



On the Classification of the Group of Units for Some Symbolic m -Plithogenic Rings

Lee Xu*

University of Chinese Academy of Sciences, CAS, Mathematics Department, Beijing, China

Email: Leexu1244@yahoo.com

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★ Corresponding author

ABSTRACT

This paper is dedicated to study and to find the symbolic m -plithogenic units in many symbolic plithogenic rings for some special values of m , where we present a full classification of many different symbolic n -plithogenic group of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

Keywords: Algebraic unit ▪ Group of units ▪ Classification ▪ Symbolic plithogenic ring

1. INTRODUCTION

The computation of invertible elements (units) in commutative algebraic rings is one of the most important problems in algebra, as well as the classification of the group generated by all units in a ring. The theory of logical rings based on neutrosophic sets and their generalizations has developed through studies of refined matrices, vector spaces, ideals, and related algebraic structures [14, 15, 20, 21, 22, 25]. Recent algebraic investigations have considered neutrosophic rings and hyperrings, group rings, near-subtraction semigroups, modules, finite fields, matrices, Lie algebras, and vector spaces [1, 2, 3, 4, 5, 6, 11, 12, 13, 17, 18, 19, 23, 24, 26, 27, 28]. Plithogenic graph structures also provide a related framework for handling multi-valued attributes and their contradictions [16]. In parallel, the group of units and associated Diophantine problems have been examined for several refined neutrosophic rings [7, 8, 9, 10]. These developments motivate us to study and find the symbolic m -plithogenic units in symbolic plithogenic rings for selected values of m , and to classify the corresponding groups of units as direct products of well-known groups by constructing suitable algebraic isomorphisms.

2. MAIN DISCUSSION

Definition. Let R be a ring with $U(R)$ as its group of units. Let

$$3 - \text{sp}R = \{x + yp_1 + zp_2 + tp_3 : x, y, z, t \in R\}$$

be the corresponding 3-plithogenic ring. Then

$$U(3 - \text{sp}R) = \{X \in 3 - \text{sp}R : \exists Y \in 3 - \text{sp}R, XY = YX = 1\}$$

is called the group of units of $3 - \text{sp}R$.

Theorem.

$$U(3 - \text{sp}R) \cong U(R) \times U(R) \times U(R) \times U(R).$$

Proof. Define $f : 3 - \text{sp}R \rightarrow R \times R \times R \times R$ by

$$f(x + yp_1 + zp_2 + tp_3) = (x, x + y, x + y + z, x + y + z + t).$$

If

$$x_0 + y_0p_1 + z_0p_2 + t_0p_3 = x_1 + y_1p_1 + z_1p_2 + t_1p_3,$$

then $x_0 = x_1$, $y_0 = y_1$, $z_0 = z_1$, and $t_0 = t_1$; hence f is well defined.

Assume that

$$\begin{aligned} X &= x_0 + y_0 p_1 + z_0 p_2 + t_0 p_3, \\ Y &= x_1 + y_1 p_1 + z_1 p_2 + t_1 p_3. \end{aligned}$$

We have

$$\begin{aligned} X + Y &= (x_0 + x_1) + (y_0 + y_1) p_1 \\ &\quad + (z_0 + z_1) p_2 + (t_0 + t_1) p_3, \end{aligned}$$

and $f(X + Y) = (a, b, c, d)$, where

$$\begin{aligned} a &= x_0 + x_1, \\ b &= x_0 + x_1 + y_0 + y_1, \\ c &= x_0 + x_1 + y_0 + y_1 + z_0 + z_1, \\ d &= x_0 + x_1 + y_0 + y_1 + z_0 + z_1 + t_0 + t_1. \end{aligned}$$

Thus

$$\begin{aligned} f(X + Y) &= (x_0, x_0 + y_0, x_0 + y_0 + z_0, x_0 + y_0 + z_0 + t_0) \\ &\quad + (x_1, x_1 + y_1, x_1 + y_1 + z_1, x_1 + y_1 + z_1 + t_1) \\ &= f(X) + f(Y). \end{aligned}$$

