Pentapartitioned Neutrosophic Vague Soft Sets and its Applications

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Abstract

The objective of this paper is to extend the concept of standard soft sets to pentapartitioned neutrosophic vague soft sets (PNVSSs) by applying soft set theory to pentapartitioned neutrosophic vague sets (PNVSs) to make them stronger and more usable. We additionally describe its null, absolute, and fundamental operations, such as complement, subset, equality, union, and intersection, using examples. In addition, we defined the Pentaprtitioned Neutrosophic Vague multiset and the Possibility Pentaprtitioned Neutrosophic Vague sets (PPNVSs). We also look at several related properties and the proofs for them. Finally, this concept is applied to a decision-making problem, and its viability is demonstrated using an example. Related properties and the proofs for them. Finally, this concept is applied to a decision-making problem, and its viability is demonstrated using an example.

Keywords: Neutrosophic sets; Pentapartitioned neutrosophic sets; Soft sets; Multiset; Neutrosophic soft sets; Vague sets

1. Introduction

Zadeh [1] proposed the fuzzy set as a mathematical tool for dealing with issues and ambiguity in everyday life. Ahmadian et al. [2], Paul, and Bhattacharya [3] have applied fuzzy sets and fuzzy logic to a variety of real-world issues in ambiguous, uncertain contexts since then. Also, some studies in fuzzy sets see [4-18] and others non-classical sets are given, like soft sets ([19-32]), permutation sets ([33-54]), nano sets [55], neutrosophic sets ([56-63]) and others [64-72].

Molodtsov [73] first presented soft set theory to deal with uncertainty and ambiguity. Since then, Alkhazaleh et al. [74], Salleh et al. [75], and others have conducted extensive research on soft set and fuzzy set methodologies, embracing the advantages of both soft set and fuzzy set methodologies.

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Furthermore, soft sets have rapidly evolved to soft expert set fuzzy parameterized single valued neutrosophic soft expert set theory [76], soft multiset theory, Q-fuzzy soft sets, vague soft sets relations and functions, intuitionistic neutrosophic soft set [77], interval valued neutrosophic soft sets and its decision making [78] interval − valued vague soft sets and its applications.

Alkhazaleh proposed the neutrosophic vague set, which combines the neutrosophic set and the vague set. The neutrosophic vague theory is a valuable tool for interpreting evidence that is ambiguous, inconsistent, or incomplete. Assume we want to improve the neutrosophic vague set so that it is more effective and beneficial in solving decision − making difficulties.

The goal of this study is to develop a more effective hybrid pent partitioned neutrosophic vague set (HPNVS) and soft set model (SSM). Pentapartitioned neutrosophic vague soft set (PNVSS) is the name given to this revised model. This study makes three major contributions. To begin, we present the pent partitioned neutrosophic vague soft set (PNVSS), which addresses incompleteness, impreciseness, uncertainty, and indeterminacy. Second, we define null, absolute, and basic operations, including complement, subset, equality, union, and intersection, and investigate several related features with accompanying proofs. Finally, we present a decision-making algorithm based on (PNVSSs).

2. Preliminaries

In this section, we will review important concepts and findings from this study.

Definition 2.1: [79] Assume that X is a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\} \text{ where } T, I, F: X \to [0,1] \text{ and}$$

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non − membership.

Definition 2.2: [80] A vague set is defined by a truth − membership function t_v and a false membership function f_v, where $t_v(x)$ s a lower bound on the grade of membership of x derived from the evidence for x, and $f(x)$ is a lower bound on the negation of x derived from the evidence against x. The values of $t_v(x)$ and $f_v(x)$ are both defined on the closed interval $[0, 1]$ with each point in a basic set X, where $t_v(x) + f_v(x) \leq 1$.

Definition 2.3: [81] A Neutrosophic vague set A_{NV} (NVS) on X written as:

$A = \{\langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle; x \in X\}$, whose truth membership, indeterminacy membership and false membership function is defined as:

$\hat{T}_A(x) = [T^-, T^+]$, $\hat{I}_A(x)[I^-, I^+]$, $\hat{F}_A(x) = [F^-, F^+]$,

Where, (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ and (3) $0 \leq T^- + I^- + F^- \leq 2^+$ when X is continuous, a NVS A can be written as

$$A = \int \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle / x, x \in X$$

When X is discrete, a NVS A can be written as

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$$A = \sum_{i=1}^{n} \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x) \rangle / x_i, \; x_i \in X$$

Definition 2.4: [82**]** Let X be a universe. A Pentapartitioned Neutrosophic set A on X is defined as

$$A = \{\langle x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x)\rangle : x \in X\}$$

Where $T, C, G, U, F: X \to [0,1]$ and $0 \le T_A(x) + C_A(x) + G_A(x) + U_A(x) + F_A(x) \le 5$. Here $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $G_A(x)$ is ignorance membership, $U_A(x)$ is unknown and $F_A(x)$ is the false membership.

Definition 2.5: [83] A Pentapartitioned Neutrosophic Vague Set $A_{\widetilde{PNV}}$ ($\widetilde{PNVS}$ in short form) on the universe of discourse written as $A_{\widetilde{PNV}} = \left\{\langle x; \hat{T}_{A_{\widetilde{PNV}}}(x); \hat{C}_{A_{\widetilde{PNV}}}(x); \hat{G}_{A_{\widetilde{PNV}}}(x); \hat{U}_{A_{PNV}}(x); \hat{F}_{A_{\widetilde{PNV}}}(x)\rangle ; x \in X\right\}$

whose truth membership, contradiction membership, ignorance membership, unknown membership, and false membership function is defined as:

$$\hat{T}_{A_{\widetilde{PNV}}}(x) = [T^-, T^+], \quad \hat{C}_{A_{\widetilde{PNV}}}(x) = [C^-, C^+], \quad \hat{G}_{A_{\widetilde{PNV}}}(x) = [G^-, G^+],$$

$$\hat{U}_{A_{\widetilde{PNV}}}(x) = [U^-, U^+], \quad \hat{F}_{A_{\widetilde{PNV}}}(x) = [F^-, F^+],$$

Where (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ (3) $C^+ = 1 - U^-$ (4) $U^+ = 1 - C^-$

(5) $0 \le T^- + C^- + G^- + U^- + F^- \le 4^+$

Definition 2.6: [84**]** Let $A_{\widetilde{PNV}}$ and $B_{\widetilde{PNV}}$ be two $\widetilde{PNVS}$ of the universe U. If $\forall\, x \in U$,

$\hat{T}_{A_{\widetilde{PNV}}}(x) = \hat{T}_{B_{\widetilde{PNV}}}(x)$; $\hat{C}_{A_{\widetilde{PNV}}}(x) = \hat{C}_{B_{\widetilde{PNV}}}(x)$; $\hat{G}_{A_{\widetilde{PNV}}}(x) = \hat{G}_{B_{\widetilde{PNV}}}(x)$; $\hat{U}_{A_{\widetilde{PNV}}}(x) = \hat{U}_{B_{\widetilde{PNV}}}(x)$; $\hat{F}_{A_{\widetilde{PNV}}}(x) = \hat{F}_{B_{\widetilde{PNV}}}(x)$ then the $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$ and $B_{\widetilde{PNV}}$ are equal.

Definition 2.7: [39] The complement of $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$ is referred as $A^C_{\widetilde{PNV}}$ and is defined by $\hat{T}^C_{A_{\widetilde{PNV}}}(x) = [1 - T^+, 1 - T^-]$, $\hat{C}^C_{A_{\widetilde{PNV}}}(x) = [1 - C^+, 1 - C^-]$, $\hat{G}^C_{A_{\widetilde{PNV}}}(x) = [1 - G^+, 1 - G^-]$, $\hat{U}^C_{A_{\widetilde{PNV}}}(x) = [1 - U^+, 1 - U^-]$, $\hat{F}^C_{A_{\widetilde{PNV}}}(x) = [1 - F^+, 1 - F^-]$.

3. Pentapartitioned Neutrosophic Vague Soft Set (PNVSS)

Definition 3.1: Assume that U is a universe, E a set of parameters and $A \subseteq E$. A family of pairs $(\hat{F}, A)$ is called a pentapartitioned neutrosophic vague soft set (PNVSS) over U where $\hat{F}$ is a mapping given by

$$\hat{F}: A \longrightarrow PNV(U)$$

and $PNV(U)$ denotes the set of all a pentapartitioned neutrosophic vague subsets of U.

