



On the Classification of the Group of Units for Some Symbolic m -Plithogenic Rings

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Abstract

This paper is dedicated to study and to find the symbolic m -plithogenic units in many symbolic plithogenic rings for some special values of m , where we present a full classification of many different symbolic n -plithogenic group of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

Keywords: Algebraic unit; Group of units; Classification; Symbolic plithogenic ring

1. Introduction

The computation of invertible elements (units) in commutative algebraic rings is one of the most important problems in algebra, as well as the classification of the group generated by all units in a ring. The theory of logical rings based on neutrosophic sets and their generalizations began with many works, see [14-16, 20-22], where we can find the concept of neutrosophic ring, neutrosophic ideals, and homomorphisms. In [1], symbolic 2-plithogenic rings were defined for the first time, and then they were generalized for higher orders [23-27] with many related algebraic substructures based on them, such as spaces, matrices, and special elements [2-6, 17-19]. In [7-13], the group of units of several logical rings was studied, such as n -cyclic refined neutrosophic units, and plithogenic units. This has motivated us study and to find the symbolic m -plithogenic units in many symbolic plithogenic rings for some special values of m , where we present a full classification of many different symbolic n -plithogenic group of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

2. Main Discussion

Definition:

Let R be a ring with $U(R)$ as its group of units.

Let $3 - sp_R = \{x + yp_1 + zp_2 + tp_3 ; x, y, z, t \in R\}$ be the corresponding 3-plithogenic ring, then:

$U(3 - sp_R) = \{X \in 3 - sp_R ; \exists Y \in 3 - sp_R : XY = YX = 1\}$ is called the group of units of $3 - sp_R$.

Theorem:

$U(3 - sp_R) \cong U(R) \times U(R) \times U(R) \times U(R)$

Proof:

Define $f: 3 - sp_R \rightarrow R \times R \times R \times R$ such that:

$f(x + yp_1 + zp_2 + tp_3) = (x, x + y, x + y + z, x + y + z + t),$

If $x_0 + y_0p_1 + z_0p_2 + t_0p_3 = x_1 + y_1p_1 + z_1p_2 + t_1p_3$, then $x_0 = x_1, y_0 = y_1, z_0 = z_1, t_0 = t_1$, hence $f(x_0 + y_0p_1 + z_0p_2 + t_0p_3) = f(x_1 + y_1p_1 + z_1p_2 + t_1p_3)$.

Assume that: $X = x_0 + y_0p_1 + z_0p_2 + t_0p_3, Y = x_1 + y_1p_1 + z_1p_2 + t_1p_3$, we have: $X + Y = (x_0 + x_1) + (y_0 + y_1)p_1 + (z_0 + z_1)p_2 + (t_0 + t_1)p_3$,

$f(X + Y) = (a, b, c, d)$; where:

$$\begin{cases} a = x_0 + x_1, b = x_0 + x_1 + y_0 + y_1 \\ c = x_0 + x_1 + y_0 + y_1 + z_0 + z_1 \\ d = x_0 + x_1 + y_0 + y_1 + z_0 + z_1 + t_0 + t_1 \end{cases}$$

Thus $f(X + Y) = (x_0, x_0 + y_0, x_0 + y_0 + z_0, x_0 + y_0 + z_0 + t_0) + (x_1, x_1 + y_1, x_1 + y_1 + z_1, x_1 + y_1 + z_1 + t_1) = f(X) + f(Y)$.

Also, $X \cdot Y = x_0x_1 + p_1(x_0y_1 + y_0x_1 + y_0y_1) + p_2(x_0z_1 + y_0z_1 + z_0x_1 + z_0y_1) + p_3(x_0t_1 + y_0t_1 + z_0t_1 + t_0x_1 + t_0y_1 + t_0z_1 + t_0t_1)$,

$f(XY) = (x_0, x_0 + y_0, x_0 + y_0 + z_0, x_0 + y_0 + z_0 + t_0)(x_1, x_1 + y_1, x_1 + y_1 + z_1, x_1 + y_1 + z_1 + t_1) = f(X)f(Y)$.

If $f(X) = 0$, then $X = 0$.

For every $(x, y, z, t) \in (R)^4$, there exists:

$X = x + (y - x)p_1 + (z - y)p_2 + (t - z)p_3 \in 3 - sp_R$ such that:

$f(X) = (x, y, z, t)$, hence (f) is a ring isomorphism.

