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FULL LENGTH ARTICLE

The Computation of Particular Roots of Nonlinear Complex Equations of the Form: $\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n=c$

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ABSTRACT

Solving polynomial equations involves finding their roots. In this respect, this idea dominates the minds of many mathematicians about how to find those roots. The Abel–Ruffini theorem emphasizes that there is no general formula involving only the coefficients of a polynomial equation of degree five or higher that allows us to compute its solutions using radicals and its associate to the Galois Theory. The mathematical need for solving polynomial equations represents the motivation for the development of systems of numbers from Natural numbers to Complex numbers throughout the history of mathematics. Complex numbers play a central role in this context. The Fundamental Theorem of Algebra tell us that every nonconstant polynomial equation with complex coefficients has at least one complex root. While the Galois group associated with a polynomial captivates its symmetries and determines whether it is solvable by radicals. From a mathematical standpoint, it is customary to visualize polynomials in the form: $P_n(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x^1+a_0$, Where the set of coefficients $\{a_n,a_{n-1},\ldots,a_0\}\in\mathbb{C}$ and $P_n(x)\in\mathbb{C}[x]$. we have reconceptualized the polynomial generated by the formula $(ax+y)^n=c$ in our previous work and computing radicals of more degree 5. In this article, we present a natural procedure formula that will lead us to find a solution for a class of polynomials nonlinear Complex numbers with degree n associated with the equation: $\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n=c$ as a particular class of Complex Polynomials.

Keywords: Binomial Theorem ▪ Complex Polynomials ▪ Exact solving of non-liner Polynomials of Complex Numbers in particular class $\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n=c$

1. INTRODUCTION

Solving polynomial equations with complex variables can be challenging, to find the general solutions. In 2012 [1], Vieira demonstrated that "polynomial equation of degree less than 5 and with real parameters can be solved by regarding the variable in which the polynomial depends as a complex variable." Once again, in 2016 [2], Vieira presented a sufficient condition for a self-inversive polynomial to have a fixed number of roots on the complex unit circle. In 2017 [3], Saghe presented "a new method to solve the general quartic equation which is different from Ferrari's method for the general quartic equation, moreover, he introduced techniques to solve an algebraic equation with degree n for some equations. In 2020 [4], Skopenkov presented a "short elementary proofs of the well-known Ruffini-Abel-Galois theorems on insolvability of algebraic equations of degree 5 in radicals." In this article, we introduce a new method to compute a particular class of nonlinear complex polynomials with coefficients or parameters reals or complex numbers not considered in the

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previous researchers and this work complete our work in [5] when polynomials belong to complex fields.

2. NEW APPROACH TO FIND THE GENERAL SOLUTION OF NONLINEAR EQUATION OF COMPLEX NUMBERS ON THE FORM: $\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n=c$

In this section, we introduced a new approach to find the general solution of Eq. (1). Let $\mathbb{R}$ denote the real field, and $\mathbb{R}[x]$ denote the ring of polynomials over $\mathbb{R}$. Consider $\mathbb{C}$ is the complex field and $\mathbb{C}[x]$ denote the ring of polynomials over $\mathbb{C}$. if we consider a new form as shown in Eq. (1.2).

$$\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n=c \tag{1.2}$$

Where $a,k,c\in\mathbb{C}$, $x,y\in\mathbb{R}$, and $n\geq 1\in\mathbb{Z}^+$. This new method depends on a new formula of radicals has been studded in [5] with string sequences of integers numbers repented the value of y and x under certain formulas of computation. In this method, if we expand the left side of equation (1) by binomial theorem of complex numbers, we get a particular polynomial (or class of polynomials) of the form as shown in the Eq. (2.2).

$$F_n(k)=a_nk^n+a_{n-1}k^{n-1}+\cdots+a_0=c. \tag{2.2}$$

This equation becomes like Eq. (3.2).

$$\begin{aligned}P_n(k)&=a_nk^n+a_{n-1}k^{n-1}+\cdots+\delta=0,\\ \delta&=a_0-c.\end{aligned} \tag{3.2}$$

The Eq. (3.2) represent the class of polynomials of complex or real values according to constants a and c. to solve the polynomial of Eq. (3.2) and to compute the roots of Eq. (1.2). We present the following method, which is called SHAD-method in [5].

Definition 2.1 Let $c\in\mathbb{C}$ bet a given complex number and $n\in\mathbb{N}$ be a given natural number, then the value of y can be determined by the Eq. (4.2):

$$y=\sqrt[n]{\frac{|c|}{10^n}}. \tag{4.2}$$

Definition 2.2 Let $c\in\mathbb{C}$ bet a given complex number and $n\in\mathbb{N}$ be a given natural number, then the value of x can be determined by the Eq. (5.2):

$$\left((x+10\cdot y)\sqrt[n]{i^s}\right)^n=c. \tag{5.2}$$

Remark. Since $x,y\in\mathbb{R}$, and any real numbers include the positive and negative integers, and the fractions made from those integers (or rational numbers) and also the irrational numbers. The principle of our techniques picks the positive integers and ignore the part of fraction.

Definition 2.3 Let $c\in\mathbb{C}$ bet a given complex number and $n\in\mathbb{N}$ be a given natural number, then value of s can be determined by the Eq. (6.2):

$$s=\begin{cases}0, & \text{if Real}(c)\in\mathbb{R}^+,\\ 2, & \text{if Real}(c)\in\mathbb{R}^-,\\ 1, & \text{if Ima}(c)\in\mathbb{R}^+,\\ 3, & \text{if Ima}(c)\in\mathbb{R}^-,\\ 0, & \text{if } c\in\mathbb{C}.\end{cases} \tag{6.2}$$

To find the nth-root of Eq. (1.2), we use the formula as shown of Eq. (7.2).

$$\begin{aligned}r_j=\sqrt[n]{c}=a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s},\\ j=1,2,\ldots,n.\end{aligned} \tag{7.2}$$

Theorem 2.1 For all $n\geq 1$, there are a set of n sequences of coefficients of complex numbers, that is, $\{a_n,a_{n-1},\ldots,\delta=a_0-c\in\mathbb{C}\}$, such that the roots of complex polynomial like in Eq. (3.2) is expressible by the set of radicals form $\{r_n,r_{n-1},\ldots,r_1\in\mathbb{C}\}$.