Example 3.2: Let $U = \{r_1, r_2, r_3, r_4\}$ be a set of universe representing residential buildings. Let $A = \{a_1, a_2, a_3\} = \{\text{large, small, medium}\}$ be a set of parameters defining the size of the residence. Define a mapping

$$\hat{F}a: A \longrightarrow PNV(U),$$

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$$\hat{F}(a_1) = \left\{ \frac{r_1}{\langle[0.1,0.3];[0.2,0.8];[0.1,0.9];[0.3,0.7];[0.2,0.8]\rangle}, \frac{r_2}{\langle[0.1,0.7];[0.2,0.5];[0.3,0.9];[0.4,0.6];[0.5,0.6]\rangle} \right. \\ \left. \frac{r_3}{\langle[0.4,0.5];[0.3,0.7];[0.5,0.6];[0.6,0.7];[0.5,0.8]\rangle}, \frac{r_4}{\langle[0,0.3];[0.2,0.4];[0.1,0.7];[0.8,0.9];[0.4,0.9]\rangle} \right\}$$

$$\hat{F}(a_2) = \left\{ \frac{r_1}{\langle[0.6,0.7];[0.3,0.8];[0.5,0.9];[0.7,0.7];[0.2,0.4]\rangle}, \frac{r_2}{\langle[0.2,0.3];[0.3,0.5];[0.5,0.9];[0.8,0.9];[0.7,0.9]\rangle} \right. \\ \left. \frac{r_3}{\langle[0.3,0.4];[0.6,0.8];[0.4,0.6];[0.7,0.8];[0.6,0.9]\rangle}, \frac{r_4}{\langle[0.1,0.3];[0.5,0.9];[0.2,0.7];[0.2,0.9];[0.4,0.6]\rangle} \right\}$$

$$\hat{F}(a_3) = \left\{ \frac{r_1}{\langle[0.3,0.4];[0.2,0.2];[0.1,0.2];[0.3,0.8];[0.1,0.8]\rangle}, \frac{r_2}{\langle[0.2,0.8];[0.5,0.7];[0.4,0.5];[0.6,0.7];[0.5,0.9]\rangle} \right. \\ \left. \frac{r_3}{\langle[0.3,0.6];[0,0.7];[0.7,0.9];[0.4,0.8];[0.2,0.6]\rangle}, \frac{r_4}{\langle[0.5,0.6];[0.3,0.8];[0.6,0.9];[0.4,0.7];[0.2,0.9]\rangle} \right\}$$

Then we can write the pentapartitioned neutrosophic vague soft set $(\hat{F}, A)$

as the following collection of approximations:

$$(\hat{F}, A) = \left\{ \begin{array}{l} \left(a_1, \left\{ \frac{r_1}{\langle[0.1,0.3];[0.2,0.8];[0.1,0.9];[0.3,0.7];[0.2,0.8]\rangle}, \frac{r_2}{\langle[0.1,0.7];[0.2,0.5];[0.3,0.9];[0.4,0.6];[0.5,0.6]\rangle}, \frac{r_3}{\langle[0.4,0.5];[0.3,0.7];[0.5,0.6];[0.6,0.7];[0.5,0.8]\rangle}, \frac{r_4}{\langle[0,0.3];[0.2,0.4];[0.1,0.7];[0.8,0.9];[0.4,0.9]\rangle} \right\} \right), \\ \left(a_2, \left\{ \frac{r_1}{\langle[0.6,0.7];[0.3,0.8];[0.5,0.9];[0.7,0.7];[0.2,0.4]\rangle}, \frac{r_2}{\langle[0.2,0.3];[0.3,0.5];[0.5,0.9];[0.8,0.9];[0.7,0.9]\rangle}, \frac{r_3}{\langle[0.3,0.4];[0.6,0.8];[0.4,0.6];[0.7,0.8];[0.6,0.9]\rangle}, \frac{r_4}{\langle[0.1,0.3];[0.5,0.9];[0.2,0.7];[0.2,0.9];[0.4,0.6]\rangle} \right\} \right), \\ \left(a_3, \left\{ \frac{r_1}{\langle[0.3,0.4];[0.2,0.2];[0.1,0.2];[0.3,0.8];[0.1,0.8]\rangle}, \frac{r_2}{\langle[0.2,0.8];[0.5,0.7];[0.4,0.5];[0.6,0.7];[0.5,0.9]\rangle}, \frac{r_3}{\langle[0.3,0.6];[0,0.7];[0.7,0.9];[0.4,0.8];[0.2,0.6]\rangle}, \frac{r_4}{\langle[0.5,0.6];[0.3,0.8];[0.6,0.9];[0.4,0.7];[0.2,0.9]\rangle} \right\} \right) \end{array} \right\}$$

Table 1 can be used to depict PNVSS. The entries are c_{ij} corresponding to the residence r_i, and the parameter a_j, where c_{ij} (true membership pentapartitioned neutrosophic vague value r_i, contradiction membership pentapartitioned neutrosophic vague value r_i, ignorance membership pentapartitioned neutrosophic vague value r_i, unknown membership pentapartitioned neutrosophic vague value r_i, false membership pentapartitioned neutrosophic vague value r_i) in $\hat{F}(a_j)$.

Table 1 shows the tabular form of the PNVSS $(\hat{F}, A)$.

Table 1: Representation of $(\hat{F}, A)$

U	a_1	a_2	a_3
(r_1)	⟨[0.1,0.3]; [0.2,0.8]; [0.1,0.9]; [0.3,0.7];	⟨[0.6,0.7]; [0.3,0.8]; [0.5,0.9]; [0.7,0.7]; [	⟨[0.3,0.4]; [0.2,0.2]; [0.1,0.2]; [0.3,0.8];
(r_2)	⟨[0.1,0.7]; [0.2,0.5]; [0.3,0.9]; [0.4,0.6];	⟨[0.2,0.3]; [0.3,0.5]; [0.5,0.9]; [0.8,0.9]; [	⟨[0.2,0.8]; [0.5,0.7]; [0.4,0.5]; [0.6,0.7];
(r_3)	⟨[0.4,0.5]; [0.3,0.7]; [0.5,0.6]; [0.6,0.7];	⟨[0.3,0.4]; [0.6,0.8]; [0.4,0.6]; [0.7,0.8]; [	⟨[0.3,0.6]; [0,0.7]; [0.7,0.9]; [0.4,0.8]; [0
(r_4)	⟨[0,0.3]; [0.2,0.4]; [0.1,0.7]; [0.8,0.9]; [0	⟨[0.1,0.3]; [0.5,0.9]; [0.2,0.7]; [0.2,0.9]; [	⟨[0.5,0.6]; [0.3,0.8]; [0.6,0.9]; [0.4,0.7];

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Definition 3.3: Let $(\hat{F},A)$ and $(\hat{G},B)$ be two (PNVSSs) over a universe U. A (PNVSS) $(\hat{F},A)$ is a subset of $(\hat{G},B)$ and referred as $(\hat{F},A) \subseteq (\hat{G},B)$ if and only if

1. $A \subseteq B$, and 2. $\forall\, a \in A,\ \hat{F}(a)$ is a pentapartitioned neutrosophic vague subset of $\hat{G}(a)$.

Definition 3.4: Assume that $(\hat{F},A)$ and $(\hat{G},B)$ are (PNVSSs) over a universe U. A (PNVSS) $(\hat{F},A)$ is equal to $(\hat{G},B)$ and referred as $(\hat{F},A) = (\hat{G},B)$ if each one is a subset of other.

Definition 3.5: For a $(PNVSS)$ $(\hat{F},A)$ over a universe U, $(\hat{F},A)$ is called a null $(PNVSS)$ referred as ψ_A, if $T_{\hat{F}(a)}(m) = [0,0]$, $C_{\hat{F}(a)}(m) = [0,0]$, $G_{\hat{F}(a)}(m) = [1,1]$, $U_{\hat{F}(a)}(m) = [1,1]$, and $F_{\hat{F}(a)}(m) = [1,1]$, $\forall\, m \in U$ and $\forall\, a \in A$.

