A Fuzzy Adaptive Control Chart as an Alternative to Neutrosophic Techniques for Handling Imprecise Data

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Abstract

Quality control (QC) charts are essential for ensuring industry process stability, but imprecise data make traditional methods unuseful in such a case. Neutrosophic control charts are available to handle the imprecise data. This article learns fuzzy logic as an approach of handling uncertainty more suitably than neutrosophic approaches. Fuzzy QC charts make use of fuzzy numbers, membership functions and fuzzy control limits and as such are more realistic compared to conventional charts. The study introduces a Fuzzy Adaptive Exponentially Weighted Moving Average (FAEWMA) chart, specifically designed for univariate data in a fuzzy atmosphere. The FAEWMA chart, incorporating α-cuts, is engineered to detect shifts in process means, showcasing its effectiveness through both theoretical development and practical applications. This approach improves decision-making in process control and represents a significant advancement over traditional QC methods.

Keywords: Adaptive control chart; Neutrosophic chart; Fuzzy control chart; Fuzzy EWMA

1. Introduction

The use of QC charts is crucial for process control in manufacturing organizations and healthcare delivery systems. However, traditional QC techniques, which focus on accurate data, can sometimes fail with imprecise or ambiguous information. This issue is solved using fuzzy logic which has been developed by Lotfi Zadeh [1], whereas fuzzy logic deals with uncertainties and it is possible to work with partially true data. Fuzzy QC charts contain fuzzy numbers, membership functions and fuzzy control limits because they depict real life conditions more accurately as compared to conventional charts. It increases flexibility, sensitivity, and effectiveness of decision-making in process control, which makes the use of fuzzy QC charts especially valuable in conditions where variabilities are inherent and measurements are imprecise. These charts apply fuzzy logic in a sophisticated way for quality control across numerous applications, making them a vast improvement over traditional QC processes. QC charts have developed from simple Shewart charts to EWMA chart by Roberts [2] that checks location parameter of a process and CUSUM chart by Page [3] that checks dispersedness of the process. This again underlines the importance of monitoring the process variance to ensure quality and productivity. Reducing variations enhances process capability, while normalization of dispersion estimators ensures unbiased average run length (ARL) performance, enabling quicker detection of out-of-control conditions. This enhances the accuracy and reliability of detecting out-of-control signals. Asalam et al. [4] developed an $\overline{X}$ chart using neutrosophic EWMA (NEWMA) statistics based on the symmetry of the normal distribution, evaluated using neutrosophic Monte Carlo simulation. The effectiveness of the recommended chart is compared with

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existing control charts in managing uncertainty. Shafqat et al. [5] designs neutrosophic double and triple EWMA (NDEWMA and NTEWMA) charts for uncertain or fuzzy data. Monte Carlo simulations assess their effectiveness, showing improved detection of small shifts compared to existing charts. The recommended charts perform better in real-life applications like monitoring road traffic crashes and electric engineering data, making them useful for minimizing road accidents and defective products under uncertain conditions. Asalam et al. [6] proposes a new NEWMA accompanied by the neutrosophic logarithmic transformation chart for variance with the use of neutrosophic numbers. Even the parameters are computed using neutrosophic Monte Carlo simulation and effectiveness of the chart is compared with the other charts existent in literature. Using multiple dependent state sample new statistic khan et al. [7] developed an effective mean chart for symmetric data. This chart overcomes the limitations of existing NEWMA charts in detecting unusual changes in manufacturing processes. The chart's performance is evaluated using neutrosophic Monte Carlo simulations, showing it to be a robust and effective solution compared to existing charts. Shah et al. [8] introduces the neutrosophic Maxwell distribution ($M\tilde{D}$) to handle imprecise data, particularly in SPC. A neutrosophic VSQ-chart ($\tilde{V}SQ$) is developed based on this distribution to address uncertainty in quality variables, outperforming conventional models. The chart's effectiveness is demonstrated through performance indicators and a real data application on the COVID-19 incubation period.