Proof. Let the Eq. (1.1) be a given equation, and by using Binomial theorem for complex numbers, where $z_1=a\sqrt[n]{i^s}k$ and $z_2=(x+10y)\sqrt[n]{i^s}$, then the Eq. (1.2) can be rewrite as shown in the Eq. (8.2).

$$(z_1+z_2)^n=c. \tag{8.2}$$

Where,

$$\left.\begin{aligned}&\sum_{r=0}^{n}\binom{n}{r}z_1^{n-r}z_2^r,\quad r=0,1,2,\ldots,n,\\ &\binom{n}{r}=\frac{n!}{(n-r)!r!}\end{aligned}\right\} \tag{9.2}$$

The left-side of Eq. (9.2) is given by Eq. (10.2).

$$(z_1+z_2)^n=\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n. \tag{10.2}$$

Eq. (10.2) expands by Eq. (11.2).

$$\left.\begin{aligned}&\binom{n}{0}\left(a\sqrt[n]{i^s}k\right)^n\\ &+\binom{n}{1}\left(a\sqrt[n]{i^s}k\right)^{n-1}\left((x+10y)\sqrt[n]{i^s}\right)^1\\ &+\binom{n}{2}\left(a\sqrt[n]{i^s}k\right)^{n-2}\left((x+10y)\sqrt[n]{i^s}\right)^2\\ &\vdots\\ &+\binom{n}{n}\left((x+10y)\sqrt[n]{i^s}\right)^n\end{aligned}\right\} \tag{11.2}$$

This led to,

$$\left.\begin{aligned}&\left(a\sqrt[n]{i^s}k\right)^n\\ &+\frac{n!}{(n-1)!1!}\left(a\sqrt[n]{i^s}k\right)^{n-1}\left((x+10y)\sqrt[n]{i^s}\right)^1\\ &+\frac{n!}{(n-2)!2!}\left(a\sqrt[n]{i^s}k\right)^{n-2}\left((x+10y)\sqrt[n]{i^s}\right)^2\\ &\vdots\\ &+\left((x+10y)\sqrt[n]{i^s}\right)^n\end{aligned}\right\} \tag{12.2}$$

Where,

$$\left.\begin{aligned} a_n &= \left(a\sqrt[n]{i^s}\right)^n, \\ a_{n-1} &= n\left(a\sqrt[n]{i^s}\right)^{n-1}\left((x+10y)\sqrt[n]{i^s}\right)^1, \\ a_{n-2} &= \frac{n(n-1)}{2!}\left(a\sqrt[n]{i^s}\right)^{n-2} \\ &\quad\times\left((x+10y)\sqrt[n]{i^s}\right)^2, \\ a_{n-3} &= \frac{n(n-1)(n-2)}{3!}\left(a\sqrt[n]{i^s}\right)^{n-3} \\ &\quad\times\left((x+10y)\sqrt[n]{i^s}\right)^3, \\ &\vdots \\ a_0 &= \left((x+10y)\sqrt[n]{i^s}\right)^n \end{aligned}\right\} \tag{13.2}$$

Then we get from the Eq. (13.2), the polynomial of complex numbers:

$$\begin{aligned} P_n(k) &= a_nk^n + a_{n-1}k^{n-1} + \cdots + \delta = 0, \\ \delta &= a_0 - c. \end{aligned} \tag{14.2}$$

Conversely, consider the polynomial of degree n in the Eq. (14.2). By completing the term:

$$a_0 = \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n = \left((x+10y)\sqrt[n]{i^s}\right)^n. \tag{15.2}$$

In the Eq. (14.2), we have,

$$\begin{aligned} &a_nk^n + a_{n-1}k^{n-1} + \cdots + \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n \\ &\quad - \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n + \delta = 0. \end{aligned} \tag{16.2}$$

$$\begin{aligned} &a_nk^n + a_{n-1}k^{n-1} + \cdots + \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n \\ &\quad = \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n - \delta. \end{aligned} \tag{17.2}$$

$$a_nk^n + a_{n-1}k^{n-1} + \cdots + a_0 = a_0 - \delta. \tag{18.2}$$

$$a_nk^n + a_{n-1}k^{n-1} + \cdots + a_0 = a_0 - (a_0 - c). \tag{19.2}$$

$$\left(\sqrt[n]{a_n}k + \sqrt[n]{a_0}\right)^n = c. \tag{20.2}$$

$$\left(a\sqrt[n]{i^s}k + (x+10y)\sqrt[n]{i^s}\right)^n = c. \tag{21.2}$$

■

Theorem 2.2 Consider the Eq. (1.2), where $a,k,c \in \mathbb{C}$, and $n \geq 1 \in \mathbb{Z}^+$. By theorem 2.1 the Eq. (1.2) has associate with the polynomial's equation of degree n on the form in the Eq. (14.2). Then the roots of polynomials Eq. (14.2) given by the following SHAD-radicals formula

$$\begin{aligned} x_{j\in J} &= -\frac{a_{n-1}}{a_n\,n} \\ &\quad + \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{a_n\,n}\,e^{2\pi i(j-1)/n}. \end{aligned} \tag{22.2}$$

Where, $j \in J = \{1,2,3,\ldots,n\}$ is index set of roots of Eq. (14.2) in [5] and the roots of Eq. (1.2) are obtaining by the formula:

$$\begin{aligned} r_n = \sqrt[n]{c} = a\sqrt[n]{i^s}\Bigg[&-\frac{a_{n-1}}{a_n\,n} \\ &+ \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{a_n\,n}\,e^{2\pi i(j-1)/n}\Bigg] \\ &+ (x+10y)\sqrt[n]{i^s}. \end{aligned} \tag{23.2}$$