Definition 3.6: For a $PNVSS$ $(\hat{F},A)$ over a universe U, $(\hat{F},A)$ is called an absolute $(PNVSS)$ referred as $\hat{\Psi}$, if $\forall\, m \in U$ and $\forall\, a \in A$, $T_{\hat{F}(a)}(m) = [1,1]$, $C_{\hat{F}(a)}(m) = [1,1]$, $G_{\hat{F}(a)}(m) = [0,0]$, $U_{\hat{F}(a)}(m) = [0,0]$, and $F_{\hat{F}(a)}(m) = [0,0]$.

4. Basic Operations

Here, we establish some fundamental $(PNVSS)$ operations including complement, union, and intersection, as well as shown examples, and then we go over certain attributes.

Definition 4.1: Let $(\hat{F},A)$ be a $PNVSS$ over a universe U. Then the *complement of* $(\hat{F},A)$ is referred as $(\hat{F},A)^C$ and is defined as $(\hat{F},A)^C = (\hat{F}^C,A)$, where $\hat{F}^C : A \to PNV(U)$ is a mapping given by

$$\hat{F}^C(\alpha) = \tilde{c}\left(\hat{F}(\alpha)\right),\ \forall\, \alpha \in A$$

where $\tilde{c}$ *is a pentapartitioned neutrosophic vague complement.*

Example 4.2: *Consider Example* 3.2, *then* $(\hat{F},A)^C$ *is given by*

$$(\hat{F},A)^C = \left\{\begin{array}{l} \left(a_1,\left\{\dfrac{r_1}{\langle[0.7,0.9];[0.2,0.8];[0.1,0.9];[0.3,0.7];[0.2,0.8]\rangle},\dfrac{r_2}{\langle[0.3,0.9];[0.5,0.8];[0.1,0.7];[0.4,0.6];[0.4,0.5]\rangle},\dfrac{r_3}{\langle[0.5,0.6];[0.3,0.7];[0.4,0.5];[0.3,0.4];[0.2,0.5]\rangle},\dfrac{r_4}{\langle[0.7,1];[0.6,0.8];[0.3,0.9];[0.1,0.2];[0.1,0.6]\rangle}\right\}\right), \\ \left(a_2,\left\{\dfrac{r_1}{\langle[0.3,0.4];[0.2,0.7];[0.1,0.5];[0.3,0.3];[0.6,0.8]\rangle},\dfrac{r_2}{\langle[0.7,0.8];[0.5,0.7];[0.1,0.5];[0.1,0.2];[0.1,0.3]\rangle},\dfrac{r_3}{\langle[0.6,0.7];[0.2,0.4];[0.4,0.6];[0.2,0.3];[0.1,0.4]\rangle},\dfrac{r_4}{\langle[0.7,0.9];[0.1,0.5];[0.3,0.8];[0.1,0.8];[0.4,0.6]\rangle}\right\}\right), \\ \left(a_3,\left\{\dfrac{r_1}{\langle[0.6,0.7];[0.8,0.8];[0.8,0.9];[0.2,0.7];[0.2,0.9]\rangle},\dfrac{r_2}{\langle[0.2,0.8];[0.3,0.5];[0.5,0.6];[0.3,0.4];[0.1,0.5]\rangle},\dfrac{r_3}{\langle[0.4,0.7];[0.3,1];[0.1,0.3];[0.2,0.6];[0.4,0.8]\rangle},\dfrac{r_4}{\langle[0.4,0.5];[0.2,0.7];[0.1,0.4];[0.3,0.6];[0.1,0.8]\rangle}\right\}\right) \end{array}\right\}$$

Definition 4.3: *Let* $(\hat{F},A)$ and $(\hat{G},B)$ be two PNVSS over a universe U. Then the union of $(\hat{F},A)$ and $(\hat{G},B)$ referred as $(\hat{F},A)\,\tilde{\cup}\,(\hat{G},B)$ and is defined as $(\hat{F},A)\,\tilde{\cup}\,(\hat{G},B) = (\hat{H},C)$ such that

$C = A \cup B\ \ \forall\, a \in C$ *is*

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$$(\hat{H},C)=\begin{cases}\hat{F}(a) & ,if\ \ a\ \in A-B\\ \hat{G}(a) & ,if\ \ a\ \in B-A\\ \hat{F}(a)\ \tilde{\cup}\ \hat{G}(a) & ,if\ \ a\ \in A\cap B\end{cases}$$

where $\tilde{\cup}$ denotes the pentapartitioned neutrosophic vague set union.

Example 4.4: Consider Example 3.2, $(\hat{F},A)$ and $(\hat{G},B)$ be two $PNVSSs$ with tabular representation as in Table 2 and Table.

Table 2: Representation of $(\hat{F},A)$

U	$a_1 = Large$	$a_2 = Medium$	$a_3 = Small$
(r_1)	⟨[0.1,0.3]; [0.2,0.8]; [0.1,0.9]; [0.3,0.7]; [	⟨[0.6,0.7]; [0.3,0.8]; [0.5,0.9]; [0.7,0.7]; [	⟨[0.3,0.4]; [0.2,0.2]; [0.1,0.2]; [0.3,0.8]; [
(r_2)	⟨[0.1,0.7]; [0.2,0.5]; [0.3,0.9]; [0.4,0.6]; [	⟨[0.2,0.3]; [0.3,0.5]; [0.5,0.9]; [0.8,0.9]; [	⟨[0.2,0.8]; [0.5,0.7]; [0.4,0.5]; [0.6,0.7]; [
(r_3)	⟨[0.4,0.5]; [0.3,0.7]; [0.5,0.6]; [0.6,0.7]; [	⟨[0.3,0.4]; [0.6,0.8]; [0.4,0.6]; [0.7,0.8]; [	⟨[0.3,0.6]; [0,0.7]; [0.7,0.9]; [0.4,0.8]; [0.
(r_4)	⟨[0,0.3]; [0.2,0.4]; [0.1,0.7]; [0.8,0.9]; [0.	⟨[0.1,0.3]; [0.5,0.9]; [0.2,0.7]; [0.2,0.9]; [	⟨[0.5,0.6]; [0.3,0.8]; [0.6,0.9]; [0.4,0.7]; [

Table 3: Representation of $(\hat{G},B)$

U	$a_1 = Small$	$a_2 = X - Large$
(r_1)	⟨[0.1,0.2]; [0.2,0.9]; [0.5,0.9]; [0.2,0.6]; [0.3,0.6]⟩	⟨[0.2,0.7]; [0.5,0.6]; [0.6,0.9]; [0.7,0.9]; [0.1,0.4]
(r_2)	⟨[0.5,0.7]; [0.2,0.8]; [0.2,0.5]; [0.3,0.7]; [0.4,0.8]⟩	⟨[0.3,0.5]; [0.5,0.7]; [0.6,0.9]; [0.1,0.8]; [0.2,0.3]
(r_3)	⟨[0.2,0.3]; [0.3,0.5]; [0.2,0.6]; [0.4,0.7]; [0.3,0.6]⟩	⟨[0.1,0.3]; [0.3,0.6]; [0.5,0.7]; [0.6,0.9]; [0.3,0.3]
(r_4)	⟨[0.3,0.3]; [0.2,0.6]; [0.5,0.7]; [0.4,0.5]; [0.5,0.6]⟩	⟨[0.4,0.6]; [0.6,0.9]; [0.1,0.3]; [0.5,0.8]; [0.3,0.5]

The union of $(\hat{F},A)$ and $(\hat{G},B)$ can be represented as in Table 4.