Maghrabi [9] proposes a Complex Proportional Assessment Based Neutrosophic Approach for Ransomware Detection in Cybersecurity (CPABNA-RDCS) in an IoT environment. The method classifies ransomware using the CPABN technique, optimized by the grey wolf optimizer (GWO), and shows superior performance on benchmark data compared to existing techniques. Neutrosophic control charts are designed to handle uncertainty by incorporating indeterminate, ambiguous, and imprecise data into the monitoring process. Similarly, fuzzy control charts also address the problem of uncertainty, allowing for the inclusion of data that is not precisely defined. Both approaches provianced flexibility in quality control by accommodating uncertainty in the data, making them effective tools for environments where precise measurements are challenging or impossible. Alipour & Noorossana [10] develops a fuzzy multivariate EWMA (F-MEWMA) chart by combining multivariate SQC control with fuzzy approach. The F-MEWMA chart's performance, demonstrated through a numerical example, outperforms the fuzzy Hotelling's T^2 control chart.

Faraz and Shapiro [11] propose a fuzzy chart to handle randomness and incomplete information without defuzzification, extending Shewhart charts into fuzzy space. It utilizes a fuzzy in-control region and a graded exclusion measure to determine out-of-control states, thereby preserving the informational content of fuzzy data. Quality control processes are crucial for monitoring and enhancing quality and productivity, yet imprecise observations are common. This study introduces a comprehensive approach to fuzzy statistical quality control, developing methods such as fuzzy control charts, process capability assessments, confidence intervals, and hypothesis testing within a fully fuzzy framework. It demonstrates their effectiveness through numerical examples by Hesamian et al. [12].

Şentürk et al. [13] introduce fuzzy EWMA chart (FEWMA) for handling uncertainties in sample data, reducing false decisions, and enhancing flexibility in detecting small shifts, demonstrated through monitoring the production process of plastic buttons in Turkey. Conventional control charts, essential for statistical process control, are unsuitable for unclear or fuzzy data common in real-world applications due to environmental uncertainties and ambiguities. Khan et al. [14] proposed a F-EWMA chart for identifying monir shifts in the location parameter using fuzzy data, supported by real-life examples.

Khademi and Amirzadeh [15] introduce two fuzzy control chart approaches for monitoring sample averages and ranges using "fuzzy mode" and "fuzzy rules" with non-symmetric triangular fuzzy numbers, avoiding defuzzification and preserving sample information. A numeric example illustrates the method's performance and results. Taylan and Darrab [16] illustrate the application of artificial intelligence in quality control by designing fuzzy charts for tip shear carpets. They describe a systematic process that includes variable selection, neural network-based membership function development, fuzzy rule base creation, and defuzzification to generate precise results. Their findings suggest that fuzzy linguistic terms improve quality assessment and effectively complement traditional statistical process control tools, providing a flexible and meaningful approach to identifying product quality distribution.

Al-Refaie et al. [17] applies fuzzy EWMA and CUSUM charts on triangular fuzzy numbers as the input data, with three real applications. The charts are able to pinpoint the blurry observations and thus to detect changes in the process means; the results presented indicate, that the decisions concerning the process condition depend on both the mean shifts and α-cut values. It is especially suitable for the control of fuzzy quality characteristics in a number of business environments.

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Ming-Hung Shu et al. [18] propose the Fuzzy-MaxGWMA (F-MaxGWMA) chart, which is the MaxGWMA chart modified for use with imprecise data because of their randomness and fuzziness. It has an index of optimism as a criterion to which different conditions of the process can be observed and then classified into various statuses, unlike the MaxGWMA chart that is only binary. This kind of approach has been illustrated by an example, which is done on actual application of coating thickness control in cutting-tool manufacturing. Kaplan Göztok et al. [19] present a FEWMA chart with α-level cuts for detecting small shifts and a fuzzy process capability index (FCpm) for performance measurement. Their method's effectiveness and versatility are illustrated through a case study at a pumice block plant. Kaya and Kahraman [20] employ fuzzy set theory to enhance process capability analysis (PCA) by incorporating fuzzy process mean and variance, leading to the development of fuzzy PCIs (FPCIs) for indices such as Cp, Cpk, Ca, Cpm, and Cpmk. A numerical example from the automotive industry demonstrates that fuzzy estimations provide richer evaluations of process performance compared to traditional crisp definitions. Zabihinpour et al. [21] develop fuzzy $\bar{X}$-S control charts using triangular fuzzy random variables to address random and vague uncertainties. A detailed study evaluates sample size and out-of-control levels, showing improved detection of process mean shifts by 0.1% to 30%. The method is validated through a case study in noodle production.

As mentioned in the introduction, there has been limited focus in the literature on detecting minor shifts under uncertain conditions. Despite the widespread use of fuzzy $\bar{X}$-R and fuzzy $\bar{X}$-S control charts, they may lack the capability to discern minor shifts in the location parameter. Additionally, these charts only trigger a response when the most recent data point falls outside the upper or lower control limits. Moreover, previous studies frequently lack a unified approach to addressing two key aspects of fuzzy control processes: fuzzy process control charts and fuzzy process capability analyses.