Proof. Consider the Eq. (1.2), then by theorem 2.1. the Eq. (1.2) can be written as shown in Eq. (14.2), by completing the term of the Eq. (15.2) in the Eq. (14.2), we get the formula in Eq. (24.2), So, we have

$$\begin{aligned} &a_nk^n + a_{n-1}k^{n-1} + \cdots + \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n \\ &\quad = \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n - \delta. \end{aligned} \tag{24.2}$$

$$\begin{aligned} &a_nk^n + a_{n-1}k^{n-1} + \cdots + \left(\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n \\ &\quad = \frac{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}{(a_n)^{n-1}n^n}. \end{aligned} \tag{25.2}$$

We get,

$$\left(\sqrt[n]{a_n}k + \frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n}\right)^n = \frac{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}{(a_n)^{n-1}n^n}. \tag{26.2}$$

By taking the n-root for both sides, so we have,

$$\begin{aligned} &\sqrt[n]{a_n}k + \frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n} \\ &\quad = \sqrt[n]{\frac{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}{(a_n)^{n-1}n^n}}\,e^{2\pi i(J-1)/n}. \end{aligned} \tag{27.2}$$

$$\begin{aligned} \sqrt[n]{a_n}k = &-\frac{a_{n-1}}{(\sqrt[n]{a_n})^{n-1}\,n} \\ &+ \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{(\sqrt[n]{a_n})^{n-1}n}\,e^{2\pi i(J-1)/n}. \end{aligned} \tag{28.2}$$

$$\begin{aligned} k = &-\frac{a_{n-1}}{\sqrt[n]{a_n}(\sqrt[n]{a_n})^{n-1}n} \\ &+ \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{\sqrt[n]{a_n}(\sqrt[n]{a_n})^{n-1}n}\,e^{2\pi i(J-1)/n}. \end{aligned} \tag{29.2}$$

$$\begin{aligned} k = &-\frac{a_{n-1}}{a_n\,n} \\ &+ \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{a_n\,n}\,e^{2\pi i(J-1)/n}. \end{aligned} \tag{30.2}$$

That is,

$$\begin{aligned} k_j = &-\frac{a_{n-1}}{a_n\,n} \\ &+ \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{a_n\,n}\,e^{2\pi i(J-1)/n}. \end{aligned} \tag{31.2}$$

Where, $j \in J = \{1,2,3,\ldots,n\}$ is index set of roots of Eq. (14.2). ■

Corollary 2.3 The formula of the Eq. (31.2), it is represented the solution of polynomials in the Eq. (14.2).

Proof. Consider the formula in the Eq. (32.2).

$$a_n = \sqrt[n]{a_n}\,(\sqrt[n]{a_n})^{n-1}. \tag{32.2}$$

And replacement Eq. (32.2) in the Eq. (31.2), we get,

$$k_j = \frac{-a_{n-1}}{\sqrt[n]{a_n}(\sqrt[n]{a_n})^{n-1}n} + \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{\sqrt[n]{a_n}(\sqrt[n]{a_n})^{n-1}n}\,e^{2\pi i(J-1)/n}. \tag{33.2}$$

$$k_j = \frac{1}{\sqrt[n]{a_n}}\left[\frac{-a_{n-1}}{(\sqrt[n]{a_n})^{n-1}n} + \frac{\sqrt[n]{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}}{(\sqrt[n]{a_n})^{n-1}n}\,e^{2\pi i(J-1)/n}\right]. \tag{34.2}$$

$$k_j = \frac{1}{\sqrt[n]{a_n}}\left[\frac{-a_{n-1}}{(\sqrt[n]{a_n})^{n-1}n} + \sqrt[n]{\frac{(a_{n-1})^n - (a_n)^{n-1}n^n\delta}{(a_n)^{n-1}n^n}}\,e^{2\pi i(J-1)/n}\right]. \tag{35.2}$$

$$k_j = \frac{1}{\sqrt[n]{a_n}}\left[\frac{-a_{n-1}}{(\sqrt[n]{a_n})^{n-1}n} + \sqrt[n]{\frac{(a_{n-1})^n}{(a_n)^{n-1}n^n} - \frac{(a_n)^{n-1}n^n\delta}{(a_n)^{n-1}n^n}} \times e^{2\pi i(J-1)/n}\right]. \tag{36.2}$$

By using the Eq. (15.2) in the Eq. (36.2), yield

$$k_j = \frac{1}{\sqrt[n]{a_n}}\left(-\sqrt[n]{a_0} + \sqrt[n]{a_0 - \delta}\right). \tag{37.2}$$

But, $\delta = a_0 - c$, therefore

$$k_j = \frac{1}{\sqrt[n]{a_n}}\left(-\sqrt[n]{a_0} + \sqrt[n]{c}\right), \quad \text{where } \sqrt[n]{a_0} = (x+10y)\sqrt[n]{i^s}. \tag{38.2}$$

The Eq. (38.2) is the solution of the Eq. (1.2). ■

3. APPLICATIONS OF THE SHAD-METHOD FORMULA

In this section, we interduces the various examples to explain the SHAD-Method for the solutions formula of Eq. (1.2) the procedure of solution depends on the degree of nonlinear polynomial of complex number in two cases when n either even or odd of integers number.

3.1 Solving the Quadratic Nonlinear Equations of Complex Numbers of the Form: $\left(a\sqrt{i^s}k + (x+10y)\sqrt{i^s}\right)^2 = c$

In this section, to illustrate our formula for the quadratic non-linear equation of complex member by the following examples.