Table 4: $(\hat{F},A)\ \tilde{\cup}\ (\hat{G},B)$

U	$a_1 = Large$	$a_2 = Medium$
(r_1)	⟨[0.1,0.3]; [0.2,0.8]; [0.1,0.9]; [0.3,0.7]; [0.2,0.8]⟩	⟨[0.6,0.7]; [0.3,0.8]; [0.5,0.9]; [0.7,0.7]; [0.2,0.4]⟩
(r_2)	⟨[0.1,0.7]; [0.2,0.5]; [0.3,0.9]; [0.4,0.6]; [0.5,0.6]⟩	⟨[0.2,0.3]; [0.3,0.5]; [0.5,0.9]; [0.8,0.9]; [0.7,0.9]⟩
(r_3)	⟨[0.4,0.5]; [0.3,0.7]; [0.5,0.6]; [0.6,0.7]; [0.5,0.8]⟩	⟨[0.3,0.4]; [0.6,0.8]; [0.4,0.6]; [0.7,0.8]; [0.6,0.9]⟩
(r_4)	⟨[0,0.3]; [0.2,0.4]; [0.1,0.7]; [0.8,0.9]; [0.4,0.9]⟩	⟨[0.1,0.3]; [0.5,0.9]; [0.2,0.7]; [0.2,0.9]; [0.4,0.6]⟩
U	$a_3 = Small$	$a_2 = X - Large$
(r_1)	⟨[0.3,0.4]; [0.2,0.2]; [0.1,0.2]; [0.3,0.8]; [0.1,0.8]⟩	⟨[0.2,0.7]; [0.5,0.6]; [0.6,0.9]; [0.7,0.9]; [0.1,0.4]⟩
(r_2)	⟨[0.2,0.8]; [0.5,0.7]; [0.4,0.5]; [0.6,0.7]; [0.5,0.9]⟩	⟨[0.3,0.5]; [0.5,0.7]; [0.6,0.9]; [0.1,0.8]; [0.2,0.3]⟩
(r_3)	⟨[0.3,0.6]; [0,0.7]; [0.7,0.9]; [0.4,0.8]; [0.2,0.6]⟩	⟨[0.1,0.3]; [0.3,0.6]; [0.5,0.7]; [0.6,0.9]; [0.3,0.3]⟩
(r_4)	⟨[0.5,0.6]; [0.3,0.8]; [0.6,0.9]; [0.4,0.7]; [0.2,0.9]⟩	⟨[0.4,0.6]; [0.6,0.9]; [0.1,0.3]; [0.5,0.8]; [0.3,0.5]⟩

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Definition 4.5: Let $(\hat{F},A)$ and $(\hat{G},B)$ be two $(PNVSSs)$ over a universe U. Then the intersection of $(\hat{F},A)$ and $(\hat{G},B)$ denoted by $(\hat{F},A)\,\tilde{\cap}\,(\hat{G},B)$ and is defined as $(\hat{F},A)\,\tilde{\cap}\,(\hat{G},B) = (\hat{K},C)$ such that $C = A \cap B\ \ \forall\, a \in C$ is $(\hat{K},C) = \begin{cases} \hat{F}(a) & ,if\ \ a \in A - B \\ \hat{G}(a) & ,if\ \ a \in B - A \\ \hat{F}(a)\,\tilde{\cap}\,\hat{G}(a) & ,if\ \ a \in A \cap B \end{cases}$, where $\tilde{\cap}$ denotes the pentapartitioned neutrosophic vague set intersection.

Example 4.6: By using basic pentapartitioned neutrosophic vague set intersection in Example (4.4), we can show that $(\hat{F},A)\,\tilde{\cap}\,(\hat{G},B) = (\hat{K},C)$, where the tabular representation of $(\hat{K},C)$ is as in Table 5.

Table 5: $(\hat{F},A)\,\tilde{\cap}\,(\hat{G},B)$

U	$a_1 = Large$	$a_2 = Medium$
(r_1)	⟨[0.1,0.3]; [0.2,0.8]; [0.1,0.9]; [0.3,0.7]; [0.2,0.8]⟩	⟨[0.6,0.7]; [0.3,0.8]; [0.5,0.9]; [0.7,0.7]; [0.2,0.4]⟩
(r_2)	⟨[0.1,0.7]; [0.2,0.5]; [0.3,0.9]; [0.4,0.6]; [0.5,0.6]⟩	⟨[0.2,0.3]; [0.3,0.5]; [0.5,0.9]; [0.8,0.9]; [0.7,0.9]⟩
(r_3)	⟨[0.4,0.5]; [0.3,0.7]; [0.5,0.6]; [0.6,0.7]; [0.5,0.8]⟩	⟨[0.3,0.4]; [0.6,0.8]; [0.4,0.6]; [0.7,0.8]; [0.6,0.9]⟩
(r_4)	⟨[0,0.3]; [0.2,0.4]; [0.1,0.7]; [0.8,0.9]; [0.4,0.9]⟩	⟨[0.1,0.3]; [0.5,0.9]; [0.2,0.7]; [0.2,0.9]; [0.4,0.6]⟩
U	$a_3 = Small$	$a_2 = X - Large$
(r_1)	⟨[0.1,0.2]; [0.2,0.9]; [0.5,0.9]; [0.2,0.6]; [0.3,0.6]⟩	⟨[0.2,0.7]; [0.5,0.6]; [0.6,0.9]; [0.7,0.9]; [0.1,0.4]⟩
(r_2)	⟨[0.5,0.7]; [0.2,0.8]; [0.2,0.5]; [0.3,0.7]; [0.4,0.8]⟩	⟨[0.3,0.5]; [0.5,0.7]; [0.6,0.9]; [0.1,0.8]; [0.2,0.3]⟩
(r_3)	⟨[0.2,0.3]; [0.3,0.5]; [0.2,0.6]; [0.4,0.7]; [0.3,0.6]⟩	⟨[0.1,0.3]; [0.3,0.6]; [0.5,0.7]; [0.6,0.9]; [0.3,0.3]⟩
(r_4)	⟨[0.3,0.3]; [0.2,0.6]; [0.5,0.7]; [0.4,0.5]; [0.5,0.6]⟩	⟨[0.4,0.6]; [0.6,0.9]; [0.1,0.3]; [0.5,0.8]; [0.3,0.5]⟩

Preposition 4.7: For any two $(PNVSSs)$ $(\hat{K},A)$ and $(\hat{L},B)$ over a universe U, we get:

1. Idempotency Laws:

a. $(\hat{K},A)\,\tilde{\cup}\,(\hat{K},A) = (\hat{K},A)$

b. $(\hat{K},A)\,\tilde{\cap}\,(\hat{K},A) = (\hat{K},A)$

2. Commutative Laws:

a. $(\hat{K},A)\,\tilde{\cup}\,(\hat{L},B) = (\hat{L},B)\,\tilde{\cup}\,(\hat{K},A)$

b. $(\hat{K},A)\,\tilde{\cap}\,(\hat{L},B) = (\hat{L},B)\,\tilde{\cap}\,(\hat{K},A)$

Proof: The proof is straightforward and thus omitted.

Preposition 4.8: For any three $(PNVSSs)$ $(\hat{K},A)$, $(\hat{L},B)$ and $(\hat{M},C)$ over a universe U, we get:

1. $(\hat{K},A)\,\tilde{\cup}\,\left((\hat{L},B)\,\tilde{\cup}\,(\hat{M},C)\right) = \left((\hat{K},A)\,\tilde{\cup}\,(\hat{L},B)\right)\tilde{\cup}\,(\hat{M},C)$

2. $(\hat{K},A)\,\tilde{\cap}\,\left((\hat{L},B)\,\tilde{\cap}\,(\hat{M},C)\right) = \left((\hat{K},A)\,\tilde{\cap}\,(\hat{L},B)\right)\tilde{\cap}\,(\hat{M},C)$

3. $(\hat{K},A)\,\tilde{\cap}\,\left((\hat{L},B)\,\tilde{\cup}\,(\hat{M},C)\right) = \left((\hat{K},A)\,\tilde{\cap}\,(\hat{L},B)\right)\tilde{\cup}\left((\hat{K},A)\,\tilde{\cap}\,(\hat{M},C)\right)$

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4. $(\hat{K},A)\,\tilde{\cup}\,\left((\hat{L},B)\,\tilde{\cap}\,(\hat{M},C)\right) = \left((\hat{K},A)\,\tilde{\cup}\,(\hat{L},B)\right)\tilde{\cap}\left((\hat{K},A)\,\tilde{\cup}\,(\hat{M},C)\right)$

Proof: The proof is obvious.

