Significant Features with M-Polar Neutrosophic Topological Spaces and Grey Wolf Optimization Algorithm for Bankruptcy Prediction Model

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Abstract

An interval neutrosophic set (INS) is an example of a NS, which is simplified from the theory of fuzzy set (FS), classical set, paradoxist set, intuitionistic FS, paraconsistent set, interval-valued FS, interval-valued intuitionistic FS, and tautological set. The association of an element to an INS is stated by 3 values such as t, i, and f. These values signify memberships of truth, indeterminacy, and false, correspondingly. Bankruptcy prediction is also called a corporate failure or bankruptcy prediction, which is a major focus in the area of finance and accounting, as the condition of a business is extremely substantial to its partners, shareholders, investors, creditors, even its suppliers, and buyers. Practitioners and researchers were reserved for emerging models and approaches to forecast the bankruptcy of companies more rapidly and precisely. With the excessive growth of contemporary information technology, it has developed to use machine learning (ML) or deep learning (DL) techniques to perform the prediction, from the preliminary study of economic statements. This study introduces an Optimized Bankruptcy Prediction using Feature Selection with m-Polar Neutrosophic Topological Spaces (OBPFS-MPNTS) method. The projected OBPFS-MPNTS system uses the parameter tuning and DL method to forecast the presence of bankruptcy. To achieve this, the OBPFS-MPNTS approach uses min-max normalization to convert input data into a uniform format. The OBPFS-MPNTS method begins with a grey wolf optimization (GWO) for selecting feature subsets. In addition, the OBPFS-MPNTS algorithm applies the m-polar neutrosophic topological space (MPNTS) system for bankruptcy prediction. To upsurge the performance of the MPNTS system, the whale optimizer algorithm (WOA) is employed. The experimentation outcome study of the OBPFS-MPNTS system is verified on a benchmark database and the outcomes pointed out the developments of the OBPFS-MPNTS algorithm over other current methodologies

Keywords: Bankruptcy Prediction; Feature Selection; Neutrosophic Set; m-Polar Neutrosophic Topological Spaces; Fuzzy Set

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1. Introduction

To cope with inconsistency and uncertainty is the most significant problem for investigators who study mathematical simulation [1]. Investigators have suggested numerous estimates to make mathematical modeling for a few difficulties comprising inconsistent and uncertain data. One of the renowned calculations is the fuzzy set (FS) concept recommended and the intuitionistic fuzzy set (IFS) notion was introduced [2]. FS is identified by a membership function and an IFS is recognized by non-membership and membership functions. However, FS and IFS do not manage inconsistent and in determinant information. Hence, neutrosophic set theory (NST) was presented as a generalization of FS and IFS according to Neutrosophy, which is a subject area of philosophy [3]. Once a corporation accepts bankruptcy, its stakeholders lose some of the value they invested in the firm. From an investor's viewpoint, it's vital to consider an organization's probability of bankruptcy such that the bankruptcy threat is properly remunerated in predictable returns [4]. Academic practitioners and researchers have advanced different methods to approximate the risk of bankruptcy.

Approaches for forecasting the bankruptcy of financial companies became a main problem in the 1960s and have been broadly examined subsequently [5]. Augmented emphasis on this subject matter can be taken as a pointer to the degree of growth and robustness of an agreed economy of the country. The higher individual, social costs, and economic inherent in business bankruptcies or failures have encouraged efforts to offer improved insight into bankruptcy prediction events [6]. The earlier bankruptcy prediction methods depend on account variables. Assuming that radical change of globalization, more precise predicting of company financial distress presents valuable information for decision-makers like creditors, governmental officials, stockholders, and even normal people [7]. Company bankruptcies are affected by several aspects including a poor investment environment, low cash flow, wrong investment decisions, etc [8]. Consequently, the various existing techniques for predicting commercial failure should be constantly improved [9]. In recent times, several researchers have established that artificial intelligence (AI) namely neural networks (NNs) is an alternate technique for classification issues to which conventional statistical approaches have no longer been utilized [10].

This article presents an Optimized Bankruptcy Prediction using Feature Selection with m-Polar Neutrosophic Topological Spaces (OBPFS-MPNTS) method. To achieve this, the OBPFS-MPNTS approach uses min-max normalization to convert input data into a uniform format. The OBPFS-MPNTS system begins with a grey wolf optimization (GWO) for selecting feature subsets. In addition, the OBPFS-MPNTS algorithm employs the m-polar neutrosophic topological space (MPNTS) method for the bankruptcies prediction. To upsurge the performance of the MPNTS methodology, the whale optimization algorithm (WOA) technique is employed. The experimentation outcome study of the OBPFS-MPNTS system is verified on the benchmark dataset.

2. Related Works

Chandok et al. [11] propose an innovative WSODL-BPFCA method. The proposed WSODL-BPFCA method uses a hyper parameter tuned DL method for predicting the presence of bankruptcy. For the prediction of bankruptcy, the WSODL-BPFCA method presents an ALSTM methodology. Finally, the parameter fine-tuning of the ALSTM process is implemented by WSO employing method. Muslim et al. [12] present a grouping technique to enhance bankruptcy prediction correctness based on a stack ensemble model and GA-SVM. This research employs the Taiwan Bankruptcy dataset from the Taiwanese Finance magazine. Later a synthetic minority over sampling system is performed for processing unbalanced datasets. The author chose the optimal feature utilizing GA-SVM, implemented an innovative method by stacked the classification, and utilized extreme gradient boosting as a meta learner. Zhao et al. [13] introduce novel company authority drivers, which harness the corporate interpersonal data utilizing a board of director's company systems and the conception of node embedding attained by mapping huge directors' systems for lower dimension spaces. The presented 2 phase approach contains developing novel drivers in the initial phase accompanied by predicting the risk class in the next phase. The author studies various intricate structures of networks, which take the variety of connection patterns detected in directors' networks.