Also,

$$\begin{aligned} X \cdot Y &= x_0 x_1 + p_1 (x_0 y_1 + y_0 x_1 + y_0 y_1) \\ &\quad + p_2 (x_0 z_1 + y_0 z_1 + z_0 z_1 + z_0 x_1 + z_0 y_1) \\ &\quad + p_3 (x_0 t_1 + y_0 t_1 + z_0 t_1 + t_0 x_1 + t_0 y_1 + t_0 z_1 + t_0 t_1). \end{aligned}$$

Moreover,

$$\begin{aligned} f(XY) &= (x_0, x_0 + y_0, x_0 + y_0 + z_0, x_0 + y_0 + z_0 + t_0) \\ &\quad \cdot (x_1, x_1 + y_1, x_1 + y_1 + z_1, x_1 + y_1 + z_1 + t_1) \\ &= f(X)f(Y). \end{aligned}$$

If $f(X) = 0$, then $X = 0$. For every $(x, y, z, t) \in R^4$, there exists

$$X = x + (y - x)p_1 + (z - y)p_2 + (t - z)p_3 \in 3 - \text{sp}R$$

such that $f(X) = (x, y, z, t)$. Hence f is a ring isomorphism. Therefore,

$$3 - \text{sp}R \cong R^4 \quad \text{and} \quad U(3 - \text{sp}R) \cong U(R)^4.$$

Example. For $R = (\mathbb{Z}_3, +, \cdot)$,

$$3 - \text{sp}\mathbb{Z}_3 = \{x + yp_1 + zp_2 + tp_3 : x, y, z, t \in \mathbb{Z}_3\},$$

and

$$U(3 - \text{sp}\mathbb{Z}_3) \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2.$$

To find all units in $3 - \text{sp}\mathbb{Z}_3$, we write the units of $(\mathbb{Z}_3)^4$:

$$\begin{aligned} e_1 &= (1, 1, 1, 1), e_2 = (1, 1, 1, 2), e_3 = (1, 1, 2, 1), e_4 = (1, 2, 1, 1), \\ e_5 &= (2, 1, 1, 1), e_6 = (2, 2, 2, 2), e_7 = (2, 2, 2, 1), e_8 = (2, 2, 1, 2), \\ e_9 &= (2, 1, 2, 2), e_{10} = (1, 2, 2, 2), e_{11} = (1, 1, 2, 2), e_{12} = (1, 2, 1, 2), \\ e_{13} &= (2, 2, 1, 1), e_{14} = (2, 1, 2, 1), e_{15} = (2, 1, 1, 2), e_{16} = \\ &= (1, 2, 2, 1). \end{aligned}$$

Thus, $f^{-1}(e_1) = 1$, $f^{-1}(e_2) = 1 + p_3$, $f^{-1}(e_3) = 1 + p_2 + 2p_3$, $f^{-1}(e_4) = 1 + p_1 + 2p_2$, $f^{-1}(e_5) = 2 + 2p_1$, $f^{-1}(e_6) = 2$, $f^{-1}(e_7) = 2 + 2p_3$, $f^{-1}(e_8) = 2 + 2p_2 + p_3$, $f^{-1}(e_9) = 2 + 2p_1 + p_2$, $f^{-1}(e_{10}) = 1 + p_1$, $f^{-1}(e_{11}) = 1 + p_2$, $f^{-1}(e_{12}) = 1 + p_1 + 2p_2 + p_3$, $f^{-1}(e_{13}) = 2 + 2p_2$, $f^{-1}(e_{14}) = 2 + 2p_1 + p_2 + 2p_3$,

$$f^{-1}(e_{15}) = 2 + 2p_1 + p_3, f^{-1}(e_{16}) = 1 + p_1 + 2p_3.$$

Example. For $R = (\mathbb{Z}_4, +, \cdot)$,

$$U(3 - \text{sp}\mathbb{Z}_4) \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2.$$

The units of $R \times R \times R \times R$ are:

$$\begin{aligned} e_1 &= (1, 1, 1, 1), e_2 = (1, 1, 1, 3), e_3 = (1, 1, 3, 1), e_4 = (1, 3, 1, 1), \\ e_5 &= (3, 1, 1, 1), e_6 = (3, 3, 3, 3), e_7 = (3, 3, 3, 1), e_8 = (3, 3, 1, 3), \\ e_9 &= (3, 1, 3, 3), e_{10} = (1, 3, 3, 3), e_{11} = (1, 1, 3, 3), e_{12} = (3, 3, 1, 1), \\ e_{13} &= (1, 3, 3, 1), e_{14} = (3, 1, 1, 3), e_{15} = (3, 1, 3, 1), e_{16} = \\ &= (1, 3, 1, 3). \end{aligned}$$