5. Pentapartitioned Neutrosophic Vague Soft Multiset

Definition 5.1: Assume $\{U_i: i \in I\}$ is a collection of universe such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i}: i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} PNV(U_i)$ where $PNV(U_i)$ denotes the set of all pentapartitioned neutrosophic vague subsets of $U_i, E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F,A) is said to be a pentapartitioned neutrosophic vague soft multiset $(PNVSM)$ over U, where $F: A \to U$.

Definition 5.2:

For any $(PNVSM)$ (F,A), a pair $\left(e_{U_{i,j}}, F_{e_{U_{i,j}}}\right)$ is said to be a U_i pentapartitioned vague soft multiset part $\forall e_{U_{i,j}} \in a_k$ and $F_{e_{U_{i,j}}} \subseteq F(A)$ is a pentapartitioned neutrosophic vague approximate value set, where $a_k \in A$, $k \in \{1,2,\dots n\}$, $i \in \{1,2,\dots m\}$ *and* $j \in \{1,2,\dots r\}$.

Definition 5.3: For two $(PNVSMs)$ (G,A) and (H,B) over U, (G,A) is said to be a pentapartitioned neutrosophic vague soft multisubset of (H,B), if

1. $A \subseteq B$, and
2. $\forall e_{U_{i,j}} \in a_k$, $\left(e_{U_{i,j}}, G_{e_{U_{i,j}}}\right)$ is a pentapartitioned neutrosophic vague subset of $\left(e_{U_{i,j}}, H_{e_{U_{i,j}}}\right)$ where $a_k \in A$, $k \in \{1,2,\dots n\}$, $i \in \{1,2,\dots m\}$ *and* $j \in \{1,2,\dots r\}$. This
3. Relationship is referred as $(G,A) \subseteq (H,B)$. In this case (H,B), is called a pentapartitioned neutrosophic vague soft multisuperset of (G,A).

Definition 5.4: Let (G,A) and (H,B) be $(PNVSMs)$ over U. They are said to be equal if each one is a pentapartitioned neutrosophic vague soft multisuperset of other.

Definition 5.5: A $(PNVSM)$ (G,A) over U is said to be a null $(PNVSM)$ and referred as $(G,A)_\emptyset$ if all of the pentapartitioned neutrosophic vague soft multiset parts of (G,A) equal $\emptyset$.

Definition 5.6: The complement of a pentapartitioned neutrosophic vague soft multiset (G,A) is referred as $(G,A)^c$ and is defined by $(G,A)^c = (G^c,A)$ where $G^c: A \to U$ is a mapping given by $G^c(\alpha) = c(G(\alpha)), \forall \alpha \in A$ where c is pentapartitioned neutrosophic vague complement.

Proposition 5.7: If (G,A) is a pentapartitioned neutrosophic vague soft multiset over , then

1. $((G,A)^c)^c = (G,A)$
2. $(G,A)^c\,\Phi = (G,A)_U$
3. $(G,A)^c U = (G,A)_\Phi$

Proof: The proof is straightforward.

Definition 5.8: The union of two pentapartitioned neutrosophic vague soft multisets (G,A) and (H,B) over U , referred as $(G,A) \cup (H,B)$ is a pentapartitioned neutrosophic vague soft multiset (I,D) where , $D = A \cup B$, and $\forall \alpha \in D$,

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$$I(\alpha) = \begin{cases} G(\alpha) & if\ \alpha \in A - B \\ H(\alpha) & if\ \alpha \in B - A \\ G(\alpha) \cup H(\alpha) & if\ \alpha \in A \cup B \end{cases}$$

and $\cup$ denotes thepentapartitioned neutrosophic vague set union.

Definition 5.9: The intersection of two pentapartitioned neutrosophic vague soft multisets (G, A) and (H, B) over U , referred as $(G, A) \cap (H, B)$ is a pentapartitioned neutrosophic vague soft multiset (I, D) where , $D = A \cap B$, and $\forall \alpha \in D$,

$$I(\alpha) = \begin{cases} G(\alpha) & if\ \alpha \in A - B \\ H(\alpha) & if\ \alpha \in B - A \\ G(\alpha) \cap H(\alpha) & if\ \alpha \in A \cap B \end{cases}$$

and $\cap$ denotes the pentapartitioned neutrosophic vague set intersection.

Proposition 5.10: If (G, A), (H, B) and (I, C) are three pentapartitioned neutrosophic vague soft multisets over U , then

1. $(G, A) \cup ((H, B) \cup (I, C)) = ((G, A) \cup (H, B)) \cup (I, C)$
2. $(G, A) \cup (G, A) = (G, A)$
3. $(G, A) \cup (H, A)_{\Phi} = (G, A)$
4. $(G, A) \cup (H, A)_{\mathrm{U}} = (H, A)_{\mathrm{U}}$

Proof: The proof is straightforward.

Proposition 5.11: If (G, A), (H, B) and (I, C) are three pentapartitioned neutrosophic vague soft multisets over U , then

1. $(G, A) \cap ((H, B) \cap (I, C)) = ((G, A) \cap (H, B)) \cap (I, C)$
2. $(G, A) \cap (G, A) = (G, A)$
3. $(G, A) \cap (H, A)_{\Phi} = (H, A)_{\Phi}$
4. $(G, A) \cap (H, A)_{\mathrm{U}} = (G, A)$

Proof: The proof is straightforward.

An Application in Decision Making

In the following paragraphs, we will review the approach that was created to solve a pentapartitioned neutrosophic soft set. Based on the technique for fuzzy soft multiset provided by Alkhazaleh and Salleh (2012), we also offer a new algorithm to address pentapartitioned neutrosophic ambiguous soft multiset decision-making issues. Then, to solve decision $-$ making difficulties, we apply this new technique to the pentapartitioned neutrosophic hazy soft multiset model. Maji′s Algorithm (Maji,2013) will be abbreviated MA, and Roy and Maji′s Algorithm (Roy & Maji,2007) will be abbreviated RMA. Maji (2013) solved a decision $-$ making problem using the following algorithm.

1. input the pentapartitioned Neutrosophic Soft Set (G, A).
2. input P, the choice parameters which is a subset of P.
3. consider the PNSS (G, P) and write it in tabular form.

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4. compute the comparison matrix of the PNSS (G, P).

5. compute the score S_i of $h_i, \forall i$.

6. find $S_k = max_i S_i$.

7. if k has more than one value then any one of h_i could be the preferable choice.

Alkhazaleh and Salleh (2012)proposed the following algorithm for fuzzy soft multiset.

1. Input the fuzzy soft multiset (M, C) which is introduced by making any operations between(G, A) and (I, B).

2. Apply RMA to the first fuzzy soft multiset part in (M, C) to get the decision S_{k_1}.

3. Redefine the fuzzy soft multiset (M, C) by keeping all values in each row where S_{k_1} is maximum and and replacing the values in the other rows by zero to get $(M, C)_1$.

4. Apply RMA to the second fuzzy soft multiset part in $(M, C)_1$ to get the decision S_{k_2}.

5. Redefine the fuzzy soft multiset $(G, A)_1$ by keeping the first and second parts and apply the method in step 3 to the third part to get $(M, C)_2$.
6. Apply RMA to the third fuzzy soft multiset part in $(M, C)_2$ to get the decision S_{k_3}.

7. The decision is $(S_{k_1}, S_{k_2}, S_{k_3})$.

Now we construct a $(PNVSM)$ decision − making method by the following algorithm. Input the $(PNVSM)$ which (M, C) is introduced by making any operation between (G, A) and (I, B).

1. Apply MA to the first pentapartitioned neutrosophic vague soft multiset part in (M, C) to get the decision S_{k_1}.

2. Redefine the pentapartitioned neutrosophic vague soft multiset (M, C) by keeping all values in each row where S_{k_1} is maximum and replacing the values in other rows by zero to get $(M, C)_1$.

3. Apply MA to the second pentapartitioned neutrosophic vague soft multiset part in $(M, C)_1$ to get the decision S_{k_2}.

4. Redefine the pentapartitioned neutrosophic vague soft multiset $(M, C)_1$ by keeping the first and s econd parts and apply the method in step 3 to the third part to get $(M, C)_2$.

5. Apply MA to the third pentapartitioned neutrosophic vague soft multiset part in $(M, C)_2$ to get the de cision S_{k_3}.