Khashei et al. [14] proposed a novel discrete direction based LR, which represents a general statistical classification technique for bankruptcy predicting has been presented. In the presented LR, on the other hand, conventionally advanced statistical classifications, the function of cost consistency, and the process of training were measured. Sen et al. [15] presented a combined technique of 4 phases that contains data rebalancing and pre-processing approaches for curating the data, performing FS, utilizing several ML and DL methods for creating a strong bankruptcy prediction method, and the usage of explainable AI to recognize the several features, which supports to the method prediction. Methods are based on the considered historical data of several non-bankrupted and bankrupted. Adisa et al. [16] introduced an innovative method of bankruptcy prediction that has been presented in this study. The new technique integrates an enhanced PSO model for identifying important features and

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improving the LSTM model for bankruptcy prediction. This study applies multi-learning and self-learning tactics to improve the intellectual and social learning parts of the PSO method. The PSO system then identifies the optimum LSTM parameters. Our concluding bankruptcy prediction technique integrates optimum FS with the LSTM method. Consequently, an enhanced technique of FS has been advanced for the LSTM method.

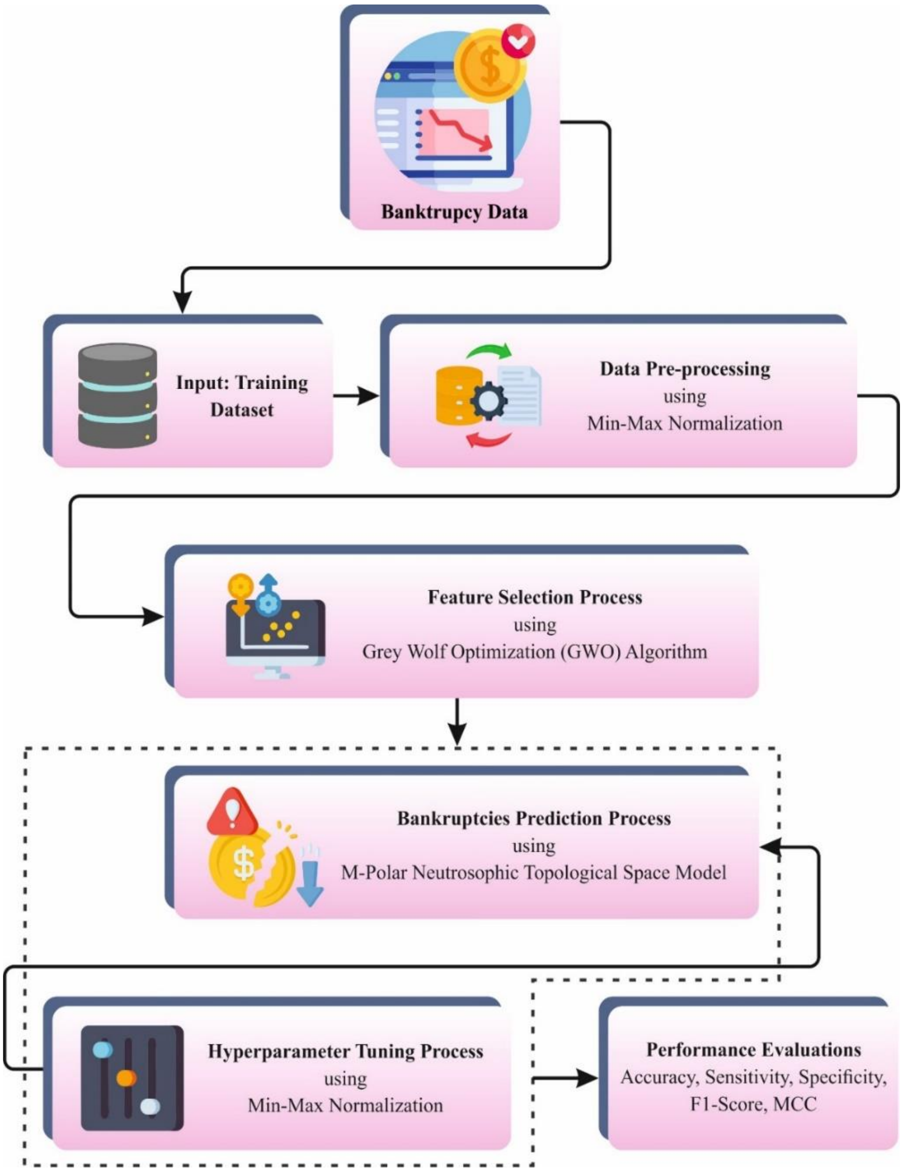

Figure 1. Workflow of OBPFS-MPNTS technique

3. Proposed Methodology

In this article, we have designed an OBPFS-MPNTS method. The projected OBPFS-MPNTS system utilizes hyperparameter-tuned DL method to forecast the presence of bankruptcy. To achieve this, the presented OBPFS-MPNTS involves data preprocessing, feature selection, bankruptcy prediction method, and hyperparameter tuning method. Fig. 1 portrays the working flow of the OBPFS-MPNTS technique.

A. Data Preprocessing

At first, the OBPFS-MPNTS method employs min max normalization to convert input data in a uniform format. Min-max normalization is a method employed to measure feature values in a fixed range, typically between 0 and 1 [17]. In bankruptcy prediction, this technique regulates economic ratios and other related features to certify that every variable donates similarly to the predictive method. By normalizing data, min-max scaling enhances the stability and performance of predictive techniques, improving the accuracy of bankruptcy risk assessments.

