



Finding the Units Tables of Finite Neutrosophic Rings Modulo Integers $Z_n(I)$ for $3 \leq n \leq 16$

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Abstract

This paper is dedicated to finding all invertible elements (algebraic units) in 14 different finite neutrosophic rings modulo integers $Z_n(I)$ for all $3 \leq n \leq 16$. We arrange the elements of the group of units of each ring in a table called the units table of the ring.

Keywords: Group of units; Units table; Finite neutrosophic ring; Integer

1. Introduction

The group of units of a commutative ring is considered one of the most important algebraic structures related to a ring. The set of all invertible elements in a commutative ring concerning the multiplication operation forms a commutative group called the group of units [1, 6]. Many authors have studied this group and its classification in many algebraic rings, see [2-7]. In [8], authors have classified the group of units of a neutrosophic ring as a direct product of two groups by using a special isomorphism they defined.

In this work, we give a powerful result, where we determine all units in 14 different finite neutrosophic rings modulo integers $Z_n(I)$ for all $3 \leq n \leq 16$. We arrange the elements of the group of units of each ring in a table called the units table of the ring.

2. Main Discussion

Remark: [8]

If $R(I) = \{x + yI; x, y \in R\}$ is a neutrosophic ring with unity, then: $U(R(I)) \cong U(R) \times U(R)$.

Also, $a + bI$ is a unit in $R(I)$ if and only if: $(a, a + b)$ is a unit in $R \times R$.

If (x, y) is a unit in $R \times R$, then $x + (y - x)I$ is a unit in $R(I)$.

Table 1: Units table of $Z_3(I)$

Unit in $Z_3 \times Z_3$	Unit in $Z_3(I)$	
(1,1)	1	1
(1,2)	1+I	2
(2,1)	2+2I	3
(2,2)	2	4

Table 2: Units table of $Z_4(I)$

Unit in $Z_4 \times Z_4$	Unit in $Z_4(I)$	
(1,3)	1+2I	1
(3,1)	3+2I	2
(3,3)	3	3
(1,1)	1	4

Table 3: Units table of $Z_5(I)$

Unit in $Z_5 \times Z_5$	Unit in $Z_5(I)$	
(1,1)	1	1
(1,2)	1+I	2
(1,3)	1+2I	3
(1,4)	1+3I	4
(2,1)	2+4I	5
(2,2)	2	6
(2,3)	2+I	7
(2,4)	2+2I	8
(3,1)	3+3I	9
(3,2)	3+4I	10
(3,3)	3	11
(3,4)	3+I	12
(4,1)	4+2I	13
(4,2)	4+3I	14
(4,3)	4+4I	15
(4,4)	4	16

Table 4: Units table of $Z_6(I)$

Unit in $(Z_6)^2$	Unit in $Z_6(I)$	
(1,5)	1+4I	1
(5,1)	5+2I	2
(1,1)	1	3
(5,5)	5	4

Table 5: Units table of $Z_8(I)$

Unit in $(Z_8)^2$	Unit in $Z_8(I)$	
(1,1)	1	1
(3,3)	3	2
(5,5)	5	3

(7,7)	7	4
(1,3)	1+2I	5
(3,1)	3+6I	6
(1,5)	1+4I	7
(5,1)	5+4I	8
(1,7)	1+6I	9
(7,1)	7+2I	10
(3,5)	3+2I	11
(5,3)	5+6I	12
(3,7)	3+4I	13
(7,3)	7+4I	14
(5,7)	5+2I	15
(7,5)	7+6I	16

Table 6: Units table of $Z_7(I)$

Unit in $(Z_7)^2$	Unit in $Z_7(I)$	
(1,1)	1	1
(2,2)	2	2
(3,3)	3	3
(4,4)	4	4
(5,5)	5	5
(6,6)	6	6
(1,2)	1+I	7
(2,1)	2+6I	8
(1,3)	1+2I	9
(3,1)	3+5I	10
(1,4)	1+3I	11
(4,1)	4+4I	12
(1,5)	1+4I	13
(5,1)	5+3I	14
(1,6)	1+5I	15
(6,1)	6+2I	16
(2,3)	2+I	17
(3,2)	3+6I	18
(2,4)	2+2I	19
(4,2)	4+5I	20
(2,5)	2+3I	21

