrt{n}}$ where we represent the number of data indicating that the series is interconnected but close to the normal distribution [note figure No.(2)].

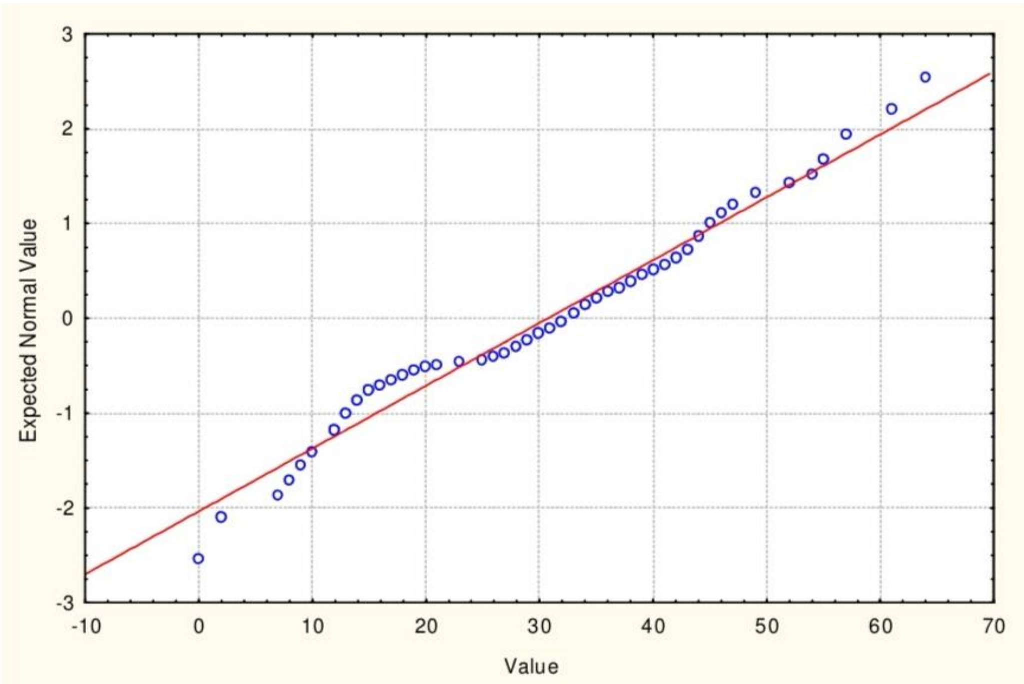

Figure 2. Natural time series graph of the number of viral hepatitis C infections pattern-B-for the period (2006-1997)

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

Modeling:
Since the series is unstable, the first difference of the original series was taken (note the figure number (3)) to convert it into a stable series, and for an increase in detail Note [2].