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B. Feature Selection

Then, the developed OBPFS-MPNTS algorithm begins with a GWO for selecting feature subsets. The GWO technique is a global random search process established by mimicking grey wolves (GW)' searching and hunting performances in the hunting [18]. There exist 4 stages of GW population on higher to lower: α, β, δ, and ω wolves. Goal seeking was strictly established on the wolf pack's order. The hunting method of the GW is divided into 3 parts such as searching and attacking prey, hunting, and encircling prey.

i) Encircled prey

GWs slowly reach and encircle prey if seeking for target and the performance is defined in Eqs. (1) and (2) that is represented below:

$$D = \left|M \cdot X_p(t) - X(f)\right| \tag{1}$$

$$X(t+1) = X_p(t) - A \cdot D \tag{2}$$

In which, t implies the existing iteration counts, $X_p(t)$ and $X(t)$ defines the position vectors of target and GW, $X(t+1)$ signifies new location of GW, and D stands for the distance among the GW and prey. M and A are correlation matrix vectors that are expressed by Eqs. (3) and (4):

$$A = 2a \cdot r_1 - a \tag{3}$$

$$M = 2 \cdot r_2 \tag{4}$$

In which, a stands for the convergence feature that linearly reduces from [2-0], under the iteration mechanism, and r_1 and r_2 signifies the random numbers among [0, 1].

ii) Hunting prey

During the direction of better wolves, wolves determine the place of possible target and continue dealing with it. This performance is expressed in Eqs. (5)- (8):

$$D_\alpha = |M_1 \cdot X_\alpha - X(t)|, X_1 = X_\alpha - A_1 \cdot D_\alpha \tag{5}$$

$$D_\beta = \left|M_2 \cdot X_\beta - X(t)\right|, X_2 = X_\beta - A_2 \cdot D_\beta \tag{6}$$

$$D_\delta = |M_3 \cdot X_\delta - X(t)|, X_3 = X_\delta - A_3 \cdot D_\delta \tag{7}$$

$$X(t+1) = \frac{X_1 + X_2 + X_3}{3} \tag{8}$$

In which, α stands for the adjacent wolf to prey, β signifies the 2nd neighboring wolf to prey, and δ stands for the 3rd neighboring wolf to prey. The advanced average of, β, and δ wolves offers the new GW position.

iii) Searching and attacking prey

GW clusters frequently hunt rely on the data of α, β, and δ wolves. Within the numerical context, the value of correlation matrix vectors A is utilized to mechanism if the GW is attacking prey or searching for prey. If $|A| > 1$, the GW in prey upsurges its searching scope to position prey more proficiently. If $|A| < 1$, the GW constrains the seeking place to hunt prey. Fig. 2 denotes the flowchart of GWO.

The FF studies the classification precision and the quantity of chosen features. It rises the classifier precision and decrease the chosen feature dimension of set. Then, the succeeding FF can be utilized to assess distinct solutions, as displayed in Eq. (9).

$$Fitness = \alpha * \ ErrorRate + (1 - \alpha) * \frac{\#SF}{\#All\_F} \tag{9}$$

whereas $ErrorRate$ represents the classification rate of error applying the chosen features. $ErrorRate$ is estimated as the percentage of improper identified to the amount of classifications created, specified as a value between 1and 0, $\#SF$ represents the value of chosen features and $\#All\_F$ denotes overall value of features in the original dataset. α can be employed to regulate the prominence of classification quality and subset length.

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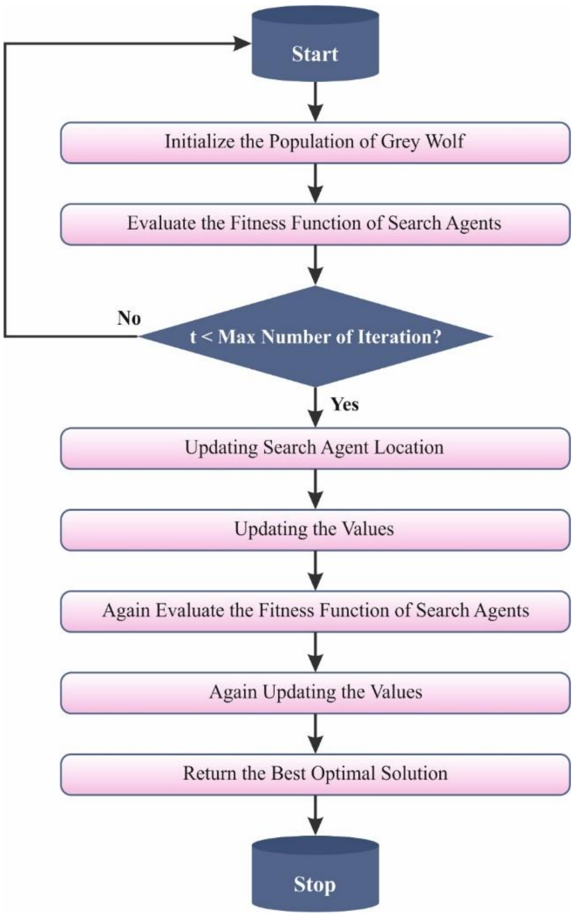

Figure 2. Flowchart of GWO

3.3. Bankruptcies Prediction Method

Furthermore, the OBPFS-MPNTS method employs the MPNTS method for the prediction of bankruptcies. In arithmetic, topology is involved with the substitutes of a geometric object, which are retained below constant bends like twisting, stretching, bending, and wrinkling, but not gluing or tearing [19]. "A space of topology is a set provided with a structure named topology that let's describing constant bend of sub-spaces and more roughly, every type of endurance. The idea of topology is definite by utilizing sets, manifolds, differential geometry, continuous functions, differentiable functions, algebra, etc.

Definition 3.1: Assume $\mathcal{Q}$ as a non-empty reference set and $\mathfrak{mpn}(\mathcal{Q})$ as a set of every MPNS in $\mathcal{Q}$. Next the set $\mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$ holding MPNSs is named MPNT if it fulfils the below mentioned assets:

- $0\mathcal{M}_{\mathfrak{R}}, 1\mathcal{M}_{\mathfrak{R}} \in \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$.
- If $(\mathcal{M}_{\mathfrak{R}})_{\wp} \in \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}, \forall \wp \in \Delta$, then $\bigcup_{\wp\in\Delta}(\mathcal{M}_{\mathfrak{R}})_{\wp} \in \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$.
- If $\mathcal{M}_{\mathfrak{R}_1}, \mathcal{M}_{\mathfrak{R}_2} \in \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$, then $\mathcal{M}_{\mathfrak{R}_1} \cap \mathcal{M}_{\mathfrak{R}_2} \in \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$.