This article introduces a fuzzy adaptive EWMA (FAEWMA) chart for univariate data within a fuzzy approach. A primary contribution of this study lies in establishing the theoretical framework for FAEWMA control chart with α-cuts for univariate data. When dealing with linguistic data and the need arises to identify small, moderate, or large shifts, the FAEWMA chart emerges as an indispensable tool for SPC. The rest of article is prepared as follows: Section 2 presents the definitions of triangular fuzzy numbers. Section 3 provides an overview of the existing fuzzy EWMA control charts utilizing alpha-cuts. In Section 4, we discuss the suggested FAEWMA chart in detail. Section 5 includes a discussion of the main findings. A real-world application is provided in Section 6, demonstrating the practical use of the recommended method. Finally, conclusions are included in Section 7.

2. Triangular Fuzzy Numbers (TFN) with an α-level Cut

This section introduced TFNs to develop the FEWMA chart. Triangular fuzzy numbers, represented by three points a, b, and c, capture uncertainty by defining a range of possible values with varying degrees of membership. The nature of the fuzzy number is represented by three constants a, b and c, where a is the lower bound, b is the expected value or mode and c as the upper bound. The triangular membership function, which is utilizied to compute the fuzzy membership values, is characterized by its simplicity and efficiency in handling fuzziness. Mathematically, the triangular membership function μ(x) can be expressed as:

$$f\left(x,a,b\right)=\begin{cases}0, x\leq a\\ \dfrac{x-a}{b-a}, a\leq x\leq b\\ \dfrac{c-x}{c-b}, b\leq x\leq c\\ 0, c\leq x\end{cases} \quad (1)$$

This function effectively models the gradual transition of membership values within the range defined by the TFNs. By employing TFNs and their corresponding triangular membership functions as described in Figure 1, the FEWMA control chart can more accurately reflect the inherent uncertainty in the data, leading to more reliable monitoring and identifying minor shifts in the process mean.

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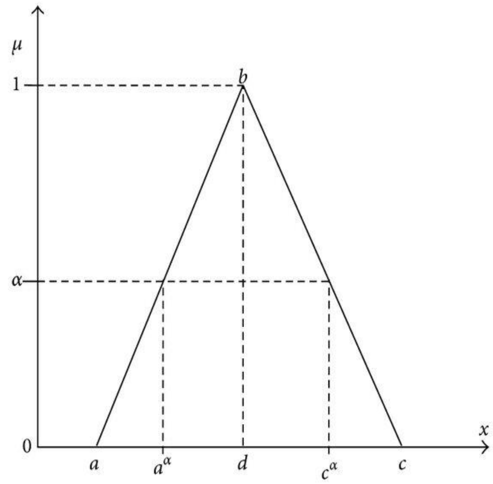

Figure 1. TFN with an α - level cut for a sample.

$$a^{\alpha} = (b-a)\alpha + a \quad (2)$$

$$b^{\alpha} = b \quad (3)$$

$$c^{\alpha} = (c-b)\alpha + b \quad (4)$$

3. Existing EWMA Control Charts based on NFRV

The following section outlines the EWMA control limits for detecting small shifts in the process using a random sample of NFRVs. This concept uses the fuzzy EWMA statistic proposed by Hesamian et al. [22]. Assume that $\tilde{X} \sim N(\tilde{\mu}, \sigma^2)$ and $\tilde{X} = \left(\tilde{X}_1, \tilde{X}_2, ..., \tilde{X}_n\right)$, is FRS of $\tilde{X}$. The FEWMA statistic is defined as follows:

$$\tilde{Z}_i = \left(\gamma \otimes \bar{\tilde{X}}\right) \oplus \left((1-\gamma) \otimes \tilde{Z}_{i-1}\right) \quad (5)$$

where $i \in \{1, 2, ..., n\}$ is the sample number, γ is the smoothing constant with $0 \le \gamma \le 1$ and $\bar{\tilde{X}}_i$ displays the fuzzy average for the *i*th FRS. It's important to note that the sample sizes for all FRSs are equal to n. The quantity $\tilde{Z}_{i-1}$ serves as the fuzzy past evidence, with its start value (i.e., $\tilde{Z}_o$) set to either the target fuzzy mean $\tilde{\mu}$ or the fuzzy average. The fuzzy lower control limit $\left(\tilde{LCL}_i\right)$, the fuzzy center line $\left(\tilde{CL}_i\right)$, and the upper control limit $\left(\tilde{UCL}_i\right)$ of the FEWMA chart for the *i*th sample are given below:

$$\tilde{LCL}_i = \tilde{\mu} - L\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\gamma}{2-\gamma}\left(1-(1-\gamma)^{i}\right)} \quad (6)$$

$$\tilde{CL}_i = \tilde{\mu} \quad (7)$$

$$\tilde{LCL}_i = \tilde{\mu} \oplus L\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\gamma}{2-\gamma}\left(1-(1-\gamma)^{i}\right)}. \quad (8)$$

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where L depict the width of the fuzzy control limits in the FEWMA chart.

4. Design of the Proposed Fuzzy Adaptive EWMA Control Charts

In this section, we offered Fuzzy AEWMA (FAEWMA) chart to monitor irregular process variations using NFRV. Following the Hesamian et al. [22], let us assume that $\tilde{X} \sim N\left(\tilde{\mu}, \sigma^2\right)$ and $\tilde{X} = \left(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n\right)$, is FRS of $\tilde{X}$

The fuzzy EWMA statistic is defined as follows:

$$\tilde{E}_i = \left(\gamma \otimes \overline{\tilde{X}_i}\right) \oplus \left((1-\gamma) \otimes \tilde{E}_{i-1}\right). \quad (9)$$

It is supposed that the process is in-control (IC) state when $\tilde{X} \sim N\left(\tilde{\mu}, \sigma^2\right)$ and after that, the process goes out of control (OOC) due to an unclear shift in its mean, indicating an OOC state, i.e., $\tilde{X}_i \sim N\left(\tilde{\mu}_1, \sigma^2\right)$ for OOC state. Let $\tilde{Z} = \frac{\tilde{X} - \tilde{\mu}}{\sigma^2}$ then $\tilde{Z}_i \sim N(\delta, 1)$. The mean shift $\delta = |\mu_1 - \mu|/\sigma$. For IC state $\tilde{Z}_i \sim N(0,1)$ and for OOC state $\tilde{Z}_i \sim N(\delta, 1)$, where $\delta \neq 0$. Let $\widehat{\delta_t}$ represent the shift estimate at time t. The shift estimator δ provided by Jiang et al. [23] is given by:

$$\hat{\delta}_i^* = \psi \tilde{Z}_i + (1-\psi)\hat{\delta}_{i-1}^* \quad (10)$$

The estimator $\hat{\delta}_t^*$ is biased for an out-of-control (OOC) process but unbiased for an in-control (IC) process state. Haq et al. [24] have provided an unbiased estimator for both states as follows:
where the ψ lies within $\psi \in (0,1]$ and $\hat{\delta}_0^* = 0$. The estimator $\hat{\delta}_t^*$ exhibits bias in an OOC state but is unbiased in an IC state. Haq et al. [24] offered an unbiased estimator applicable to both states as follows:

$$\hat{\delta}_t^{**} = \frac{\hat{\delta}_t^*}{1-(1-\psi)^t} \quad (11)$$

It is recommended to use $\hat{\delta}_t = \left|\hat{\delta}_t^{**}\right|$.
Therefore, the plotting statistic of offered FAEWMA control chart, which utilizes the sequence $\{\tilde{Z}_i\}$ to track the process mean, is given by:

$$F_t = g(\hat{\delta}_t)\tilde{Z}_i + \left(1 - g(\hat{\delta}_t)\right)F_{t-1} \quad (12)$$

where $F_0 = 0$ and $g(\hat{\delta}_t) \in (0,1]$. Sarwar et al. [25] proposed the function in (13) to adjust the smoothing constant utilizing estimated shift. We have applied this same function for adapting the smoothing constant.

$$g(\hat{\delta}_t) = \begin{cases} \frac{1}{a\left[1+(\hat{\delta}_t)^{-c}\right]} & if \quad 0 < \hat{\delta}_t \leq 2.7 \\ 1 & if \quad \hat{\delta}_t > 2.7 \end{cases}. \quad (13)$$

In the suggested function, a=7 and the c is recommended to be 2 for $\hat{\delta}_t \leq 1$ and 1 for $1 < \hat{\delta}_t \leq 2.7$. The suggested values of a and c to improve the performance of the function with respect to run length profiles. The function $g(\hat{\delta}_t)$ calculates the results of the random variable Ft, the statistic, ensuring that the offered FAEWMA control chart is nearly optimal for early identifying a shifts. The operation of the FAEWMA control chart is similar to that of the FEWMA chart recommended by Hesamian et al. [22]. A notable feature of the proposed FAEWMA chart is its ability to further improve RL profiles for a range of mean shifts. In the one-sided FAEWMA control chart, an OOC signal is triggered if the statistic $|F_t| > h$, For the two-sided FAEWMA control chart, an OOC signal is triggered if statistic $F_t > h$ or $F_t < -h$.