(5,2)	5+4I	22
(2,6)	2+4I	23
(6,2)	6+3I	24
(3,4)	3+I	25
(4,3)	4+6I	26
(3,5)	3+2I	27
(5,3)	5+5I	28
(3,6)	3+3I	29
(6,3)	6+4I	30
(4,5)	4+I	31
(5,4)	5+6I	32
(4,6)	4+2I	33
(6,4)	6+5I	34
(5,6)	5+I	35
(6,5)	6+6I	36

Table 7: Units table of $Z_9(I)$

Units in $(Z_9)^2$	Units in $Z_9(I)$	
(1,1)	1	1
(2,2)	2	2
(4,4)	4	3
(5,5)	5	4
(7,7)	7	5
(8,8)	8	6
(1,2)	1+I	7
(2,1)	2+8I	8
(1,4)	1+3I	9
(4,1)	4+6I	10
(1,5)	1+4I	11
(5,1)	5+5I	12
(1,7)	1+6I	13
(7,1)	7+3I	14
(1,8)	1+7I	15
(8,1)	8+2I	16
(2,4)	2+2I	17
(4,2)	4+7I	18
(2,5)	2+3I	19

(5,2)	5+6I	20
(2,7)	2+5I	21
(7,2)	7+4I	22
(2,8)	2+6I	23
(8,2)	8+3I	24
(4,5)	4+I	25
(5,4)	5+8I	26
(4,7)	4+3I	27
(7,4)	7+6I	28
(4,8)	4+4I	29
(8,4)	8+5I	30
(5,7)	5+2I	31
(7,5)	7+7I	32
(5,8)	5+3I	33
(8,5)	8+6I	34
(7,8)	7+I	35
(8,7)	8+8I	36

Table 8: Units table of $Z_{10}(I)$

Unit in $(Z_{10})^2$	Unit in $Z_{10}(I)$	
(1,1)	1	1
(3,3)	3	2
(7,7)	7	3
(9,9)	9	4
(1,3)	1+2I	5
(3,1)	3+8I	6
(1,7)	1+6I	7
(7,1)	7+4I	8
(1,9)	1+8I	9
(9,1)	9+2I	10
(3,7)	3+4I	11
(7,3)	7+6I	12
(3,9)	3+6I	13
(9,3)	9+4I	14
(7,9)	7+2I	15
(9,7)	9+8I	16

Table 9: Units table of $Z_{11}(I)$

Units in $Z_{11} \times Z_{11}$	Units in $Z_{11}(I)$	
(1,1)	1	1
(2,2)	2	2
(3,3)	3	3
(4,4)	4	4
(5,5)	5	5
(6,6)	6	6
(7,7)	7	7
(8,8)	8	8
(9,9)	9	9
(10,10)	10	10
(1,2)	1+I	11
(2,1)	2+10I	12
(1,3)	1+2I	13
(3,1)	3+9I	14
(1,4)	1+3I	15
(4,1)	4+8I	16
(1,5)	1+4I	17
(5,1)	5+7I	18
(1,6)	1+5I	19
(6,1)	6+6I	20
(1,7)	1+6I	21
(7,1)	7+5I	22
(1,8)	1+7I	23
(8,1)	8+4I	24
(1,9)	1+8I	25
(9,1)	9+3I	26
(1,10)	1+9I	27
(10,1)	10+2I	28
(2,3)	2+I	29
(3,2)	3+10I	30
(2,4)	2+2I	31
(4,2)	4+9I	32
(2,5)	2+3I	33
(5,2)	5+8I	34

(2,6)	2+4I	35
(6,2)	6+7I	36
(2,7)	2+5I	37
(7,2)	7+6I	38
(2,8)	2+6I	39
(8,2)	8+5I	40
(2,9)	2+7I	41
(9,2)	9+4I	42
(2,10)	2+8I	43
(10,2)	10+3I	44
(3,4)	3+I	45
(4,3)	4+10I	46
(3,5)	3+2I	47
(5,3)	5+9I	48
(3,6)	3+3I	49
(6,3)	6+8I	50
(3,7)	3+4I	51
(7,3)	7+7I	52
(3,8)	3+5I	53
(8,3)	8+6I	54
(3,9)	3+6I	55
(9,3)	9+5I	56
(3,10)	3+7I	57
(10,3)	10+4I	58
(4,5)	4+I	59
(5,4)	5+10I	60
(4,6)	4+2I	61
(6,4)	6+9I	62
(4,7)	4+3I	63
(7,4)	7+8I	64
(4,8)	4+4I	65
(8,4)	8+7I	66
(4,9)	4+5I	67
(9,4)	9+6I	68
(4,10)	4+6I	69
(10,4)	10+5I	70