Example 3.1.1 Consider the quadratic non-linear equation of complex member is given by the Following equation:

$$\left(4\sqrt{i^s}k + (x+10y)\sqrt{i^s}\right)^2 = 70. \tag{1.3.1}$$

By using Eq. (6.2), we have $s = 0$, because $c \in \mathbb{R}^+$. So,

$$\left(4\sqrt{i^0}k + (x+10y\sqrt{i^0})\right)^2 = 70. \tag{2.3.1}$$

By using Eq. (9.2), we get

$$16k^2 + 8k^1(x+10\cdot y)^1 + (x+10\cdot y)^2 = 70. \tag{3.3.1}$$

By using Eq. (4.2), we have

$$y = \sqrt[n]{\frac{|c|}{10^n}} = \sqrt[2]{\frac{|70|}{10^2}} = 0.836660265. \tag{4.3.1}$$

So, take the positive integer in y, that is $y = 0$, by using Eq. (5.2), we conclude that

$$\left.\begin{aligned}(x+10\cdot 0)^2 &= 70,\\ x &= 8.36660025, \quad \text{i.e.,}\\ x &= 8\end{aligned}\right\} \tag{5.3.1}$$

Put Eq. (4.3.1) and Eq. (5.3.1) in Eq. (3.3.1). We get,

$$16k^2 + 8k^1(8+10\cdot 0)^1 + (8+10\cdot 0)^2 = 70. \tag{6.3.1}$$

$$16k^2 + 64k - 6 = 0. \tag{7.3.1}$$

The Eq. (7.3.1) has the solution by the formula in [5] which is given by Eq. (22.2).

Further, the radical roots of the Eq. (7.3.1) looks like:

$$k_1 = \frac{-64}{16\cdot 2} + \frac{\sqrt[2]{(64)^2 - (16)^{2-1}2^2(-6)}}{16\cdot 2}\,e^{2\pi i(1-1)/2} = 0.09165006634. \tag{8.3.1}$$

$$k_2 = \frac{-64}{16\cdot 2} + \frac{\sqrt[2]{(64)^2 - (16)^{2-1}2^2(-6)}}{16\cdot 2}\,e^{2\pi i(2-1)/2} = -4.09165006634. \tag{9.3.1}$$

Eq. (8.3.1) and Eq. (9.3.1) are solution of the Eq. (7.3.1), then the general solution of Eq. (1.3.1) are given by:

$$r_1 = \sqrt{70} = 4\sqrt{i^0}\left[\frac{-64}{16\cdot 2} + \frac{\sqrt[2]{(64)^2\,2^2(-6)}}{16\cdot 2}\,e^{2\pi i(1-1)/2}\right] + (8+10\cdot 0)\sqrt{i^0} = 8.36660026534. \tag{10.3.1}$$

$$
\begin{aligned}
r_2 &= \sqrt{70} \\
&= 4\sqrt{i^0}\Big[\frac{-64}{16\cdot 2} \\
&\quad + \frac{\sqrt[2]{(64)^2-(16)^2 2^2(-6)}}{16\cdot 2}\, e^{2\pi i(1-1)/2}\Big] \\
&\quad + (8+10\cdot 0)\sqrt{i^0} \\
&= -8.36660026534.
\end{aligned} \tag{11.3.1}
$$

Example 3.1.2 Consider the quadratic non-linear equation of complex member is given by the Following equation:

$$
\left(4\sqrt{i^s}k + (x+10y)\sqrt{i^s}\right)^2 = 70i. \tag{12.3.1}
$$

By using Eq. (6.2), we have $s = 1$, because $\mathrm{Im}(c) \in \mathbb{R}^+$. hence,

$$
\left(4\sqrt{i^1}k + (x+10y)\sqrt{i^1}\right)^2 = 70i. \tag{13.3.1}
$$

By using Eq. (9.2), we get

$$
\begin{aligned}
&16ik^2 + 8ik^1(x+10\cdot y)^1 \\
&\qquad + i(x+10\cdot y)^2 = 70i.
\end{aligned} \tag{14.3.1}
$$

By using Eq. (4.2), we have

$$
y = \sqrt[n]{\frac{|c|}{10^n}} = \sqrt[2]{\frac{|70i|}{10^2}} = 0.836660265. \tag{15.3.1}
$$

Thus, $y = 0$, and by using Eq. (5.2), we deduced that,

$$
\begin{aligned}
&\left((x+10\cdot 0)\sqrt{i^1}\right)^2 = 70i \\
&\qquad \Rightarrow \quad x\sqrt{i^1} = 8.36660025\sqrt{i^1}, \\
&\qquad\quad \text{so } x = 8.
\end{aligned} \tag{16.3.1}
$$

Put Eq. (15.3.1), and Eq. (16.3.1) in Eq. (14.3.1), we get,

$$
\begin{aligned}
&16ik^2 + 8ik^1(8+10\cdot 0)^1 \\
&\qquad + i(8+10\cdot 0)^2 = 70i.
\end{aligned} \tag{17.3.1}
$$

$$
16ik^2 + 64ik - 6i = 0. \tag{18.3.1}
$$

The Eq. (18.3.1) has the solution by the formula in [5], which is given by the general solution of Eq. (12.3.1) are given by:

The radical roots of the Eq. (18.3.1) looks like:

$$
\begin{aligned}
k_1 &= \frac{-64i}{16i\cdot 2} \\
&\quad + \frac{\sqrt[2]{(64i)^2-(16i)^{2-1}2^2(-6i)}}{16i\cdot 2}\, e^{2\pi i(1-1)/2} \\
&= 0.09165006634.
\end{aligned} \tag{19.3.1}
$$

$$
\begin{aligned}
k_2 &= \frac{-64i}{16i\cdot 2} \\
&\quad + \frac{\sqrt[2]{(64i)^2-(16i)^{2-1}2^2(-6i)}}{16i\cdot 2}\, e^{2\pi i(2-1)/2} \\
&= -4.09165006634.
\end{aligned} \tag{20.3.1}
$$