The units of $3 - \text{sp}\mathbb{Z}_4$ are: $f^{-1}(e_1) = 1$, $f^{-1}(e_2) = 1 + 2p_3$, $f^{-1}(e_3) = 1 + 2p_2 + 2p_4$, $f^{-1}(e_4) = 1 + 2p_1 + 2p_2$, $f^{-1}(e_5) = 3 + 2p_1$, $f^{-1}(e_{11}) = 1 + 2p_2$, $f^{-1}(e_7) = 3 + 2p_4$, $f^{-1}(e_8) = 3 + 2p_2 + 2p_4$, $f^{-1}(e_9) = 3 + 2p_1 + 2p_2$, $f^{-1}(e_{10}) = 1 + 2p_1$, $f^{-1}(e_6) = 3$, $f^{-1}(e_{12}) = 3 + 2p_2$, $f^{-1}(e_{13}) = 1 + 2p_1 + 2p_4$, $f^{-1}(e_{14}) = 3 + 2p_1 + 2p_4$, $f^{-1}(e_{15}) = 3 + 2p_1 + 2p_2 + 2p_4$, $f^{-1}(e_{16}) = 1 + 2p_1 + 2p_2 + 2p_4$.

Example. For $R = (\mathbb{Z}, +, \cdot)$,

$$U(3 - \text{sp}\mathbb{Z}) \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2.$$

The units of $\mathbb{Z}^4$ are:

$$\begin{aligned} e_1 &= (1, 1, 1, 1), e_2 = (1, 1, 1, -1), e_3 = (1, 1, -1, 1), e_4 = \\ &= (1, -1, 1, 1), e_5 = (-1, 1, 1, 1), e_6 = (-1, -1, -1, -1), e_7 = \\ &= (-1, -1, -1, 1), e_8 = (-1, -1, 1, -1), e_9 = (-1, 1, -1, -1), e_{10} = \\ &= (1, -1, -1, -1), e_{11} = (1, 1, -1, -1), e_{12} = (-1, -1, 1, 1), e_{13} = \\ &= (1, -1, -1, 1), e_{14} = (-1, 1, 1, -1), e_{15} = (-1, 1, -1, 1), e_{16} = \\ &= (1, -1, 1, -1). \end{aligned}$$

The units of $3 - \text{sp}\mathbb{Z}$ are: $f^{-1}(e_1) = 1$, $f^{-1}(e_2) = 1 - 2p_3$, $f^{-1}(e_3) = 1 - 2p_2 + 2p_3$, $f^{-1}(e_4) = 1 - 2p_1 + 2p_2$, $f^{-1}(e_5) = -1 + 2p_1$, $f^{-1}(e_6) = -1$, $f^{-1}(e_7) = -1 + 2p_3$, $f^{-1}(e_8) = -1 + 2p_2 - 2p_3$, $f^{-1}(e_9) = -1 + 2p_1 - 2p_2$, $f^{-1}(e_{10}) = 1 - 2p_1$, $f^{-1}(e_{11}) = 1 - 2p_2$, $f^{-1}(e_{12}) = -1 + 2p_2$, $f^{-1}(e_{13}) = 1 - 2p_1 + 2p_3$, $f^{-1}(e_{14}) = -1 + 2p_1 - 2p_3$, $f^{-1}(e_{15}) = -1 + 2p_1 - 2p_2 + 2p_3$, $f^{-1}(e_{16}) = 1 - 2p_1 + 2p_2 - 2p_3$.