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Table 1: Run length outcomes of suggested EWMA chart under Fuzzy approach with ψ = 0.10.

Shift	***ARL***	***SDRL***	P_{05}	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	P_{95}
0.00	369.45	397.99	4	12	79	243	518	883	1151
0.10	68.89	72.02	2	5	16	47	98	163	210
0.20	25.94	24.81	2	3	7	18	37	59	76
0.30	13.75	12.50	1	2	5	10	19	30	39
0.40	8.58	7.22	1	2	3	6	12	18	23
0.50	6.04	4.75	1	2	3	5	8	12	15
0.75	3.28	2.18	1	1	2	3	4	6	8
0.90	2.51	1.51	1	1	1	2	3	4	5
1.0	2.21	1.26	1	1	1	2	3	4	5
1.50	1.37	0.59	1	1	1	1	2	2	2
2.0	1.09	0.30	1	1	1	1	1	1	2
2.50	1.01	0.12	1	1	1	1	1	1	1
3.0	1.00	0.03	1	1	1	1	1	1	1
4.0	1.00	0.0	1	1	1	1	1	1	1

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Table 2: ARL and SDRL results of the suggested chart under Fuzzy approach at $ARL_0 = 370$ with $\psi = 0.20$.

Shift	***ARL***	***SDRL***	P_{05}	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	P_{95}
0.00	369.77	378.73	11.00	30.00	98.00	256.00	505.00	845.00	1101.45
0.10	81.53	77.79	5	10	26	58	113	184	237
0.20	32.46	28.07	3	5	12	25	45	68	87
0.30	17.51	13.96	2	3	7	14	24	36	45
0.40	11.05	8.35	2	3	5	9	15	22	27
0.50	7.77	5.55	2	2	4	6	10	15	19
0.75	4.10	2.52	1	1	2	4	5	7	9
0.90	3.13	1.80	1	1	2	3	4	6	7
1.0	2.67	1.46	1	1	2	2	3	5	5
1.5	1.58	0.71	1	1	1	1	2	2	3
2.0	1.18	0.39	1	1	1	1	1	2	2
2.5	1.03	0.19	1	1	1	1	1	1	1
3.0	1.00	0.06	1	1	1	1	1	1	1
4.0	1.00	0.0	1	1	1	1	1	1	1

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Table 3: ARL and SDRL outcomes of offered chart under Fuzzy approach at $ARL_0 = 500$ with $\psi = 0.10$.

Shift	***ARL***	***SDRL***	P_{05}	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	P_{95}
0.00	499.49	536.51	5.00	19.00	115.00	331.00	702.25	1193.00	1581.10
0.10	84.03	86.03	3	6	21	59	118	197	258
0.20	30.57	29.16	2	3	9	22	43	69	88
0.30	15.55	13.95	2	2	5	12	22	34	43
0.40	9.45	7.81	1	2	4	7	13	20	25
0.50	6.60	5.11	1	2	3	5	9	13	17
0.75	3.50	2.28	1	1	2	3	5	7	8
0.90	2.66	1.60	1	1	1	2	3	5	6
1.0	2.33	1.32	1	1	1	2	3	4	5
1.50	1.43	0.62	1	1	1	1	2	2	3
2.0	1.11	0.32	1	1	1	1	1	2	2
2.50	1.01	0.13	1	1	1	1	1	1	1
3.0	1.00	0.04	1	1	1	1	1	1	1
4.0	1.00	0.0	1	1	1	1	1	1	1

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Table 4: ARL and SDRL outcomes of suggested EWMA control chart under Fuzzy approach at $ARL_0 = 500$ with $\psi = 0.20$.