Eq. (19.3.1), and Eq. (20.3.1) are solution of the Eq. (18.3.1), and the general solution of Eq. (7.2) are given by:

$$
\begin{aligned}
r_1 &= \sqrt{70i} \\
&= 4\sqrt{i^1}\Big[\frac{-64i}{16i\cdot 2} \\
&\quad + \frac{\sqrt[2]{(64i)^2-(16i)^2 2^2(-6i)}}{16i\cdot 2}\, e^{2\pi i(j-1)/2}\Big] \\
&\quad + (8+10\cdot 0)\sqrt{i^1} \\
&= 5.9160797831 + 5.9160797831i.
\end{aligned} \tag{21.3.1}
$$

$$
\begin{aligned}
r_2 &= \sqrt{70i} \\
&= 4\sqrt{i^1}\Big[\frac{-64i}{16i\cdot 2} \\
&\quad + \frac{\sqrt[2]{(64i)^2-(16i)^2 2^2(-6i)}}{16i\cdot 2}\, e^{2\pi i(2-1)/2}\Big] \\
&\quad + (8+10\cdot 0)\sqrt{i^1} \\
&= -5.9160797831 - 5.9160797831i.
\end{aligned} \tag{22.3.1}
$$

3.2 Solving the Cubic Non-Linear Equations of Complex Numbers of the Form: $\left(a\sqrt[3]{i^s}k + (x+10y)\sqrt[3]{i^s}\right)^3 = c$

In this section, we presented just one example when the degree of the complex number is 3 with coefficients in $\mathbb{C}$, to illustrate our method.

Example 3.2.1 Consider the quadratic non-linear equation of complex member which is given by:

$$
\left(2\sqrt[3]{i^s}k + (x+10y)\sqrt[3]{i^s}\right)^3 = 1005. \tag{1.3.2}
$$

By using Eq. (6.2), we have $s = 0$, because $\mathrm{Real}(c) \in \mathbb{R}^+$. So,

$$
\left(2\sqrt[3]{i^0}k + (x+10y)\sqrt[3]{i^0}\right)^3 = 1005. \tag{2.3.2}
$$

By using Eq. (9.2), we get

$$
\begin{aligned}
&8k^3 + 12k^2(x+10\cdot y)^1 \\
&\quad + 6ki(x+10\cdot y)^2 + (x+10\cdot y)^3 = 1005.
\end{aligned} \tag{3.3.2}
$$

By using Eq. (4.2), we have

$$
y = \sqrt[n]{\frac{|c|}{10^n}} = \sqrt[3]{\frac{|1005|}{10^3}} = 1.001663897. \tag{4.3.2}
$$

So, take the non-decimal part in the number, that is $y = 1$. By using Eq. (5.2), we conclude that

$$
\left.
\begin{aligned}
&\left((x+10\cdot 1)\sqrt[3]{i^0}\right)^3 = 1005, \\
&\qquad x\sqrt[3]{i^0} = -10\sqrt[3]{i^0} + \sqrt[3]{1005}, \\
&\qquad\quad x = 0.01663895579, \quad \text{i.e.,} \\
&\qquad\quad x = 0
\end{aligned}
\right\} \tag{5.3.2}
$$

Put Eq. (4.3.2), and Eq. (5.3.2) in Eq. (3.3.2), we get,

$$
\begin{aligned}
&8k^3 + 12k^2(0+10\cdot 1)^1 \\
&\quad + 6ki(0+10\cdot 1)^2 + (0+10\cdot 1)^3 = 1005.
\end{aligned} \tag{6.3.2}
$$

$$
8k^3 + 120k^2 + 600k - 5 = 0. \tag{7.3.2}
$$

The radical roots of the Eq. (7.3.2) looks like:

$$\begin{aligned} k_1 &= \frac{-120}{8\cdot 3} \\ &\quad + \frac{\sqrt[3]{(120)^3-(8)^{3-1}3^3(-5)}}{8\cdot 3}\, e^{2\pi i(1-1)/3} \\ &= 0.0083194829. \end{aligned} \tag{8.3.2}$$

$$\begin{aligned} k_2 &= \frac{-120}{8\cdot 3} \\ &\quad + \frac{\sqrt[3]{(120)^3-(8)^{3-1}3^3(-5)}}{8\cdot 3}\, e^{2\pi i(2-1)/3} \\ &= -7.50415974145 + 4.33733190246i. \end{aligned} \tag{9.3.2}$$

$$\begin{aligned} k_3 &= \frac{-120}{8\cdot 3} \\ &\quad + \frac{\sqrt[3]{(120)^3-(8)^{3-1}3^3(-5)}}{8\cdot 3}\, e^{2\pi i(3-1)/3} \\ &= -7.50415974145 - 4.33733190246i. \end{aligned} \tag{10.3.2}$$

The Eq. (8.3.2), Eq. (9.3.2), and Eq. (10.3.2) are solution of the Eq. (7.3.2), the general solution of Eq. (1.3.2) are given by:

$$\begin{aligned} r_1 &= \sqrt[3]{1005} \\ &= 2\sqrt[3]{i^0}\Big[\frac{-120}{8\cdot 3} \\ &\quad + \frac{\sqrt[3]{(120)^3-(8)^{3-1}3^3(-5)}}{8\cdot 3}\, e^{2\pi i(1-1)/3}\Big] \\ &\quad + (0+10\cdot 1)\sqrt[3]{i^0} \\ &= 10.01663896579. \end{aligned} \tag{11.3.2}$$

$$\begin{aligned} r_2 &= \sqrt[3]{1005} \\ &= 2\sqrt[3]{i^0}\Big[\frac{-120}{8\cdot 3} \\ &\quad + \frac{\sqrt[3]{(120)^3-(8)^{3-1}3^3(-5)}}{8\cdot 3}\, e^{2\pi i(2-1)/3}\Big] \\ &\quad + (0+10\cdot 1)\sqrt[3]{i^0} \\ &= -5.0083194829 + 8.67466380491i. \end{aligned} \tag{12.3.2}$$

$$\begin{aligned} r_3 &= \sqrt[3]{1005} \\ &= 2\sqrt[3]{i^0}\Big[\frac{-120}{8\cdot 3} \\ &\quad + \frac{\sqrt[3]{(120)^3-(8)^{3-1}3^3(-5)}}{8\cdot 3}\, e^{2\pi i(3-1)/3}\Big] \\ &\quad + (0+10\cdot 1)\sqrt[3]{i^0} \\ &= -5.0083194829 - 8.67466380491i. \end{aligned} \tag{13.3.2}$$