Example. For $R = (\mathbb{Z}_5, +, \cdot)$, we have $U(R) \cong \mathbb{Z}_4$ and

$$U(3 - \text{sp}\mathbb{Z}_5) \cong \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_4.$$

We will find 101 units of 256 possible units:

$$\begin{aligned} e_1 &= (1, 1, 1, 1), e_2 = (1, 1, 1, 2), e_3 = (1, 1, 2, 1), e_4 = (1, 2, 1, 1), \\ e_5 &= (2, 1, 1, 1), e_6 = (1, 1, 1, 3), e_7 = (1, 1, 3, 1), e_8 = (1, 3, 1, 1), \\ e_9 &= (3, 1, 1, 1), e_{10} = (3, 3, 3, 3), e_{11} = (1, 1, 1, 4), e_{12} = (1, 1, 4, 1), \\ e_{13} &= (1, 4, 1, 1), e_{14} = (4, 1, 1, 1), e_{15} = (4, 4, 4, 4), e_{16} = (2, 2, 2, 1), \\ e_{17} &= (2, 2, 1, 2), e_{18} = (2, 1, 2, 2), e_{19} = (1, 2, 2, 2), e_{20} = (3, 3, 3, 1), \\ e_{21} &= (3, 3, 1, 3), e_{22} = (3, 1, 3, 3), e_{23} = (1, 3, 3, 3), e_{24} = (4, 4, 4, 1), \\ e_{25} &= (4, 4, 1, 4), e_{26} = (4, 1, 4, 4), e_{27} = (1, 4, 4, 4), e_{28} = (2, 2, 2, 3), \\ e_{29} &= (2, 2, 3, 2), e_{30} = (2, 3, 2, 2), e_{31} = (3, 2, 2, 2), e_{32} = (2, 2, 2, 4), \\ e_{33} &= (2, 2, 4, 2), e_{34} = (2, 4, 2, 2), e_{35} = (4, 2, 2, 2), e_{36} = (3, 3, 3, 4), \\ e_{37} &= (3, 3, 4, 3), e_{38} = (3, 4, 3, 3), e_{39} = (4, 3, 3, 3), e_{40} = (1, 1, 2, 2), \\ e_{41} &= (2, 2, 1, 1), e_{42} = (1, 2, 2, 1), e_{43} = (2, 1, 1, 2), e_{44} = (1, 2, 1, 2), \\ e_{45} &= (2, 1, 2, 1), e_{46} = (3, 3, 1, 1), e_{47} = (1, 1, 3, 3), e_{48} = (3, 1, 1, 3), \\ e_{49} &= (1, 3, 3, 1), e_{50} = (3, 1, 3, 1), e_{51} = (1, 3, 1, 3), e_{52} = (4, 4, 1, 1), \\ e_{53} &= (1, 1, 4, 4), e_{54} = (1, 4, 4, 1), e_{55} = (4, 1, 1, 4), e_{56} = (1, 4, 1, 4), \\ e_{57} &= (4, 1, 4, 1), e_{58} = (2, 2, 3, 3), e_{59} = (3, 3, 2, 2), e_{60} = (2, 3, 3, 2), \\ e_{61} &= (3, 2, 2, 3), e_{62} = (3, 2, 3, 2), e_{63} = (2, 3, 2, 3), e_{64} = (2, 2, 4, 4), \\ e_{65} &= (4, 4, 2, 2), e_{66} = (2, 4, 4, 2), e_{67} = (4, 2, 2, 4), e_{68} = (2, 4, 2, 4), \\ e_{69} &= (4, 2, 4, 2), e_{70} = (3, 3, 4, 4), e_{71} = (4, 4, 3, 3), e_{72} = (4, 3, 3, 4), \\ e_{73} &= (3, 4, 4, 3), e_{74} = (3, 4, 3, 4), e_{75} = (4, 3, 4, 3), e_{76} = (2, 2, 2, 2), \\ e_{77} &= (4, 4, 4, 4), e_{78} = (1, 2, 3, 4), e_{79} = (2, 1, 3, 4), e_{80} = (1, 2, 4, 3), \\ e_{81} &= (2, 1, 4, 3), e_{82} = (1, 3, 2, 4), e_{83} = (1, 3, 4, 2), e_{84} = (3, 1, 2, 4), \\ e_{85} &= (3, 1, 4, 2), e_{86} = (1, 4, 2, 3), e_{87} = (1, 4, 3, 2), e_{88} = (4, 1, 2, 3), \\ e_{89} &= (4, 1, 3, 2), e_{90} = (2, 3, 1, 4), e_{91} = (2, 3, 4, 1), e_{92} = (3, 2, 1, 4), \\ e_{93} &= (3, 2, 4, 1), e_{94} = (2, 4, 1, 3), e_{95} = (2, 4, 3, 1), e_{96} = (4, 2, 1, 3), \\ e_{97} &= (4, 2, 3, 1), e_{98} = (3, 4, 1, 2), e_{99} = (3, 4, 2, 1), e_{100} = (4, 3, 1, 2), \\ e_{101} &= (4, 3, 2, 1), \end{aligned}$$