Shift	***ARL***	***SDRL***	P_{05}	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}	P_{95}
0.00	500.83	505.60	20	49	140	342	694	1164	1536
0.10	102.89	98.08	6	13	33	74	142	230	297
0.20	38.44	33.02	4	6	15	30	53	81	103
0.30	19.98	15.85	3	4	8	16	27	41	51
0.40	12.50	9.32	2	3	6	10	17	25	31
0.50	8.49	6.07	2	2	4	7	11	17	20
0.75	4.41	2.73	1	2	2	4	6	8	10
0.90	3.31	1.90	1	1	2	3	4	6	7
1.0	2.84	1.54	1	1	2	3	4	5	6
1.50	1.65	0.74	1	1	1	2	2	3	3
2.0	1.21	0.43	1	1	1	1	1	2	2
2.50	1.04	0.21	1	1	1	1	1	1	1
3.0	1.00	0.07	1	1	1	1	1	1	1
4.0	1.00	0.0	1	1	1	1	1	1	1

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Table 5. ARL and SDRL results under different alpha cuts for the EWMA and AEWMA control chart under Fuzzy approach.

Shift	Fuzzy EWMA ψ = 0.20, Cut = 0.50		Fuzzy AEWMA ψ = 0.20, Cut = 0.50		Fuzzy EWMA ψ = 0.20, Cut = 0.65		Fuzzy AEWMA ψ = 0.20, Cut = 0.65		Fuzzy EWMA ψ = 0.20, Cut = 0.80		Fuzzy AEWMA ψ = 0.20, Cut = 0.80	
	ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
0.0	500.71	498.84	499.88	505.20	499.55	498.55	500.70	513.10	501.33	496.69	499.39	506.69
0.10	148.36	144.02	101.58	96.63	145.21	139.94	102.68	98.25	145.38	141.27	102.84	98.70
0.20	55.71	50.46	38.46	33.25	56.74	51.50	38.28	33.03	56.69	51.49	38.14	33.08
0.30	27.58	22.84	20.18	15.99	27.77	22.95	20.09	15.98	27.77	23.07	19.83	15.70
0.40	16.43	12.01	12.49	9.36	16.51	12.13	12.36	9.30	16.65	12.23	12.38	9.30
0.50	11.24	7.25	8.61	6.08	11.24	7.28	8.48	6.06	11.29	7.20	8.56	6.14
0.60	8.31	4.76	6.29	4.28	8.32	4.77	6.31	4.23	8.31	4.76	6.30	4.24
0.70	6.56	3.38	4.91	3.12	6.56	3.35	4.88	3.12	6.55	3.36	4.90	3.13
0.80	5.42	2.55	3.99	2.40	5.42	2.51	3.97	2.40	5.41	2.52	3.98	2.38
0.90	4.62	1.98	3.31	1.89	4.61	1.96	3.33	1.91	4.60	1.95	3.31	1.90
1.0	4.03	1.59	2.84	1.55	4.05	1.60	2.84	1.56	4.02	1.60	2.84	1.56
1.5	2.53	0.76	1.65	0.74	2.54	0.76	1.65	0.73	2.53	0.76	1.65	0.74
2.0	1.95	0.49	1.21	0.42	1.95	0.48	1.21	0.43	1.95	0.48	1.21	0.43
2.5	1.58	0.50	1.04	0.20	1.58	0.50	1.04	0.20	1.57	0.50	1.04	0.20
3	1.25	0.43	1.00	0.07	1.24	0.43	1.00	0.07	1.24	0.43	1.00	0.07

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5. Performance Evaluation and Discussion