Theorem3.2: Assume $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}})$ as an MPNTS. Next, the below mentioned criteria are fulfilled:

- $0\mathcal{M}_{\mathfrak{R}}$ and $1\mathcal{M}_{\mathfrak{R}}$ are open MPNSs.
- Union of any amount of open MPNSs is open.
- Intersection of a finite amount of closed MPNSs can be closed.

Definition3.4: Assume $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}})$ and $(\mathcal{Q}, \mathcal{T}'_{\mathcal{M}_{\mathfrak{R}}})$ as a dual MPNTSs in $\mathcal{Q}$. They are similar when $\mathcal{T}_{\mathcal{M}_{\mathfrak{R}}} \subseteq \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$ or $\mathcal{T}_{\mathcal{M}_{\mathfrak{R}}} \subseteq \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$.

When $\mathcal{T}_{\mathcal{M}_{\mathfrak{R}}} \subseteq \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$, then $\mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$ is weaker or worse than $\mathcal{T}'_{\mathcal{M}_{\mathfrak{R}}}$ and $\mathcal{T}'_{\mathcal{M}_{\mathfrak{R}}}$ is robust and better than $\mathcal{T}_{\mathcal{M}_{\mathfrak{R}}}$.

Theorem3.5: Assume $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\mathfrak{R}}})$ as an MPNTS. Thus the below mentioned criteria are fulfilled:

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1) $0\mathcal{M}_{\Re}$ and $1\mathcal{M}_{\Re}$ are closed MPNSs.
2) Intersection of any quantity of closed MPNSs is closed.
3) Union of a finite amount of closed MPNSs is closed.

- $({}^{1}\mathcal{M}_{\Re})^{c} = 0\mathcal{M}_{\Re}$ and $({}^{0}\mathcal{M}_{\Re})^{c} = 1\mathcal{M}_{\Re}$ are both closed and open MPNSs.
- If $\{\mathcal{M}_{\Re_\alpha} : \mathcal{M}_{\Re_\alpha}^{c} \in \mathcal{T}_{\mathcal{M}_{\Re}}, \alpha \in \Delta\}$ is a collection of closed MPNSs then $(\bigcap_{\alpha\in\Delta} \mathcal{M}_{\Re_\alpha})^{c} = \bigcup_{\alpha\in\Delta} \mathcal{M}_{\Re_\alpha}^{c}$ is open. This displays that $\bigcap_{\alpha\in\Delta} \mathcal{M}_{\Re_\alpha}$ is closed MPNS.
- Then $\mathcal{M}_{\Re_\beta}$ is closed for $\beta = 1,2,\dots,z$, next $(\bigcup_{\beta=1}^{z} \mathcal{M}_{\Re_\beta})^{c} = \bigcap_{\beta=1}^{z} \mathcal{M}_{\Re_\beta}^{\ c}$ is open MPNS. Thus $\bigcup_{\beta=1}^{z} \mathcal{M}_{\Re_\beta}$ is closed MPNS.□

Definition3.6: Assume $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\Re}})$ as a MPNTS and

$\mathcal{M}_{\Re} \in \mathfrak{mpn}({}^{1}\mathcal{M}_{\Re})$, then interior of $\mathcal{M}_{\Re}$ is signified by $\mathcal{M}_{\Re}^{o}$ and definite as the union of every open MPN sub-set restricted in $\mathcal{M}_{\Re}$. It is the highest open MPNS limited in $\mathcal{M}_{\Re}$.

Theorem3.8: Assume $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\Re}})$ as a MPNTS and $\mathcal{M}_{\Re} \in \mathfrak{mpn}(\mathcal{Q})$. Then $\mathcal{M}_{\Re}$ is open $MPNS \Leftrightarrow \mathcal{M}_{\Re}^{o} = \mathcal{M}_{\Re}$.

Proof: When $\mathcal{M}_{\Re}$ is open MPNS then the highest open MPNS limited in $\mathcal{M}_{\Re}$ is itself $\mathcal{M}_{\Re}$. Therefore $\mathcal{M}_{\Re}^{o} = \mathcal{M}_{\Re}$.

Theorem3.9: Consider $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\Re}})$ as an MPNTS and $\mathcal{M}_{\Re_1}, \mathcal{M}_{\Re_2} \in \mathfrak{mpn}({}^{1}\mathcal{M}_{\Re})$, then

- $(\mathcal{M}_{\Re_1}^{o})^{o} = \mathcal{M}_{\Re_1}^{o}$,
- $\mathcal{M}_{\Re_1} \subseteq \Lambda4_{\Re_2} \Rightarrow \mathcal{M}_{\Re_1}^{o} \subseteq \mathcal{M}_{\Re_2}^{o}$,
- $(\mathcal{M}_{\Re_1} \cap \mathcal{M}_{\Re_2})^{o} = \mathcal{M}_{\Re_1}^{o} \cap \mathcal{M}_{\Re_2}^{o}$,
- $(\mathcal{M}_{\Re_1} \cup \mathcal{M}_{\Re_2})^{o} \subseteq \mathcal{M}_{\Re_1}^{o} \cup \mathcal{M}_{\Re_2}^{o}$.

Proof: The proof is understandable.

Definition3.10: Assume $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\Re}})$ as an MPNTS and

$\mathcal{M}_{\Re} \in \mathfrak{mpn}(\mathcal{Q})$, then the closure of $\mathcal{M}_{\Re}$ is represented as $\overline{\mathcal{M}_{\Re}}$ and definite by the connection of every closed MPN $\mathcal{M}_{\Re}$ super-sets of $\mathcal{M}_{\Re}$. It is the least closed-MPN super-set of

Theorem3.12: Consider $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\Re}})$ as an MPNTS and $\mathcal{M}_{\Re} \in \mathfrak{mpn}(\mathcal{Q}).\mathcal{M}_{\Re}$ is closed $MPNS \Leftrightarrow \overline{\mathcal{M}_{\Re}} = \mathcal{M}_{\Re}$.