(5,6)	5+I	71
(6,5)	6+10I	72
(5,7)	5+2I	73
(7,5)	7+9I	74
(5,8)	5+3I	75
(8,5)	8+8I	76
(5,9)	5+4I	77
(9,5)	9+7I	78
(5,10)	5+5I	79
(10,5)	10+6I	80
(6,7)	6+I	81
(7,6)	7+10I	82
(6,8)	6+2I	83
(8,6)	8+9I	84
(6,9)	6+3I	85
(9,6)	9+8I	86
(6,10)	6+4I	87
(10,6)	10+7I	88
(7,8)	7+I	89
(8,7)	8+10I	90
(7,9)	7+2I	91
(9,7)	9+9I	92
(7,10)	7+3I	93
(10,7)	10+8I	94
(8,9)	8+I	95
(9,8)	9+10I	96
(8,10)	8+2I	97
(10,8)	10+9I	98
(9,10)	9+I	99
(10,9)	10+10I	100

Table 10: Units table of $Z_{12}(I)$

Units in $(Z_{12})^2$	Units in $Z_{12}(I)$	
(1,1)	1	1
(1,5)	1+4I	2
(5,1)	5+8I	3
(5,5)	5	4

(1,7)	1+6I	5
(7,1)	7+6I	6
(7,7)	7	7
(5,7)	5+2I	8
(7,5)	7+10I	9
(1,11)	1+10I	10
(11,1)	11+2I	11
(5,11)	5+6I	12
(11,5)	11+6I	13
(7,11)	7+4I	14
(11,7)	11+8I	15
(11,11)	11	16

Table 11: Units table of $Z_{13}(I)$

Units in $Z_{13} \times Z_{13}$	Units in $Z_{13}(I)$	
(1,1)	1	1
(2,2)	2	2
(3,3)	3	3
(4,4)	4	4
(5,5)	5	5
(6,6)	6	6
(7,7)	7	7
(8,8)	8	8
(9,9)	9	9
(10,10)	10	10
(11,11)	11	11
(12,12)	12	12
(1,2)	1+I	13
(2,1)	2-I	14
(1,3)	1+2I	15
(3,1)	3-2I	16
(1,4)	1+3I	17
(4,1)	4-3I	18
(1,5)	1+4I	19
(5,1)	5-4I	20
(1,6)	1+5I	21
(6,1)	6-5I	22

(1,7)	1+6I	23
(7,1)	7-6I	24
(1,8)	1+7I	25
(8,1)	8-7I	26
(1,9)	1+8I	27
(9,1)	9-8I	28
(1,10)	1+9I	29
(10,1)	10-9I	30
(1,11)	1+10I	31
(11,1)	11-10I	32
(1,12)	1+11I	33
(12,1)	12-11I	34
(2,3)	2+I	35
(3,2)	3-I	36
(2,4)	2+2I	37
(4,2)	4-2I	38
(2,5)	2+3I	39
(5,2)	5-3I	40
(2,6)	2+4I	41
(6,2)	6-4I	42
(2,7)	2+5I	43
(7,2)	7-5I	44
(2,8)	2+6I	45
(8,2)	8-6I	46
(2,9)	2+7I	47
(9,2)	9-7I	48
(2,10)	2+8I	49
(10,2)	10-8I	50
(2,11)	2+9I	51
(11,2)	11-9I	52
(2,12)	2+10I	53
(12,2)	12-10I	54
(3,4)	3+I	55
(4,3)	4-I	56
(3,5)	3+2I	57
(5,3)	5-2I	58