Performance assessment of a control chart may be based on RL statistics viz., mean, standard deviation, and RL percentiles. It is possible to estimate the discussed attributes of RL using a number of approaches, such as the Probability method, Markov chain, Integral equations and Monte Carlo methods. The run length profiles of the suggested FAEWMA control chart were computed utilizing the Monte Carlo simulation approach. This approach involved using samples drawn from a normal distribution with 'δ' mean shift and variance of '1' in determining the performance of the chart. Thus we keep δ at 0 has being determined earlier, by our prior fixed values above. 00, 0. 1, 0. 20, 0. 30, 0. 40, 0. 50, 0. 60, 0. 70, 80, 0. 90, 1. 0, 1. 5, 2. 0, 2. 5 and 3. For the simulation study, we have used the following simulation steps that were developed by Sarwar et al. [25] for the FNRV. The run length profiles of the FAEWMA control chart are displayed in Table 1 when ψ =0. 15 ARL0 =370. In Table 1 and Table 4, the ARL and SDRL have been determined in the context of the FAEWMA control chart with ϕ values set at 0. 10 and 0. 20. These tables detail the effectiveness of chart under various shifts in the process mean. Tables 1-4 indicate the result of ψ significantly impacts the RL profiles for both in-control and out-of-control states. As ψ varies, there are noticeable changes in the ARL and SDRL, highlighting its influence on the sensitivity and performance of the control chart in detecting shifts in the process mean. For a fixed ψ, increasing the value of δ causes the RL profiles to decrease, and vice versa. For example, in Table 1, when δ = 0.50 at ϕ = 0.10, the corresponding ARL is 6.04, and the SDRL is 4.75. Conversely, with δ = 2.5, the ARL drops to 1.01 and the SDRL to 0.12. For a fixed δ, increasing the value of ϕ causes the ARL and SDRL to increase. For instance, in Table 1, with δ = 0.75 and ψ = 0.10, the respective ARL is 3.28, and the SDRL is 2.18. From Table 2, with the same δ but at ϕ = 0.20, the ARL increases to 4.10 and the SDRL to 2.52, with ARL0 set at 370. Similarly, the same result is observed in Tables 1-4, where ARL0 = 370 and 500, and ψ = 0.10 and 0.20. It is evident from Table 5 that the FAEWMA chart consistently outperforms the FEWMA control chart across all cases. This demonstrates the superior performance of the FAEWMA chart over its counterpart, providing more reliable detection of process shifts. The FAEWMA chart's enhanced sensitivity to shifts, regardless of the value of ψ, confirms its effectiveness as a robust statistical process control tool. We have also evaluated the performance of the proposed FAEWMA control chart using different alpha-cuts and compared it with the existing FEWMA control chart under various smoothing constants, as detailed in Table 5. The results indicate that as the alpha-cut value increases, the ARL values tend to increase, affecting the performance of the control charts. Additionally, Table 5 shows that the FAEWMA control chart exhibits higher sensitivity in detecting out-of-control signals than the FEWMA chart. This enhanced sensitivity underscores the effectiveness of the FAEWMA chart in monitoring process shifts more accurately. The following are the main findings of the proposed study.

- The FAEWMA control chart is particularly adept at quickly identifying small shifts in the process mean under a fuzzy approach. This is evident from Table 5, where, for a 20% shift and a smoothing constant ψ=0.20, ARL for FAEWMA chart is 101.58. In contrast, under the same parameters, the ARL for the FEWMA chart is 148.36. This significant difference highlights the FAEWMA chart's superior ability to rapidly detect small shifts, making it a more effective tool for monitoring the process mean in environments with inherent fuzziness. Promptly identifying these shifts ensures better process control and timely corrective actions.
- Table 3 demonstrates that as the shift increases, the ARL values decrease, indicating that larger changes in the mean are detected earlier. For example, from Table 3, with a 10% shift, the ARL is 84.03 when ψ = 0.10. However, at a 200% scale shift with the same parameters, the ARL drops significantly to 1.11. This shows that the chart effectively detects small and large shifts in the mean, highlighting its robust performance across different magnitudes of process shifts.
- For a fixed sample size and smoothing constant, the ARL profiles of the proposed FAEWMA chart are consistently lower than those of its competitor across all scale shifts. For instance, from Table 5, at a 20% scale shift with ψ = 0.20, ARL of the FAEWMA chart is 38.46, whereas the ARL of the FEWMA chart is 55.71 under the same parameters. This demonstrates the superior proficiency of the FAEWMA chart compared to the FEWMA chart.
- For the fixed smoothing constant and with different alpha-cuts, the ARL profiles of the proposed FAEWMA control chart are consistently lower than those of the FEWMA chart across all scale shifts. For instance, Table 5, at a 40% scale shift with an alpha-cut of 0.65, the ARL of the FAEWMA chart is 12.36 with ψ = 0.20, whereas under similar parameters, the ARL of the FEWMA chart is 16.51. This demonstrates the superior proficiency of the FAEWMA chart compared to its predecessor.

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6. Illustrative Example

In this section, an application of the offered chart is presented. A total of 25 samples, each comprising one observation, were simulated. Data were generated for the initial 15 samples assuming the process is in the in-control state. Subsequently, the remaining 10 samples were simulated with a mean shift of 0.75 to represent an out-of-control process. Furthermore, the Fuzzy Adjusted EWMA (AEWMA) control chart is presented in Figure 2 with ψ =0.15. The following steps were used to construct the charts shown in Figures 2 and 3:

i. The control constant (L) is used to achieve ARL0 = 500 using the in-control data set.
ii. Based on the design of the proposed control chart, compute the plotting statistic using the simulated data.
iii. Plot the control limits and the calculated plotting statistic to construct the control chart, as shown in Figures 2 and 3.