Proof: The proof is obvious.

Definition3.13: Examine $\mathcal{M}_{\Re}$ as an MPN subset of $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\Re}})$, then its border is determined by $F_r(\Lambda4_{\Re}) = \overline{\mathcal{M}_{\Re}} \cap \overline{\mathcal{M}_{\Re_4}^{c}}$.

Definition3.14: Assume $\mathcal{M}_{\Re}$ as an MPN subset of $(\mathcal{Q}, \mathcal{T}_{\mathcal{M}_{\Re}})$, then its external is signified as $Ext(\mathcal{M}_{\Re})$ and determined as $Ext(\overline{\mathcal{M}_{\mathcal{R}}}) = (\overline{\mathcal{M}_{\Re}})^{c} = (\mathcal{M}_{\Re}^{c})^{o}$

Example3.15: Let's assume the MPNTS built-in

Example3.3 and assume the MPNSs $\mathcal{M}_{\Re_3}$ and $\mathcal{M}_{\Re_4}$ set in Examples 3.7 and 3.11. Next, by utilizing the preceding descriptions we can inscribe that

$\mathcal{M}_{\Re_3}^{o} = \mathcal{M}_{\Re_2}, \overline{\mathcal{M}_{\Re_3}} =^{1} \mathcal{M}_{\Re}$,

$$F_r(\mathcal{M}_{\Re_3}) =^{1} \mathcal{M}_{\Re}, Ext(\mathcal{M}_{\Re_3}) =^{0} \mathcal{M}_{\Re},$$

$$\mathcal{M}_{\Re_4}^{o} =^{0} \mathcal{M}_{\Re}, \overline{\mathcal{M}_{\Re_4}} = \mathcal{M}_{\Re_1}^{c},$$

$$F_r(\mathcal{M}_{\Re_4}) = \mathcal{M}_{\Re_1}^{c}, Ext(\mathcal{M}_{\Re_4}) = \mathcal{M}_{\Re_1}.$$

Now, we introduce a few outcomes that don't grip in MPNTS nevertheless embrace in crisp set theory owing to the laws of excluded central and contradiction.

Theorem3.16: Consider $\Lambda4_{\Re} \in \mathfrak{mpn}({}^{1}\Lambda4_{\Re})$, then

1) $(\mathcal{M}_{\Re}^{o})^{c} = \overline{(\mathcal{M}_{\Re}^{c})}$,
2) $(\overline{\mathcal{M}}_{\mathcal{R}})^{c} = (\mathcal{M}_{\Re}^{c})^{o}$,
3) $Ext(\mathcal{M}_{\Re}^{c}) = \mathcal{M}_{\Re}^{o}$,
4) $Ext(\mathcal{M}_{\Re}) = (\mathcal{M}_{\Re}^{c})^{o}$,

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5) $Ext(\mathcal{M}_{\Re}) \cup F_r(\mathcal{M}_{\Re}) \cup \mathcal{M}_{\Re}^{O} \neq 1\mathcal{M}_{\Re}$,
6) $F_r(\mathcal{M}_{\Re}) = F_r(\mathcal{M}_{\Re}^{c})$,
7) $\mathcal{M}_{\Re}^{o} \cap F_r(\mathcal{M}_{\Re}) \neq 0\mathcal{M}_{\Re}$.

Proof

(1) and (2): are obvious.

(3) $Ext(\mathcal{M}_{\Re}^{c}) = (\overline{\mathcal{M}_{\mathcal{R}}})^{c}$

$$\Rightarrow Ext(\mathcal{M}_{\Re}^{c}) = [(\mathcal{M}_{\Re}^{c})^{c}]^{o}$$

$$\Rightarrow Ext(\mathcal{M}_{\Re}^{c}) = \mathcal{M}_{\Re}^{o}.$$

(4) $Ext(\mathcal{M}_{\Re}) = (\overline{\mathcal{M}_{\mathcal{R}}})^{c}$

$$\Rightarrow Ext(\mathcal{M}_{\Re}) = (\mathcal{M}_{\Re}^{c})^{o}$$

(5) $Ext(\mathcal{M}_{\Re}) \cup F_r(\mathcal{M}_{\Re}) \cup \mathcal{M}_{\Re}^{o} \neq 1\mathcal{M}_{\Re}$.

By Example3.15: we can perceive that $\mathcal{M}_{\Re_1} \cup \mathcal{M}_{\Re_1}^{c} \cup^{0} \mathcal{M}_{\Re} \neq 1\mathcal{M}_{\Re}$.

(6) $F_r(\mathcal{M}_{\Re}^{c}) = \overline{(\mathcal{M}_{\Re}^{c})} \cap \overline{[(\mathcal{M}_{\Re}^{c})]^{c}}$

$\Rightarrow F_r(\mathcal{M}_{\Re}^{c}) = \overline{(\mathcal{M}_{\Re}^{c})} \cap \overline{(\mathcal{M}_{\Re})} = F_r(\mathcal{M}_{\Re})$.

(7) $\mathcal{M}_{\Re}^{o} \cap F_r(\mathcal{M}_{\Re}) \neq 0\mathcal{M}_{\Re}$. Example3.15 displays that $\mathcal{M}_{\Re_2} \cap^{1} \mathcal{M}_{\Re} \neq 0\mathcal{M}_{\Re}$.