6. The decision is $(S_{k_1}, S_{k_2}, S_{k_3})$.

We will apply this algorithm to the pentapartitioned neutrosophic vague soft multiset model to solve a decision making problem in the following example.

Example 5.12: Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c, c_3\}, and\ U_1 = \{v_1, v_2,\}$ be the sets of "houses", "cars", and "hotels" respectively.

Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

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$E_{U_1} = \{e_{U_1,1} = Bungalow\,, \quad e_{U_1,2} = Apartment \quad , \quad e_{U_1,3} = \ Cottage\}$
$E_{U_2} = \{e_{U_2,1} = Coupe\,, \quad e_{U_2,2} = Hatchback \quad , \quad e_{2,3} = \ Convertible\}$
$E_{U_3} = \{e_{U_3,1} = Resort\,, \quad e_{U_3,2} = Suite \quad , \quad e_{U_3,3} = \ Motel\}$
Let $A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,2}), a_3 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,3})\}$ and $B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ b_2 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,2}), b_3 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,3})\}$

Suppose a person wants to choose objects from the sets of given objects with respect to the sets of choice parameters. Let there be two observations (G, A) and (I, B) by two experts X_1 and X_2 respectively. Suppose

$(G, A) =$

$$\left\{\begin{array}{l}
\left(a_1, \left(\begin{array}{c}
\left\{\frac{h_1}{[0.2,0.5],[0.3,0.6],[0.4,0.7],[0.4,0.8],[0.2,0.3]}, \frac{h_2}{[0.4,0.6],[0.5,0.5],[0.6,0.7],[0.2,0.9],[0.2,0.4]}, \frac{h_3}{[0.3,0.7],[0.6,0.9],[0.2,1],[0.1,0.2],[0.5,0.8]}\right\}, \\
\left\{\frac{c_1}{[0.4,0.9],[0.7,0.7],[0.3,0.7],[0.1,0.4],[0.5,0.6]}, \frac{c_2}{[0.2,0.4],[0.3,0.9],[0.7,0.7],[0.8,0.9],[0.4,0.6]}, \frac{c_3}{[0.4,0.8],[0.3,0.8],[0.1,0.6],[0.4,0.7],[0.4,0.6]}\right\}, \\
\left\{\frac{v_1}{[0.5,0.7],[0.6,0.8],[0.2,0.7],[0.3,0.4],[0.8,0.9]}, \frac{v_2}{[0.4,0.6],[0.2,0.5],[0.7,0.8],[0.4,0.7],[0.1,0.8]}\right\}
\end{array}\right)\right) \\
a_2, \left(\begin{array}{c}
\left\{\frac{h_1}{[0.3,0.6],[0.4,0.7],[0.5,0.8],[0.5,0.9],[0.2,0.2]}, \frac{h_2}{[0.5,0.7],[0.6,0.7],[0.7,0.9],[0.3,1],[0.5,0.7]}, \frac{h_3}{[0.2,0.7],[0.4,0.5],[0.3,0.5],[0.6,0.8],[0.7,0.8]}\right\}, \\
\left\{\frac{c_1}{[0.1,0.8],[0.2,0.7],[0.3,.6],[0.4,0.5],[0.6,0.6]}, \frac{c_2}{[0.3,0.9],[0.4,0.8],[0.5,0.7],[0.6,0.9],[0.2,0.5]}, \frac{c_3}{[0.2,0.4],[0.4,0.6],[0.6,0.8],[0.3,0.5],[0.5,0.6]}\right\}, \\
\left\{\frac{v_1}{[0.4,0.8],[0.1,0.2],[0.2,0.4],[0.5,0.5],[0.3,0.6]}, \frac{v_2}{[0.6,0.7],[0.7,0.9],[0.5,0.8],[0.3,0.6],[0.2,0.3]}\right\}
\end{array}\right) \\
a_3, \left(\begin{array}{c}
\left\{\frac{h_1}{[0.4,0.8],[0.5,0.6],[0.3,0.9],[0.7,0.8],[0.2,0.3]}, \frac{h_2}{[0.2,0.4],[0.7,0.8],[0.4,0.5],[0.1,0.9],[0.3,0.5]}, \frac{h_3}{[0.5,0.8],[0.3,0.7],[0.1,0.3],[0.4,0.6],[0.6,0.9]}\right\}, \\
\left\{\frac{c_1}{[0.2,0.6],[0.3,0.6],[0.4,0.7],[0.2,0.5],[0.4,0.7]}, \frac{c_2}{[0.2,0.5],[0.5,0.9],[0.2,0.3],[0.6,0.8],[0.5,0.9]}, \frac{c_3}{[0.2,0.6],[0.1,0.8],[0.2,0.7],[0.5,0.8],[0.3,0.8]}\right\}, \\
\left\{\frac{v_1}{[0.2,0.5],[0.3,0.7],[0.4,0.6],[0.6,0.7],[0.4,0.9]}, \frac{v_2}{[0.1,0.5],[0.2,0.6],[0.4,0.6],[0.8,0.8],[0.4,0.9]}\right\}
\end{array}\right)
\end{array}\right\}$$

$(I, B) =$

$$\left\{\begin{array}{l}
\left(b_1, \left(\begin{array}{c}
\left\{\frac{h_1}{[0.4,0.9],[0.8,0.8],[0.4,0.6],[0.2,0.8],[0.1,0.5]}, \frac{h_2}{[0.4,0.9],[0.6,0.7],[0.4,0.6],[0.3,0.7],[0.4,0.5]}, \frac{h_3}{[0.4,0.8],[0.5,0.8],[0.3,0.7],[0.2,0.8],[0.3,0.6]}\right\}, \\
\left\{\frac{c_1}{[0.5,0.8],[0.6,0.7],[0.2,0.3],[0.5,0.9],[0.2,0.5]}, \frac{c_2}{[0.4,0.7],[0.2,0.5],[0.4,0.7],[0.3,0.6],[0.5,0.8]}, \frac{c_3}{[0.6,0.9],[0.4,0.6],[0.1,0.3],[0.5,0.8],[0.3,0.7]}\right\}, \\
\left\{\frac{v_1}{[0.3,0.5],[0.1,0.9],[0.4,0.5],[0.7,0.8],[0.2,0.4]}, \frac{v_2}{[0.2,0.3],[0.7,0.7],[0.3,0.9],[0.5,0.6],[0.4,0.8]}\right\}
\end{array}\right)\right) \\
b_2, \left(\begin{array}{c}
\left\{\frac{h_1}{[0.2,0.5],[0.3,0.6],[0.7,0.9],[0.6,0.7],[0.4,0.5]}, \frac{h_2}{[0.3,0.6],[0.5,0.6],[0.2,0.4],[0.1,0.2],[0.4,0.8]}, \frac{h_3}{[0.5,0.6],[0.3,0.5],[0.6,0.8],[0.4,0.6],[0.2,0.4]}\right\}, \\
\left\{\frac{c_1}{[0.2,0.5],[0.6,0.9],[0.5,0.6],[0.3,0.8],[0.3,0.9]}, \frac{c_2}{[0.5,0.9],[0.3,0.5],[0.4,0.6],[0.2,0.7],[0.1,0.8]}, \frac{c_3}{[0.7,0.8],[0.6,0.6],[0.3,0.5],[0.4,0.5],[0.2,0.7]}\right\}, \\
\left\{\frac{v_1}{[0.5,0.7],[0.3,0.4],[0.7,0.9],[0.6,0.7],[0.5,0.7]}, \frac{v_2}{[0.2,0.2],[0.5,0.9],[0.7,0.9],[0.4,0.7],[0.3,0.6]}\right\}
\end{array}\right) \\
b_3, \left(\begin{array}{c}
\left\{\frac{h_1}{[0.5,0.6],[0.4,0.7],[0.7,0.8],[0.1,0.6],[0.5,0.7]}, \frac{h_2}{[0.8,0.9],[0.3,0.4],[0.2,0.7],[0.6,0.8],[0.5,0.7]}, \frac{h_3}{[0.4,0.6],[0.8,0.9],[0.2,0.6],[0.3,0.8],[0.2,0.7]}\right\}, \\
\left\{\frac{c_1}{[0.4,0.7],[0.7,0.8],[0.1,0.7],[0.6,0.7],[0.3,0.9]}, \frac{c_2}{[0.5,0.6],[0.1,0.4],[0.3,0.7],[0.1,0.5],[0.4,0.8]}, \frac{c_3}{[0.5,0.8],[0.1,0.2],[0.4,0.6],[0.6,0.9],[0.4,0.6]}\right\}, \\
\left\{\frac{v_1}{[0.3,0.4],[0.2,0.9],[0.6,0.7],[0.3,0.8],[0.5,0.6]}, \frac{v_2}{[0.2,0.3],[0.4,0.8],[0.3,0.7],[0.4,0.5],[0.1,0.3]}\right\}
\end{array}\right)
\end{array}\right\}$$