This means that $3 - sp_R \cong R \times R \times R \times R$, and:

$$U(3 - sp_R) \cong U(R) \times U(R) \times U(R) \times U(R).$$

Example:

For $R = (Z_3, +, \cdot)$, then $3 - sp_{Z_3} = \{x + yp_1 + zp_2 + tp_3 \mid x, y, z, t \in Z_3\}$.

$$U(3 - sp_{Z_3}) \cong Z_2 \times Z_2 \times Z_2 \times Z_2.$$

To find all units in $3 - sp_{Z_3}$, we write the units of $(Z_3)^4$:

$$e_1 = (1, 1, 1, 1), e_2 = (1, 1, 1, 2),$$

$$e_3 = (1, 1, 2, 1), e_4 = (1, 2, 1, 1),$$

$$e_5 = (2, 1, 1, 1), e_6 = (2, 2, 2, 2),$$

$$e_7 = (2, 2, 2, 1), e_8 = (2, 2, 1, 2),$$

$$e_9 = (2, 1, 2, 2), e_{10} = (1, 2, 2, 2),$$

$$e_{11} = (1, 1, 2, 2), e_{12} = (1, 2, 1, 2),$$

$$e_{13} = (2, 2, 1, 1), e_{14} = (2, 1, 2, 1),$$

$$e_{15} = (2, 1, 1, 2), e_{16} = (1, 2, 2, 1).$$

Thus: $U(3 - sp_R) = \{f^{-1}(e_1) = 1, f^{-1}(e_2) = 1 + p_3,$

$$f^{-1}(e_3) = 1 + p_2 + 2p_3, f^{-1}(e_4) = 1 + p_1 + 2p_2,$$

$$f^{-1}(e_5) = 2 + 2p_1, f^{-1}(e_6) = 2,$$

$$f^{-1}(e_7) = 2 + 2p_3, f^{-1}(e_8) = 2 + 2p_2 + p_3,$$

$$f^{-1}(e_9) = 2 + 2p_1 + p_2, f^{-1}(e_{10}) = 1 + p_1,$$

$$f^{-1}(e_{11}) = 1 + p_2, f^{-1}(e_{12}) = 1 + p_1 + 2p_2 + p_3,$$

$$f^{-1}(e_{13}) = 2 + 2p_2, f^{-1}(e_{14}) = 2 + 2p_1 + p_2 + 2p_3,$$

$$f^{-1}(e_{15}) = 2 + 2p_1 + p_3, f^{-1}(e_{16}) = 1 + p_1 + 2p_3\}.$$

Example:

For $R = (Z_4, +, \cdot)$, $U(3 - sp_{Z_4}) \cong Z_2 \times Z_2 \times Z_2 \times Z_2$.

The units of $R \times R \times R \times R$ are:

$$e_1 = (1, 1, 1, 1), e_2 = (1, 1, 1, 3),$$

$$e_3 = (1, 1, 3, 1), e_4 = (1, 3, 1, 1),$$

$$e_5 = (3, 1, 1, 1), e_6 = (3, 3, 3, 3),$$

$$e_7 = (3, 3, 3, 1), e_8 = (3, 3, 1, 3),$$

$$e_9 = (3, 1, 3, 3), e_{10} = (1, 3, 3, 3),$$

$$e_{11} = (1, 1, 3, 3), e_{12} = (3, 3, 1, 1),$$

$$e_{13} = (1, 3, 3, 1), e_{14} = (3, 1, 1, 3),$$

$$e_{15} = (3, 1, 3, 1), e_{16} = (1, 3, 1, 3).$$

The units of $3 - sp_{Z_4}$ are:

$$U(3 - sp_{Z_4}) = \{f^{-1}(e_1) = 1, f^{-1}(e_2) = 1 + 2p_3,$$

$$f^{-1}(e_3) = 1 + 2p_2 + 2p_4, f^{-1}(e_4) = 1 + 2p_1 + 2p_2,$$

$$f^{-1}(e_5) = 3 + 2p_1, f^{-1}(e_{11}) = 1 + 2p_2, f^{-1}(e_7) = 3 + 2p_4,$$

$$\begin{aligned}
f^{-1}(e_8) &= 3 + 2p_2 + 2p_4, f^{-1}(e_9) = 3 + 2p_1 + 2p_2, \\
f^{-1}(e_{10}) &= 1 + 2p_1, f^{-1}(e_6) = 3, f^{-1}(e_{12}) = 3 + 2p_2, \\
f^{-1}(e_{13}) &= 1 + 2p_1 + 2p_4, f^{-1}(e_{14}) = 3 + 2p_1 + 2p_4, \\
f^{-1}(e_{15}) &= 3 + 2p_1 + 2p_2 + 2p_4, f^{-1}(e_{16}) = 1 + 2p_1 + 2p_2 + 2p_4\}.
\end{aligned}$$

Example:

For $R = (Z_1, +, \cdot)$, $U(3 - sp_Z) \cong Z_2 \times Z_2 \times Z_2 \times Z_2$.

The units of Z^4 are:

$$\begin{aligned}
e_1 &= (1,1,1,1), e_2 = (1,1,1,-1), \\
e_3 &= (1,1,-1,1), e_4 = (1,-1,1,1), e_5 = (-1,1,1,1), \\
e_6 &= (-1,-1,-1,-1), e_7 = (-1,-1,-1,1), \\
e_8 &= (-1,-1,1,-1), e_9 = (-1,1,-1,-1), \\
e_{10} &= (1,-1,-1,-1), e_{11} = (1,1,-1,-1), \\
e_{12} &= (-1,-1,1,1), e_{13} = (1,-1,-1,1), e_{14} = (-1,1,1,-1), \\
e_{15} &= (-1,1,-1,1), e_{16} = (1,-1,1,-1).
\end{aligned}$$