3.4. Hyperparameter Tuning Process

Eventually, to increase the MPNTS model performance, the WOA model can be used. The WOA is a nature-inspired MH method that has been developed by S. Mirjalili et al. and emulates the natural accommodating behaviors of humpback whales [20]. In WOA, the optimization algorithm can be achieved utilizing ERP and EIP. Bubble-net and Encircling bait attacking, are two approaches of WOA used to upgrade the optimal position of whales. Generally, WOA can be performed in three phases that is represented below;

i) Encircling bait

After the finest solution or Search Agent (SA) in SS can be determined, other SAs try to upgrade its position in the SS to the optimum SA as follows:

$$\vec{D} = \left|\vec{C}.\vec{X} * (t) - \vec{X}(t)\right| \tag{10}$$

$$\vec{A} = 2\vec{a}.\vec{r} - \vec{a} \tag{11}$$

$$\vec{C} = 2\vec{r} \tag{12}$$

Where$\vec{D}$ refers to the distance among the locations of $\vec{X} * (t)$ and $\vec{X}(t)$, t indicates the existing iteration, $\vec{a}$ linearly dropped from 2 to 0, $\vec{A}$ and $\vec{C}$ are the vectors of coefficient, the $X^{*}(t)$ and $\vec{X}(t)$ are location vectors from which the optimum solution is attained, and $\vec{r}$ is a arbitrary vector within [0,1].

ii) Exploitation Phase

In this stage, an equation of spiral is generated among the whales position and the bait to stimulate the humpback whales spiral shaped movement:

$$\vec{X_{\iota}}(t+1) = \vec{D}'(t).e^{bl}.cos(2\pi l) + \vec{X} * (t) \tag{13}$$

Where $\vec{D}'$ denotes the distance from $1 - th$ SA to the bait (the optimum solution attained), the constant b defines the shape of a logarithmic spiral, and $l \in [-1,1]$.

$$\vec{X_{\iota}}(t+1) = \begin{cases} \vec{X}(t). -\vec{A} \cdot \vec{D}, & p < 05 \\ \vec{D}'(t)e^{bl}.cos(2\pi l) + \vec{X} * (t), & p \geq 0.5 \end{cases} \tag{14}$$

iii) Exploration Phase

The mathematical modeling of these behaviors is given below:

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$$\vec{D} = \left|\vec{C}.\overrightarrow{X_{rand}} - \vec{X}\right| \tag{15}$$

$$\vec{X}(t+1) = \overrightarrow{X_{rand}} - \vec{A}.\vec{D} \tag{16}$$

Where the arbitrarily selected location vector of the current population is $\overrightarrow{X_{rand}}$. If $\left|\vec{A}\right| > 1$, the random SA is selected, if $\left|\vec{A}\right| < 1$ Then optimum solution is selected for updating the location of SAs. The process of improving the solution is done until the ending criteria are met.

The WOA method develops an FF to acquire enhanced classification performance. It identifies a positive integer to signify the greater performances of the candidate solutions. In this research, the reduction of the classification rate of error can be examined as the FF, as provided in Eq. (17).

$$fitness(x_i) = ClassifierErrorRate(x_i)$$

$$= \frac{no.of\ misclassified\ samples}{Total\ no.of\ samples} * 100 \tag{17}$$

4. Performance Analysis

The performance validation of the OBPFS-MPNTS approach is tested utilizing the database [21]. The database comprises 1000 examples under two classes as displayed in Table 1. The total number of features is 24 but selected features are 13.

Table 1: Details of Dataset

Class	No. of Instances
Bankrupt	300
Non-Bankrupt	700
Total Instances	1000

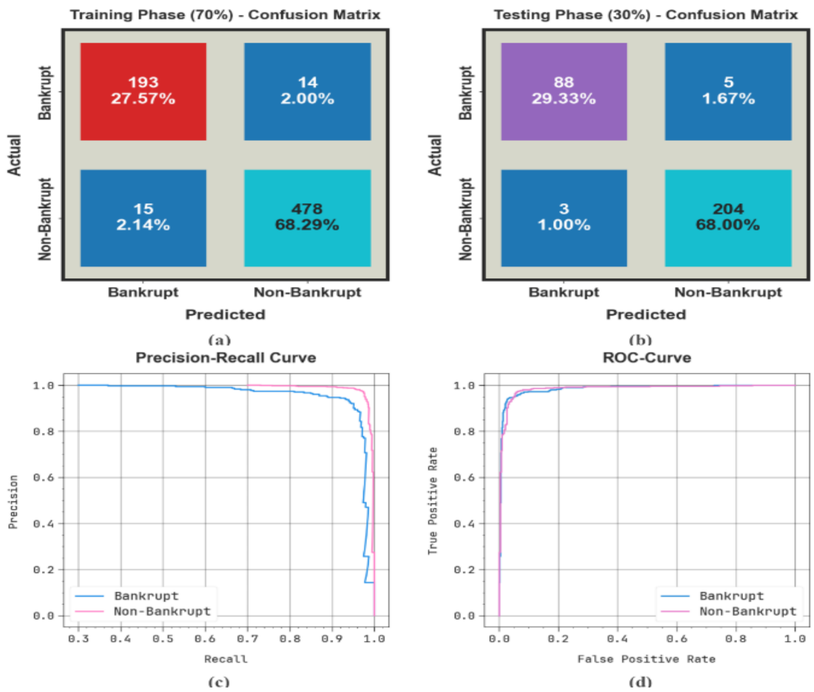

Figure 3. Classifier outcomes of (a-b) confusion matrices and (c-d) PR and ROC curves

Fig. 3 represents the classification outcomes of the OBPFS-MPNTS algorithm on the test dataset. Figs. 3a-3b displays the confusion matrix with accurate identification and classification of all class labels on a 70:30 TRAP/TESP. Fig. 3c represents the PR analysis, demonstrating maximum performance across all classes. Lastly, Fig. 3d shows the ROC analysis, representing efficient outcomes with high ROC values for distinct class labels.