The corresponding units of $U(3 - \text{sp}\mathbb{Z}_5)$ are:

$$\begin{aligned}
 f^{-1}(e_1) &= 1, f^{-1}(e_2) = 1 + p_3, f^{-1}(e_3) = 1 + p_2 + 4p_3, \\
 f^{-1}(e_4) &= 1 + p_1 + 4p_2, f^{-1}(e_5) = 2 + 4p_1, f^{-1}(e_6) = 1 + 2p_3, \\
 f^{-1}(e_7) &= 1 + 2p_2 + 3p_3, f^{-1}(e_8) = 1 + 2p_1 + 3p_2, f^{-1}(e_9) = 3 + 3p_1, \\
 f^{-1}(e_{10}) &= 3, f^{-1}(e_{11}) = 1 + 3p_3, f^{-1}(e_{12}) = 1 + 3p_2 + 2p_3, \\
 f^{-1}(e_{13}) &= 1 + 3p_1 + 2p_2, f^{-1}(e_{14}) = 4 + 2p_1, f^{-1}(e_{15}) = 4, \\
 f^{-1}(e_{16}) &= 2 + 4p_3, f^{-1}(e_{17}) = 2 + 4p_2 + p_3, f^{-1}(e_{18}) = 2 + 4p_1 + p_2, \\
 f^{-1}(e_{19}) &= 1 + p_1, f^{-1}(e_{20}) = 3 + 3p_3, f^{-1}(e_{21}) = 3 + 3p_2 + 2p_3, \\
 f^{-1}(e_{22}) &= 3 + 3p_1 + 2p_2, f^{-1}(e_{23}) = 1 + 2p_1, f^{-1}(e_{24}) = 4 + 2p_3, \\
 f^{-1}(e_{25}) &= 4 + 2p_2 + 3p_3, f^{-1}(e_{26}) = 4 + 2p_1 + 3p_2, f^{-1}(e_{27}) = 1 + 3p_1, \\
 f^{-1}(e_{28}) &= 2 + p_3, f^{-1}(e_{29}) = 2 + p_2 + 4p_3, f^{-1}(e_{30}) = 2 + p_1 + 4p_2, \\
 f^{-1}(e_{31}) &= 3 + 4p_1, f^{-1}(e_{32}) = 2 + 2p_3, f^{-1}(e_{33}) = 2 + 2p_2 + 3p_3, \\
 f^{-1}(e_{34}) &= 2 + 2p_1 + 3p_2, f^{-1}(e_{35}) = 4 + 3p_1, f^{-1}(e_{36}) = 3 + p_3, \\
 f^{-1}(e_{37}) &= 3 + p_2 + 4p_3, f^{-1}(e_{38}) = 3 + p_1 + 4p_2, f^{-1}(e_{39}) = 4 + 4p_1, \\
 f^{-1}(e_{40}) &= 1 + p_2, f^{-1}(e_{41}) = 2 + 4p_2, f^{-1}(e_{42}) = 1 + p_1 + 4p_3, \\
 f^{-1}(e_{43}) &= 2 + 4p_1 + p_3, f^{-1}(e_{44}) = 1 + p_1 + 4p_2 + p_3, \\
 f^{-1}(e_{45}) &= 2 + 4p_1 + p_2 + 4p_3, f^{-1}(e_{46}) = 3 + 3p_2, f^{-1}(e_{47}) = 1 + 2p_2, \\
 f^{-1}(e_{48}) &= 3 + 3p_1 + 2p_3, f^{-1}(e_{49}) = 1 + 2p_1 + 3p_3, \\
 f^{-1}(e_{50}) &= 3 + 3p_1 + 2p_2 + 3p_3, f^{-1}(e_{51}) = 1 + 2p_1 + 3p_2 + 2p_3, \\
 f^{-1}(e_{52}) &= 4 + 2p_2, f^{-1}(e_{53}) = 1 + 3p_2, f^{-1}(e_{54}) = 1 + 3p_1 + 2p_3, \\
 f^{-1}(e_{55}) &= 4 + 2p_1 + 3p_3, f^{-1}(e_{56}) = 1 + 3p_1 + 2p_2 + 3p_3, \\