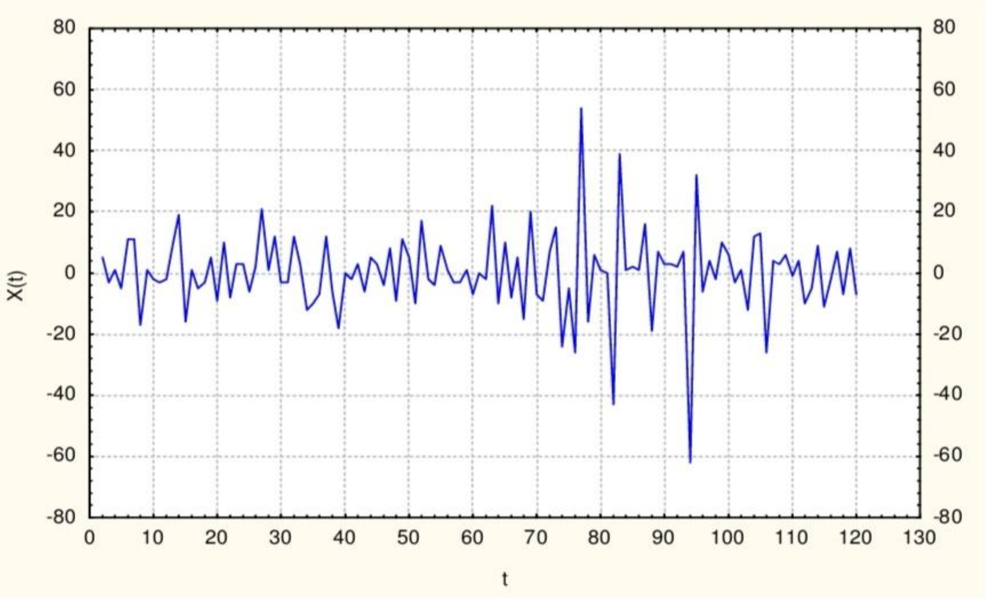

Figure 3. Altered Time series graph of the number of viral hepatitis disease infections pattern-B-for the period (2006-1997)

The Box & Jenkins method was used to analyze and model the time series of the number of cases of the disease represented by the three basic stages (diagnosis - Identification, Estimation, Diagnostic Checking) [4].

Using the ready-made statistical program (Statistica-99) for the purpose of estimating the parameters of the diagnosed model in the approximate maximum likelihood method and for an increase in detail Note [4], we obtained an auto- regression on model of the first rank AR(1) for the time series and using both AIC and BIC (Bayesian Information Criterion) as:

$$AIC(m) = nln(\sigma_z^2) + 2m$$

m represents the number of parameters in the model.

n number of data.

σ_z^2 the residual variance

The best rank of the model is elected by the lowest value of AIC(m)

The BIC standard is a development of the AIC standard, which is called the Bayesian Information Criterion and is defined as follows [10].

$$BIC(m) = nln(\sigma_z^2) + mln(n)$$

We note that the model AR(1) has the lowest value within the above two criteria, so it was chosen to represent the series,the following table shows the models that have been adapted to the series according to the AIC, BIC and MSE standards.

Table 3: Convenient mathematical models of the Series ARIMA (p, d, q)

p	d	MSE	AIC	BIC
1	1	144	550	556
2	1	140	552	560
3	1	136	554	565
4	1	137	556	570
5	1	131	558	574
6	1	132	560	579

DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

The form AR (1) is expressed by the following formula:

$X_t - 0.4498X_{t-1} = Z_t$ …(14)

(0.08269)

BIC= 556

3-2 inverting the discrete-time model to a continuous-time model:

Using tables (1,2) and relations (4, 5, 6, 7), and since the model we obtained is a auto- regression model in discontinuous time, we obtained the following model, which represents an auto- regression model in continuous time:

$$X^{(1)}(t) + \alpha_1 X(t) =\in (t)$$

Assuming that $\Delta t = 0.1$, and from equation (14) we obtain the following values:

$a_0 = 1$

$a_1 = -0.4498$

Where:

$$\alpha_1 = \frac{(1 + a_1)}{a_1(\Delta t)} = -12.2321$$

$$\in (t) = \frac{Y_n}{a_1(\Delta t)}$$

The staionarity condition of the proposed model AR(1) in discontinuous time is to be |μ|<1.

The characteristic equation of the model AR(1) in discrete time has the following form:

Where the root $\lambda_1 = \mu_1$ represents $\lambda - 0.4498 = 0$

So, we get: $\lambda_1 = 0.4498$

We note that $|\mu_1| < 1$, which indicates that the series represented by the model AR(1) is considered stable.

To test staionarity the auto- regression model in continuous time.

We find the characteristic equation of the model AR(1) in continuous time, which has the following form:

$\lambda - 12.2321 = 0$

Where $\lambda_1 = \mu_1$ represents the root.

We get:

$\lambda_1 = 12.2321$

And that the stationarity condition for the inverse model AR(1) in continuous time is for the real part of the root of the equation to be a negative amount, that is, $R(\mu_1) < 0$

Where we notice that the real part of the root of the equation is not a negative value, which indicates that the inverse model is unstable.

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DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024

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DOI: https://doi.org/10.54216/NIF.040205
Received: January 17, 2024 Accepted: July 12, 2024