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Table 2 and Fig. 4 signify the bankruptcy prediction results of the OBPFS-MPNTS algorithm under 70%TRAP and 30%TESP. The results denote that the OBPFS-MPNTS process properly identified the samples. With 70%TRAP, the OBPFS-MPNTS technique provides average $accu_y$, $sens_y$, $spec_y$, $F1_{score}$ and MCC of 95.10%, 95.10%, 95.10%, 95.03%, and 90.07%, respectively. At the same time, With 30%TESP, the OBPFS-MPNTS approach provides average $accu_y$, $sens_y$, $spec_y$, $F1_{score}$ and MCC of 96.59%, 96.59%, 96.59%, 96.86%, and 93.74%, correspondingly.

Table 2: Bankruptcy Prediction result of OBPFS-MPNTS method under 70%TRAP and 30%TESP

Class	Accuracy	Sensitivity	Specificity	F1-Score	MCC
TRAP (70%)					
Bankrupt	93.24	93.24	96.96	93.01	90.07
Non-Bankrupt	96.96	96.96	93.24	97.06	90.07
Average	95.10	95.10	95.10	95.03	90.07
TESP (30%)					
Bankrupt	94.62	94.62	98.55	95.65	93.74
Non-Bankrupt	98.55	98.55	94.62	98.08	93.74
Average	96.59	96.59	96.59	96.86	93.74

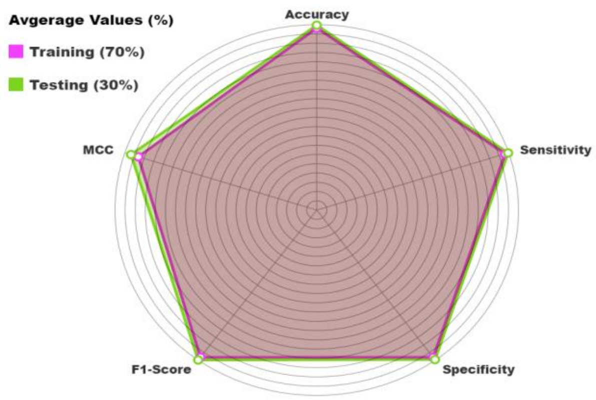

Figure 4. Average of OBPFS-MPNTS technique under 70%TRAP and 30%TESP

In Fig. 5, the training (TRA) and validation (VLA) accuracy outcomes of the OBPFS-MPNTS process are illustrated. The accuracy values are computed for 0-25 epochs. The figure emphasized that the TRA and VLA accuracy values display a rising trend which notified the capability of the OBPFS-MPNTS model with improved performance over several iterations. Moreover, the TRA accuracy and VLA accuracy remain closer over the epochs, which specifies low minimal overfitting and exhibits enhanced performance of the OBPFS-MPNTS methodology, guaranteeing consistent prediction on unseen samples.

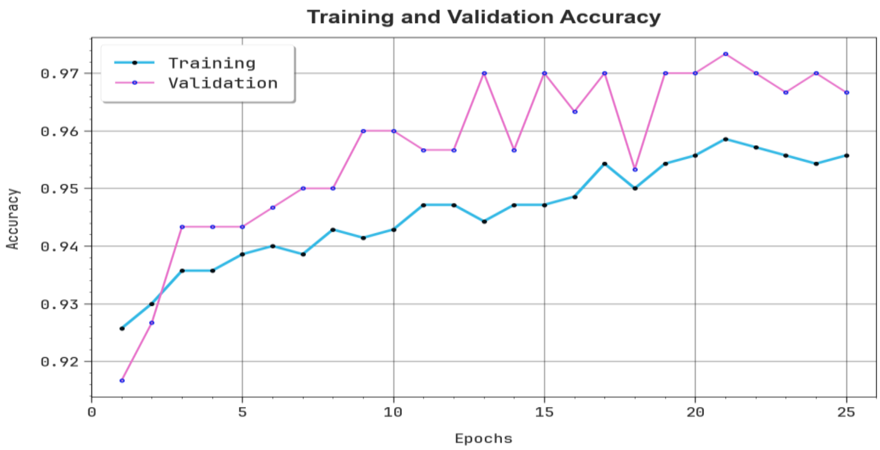

Figure 5. $Accu_y$ Curve of the OBPFS-MPNTS technique

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Figure 6. Loss curve of OBPFS-MPNTS technique

In Fig. 6, the TRA and VLA loss graph of the OBPFS-MPNTS system is shown. The loss values are computed for 0-25 epochs. It is signified that the TRA and VLA accuracy values demonstrate a lower trend, notifying the ability of the OBPFS-MPNTS approach to balance a trade-off between data fitting and generalization. The continual reduction in loss values furthermore guarantees the enhanced performance of the OBPFS-MPNTS method and tunes the prediction results over time.

In Fig. 7, the comparative study of the OBPFS-MPNTS algorithm with current methods is displayed in terms of $accu_y$ and $F1_{score}$ [22]. The simulation result indicated that the OBPFS-MPNTS model outperformed better performances. Based on $accu_y$, the OBPFS-MPNTS process has higher $accu_y$ of 96.59% while the HHPO-DLFCP algorithm, QABO-LSTM-RNN method, LSTM-RNN system, ACO system, MLP methodology, SVM technique, and AdaBoost approaches have lesser $accu_y$ of 95.11%, 92.27%, 84.98%, 76.09%, 71.22%, 71.53%, and 67.75%, correspondingly. Based on $F1_{score}$, the OBPFS-MPNTS methodology has a higher $F1_{score}$ of 96.86% while the HHPO-DLFCP algorithm, QABO-LSTM-RNN method, LSTM-RNN system, ACO system, MLP methodology, SVM technique, and AdaBoost approaches have lesser $F1_{score}$ of 94.14%, 90.53%, 89.15%, 85.71%, 75.53%, 72.14%, and 71.65%, correspondingly.