3.3 Solving the Quartic Nonlinear Equations of Complex Numbers of the Form: $\left(a\sqrt[4]{i^s}k+(x+10y)\sqrt[4]{i^s}\right)^4 = c$

In this section, we introduce one example of quartic nonlinear of complex numbers, to illustrate the method.

Example 3.3.1 Consider the quadratic nonlinear equation of complex member is given by:

$$\left(2\sqrt[4]{i^s}k+(x+10y)\sqrt[4]{i^s}\right)^4 = -279880i. \tag{1.3.3}$$

By using Eq. (6.2), we have $s=3$, because $\text{Real}(c)\in\mathbb{R}^-$.

So,

$$\left(2\sqrt[4]{i^3}k+(x+10y)\sqrt[4]{i^3}\right)^4 = -279880i. \tag{2.3.3}$$

By using Eq. (9.2), we get

$$\left.\begin{aligned} &-16ik^4 \\ &-32ik^3(x+10\cdot y)^1 \\ &-24ik^2(x+10\cdot y)^2 \\ &-8ik(x+10\cdot y)^3 \\ &-i(x+10\cdot y)^4 \\ &= -279880i \end{aligned}\right\} \tag{3.3.3}$$

By using Eq. (4.2), we have

$$\begin{aligned} y &= \sqrt[n]{\frac{|c|}{10^n}} = \sqrt[4]{\frac{|-279880i|}{10^4}} \\ &= 2.300080131. \end{aligned} \tag{4.3.3}$$

So, $y=2$, and by using Eq. (5.2), we conclude that

$$\left.\begin{aligned} \left((x+10\cdot 2)\sqrt[4]{i^3}\right)^4 &= -279880i, \\ x\sqrt[4]{i^3} &= -20\sqrt[4]{i^3}+\sqrt[4]{-279880i}, \\ x &= 3.000801306, \quad \text{i.e.,} \\ x &= 3 \end{aligned}\right\} \tag{5.3.3}$$

Put Eq. (4.3.3) and Eq. (5.3.3) in Eq. (3.3.3), we get,

$$\left.\begin{aligned} &-16ik^4 \\ &-32ik^3(3+10\cdot 2)^1 \\ &-24ik^2(3+10\cdot 2)^2 \\ &-8ik(3+10\cdot 2)^3 \\ &-i(3+10\cdot 2)^4 \\ &= -279880i \end{aligned}\right\} \tag{6.3.3}$$

$$\begin{aligned} &-16ik^4 - 736ik^3 - 12696ik^2 \\ &\quad -97336ik + 39i = 0. \end{aligned} \tag{7.3.3}$$

The Eq. (7.3.3) has solution by the formula in [5] which is given by the radical roots of the Eq. (22.2) looks like:

$$\begin{aligned} k_1 &= \frac{-(-736i)}{-16i\cdot 4} \\ &\quad + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\, e^{2\pi i(1-1)/4} \\ &= -11.5 + 11.50040065302i. \end{aligned} \tag{8.3.3}$$

$$\begin{aligned} k_2 &= \frac{-(-736i)}{-16i\cdot 4} \\ &\quad + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\, e^{2\pi i(2-1)/4} \\ &= -23.00040065302. \end{aligned} \tag{9.3.3}$$

$$k_3 = \frac{-(-736i)}{-16i\cdot 4} + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\,e^{2\pi i(3-1)/4} = -11.5-11.50040065302i. \tag{10.3.3}$$

$$k_4 = \frac{-(-736i)}{-16i\cdot 4} + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\,e^{2\pi i(4-1)/4} = 0.00040065302. \tag{11.3.3}$$

By using Eq. (8.3.3), Eq. (9.3.3), Eq. (10.3.3), and Eq. (11.3.3), in the Eq. (7.2), to find the general solution of Eq. (1.3.3) as following:

$$r_1 = \sqrt[4]{-279880i} = 2\sqrt[4]{i^3}\left[\frac{-(-736i)}{-16i\cdot 4} + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\,e^{2\pi i(1-1)/4}\right] + (3+10\cdot 2)\sqrt[4]{i^3} = 8.80202559094+21.249969558i. \tag{12.3.3}$$

$$r_2 = \sqrt[4]{-279880i} = 2\sqrt[4]{i^3}\left[\frac{-(-736i)}{-16i\cdot 4} + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\,e^{2\pi i(2-1)/4}\right] + (3+10\cdot 2)\sqrt[4]{i^3} = -21.249969558+8.80202559094i. \tag{13.3.3}$$

$$r_3 = \sqrt[4]{-279880i} = 2\sqrt[4]{i^3}\left[\frac{-(-736i)}{-16i\cdot 4} + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\,e^{2\pi i(3-1)/4}\right] + (3+10\cdot 2)\sqrt[4]{i^3} = -8.80202559094-21.249969558i. \tag{14.3.3}$$

$$r_4 = \sqrt[4]{-279880i} = 2\sqrt[4]{i^3}\left[\frac{-(-736i)}{-16i\cdot 4} + \frac{\sqrt[4]{(-736i)^4-(-16i)^{4-1}4^4(39i)}}{-16i\cdot 4}\,e^{2\pi i(4-1)/4}\right] + (3+10\cdot 2)\sqrt[4]{i^3} = +21.249969558-8.80202559094i. \tag{14.3.3}$$

3.4 Solving the Quartic Nonlinear Equations of Complex Numbers of the Form: $\left(a\sqrt[5]{i^s}k+(x+10y)\sqrt[4]{i^s}\right)^5 = c$

In this section, we introduce two examples of quartic nonlinear of complex numbers, as application method of formula.