(3,6)	3+3I	59
(6,3)	6-3I	60
(3,7)	3+4I	61
(7,3)	7-4I	62
(3,8)	3+5I	63
(8,3)	8-5I	64
(3,9)	3+6I	65
(9,3)	9-6I	66
(3,10)	3+7I	67
(10,3)	10-7I	68
(3,11)	3+8I	69
(11,3)	11-8I	70
(3,12)	3+9I	71
(12,3)	12-9I	72
(4,5)	4+I	73
(5,4)	5-I	74
(4,6)	4+2I	75
(6,4)	6-2I	76
(4,7)	4+3I	77
(7,4)	7-3I	78
(4,8)	4+4I	79
(8,4)	8-4I	80
(4,9)	4+5I	81
(9,4)	9-5I	82
(4,10)	4+6I	83
(10,4)	10-6I	84
(4,11)	4+7I	85
(11,4)	11-7I	86
(4,12)	4+8I	87
(12,4)	12-8I	88
(5,6)	5+I	89
(6,5)	6-I	90
(5,7)	5+2I	91
(7,5)	7-2I	92
(5,8)	5+3I	93
(8,5)	8-3I	94

(5,9)	5+4I	95
(9,5)	9-4I	96
(5,10)	5+10I	97
(10,5)	10-5I	98
(5,11)	5+6I	99
(11,5)	11-6I	100
(5,12)	5+7I	101
(12,5)	12-7I	102
(6,7)	6+I	103
(7,6)	7-I	104
(6,8)	6+2I	105
(8,6)	8-2I	106
(6,9)	6+3I	107
(9,6)	9-3I	108
(6,10)	6+4I	109
(10,6)	10-4I	110
(6,11)	6+5I	111
(11,6)	11-5I	112
(6,12)	6+6I	113
(12,6)	12-6I	114
(7,8)	7+I	115
(8,7)	8-I	116
(7,9)	7+2I	117
(9,7)	9-2I	118
(7,10)	7+3I	119
(10,7)	10-3I	120
(7,11)	7+4I	121
(11,7)	11-4I	122
(7,12)	7+5I	123
(12,7)	12-5I	124
(8,9)	8+I	125
(9,8)	9-I	126
(8,10)	8+2I	127
(10,8)	10-2I	128
(8,11)	8+3I	129
(11,8)	11-3I	130

(8,12)	8+4I	131
(12,8)	12-4I	132
(9,10)	9+I	133
(10,9)	10-I	134
(9,11)	9+2I	135
(11,9)	11-2I	136
(9,12)	9+3I	137
(12,9)	12-3I	138
(10,11)	10+I	139
(11,10)	11-I	140
(10,12)	10+2I	141
(12,10)	12-2I	142
(11,12)	11+I	143
(12,11)	12-I	144

Table 12: Units table of $Z_{14}(I)$

Units in $Z_{14} \times Z_{14}$	Units in $Z_{14}(I)$	
(1,1)	1	1
(3,3)	3	2
(5,5)	5	3
(9,9)	9	4
(11,11)	11	5
(13,13)	13	6
(1,3)	1+2I	7
(3,1)	3-2I	8
(1,5)	1+4I	9
(5,1)	5-4I	10
(1,9)	1+8I	11
(9,1)	9-8I	12
(1,11)	1+10I	13
(11,1)	11-10I	14
(1,13)	1+12I	15
(13,1)	13-12I	16
(3,5)	3+2I	17
(5,3)	5-2I	18
(3,9)	3+6I	19
(9,3)	9-6I	20

(3,11)	3+8I	21
(11,3)	11-8I	22
(3,13)	3+10I	23
(13,3)	13-10I	24
(5,9)	5+4I	25
(9,5)	9-4I	26
(5,11)	5+6I	27
(11,5)	11-6I	28
(5,13)	5+8I	29
(13,5)	13-8I	30
(9,11)	9+2I	31
(11,9)	11-2I	32
(9,13)	9+4I	33
(13,9)	13-4I	34
(11,13)	11+2I	35
(13,11)	13-2I	36

Table 13: Units table of $Z_{15}(I)$

Units in $(Z_{15})^2$	Units in $Z_{15}(I)$	
(1,1)	1	1
(2,2)	2	2
(4,4)	4	3
(7,7)	7	4
(8,8)	8	5
(11,11)	11	6
(13,13)	13	7
(14,14)	14	8
(1,2)	1+I	9
(2,1)	2-I	10
(1,4)	1+3I	11
(4,1)	4-3I	12
(1,7)	1+6I	13
(7,1)	7-6I	14
(1,8)	1+7I	15
(8,1)	8-7I	16
(1,11)	1+10I	17
(11,1)	11-10I	18