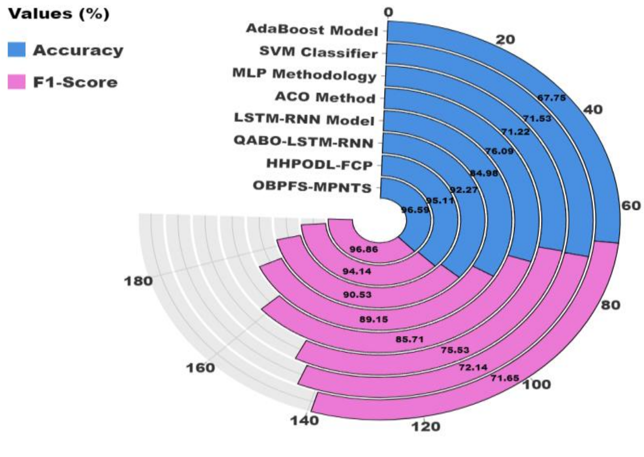

Figure 7. $Accu_y$ and $F1_{score}$ of OBPFS-MPNTS technique with existing models

In Fig. 8, the comparative study of the OBPFS-MPNTS algorithm with recent models is reported in terms of $sens_y$ and $spec_y$. Based on $sens_y$, the OBPFS-MPNTS model has higher $sens_y$ of 96.59% while the HHPO-DLFCP algorithm, QABO-LSTM-RNN method, LSTM-RNN system, ACO system, MLP methodology, SVM technique, and AdaBoost approaches have lesser $sens_y$ of 93.75%, 87.61%, 82.35%, 78.54%, 74.23%, 73.02%, and 71.61%, correspondingly. Based on $spec_y$, the OBPFS-MPNTS method has higher $spec_y$ of 96.59% while the HHPO-DLFCP algorithm, QABO-LSTM-RNN method, LSTM-RNN system, ACO system, MLP methodology, SVM technique, and AdaBoost processes have lesser $spec_y$ of 94.28%, 93.94%, 88.72%, 69.56%, 67.11%, 66.83%, and 61.5%, respectively.

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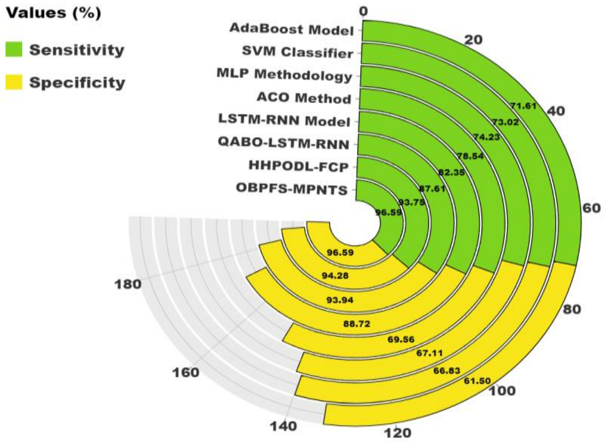

Figure 8. $Sens_y$ and $Spec_y$ of OBPFS-MPNTS technique with recent methods

In Table 3 and Fig. 9, the comparative results of the OBPFS-MPNTS system are specified in terms of computational time (CT). The result suggests that the OBPFS-MPNTS model gets better performance. Based on CT, the OBPFS-MPNTS technique offers lesser CT of 2.55s whereas the HHPO-DLFCP algorithm, QABO-LSTM-RNN method, LSTM-RNN system, ACO system, MLP methodology, SVM technique, and AdaBoost method obtain greater CT values of 5.40s, 4.54s, 7.12s, 4.83s, 6.63s, 3.37s, and 6.45s, respectively.

Table 3: CT outcome of OBPFS-MPNTS method with recent methods

Classifiers	Computational Time (sec)
OBPFS-MPNTS	2.55
HHPO-DLFCP	5.40
QABO-LSTM-RNN	4.54
LSTM-RNN Model	7.12
ACO Method	4.83
MLP Methodology	6.63
SVM Classifier	3.37
AdaBoost Model	6.45

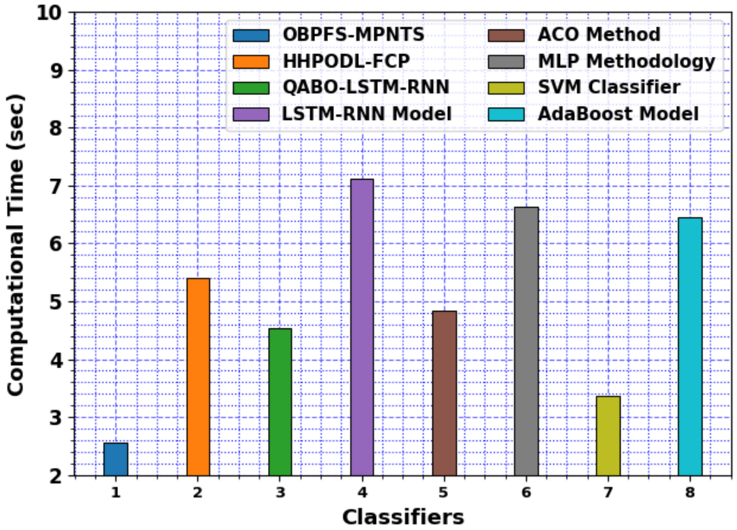

Figure 9. CT outcome of OBPFS-MPNTS approach with recent methods

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5. Conclusion

In this article, we presented an OBPFS-MPNTS method. The projected OBPFS-MPNTS system utilizes a hyperparameter-tuned DL method to forecast the presence of bankruptcy. To achieve this, the presented OBPFS-MPNTS involves data preprocessing, feature selection, bankruptcy prediction method, and hyperparameter tuning process. At first, the presented OBPFS-MPNTS algorithm applies min-max normalization to convert input data into a uniform format. Next, the OBPFS-MPNTS approach begins with a GWO for electing feature subsets. Furthermore, the OBPFS-MPNTS methodology employs the MPNTS method for the bankruptcy prediction. Eventually, to optimum the performance of the MPNTS model, the WOA model is utilized. The experimentation outcome analysis of the OBPFS-MPNTS system is verified on the benchmark dataset.

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