Example 3.4.1 Consider the quadratic nonlinear equation of complex member is given by:

$$\left(2i\sqrt[5]{i^s}k+(x+10y)\sqrt[4]{i^s}\right)^5 = 4i. \tag{1.3.4}$$

By using Eq. (6.2), we have $s=1$, because $\text{Real}(c)\in\mathbb{R}^+$. So,

$$\left(2i\sqrt[5]{i^1}k+(x+10y)\sqrt[5]{i^1}\right)^5 = 4i. \tag{2.3.4}$$

By using Eq. (9.2), we get

$$\left.\begin{aligned} &-32k^5\\ &+80ik^4(x+10\cdot y)^1\\ &+80k^3(x+10\cdot y)^2\\ &-40ik^2(x+10\cdot y)^3\\ &-10k(x+10\cdot y)^4\\ &+i(x+10\cdot y)^5\\ &=4i \end{aligned}\right\} \tag{3.3.4}$$

By using Eq. (4.2), we have

$$y = \sqrt[n]{\frac{|c|}{10^n}} = \sqrt[5]{\frac{|4i|}{10^5}} = 0.1319507911. \tag{4.3.4}$$

So, in this case we take $y=0$, and by using Eq. (5.2), we conclude that

$$\left.\begin{aligned} \left((x+10\cdot 0)\sqrt[5]{i^1}\right)^5 &= 4i,\\ x\sqrt[5]{i^1} &= \sqrt[5]{4i},\\ x &= \frac{\sqrt[5]{4i}}{\sqrt[5]{i^1}} = 1.319507911,\\ x &= 1 \end{aligned}\right\} \tag{5.3.4}$$

Put Eq. (4.3.4) and Eq. (5.3.4) in Eq. (3.3.4). we get,

$$\left.\begin{aligned} &-32k^5\\ &+80ik^4(1+10\cdot 0)^1\\ &+80k^3(1+10\cdot 0)^2\\ &-40ik^2(1+10\cdot 0)^3\\ &-10k(1+10\cdot 0)^4\\ &+i(1+10\cdot 0)^5\\ &=4i \end{aligned}\right\} \tag{6.3.4}$$

From Eq. (6.3.4), we conclude that

$$-32k^5+80ik^4+80k^3 -40ik^2-10k-3i=0. \tag{7.3.4}$$

The Eq. (7.3.4) has the solution by the formula of Eq. (22.2), and the radical roots of the Eq. (7.3.4) looks like:

$$k_1 = \frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}\,e^{2\pi i(1-1)/5} = -0.62746329842+0.29612481568i. \tag{8.3.4}$$

$$k_2 = \frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(2-1)/5} = -0.15975395539i. \quad (9.3.4)$$

$$k_3 = \frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(3-1)/5} = 0.62746329842+0.29612481568i. \quad (10.3.4)$$

$$k_4 = \frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(4-1)/5} = 0.38779364512+1.03375216201i. \quad (11.3.4)$$

$$k_5 = \frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(5-1)/5} = -0.38779364512+1.03375216201i. \quad (12.3.4)$$

We put Eq. (8.3.4), Eq. (9.3.4), Eq. (10.3.4), Eq. (11.3.4), and Eq. (12.3.4), in the Eq. (22.2) to get the general solution of Eq. (1.3.4) as the following roots:

$$r_1 = \sqrt[5]{4i} = 2i\sqrt[5]{i}\Big[\frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(1-1)/5}\Big] + (1+10\cdot 0)\sqrt[5]{i} = 0.775587290224-1.06750432403i. \quad (13.3.4)$$

$$r_2 = \sqrt[5]{4i} = 2i\sqrt[5]{i}\Big[\frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(2-1)/5}\Big] + (1+10\cdot 0)\sqrt[5]{i} = 1.25492659684+0.40775036864i. \quad (14.3.4)$$

$$r_3 = \sqrt[5]{4i} = 2i\sqrt[5]{i}\Big[\frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(3-1)/5}\Big] + (1+10\cdot 0)\sqrt[5]{i} = 2.22044604925\times 10^{-16}+1.31950791077i. \quad (15.3.4)$$

$$r_4 = \sqrt[5]{4i} = 2i\sqrt[5]{i}\Big[\frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(4-1)/5}\Big] + (1+10\cdot 0)\sqrt[5]{i} = -1.25492659684+0.40775036864i. \quad (16.3.4)$$

$$r_5 = \sqrt[5]{4i} = 2i\sqrt[5]{i}\Big[\frac{-80i}{-32\cdot 5} + \frac{\sqrt[5]{(80i)^5-(-32)^{5-1}5^5(-3i)}}{-32\cdot 5}e^{2\pi i(5-1)/5}\Big] + (1+10\cdot 0)\sqrt[5]{i} = -0.775587290224-1.06750432403i. \quad (17.3.4)$$

Example 3.4.2 Consider the quadratic nonlinear equation of complex member is given by:

$$\left(3i\sqrt[5]{i^s}k+(x+10y)\sqrt[4]{i^s}\right)^5 = 2+3i. \quad (18.3.4)$$

By using Eq. (6.2), we have $s=0$, because $c\in\mathbb{C}$. So,

$$\left(3i\sqrt[5]{i^0}k+(x+10y)\sqrt[5]{i^0}\right)^5 = 2+3i. \quad (19.3.4)$$

By using Eq. (9), we get

$$\left.\begin{aligned} &243ik^5\\ &+405k^4(x+10\cdot y)^1\\ &-270ik^3(x+10\cdot y)^2\\ &-90k^2(x+10\cdot y)^3\\ &+15ik(x+10\cdot y)^4\\ &+(x+10\cdot y)^5\\ &=2+3i \end{aligned}\right\} \quad (20.3.4)$$