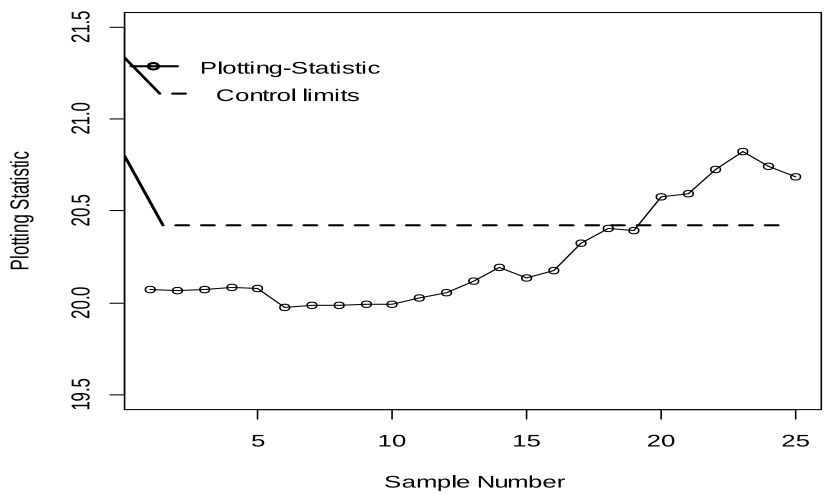

Figure 2. Proposed control chart for ψ =0.15.

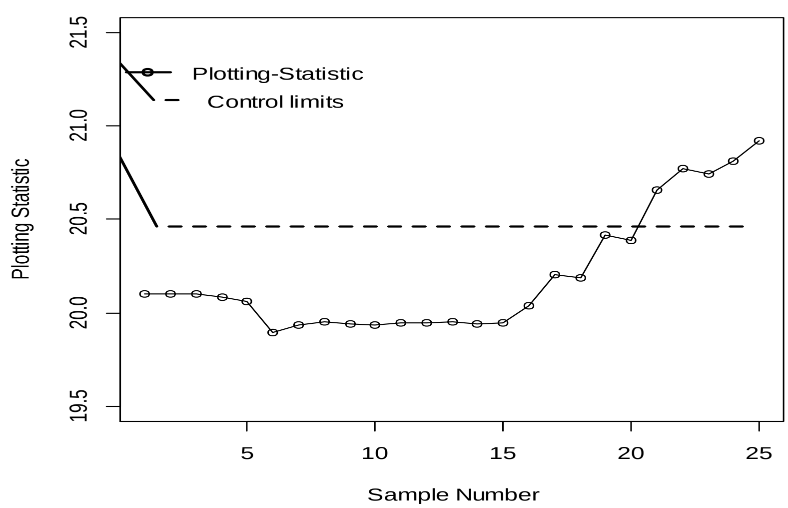

Figure 3. Proposed control chart for ψ =0.2.

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Figure 2 identifies the out-of-control signal at the 20th observation, demonstrating the chart's ability to detect process shifts promptly. In Figure 3, another Fuzzy AEWMA control chart is displayed, constructed with λ=0.2. In this instance, the out-of-control point is detected at the 26th observation. This further illustrates the efficient performance of the proposed FAEWMA control chart in identifying process anomalies. The consistent detection of out-of-control signals at different observations highlights the chart's robustness and sensitivity in monitoring process stability.

7. Conclusion

SPC within a fuzzy environment is a well-developed research field, for which many articles and various applications were reported in the literature. This paper contributes to this field by introducing a new SPC tool: another form of control chart is the fuzzy AEWMA control chart. It is especially helpful when used by process engineers wanting to identify changes in processes with measurement system or process related variability built into the system. Thereby, consistent, using of the fuzzy FAEWMA control charts by engineers allows for monitoring of processes under these conditions. In this paper, we introduces the basic statistical theory of the new control charts under study: fuzzy AEWMA control charts, and apply them to an actual case. These charts are particularly effective when fine changes in processes under a relatively imprecise environment are needed, and when the flexibility of control limits is to be improved to minimise the generation of false alarms. Additional future research works might apply this fuzzy approach to other advanced control charts to broaden horizon of applying the proposed approach in monitoring processes.

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