 f^{-1}(e_{57}) &= 4 + 2p_1 + 3p_2 + 2p_3, f^{-1}(e_{58}) = 2 + p_2, f^{-1}(e_{59}) = 3 + 4p_2, \\
 f^{-1}(e_{60}) &= 2 + p_1 + 4p_3, f^{-1}(e_{61}) = 3 + 4p_1 + p_3, \\
 f^{-1}(e_{62}) &= 3 + 4p_1 + p_2 + 4p_3, f^{-1}(e_{63}) = 2 + p_1 + 4p_2 + p_3, \\
 f^{-1}(e_{64}) &= 2 + 2p_2, f^{-1}(e_{65}) = 4 + 3p_2, f^{-1}(e_{66}) = 2 + 2p_1 + 3p_3, \\
 f^{-1}(e_{67}) &= 4 + 3p_1 + 2p_3, f^{-1}(e_{68}) = 2 + 2p_1 + 3p_2 + 2p_3, \\
 f^{-1}(e_{69}) &= 4 + 3p_1 + 2p_2 + 3p_3, f^{-1}(e_{70}) = 3 + p_2, f^{-1}(e_{71}) = 4 + 4p_2, \\
 f^{-1}(e_{72}) &= 4 + 4p_1 + p_3, f^{-1}(e_{73}) = 3 + p_1 + 4p_3, \\
 f^{-1}(e_{74}) &= 3 + p_1 + 4p_2 + p_3, f^{-1}(e_{75}) = 4 + 4p_1 + p_2 + 4p_3, \\
 f^{-1}(e_{76}) &= 2, f^{-1}(e_{77}) = 4, f^{-1}(e_{78}) = 1 + p_1 + p_2 + p_3, \\
 f^{-1}(e_{79}) &= 2 + 4p_1 + 2p_2 + p_3, f^{-1}(e_{80}) = 1 + p_1 + 2p_2 + 4p_3, \\
 f^{-1}(e_{81}) &= 2 + 4p_1 + 3p_2 + 4p_3, f^{-1}(e_{82}) = 1 + 2p_1 + 4p_2 + 2p_3, \\
 f^{-1}(e_{83}) &= 1 + 2p_1 + p_2 + 3p_3, f^{-1}(e_{84}) = 3 + 3p_1 + p_2 + 2p_3, \\
 f^{-1}(e_{85}) &= 3 + 3p_1 + 3p_2 + 3p_3, f^{-1}(e_{86}) = 1 + 3p_1 + 3p_2 + p_3, \\
 f^{-1}(e_{87}) &= 1 + 3p_1 + 4p_2 + 4p_3, f^{-1}(e_{88}) = 4 + 2p_1 + p_2 + p_3, \\
 f^{-1}(e_{89}) &= 4 + 2p_1 + 2p_2 + 4p_3, f^{-1}(e_{90}) = 2 + p_1 + 3p_2 + 3p_3, \\
 f^{-1}(e_{91}) &= 2 + p_1 + p_2 + 2p_3, f^{-1}(e_{92}) = 3 + 4p_1 + 4p_2 + 3p_3, \\
 f^{-1}(e_{93}) &= 3 + 4p_1 + 2p_2 + 2p_3, f^{-1}(e_{94}) = 2 + 2p_1 + 2p_2 + 2p_3, \\
 f^{-1}(e_{95}) &= 2 + 2p_1 + 4p_2 + 3p_3, f^{-1}(e_{96}) = 4 + 3p_1 + 4p_2 + 2p_3, \\
 f^{-1}(e_{97}) &= 4 + 3p_1 + p_2 + 3p_3, f^{-1}(e_{98}) = 3 + p_1 + 2p_2 + p_3, \\
 f^{-1}(e_{99}) &= 3 + p_1 + 3p_2 + 4p_3, f^{-1}(e_{100}) = 4 + 4p_1 + 3p_2 + p_3, \\
 f^{-1}(e_{101}) &= 4 + 4p_1 + 4p_2 + 4p_3,
 \end{aligned}$$

By continuing the same argument, we get the rest of the units.

3. RESULTS

We will write all the computed units in tables called the units tables of the symbolic plithogenic ring.