(1,13)	1+12I	19
(13,1)	13-12I	20
(1,14)	1+13I	21
(14,1)	14-13I	22
(2,4)	2+2I	23
(4,2)	4-2I	24
(2,7)	2+5I	25
(7,2)	7-5I	26
(2,8)	2+6I	27
(8,2)	8-6I	28
(2,11)	2+9I	29
(11,2)	11-9I	30
(2,13)	2+11I	31
(13,2)	13-11I	32
(2,14)	2+12I	33
(14,2)	14-12I	34
(4,7)	4+3I	35
(7,4)	7-3I	36
(4,8)	4+4I	37
(8,4)	8-4I	38
(4,11)	4+7I	39
(11,4)	11-7I	40
(4,13)	4+9I	41
(13,4)	13-9I	42
(4,14)	4+10I	43
(14,4)	14-10I	44
(7,8)	7+I	45
(8,7)	8-I	46
(7,11)	7+4I	47
(11,7)	11-4I	48
(7,13)	7+6I	49
(13,7)	13-6I	50
(7,14)	7+7I	51
(14,7)	14-7I	52
(8,11)	8+3I	53
(11,8)	11-3I	54

(8,13)	8+5I	55
(13,8)	13-5I	56
(8,14)	8+6I	57
(14,8)	14-6I	58
(11,13)	11+2I	59
(13,11)	13-2I	60
(11,14)	11+3I	61
(14,11)	14-3I	62
(13,14)	13+I	63
(14,13)	14-I	64

Table 14: Units table of $Z_{16}(I)$

Units in $Z_{16} \times Z_{16}$	Units in $Z_{16}(I)$	
(1,1)	1	1
(3,3)	3	2
(5,5)	5	3
(7,7)	7	4
(9,9)	9	5
(11,11)	11	6
(13,13)	13	7
(15,15)	15	8
(1,3)	1+2I	9
(3,1)	3-2I	10
(1,5)	1+4I	11
(5,1)	5-4I	12
(1,7)	1+6I	13
(7,1)	7-6I	14
(1,9)	1+8I	15
(9,1)	9-8I	16
(1,11)	1+10I	17
(11,1)	11-10I	18
(1,13)	1+12I	19
(13,1)	13-12I	20
(1,15)	1+14I	21
(15,1)	15-14I	22
(3,5)	3+2I	23
(5,3)	5-2I	24

(3,7)	3+4I	25
(7,3)	7-4I	26
(3,9)	3+6I	27
(9,3)	9-6I	28
(3,11)	3+8I	29
(11,3)	11-8I	30
(3,13)	3+10I	31
(13,3)	13-10I	32
(3,15)	3+12I	33
(15,3)	15-12I	34
(5,7)	5+2I	35
(7,5)	7-2I	36
(5,9)	5+4I	37
(9,5)	9-4I	38
(5,11)	5+6I	39
(11,5)	11-6I	40
(5,13)	5+8I	41
(13,5)	13-8I	42
(5,15)	5+10I	43
(15,5)	15-10I	44
(7,9)	7+2I	45
(9,7)	9-2I	46
(7,11)	7+4I	47
(11,7)	11-4I	48
(7,13)	7+6I	49
(13,7)	13-6I	50
(7,15)	7+8I	51
(15,7)	15-8I	52
(9,11)	9+2I	53
(11,9)	11-2I	54
(9,13)	9+4I	55
(13,9)	13-4I	56
(9,15)	9+6I	57
(15,9)	15-6I	58
(11,13)	11+2I	59
(13,11)	13-2I	60

(11,15)	11+4I	61
(15,11)	15-4I	62
(13,15)	13+2I	63
(15,13)	15-2I	64

3. Conclusion

In this paper we listed all invertible elements (algebraic units) in 14 different finite neutrosophic rings modulo integers $Z_n(I)$ for all $3 \leq n \leq 16$. We arranged the elements of the group of units of each ring in a table called the units table of the ring.

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