By using Eq. (4.2), we have

$$y = \sqrt[n]{\frac{|c|}{10^n}} = \sqrt[5]{\frac{\sqrt{13}}{10^5}} = 0.1292392221. \quad (21.3.4)$$

For this case, $y=0$, and using Eq. (5.2) to find the value of x.

$$\left.\begin{aligned} \left((x+10\cdot 0)\sqrt[5]{i^0}\right)^5 &= 2+3i,\\ x\sqrt[5]{i^0} &= \sqrt[5]{\sqrt{13}},\\ x &= \frac{\sqrt[5]{\sqrt{13}}}{\sqrt[5]{i^0}} = 1.293292221,\\ x &= 1 \end{aligned}\right\} \quad (22.3.4)$$

Put Eq. (21.3.4) and Eq. (22.3.4) in Eq. (20.3.4). we deduced

that,

$$\left.\begin{aligned} &243ik^5 \\ &+405k^4(1+10\cdot 0)^1 \\ &-270ik^3(1+10\cdot 0)^2 \\ &-90k^2(1+10\cdot 0)^3 \\ &+15ik(1+10\cdot 0)^4 \\ &+(1+10\cdot 0)^5 \\ &=2+3i \end{aligned}\right\} \tag{23.3.4}$$

$$\begin{aligned} &243ik^5+405k^4-270ik^3 \\ &\quad -90k^2+15ik-1-3i=0. \end{aligned} \tag{24.3.4}$$

The Eq. (24.3.4) has the solution by the formula in Eq. (22.2) to get the radical roots of Eq. (24.3.4), we get

$$\begin{aligned} k_1 &= \frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(1-1)/5} \\ &= 0.08413279574-0.08916883056i. \end{aligned} \tag{25.3.4}$$

$$\begin{aligned} k_2 &= \frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(2-1)/5} \\ &= 0.42782189979+0.28278802815i. \end{aligned} \tag{26.3.4}$$

$$\begin{aligned} k_3 &= \frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(3-1)/5} \\ &= 0.18027567946+0.724596780650i. \end{aligned} \tag{26.3.4}$$

$$\begin{aligned} k_4 &= \frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(4-1)/5} \\ &= -0.31640540254+0.62569274751i. \end{aligned} \tag{27.3.4}$$

$$\begin{aligned} k_5 &= \frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(5-1)/5} \\ &= -0.37582497245+0.12275794091i. \end{aligned} \tag{28.3.5}$$

The pervious Eq. (25.3.4), Eq. (26.3.4), Eq. (27.3.4), Eq. (27.3.4), and Eq. (28.3.4) are putting in Eq. (22.2) To computing the radical's roots of Eq. (18.3.4),

$$\begin{aligned} r_1 &= \sqrt[5]{2+3i} \\ &= 3i\sqrt[5]{i^0}\Big[\frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(1-1)/5}\Big] \\ &\quad + (1+10\cdot 0)\sqrt[5]{i^0} \\ &= 1.26750649169+0.25239838722i. \end{aligned} \tag{29.3.4}$$

$$\begin{aligned} r_2 &= \sqrt[5]{2+3i} \\ &= 3i\sqrt[5]{i^0}\Big[\frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(2-1)/5}\Big] \\ &\quad + (1+10\cdot 0)\sqrt[5]{i^0} \\ &= 0.15163591554+1.28346569937i. \end{aligned} \tag{30.3.4}$$

$$\begin{aligned} r_3 &= \sqrt[5]{2+3i} \\ &= 3i\sqrt[5]{i^0}\Big[\frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(3-1)/5}\Big] \\ &\quad + (1+10\cdot 0)\sqrt[5]{i^0} \\ &= -1.17379034196+0.54082703838i. \end{aligned} \tag{31.3.4}$$

$$\begin{aligned} r_4 &= \sqrt[5]{2+3i} \\ &= 3i\sqrt[5]{i^0}\Big[\frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(4-1)/5}\Big] \\ &\quad + (1+10\cdot 0)\sqrt[5]{i^0} \\ &= -0.87707824254-0.94921620761i. \end{aligned} \tag{32.3.4}$$

$$\begin{aligned} r_5 &= \sqrt[5]{2+3i} \\ &= 3i\sqrt[5]{i^0}\Big[\frac{-405}{243i\cdot 5} \\ &\quad + \frac{\sqrt[5]{(405)^5-(243i)^{5-1}5^5(-1-3i)}}{243i\cdot 5}\, e^{2\pi i(5-1)/5}\Big] \\ &\quad + (1+10\cdot 0)\sqrt[5]{i^0} \\ &= 0.63172617728-1.12747491736i. \end{aligned} \tag{32.3.4}$$

4. CONCLUSION

To solve the equation $\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n=c$, after expanding the left side of equation by the Binomial theorem of complex numbers, the SHAS-formula is depending on the terms: a_nk^n, $a_{n-1}k^{n-1}$, a_0 and $\delta=a_0-c$, respectively for computing all roots. In fact, if we have:

$$\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n=c,$$

thus:

$$\left(a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}\right)^n\cdot 1^n=c\cdot 1,$$

where $1=e^{2\pi i(J-1)}$, therefore,

$$a\sqrt[n]{i^s}k+(x+10y)\sqrt[n]{i^s}=\sqrt[n]{c}\,e^{2\pi i(J-1)/n}.$$

The procedure in this article is a different method and a new approach to finding radicals.

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