$(M, D) =$

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$$\left\{\begin{array}{l}\left(d_1,\left(\begin{array}{c}\left\{\frac{h_1}{[0.4,0.9],[0.8,0.8],[0.4,0.6],[0.2,0.8],[0.1,0.3]},\frac{h_2}{[0.4,0.9],[0.6,0.7],[0.4,0.6],[0.2,0.7],[0.2,0.4]},\frac{h_3}{[0.4,0.8],[0.6,0.9],[0.2,0.7],[0.1,0.2],[0.3,0.6]}\right\},\\ \left\{\frac{c_1}{[0.5,0.9],[0.6,0.7],[0.2,0.3],[0.1,0.4],[0.2,0.5]},\frac{c_2}{[0.4,0.7],[0.3,0.9],[0.4,0.7],[0.3,0.6],[0.4,0.6]},\frac{c_3}{[0.6,0.9],[0.4,0.8],[0.1,0.3],[0.4,0.7],[0.3,0.6]}\right\},\\ \left\{\frac{v_1}{[0.5,0.7],[0.6,0.9],[0.2,0.5],[0.3,0.4],[0.2,0.4]},\frac{v_2}{[0.4,0.6],[0.7,0.7],[0.3,0.8],[0.4,0.6],[0.1,0.8]}\right\}\end{array}\right)\right)\\ d_2,\left(\begin{array}{c}\left\{\frac{h_1}{[0.3,0.6],[0.4,0.7],[0.5,0.8],[0.5,0.7],[0.2,0.2]},\frac{h_2}{[0.5,0.7],[0.6,0.7],[0.2,0.4],[0.1,0.2],[0.4,0.7]},\frac{h_3}{[0.5,0.7],[0.4,0.5],[0.3,0.5],[0.4,0.6],[0.2,0.4]}\right\},\\ \left\{\frac{c_1}{[0.2,0.8],[0.6,0.9],[0.3,0.6],[0.3,0.5],[0.3,0.6]},\frac{c_2}{[0.5,0.9],[0.4,0.8],[0.4,0.6],[0.2,0.7],[0.1,0.5]},\frac{c_3}{[0.7,0.8],[0.6,0.6],[0.3,0.5],[0.3,0.5],[0.2,0.6]}\right\},\\ \left\{\frac{v_1}{[0.5,0.8],[0.3,0.4],[0.2,0.4],[0.5,0.5],[0.3,0.6]},\frac{v_2}{[0.6,0.7],[0.7,0.9],[0.5,0.8],[0.3,0.6],[0.2,0.3]}\right\}\end{array}\right)\\ d_3,\left(\begin{array}{c}\left\{\frac{h_1}{[0.5,0.8],[0.5,0.7],[0.3,0.8],[0.1,0.6],[0.2,0.3]},\frac{h_2}{[0.8,0.9],[0.7,0.8],[0.2,0.5],[0.1,0.8],[0.3,0.5]},\frac{h_3}{[0.5,0.8],[0.8,0.9],[0.1,0.3],[0.3,0.6],[0.2,0.7]}\right\},\\ \left\{\frac{c_1}{[0.4,0.7],[0.7,0.8],[0.1,0.7],[0.2,0.5],[0.3,0.7]},\frac{c_2}{[0.5,0.6],[0.5,0.9],[0.2,0.3],[0.1,0.5],[0.4,0.8]},\frac{c_3}{[0.5,0.8],[0.1,0.8],[0.2,0.6],[0.5,0.8],[0.3,0.6]}\right\},\\ \left\{\frac{v_1}{[0.3,0.5],[0.3,0.9],[0.4,0.6],[0.3,0.7],[0.4,0.6]},\frac{v_2}{[0.2,0.5],[0.4,0.8],[0.3,0.6],[0.4,0.5],[0.1,0.3]}\right\}\end{array}\right)\\ d_4,\left(\begin{array}{c}\left\{\frac{h_1}{[0.4,0.5],[0.1,0.4],[0.5,0.6],[0.2,0.7],[0.3,0.4]},\frac{h_2}{[0.2,0.3],[0.2,0.5],[0.7,0.8],[0.2,0.9],[0.4,0.6]},\frac{h_3}{[0.4,0.5],[0.6,0.6],[0.5,0.6],[0.4,0.7],[0.3,0.4]}\right\},\\ \left\{\frac{c_1}{[0.5,0.5],[0.2,0.4],[0.5,0.5],[0.1,0.4],[0.2,0.6]},\frac{c_2}{[0.7,0.8],[0.5,0.9],[0.2,0.3],[0.3,0.6],[0.1,0.7]},\frac{c_3}{[0.4,0.7],[0.4,0.5],[0.3,0.6],[0.2,0.4],[0.6,0.7]}\right\},\\ \left\{\frac{v_1}{[0.2,0.5],[0.5,0.9],[0.5,0.8],[0.3,0.4],[0.2,0.7]},\frac{v_2}{[0.4,0.6],[0.8,0.9],[0.4,0.6],[0.3,0.7],[0.6,0.8]}\right\}\end{array}\right)\end{array}\right\}$$

Now we apply MA to the first pentapartitioned neutrosophic vague soft multiset part in (M,D) to take the decision from the availability set U_1 and find the values of $T_*(x_i) = \bar{T}(x_i) - \bar{F}(x_i)$ for interval truth-membership part $\hat{T}_{A_{PNV}}(x_i) = [\bar{T}(x_i), T^+(x_i)]$, where $T^+(x_i) = 1 - \bar{F}(x_i), \forall x_i \in U_1$, $F_*(x_i) = \bar{F}(x_i) - \bar{T}(x_i)$ for interval falsity-membership part $\hat{F}_{A_{PNV}}(x_i) = [\bar{F}(x_i), F^+(x_i)]$, where $F^+(x_i) = 1 - \bar{T}(x_i), \forall x_i \in U_1$ and take the arithmetic average $C_*(x_i), G_*(x_i)$ and $U_*(x_i)$ of the end points of the interval indeterminacy $-$ membership part $\hat{C}_{A_{PNV}}(x_i) = [\bar{C}(x_i), C^+(x_i)]$, $\hat{G}_{A_{PNV}}(x_i) = [\bar{G}(x_i), G^+(x_i)]$ and $\hat{U}_{A_{PNV}}(x_i)$ =$[\bar{U}(x_i), U^+(x_i)]$. Then find the values of $T_*(x_i) + C_*(x_i) + G_*(x_i) + U_*(x_i) - F_*(x_i)$ $\forall x_i \in U_1$.

The $(PNVSM)$ is a generalization of vague soft multiset. It can explain the universal U in more detail with three membership functions, especially when there are many parameters involved, whereas vague soft multiset can tell us limited information about the universal U. It can only h$andl$e the incomplete information considering both the truth $-$ membership and falsity $-$ membership values, while $(PNVSM)$ can handle problems involving imprecise, indeterminacy and inconsistent data, which makes it more accurate and realistic than vague soft multiset. Furthermore, vague set is an intutionistic fuzzy set (IFS) (Atanassov, 1986) which is a a generalization of fuzzy set, therefore $(PNVSM)$ can better handle the elements of imprecision and uncertainty compared to soft multiset, fuzzy soft multiset, vague soft multiset and intuitionistic fuzzy multiset.