Table 1. Units table of $3 - \text{sp}\mathbb{Z}_3$

Unit in $(\mathbb{Z}_3)^4$	Corresponding Unit in $3 - \text{sp}\mathbb{Z}_3$	No.
(1, 1, 1, 1)	1	1
(1, 1, 1, 2)	$1 + p_3$	2
(1, 1, 2, 1)	$1 + p_2 + 2p_3$	3
(1, 2, 1, 1)	$1 + p_1 + 2p_2$	4
(2, 1, 1, 1)	$2 + 2p_1$	5
(2, 2, 2, 2)	2	6
(2, 2, 2, 1)	$2 + 2p_3$	7
(2, 2, 1, 2)	$2 + 2p_2 + p_3$	8
(2, 1, 2, 2)	$2 + 2p_1 + p_2$	9
(1, 2, 2, 2)	$1 + p_1$	10
(1, 1, 2, 2)	$1 + p_2$	11
(1, 2, 1, 2)	$1 + p_1 + 2p_2 + p_3$	12
(2, 2, 1, 1)	$2 + 2p_2$	13
(2, 1, 2, 1)	$2 + 2p_1 + p_2 + 2p_3$	14
(2, 1, 1, 2)	$2 + 2p_1 + p_3$	15
(1, 2, 2, 1)	$1 + p_1 + 2p_3$	16

Table 2. Units table of $3 - \text{sp}\mathbb{Z}_4$

Unit in $(\mathbb{Z}_4)^4$	Corresponding Unit in $3 - \text{sp}\mathbb{Z}_4$	No.
(1, 1, 1, 1)	1	1
(1, 1, 1, 3)	$1 + 2p_3$	2
(1, 1, 3, 1)	$1 + 2p_2 + 2p_4$	3
(1, 3, 1, 1)	$1 + 2p_1 + 2p_2$	4
(3, 1, 1, 1)	$3 + 2p_1$	5
(3, 3, 3, 3)	3	6
(3, 3, 3, 1)	$3 + 2p_4$	7
(3, 3, 1, 3)	$3 + 2p_2 + 2p_4$	8
(3, 1, 3, 3)	$3 + 2p_1 + 2p_2$	9
(1, 3, 3, 3)	$1 + 2p_1$	10
(1, 1, 3, 3)	$1 + 2p_2$	11
(3, 3, 1, 1)	$3 + 2p_2$	12
(1, 3, 3, 1)	$1 + 2p_1 + 2p_4$	13
(3, 1, 1, 3)	$3 + 2p_1 + 2p_4$	14
(3, 1, 3, 1)	$3 + 2p_1 + 2p_2 + 2p_4$	15
(1, 3, 1, 3)	$1 + 2p_1 + 2p_2 + 2p_4$	16

Table 3. Units table of $3 - \text{sp}\mathbb{Z}$

Unit in $\mathbb{Z}^4$	Corresponding Unit in $3 - \text{sp}\mathbb{Z}$	No.
(1, 1, 1, 1)	1	1
(1, 1, 1, -1)	$1 - 2p_3$	2
(1, 1, -1, 1)	$1 - 2p_2 + 2p_3$	3
(1, -1, 1, 1)	$1 - 2p_1 + 2p_2$	4
(-1, 1, 1, 1)	$-1 + 2p_1$	5
(-1, -1, -1, -1)	-1	6
(-1, -1, -1, 1)	$-1 + 2p_3$	7
(-1, -1, 1, -1)	$-1 + 2p_2 - 2p_3$	8
(-1, 1, -1, -1)	$-1 + 2p_1 - 2p_2$	9
(1, -1, -1, -1)	$1 - 2p_1$	10
(1, 1, -1, -1)	$1 - 2p_2$	11
(-1, -1, 1, 1)	$-1 + 2p_2$	12
(1, -1, -1, 1)	$1 - 2p_1 + 2p_3$	13
(-1, 1, 1, -1)	$-1 + 2p_1 - 2p_3$	14
(-1, 1, -1, 1)	$-1 + 2p_1 - 2p_2 + 2p_3$	15
(1, -1, 1, -1)	$1 - 2p_1 + 2p_2 - 2p_3$	16

4. CONCLUSION

In this paper, we study and find the symbolic m -plithogenic units in many symbolic plithogenic rings for some special values of m , where we presented a full classification of many different symbolic n -plithogenic groups of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

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