6. Possibility Pentapartitioned Neutrosophic Vague Soft Set

In this part, we introduce the notion of possibility pentapartitioned neutrosophic vague soft sets as an extension of (PNVSSs) and describe certain operations that can be performed on them, namely, subset, equality, null, absolute, complement, union, intersection,

Definition 6.1: Let $U = \{x_1, x_2, \dots \dots . x_n\}$ be the universal set of elements and $E = \{x_1, x_2, \dots \dots . x_n\}$ be the universal set of parameters. Let $F: E \to PNV(U)$ where $PNV(U)$ is the collection of all

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pentapartitioned neutrosophic vague subsets of U and μ be a fuzzy subset of E, that is $\mu : E \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu: E \to PNV(U) \times I^U$ be a function defined as follows:

$$F_\mu(e) = \big(F(e)(x),\ \mu(e)(x)\big), \forall x \in U.$$

Then F_μ is called a possibility pentapartitioned neutrosophic vague soft set (PPNVSS) over the soft universe (U, E). For any parameter $(e_i, F_\mu(e_i)\big(F(e_i)(x),\ \mu(e_i)(x)\big)$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by μ(ei). So we $can\ write\ F\mu ei\ as\ follows:$

$$F_\mu(e_i) = \left\{\left(\frac{x_1}{F(e_i)(x_1)}, \mu(e_i)(x_1)\right), \left(\frac{x_2}{F(e_i)(x_2)}, \mu(e_i)(x_2)\right), \ldots\ldots \left(\frac{x_n}{F(e_i)(x_n)}, \mu(e_i)(x_n)\right)\right\}.$$

We can write F_μ as (F_μ, E). If $A \subseteq E$, we can also have a $PPNVSS\ (F_\mu, A)$.

Example 6.2: Let $U = \{x_1, x_2\}$ be a universe set. Let $H = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters and let $\mu: H \to I^U$.

We define a function $F_\mu: H \to PNV(U) \times I^U$ as follows:

$$F_\mu(e_1) = \left\{\left(\frac{x_1}{([0.2,0.4],[0.4,0.6],[0.5,0.7],[0.3,0.8],[0.3,0.5])}, 0.7\right), \left(\frac{x_1}{([0.5,0.7],[0.2,0.6],[0.3,0.9],[0.4,0.5],[0.1,0.7])}, 0.5\right)\right\}$$

$$F_\mu(e_2) = \left\{\left(\frac{x_1}{([0.3,0.4],[0.5,0.7],[0.7,0.8],[0.2,0.9],[0.5,0.5])}, 0.2\right), \left(\frac{x_1}{([0.1,0.3],[0.4,0.7],[0.7,0.9],[0.3,0.6],[0.5,0.8])}, 0.3\right)\right\}$$

$$F_\mu(e_3) = \left\{\left(\frac{x_1}{([0.7,0.8],[0.4,0.5],[0.2,0.3],[0.5,0.8],[0.4,0.6])}, 0.9\right), \left(\frac{x_1}{([0.3,0.4],[0.6,0.8],[0.3,0.7],[0.4,0.8],[0.2,0.7])}, 0.6\right)\right\}$$

$$F_\mu(e_4) = \left\{\left(\frac{x_1}{([0.4,0.5],[0.5,0.7],[0.3,0.3],[0.4,0.6],[0.5,0.8])}, 0.4\right), \left(\frac{x_1}{([0.1,0.4],[0.4,0.7],[0.5,0.6],[0.2,0.6],[0.4,0.6])}, 0.7\right)\right\}$$

$$F_\mu(e_5) = \left\{\left(\frac{x_1}{([0.3,0.6],[0.2,0.4],[0.4,0.8],[0.1,0.2],[0.5,0.5])}, 0.5\right), \left(\frac{x_1}{([0.4,0.6],[0.2,0.5],[0.6,0.7],[0.4,0.9],[0.3,0.8])}, 0.8\right)\right\}$$

Then F_μ is a $PPNVSS$ over (U, H). In matrix notation we can write

$$F_\mu = \begin{pmatrix} [0.2,0.4],[0.4,0.6],[0.5,0.7],[0.3,0.8],[0.3,0.5],\ 0.7 & [0.5,0.7],[0.2,0.6],[0.3,0.9],[0.4,0.5],[0.1,0.7],\ 0.5 \\ [0.3,0.4],[0.5,0.7],[0.7,0.8],[0.2,0.9],[0.5,0.5],\ 0.2 & [0.1,0.3],[0.4,0.7],[0.7,0.9],[0.3,0.6],[0.5,0.8], 0.3 \\ [0.7,0.8],[0.4,0.5],[0.2,0.3],[0.5,0.8],[0.4,0.6], 0.9 & [0.3,0.4],[0.6,0.8],[0.3,0.7],[0.4,0.8],[0.2,0.7],\ 0.6 \\ [0.4,0.5],[0.5,0.7],[0.3,0.3],[0.4,0.6],[0.5,0.8],\ 0.4 & [0.1,0.4],[0.4,0.7],[0.5,0.6],[0.2,0.6],[0.4,0.6],\ 0.7 \\ [0.3,0.6],[0.2,0.4],[0.4,0.8],[0.1,0.2],[0.5,0.5],\ 0.5 & [0.4,0.6],[0.2,0.5],[0.6,0.7],[0.4,0.9],[0.3,0.8],\ 0.8 \end{pmatrix}$$

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Definition 6.3: let F_μ and G_δ be $PPNVSSs$ over (U, E). Then F_μ is said to be possibility pentapartitioned neutrosophic vague soft subset of G_δ then $F_\mu \subseteq G_\delta$ if

a) $\mu(e)$ is a fuzzy subset of $\delta(e), \forall\, e \in E$

b) $F(e)$ is a pentapartitioned neutrosophic vague subset of $G(e)$, $\forall\, e \in E$

Proposition 6.4: Let $F_\mu,\ G_\delta,\ H_\nu$ be any three $PPNVSSs\ over\ (U, E)$. Then the following results are true:

a) $F_\mu \,\tilde{\cup}\, G_\delta = G_\delta \,\tilde{\cup}\, F_\mu$

b) $F_\mu \,\tilde{\cap}\, G_\delta = G_\delta \,\tilde{\cap}\, F_\mu$

c) $F_\mu \,\tilde{\cup}\, (G_\delta \,\tilde{\cup}\, H_\nu) = \left(F_\mu \,\tilde{\cup}\, G_\delta\right) \tilde{\cup}\, H_\nu\ F_\mu$

d) $F_\mu \,\tilde{\cap}\, (G_\delta \,\tilde{\cap}\, H_\nu) = \left(F_\mu \,\tilde{\cap}\, G_\delta\right) \tilde{\cap}\, H_\nu\ F_\mu$

Proof: The proofs can be obtained by the fact thatpentapartitioned neutrosophic vague sets are commutative and associative.

Definition 6.5: let F_μ and G_δ be $PPNVSSs$ over (U, E). We say that F_μ and G_δ are significantly similar if

$S(\,G_\delta) \geq \frac{1}{2}$.

Proposition 6.6: F_μ and G_δ be $PPNVSSs$ over (U, E) such that F_μ or G_δ is a non-zero $PPNVSS$. Then the following holds:

a. $S(F_\mu, G_\delta) = S(G_\delta, F_\mu\,)$

b. $0 \leq S(F_\mu, G_\delta) \leq 1$

c. $F_\mu = G_\delta \Longrightarrow\ S(F_\mu, G_\delta) = 1.$

d. $F_\mu \subseteq G_\delta \subseteq H_\lambda \Longrightarrow\ S(F_\mu, H_\lambda) \leq\ S(G_\delta, H_\lambda)$

e. $F_\mu \,\tilde{\cap}\, G_\delta = \emptyset \Longrightarrow\ S(F_\mu, G_\delta) = 0$

Proof: The proof is straightforward and follows from the definition.

7. Conclusion

By applying the notion ofSoft Set to Pentaprtitioned Neutrosophic Vague Set, we came up with the concept of thePentaprtitioned Neutrosophic Vague Set. We have established the essential operations on the Pentaprtitioned Neutrosophic Vague Soft Set, namely subset, complement, union, and intersection, Pentaprtitioned Neutrosophic Vague multiset, and Possibility Pentaprtitioned Neutrosophic Vague sets, with the aid of several instances.

The basic properties of these operations also were proven. A generalized algorithm is introduced and applied to solve a hypothetical decision making problem. With the help of this new extension, existing theories for handling issues involving imprecise, indeterminacy, and inconsistent data will be significantly improved.

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