The units of $3 - sp_Z$ are:

$$\begin{aligned}
U(3 - sp_Z) &= \{f^{-1}(e_1) = 1, f^{-1}(e_2) = 1 - 2p_3, \\
f^{-1}(e_3) &= 1 - 2p_2 + 2p_3, f^{-1}(e_4) = 1 - 2p_1 + 2p_2, \\
f^{-1}(e_5) &= -1 + 2p_1, f^{-1}(e_6) = -1, \\
f^{-1}(e_7) &= -1 + 2p_3, f^{-1}(e_8) = -1 + 2p_2 - 2p_3, \\
f^{-1}(e_9) &= -1 + 2p_1 - 2p_2, f^{-1}(e_{10}) = 1 - 2p_1, \\
f^{-1}(e_{11}) &= 1 - 2p_2, f^{-1}(e_{12}) = -1 + 2p_2, f^{-1}(e_{13}) = 1 - 2p_1 + 2p_3, \\
f^{-1}(e_{14}) &= -1 + 2p_1 - 2p_3, f^{-1}(e_{15}) = -1 + 2p_1 - 2p_2 + 2p_3, f^{-1}(e_{16}) = 1 - 2p_1 + 2p_2 - 2p_3\}.
\end{aligned}$$

Example:

For $R = (Z_5, +, \cdot)$, then $U(R) \cong Z_4$ and $U(3 - sp_{Z_5}) \cong Z_4 \times Z_4 \times Z_4 \times Z_4$.

We will find 101 units of 256 possible units:

$$\begin{aligned}
e_1 &= (1,1,1,1), e_2 = (1,1,1,2), e_3 = (1,1,2,1), \\
e_4 &= (1,2,1,1), e_5 = (2,1,1,1), e_6 = (1,1,1,3), \\
e_7 &= (1,1,3,1), e_8 = (1,3,1,1), e_9 = (3,1,1,1), \\
e_{10} &= (3,3,3,3), e_{11} = (1,1,1,4), e_{12} = (1,1,4,1), \\
e_{13} &= (1,4,1,1), e_{14} = (4,1,1,1), e_{15} = (4,4,4,4), \\
e_{16} &= (2,2,2,1), e_{17} = (2,2,1,2), e_{18} = (2,1,2,2), \\
e_{19} &= (1,2,2,2), e_{20} = (3,3,3,1), e_{21} = (3,3,1,3), \\
e_{22} &= (3,1,3,3), e_{23} = (1,3,3,3), e_{24} = (4,4,4,1), \\
e_{25} &= (4,4,1,4), e_{26} = (4,1,4,4), e_{27} = (1,4,4,4), \\
e_{28} &= (2,2,2,3), e_{29} = (2,2,3,2), e_{30} = (2,3,2,2), \\
e_{31} &= (3,2,2,2), e_{32} = (2,2,2,4), e_{33} = (2,2,4,2), \\
e_{34} &= (2,4,2,2), e_{35} = (4,2,2,2), e_{36} = (3,3,3,4), \\
e_{37} &= (3,3,4,3), e_{38} = (3,4,3,3), e_{39} = (4,3,3,3), \\
e_{40} &= (1,1,2,2), e_{41} = (2,2,1,1), e_{42} = (1,2,2,1), \\
e_{43} &= (2,1,1,2), e_{44} = (1,2,1,2), e_{45} = (2,1,2,1), \\
e_{46} &= (3,3,1,1), e_{47} = (1,1,3,3), e_{48} = (3,1,1,3), \\
e_{49} &= (1,3,3,1), e_{50} = (3,1,3,1), e_{51} = (1,3,1,3), \\
e_{52} &= (4,4,1,1), e_{53} = (1,1,4,4), e_{54} = (1,4,4,1), \\
e_{55} &= (4,1,1,4), e_{56} = (1,4,1,4), e_{57} = (4,1,4,1), \\
e_{58} &= (2,2,3,3), e_{59} = (3,3,2,2), e_{60} = (2,3,3,2), \\
e_{61} &= (3,2,2,3), e_{62} = (3,2,3,2), e_{63} = (2,3,2,3), \\
e_{64} &= (2,2,4,4), e_{65} = (4,4,2,2), e_{66} = (2,4,4,2), \\
e_{67} &= (4,2,2,4), e_{68} = (2,4,2,4), e_{69} = (4,2,4,2), \\
e_{70} &= (3,3,4,4), e_{71} = (4,4,3,3), e_{72} = (4,3,3,4), \\
e_{73} &= (3,4,4,3), e_{74} = (3,4,3,4), e_{75} = (4,3,4,3), \\
e_{76} &= (2,2,2,2), e_{77} = (4,4,4,4), e_{78} = (1,2,3,4), \\
e_{79} &= (2,1,3,4), e_{80} = (1,2,4,3), e_{81} = (2,1,4,3), \\
e_{82} &= (1,3,2,4), e_{83} = (1,3,4,2), e_{84} = (3,1,2,4),
\end{aligned}$$

$$\begin{aligned}
e_{85} &= (3,1,4,2), e_{86} = (1,4,2,3), e_{87} = (1,4,3,2), \\
e_{88} &= (4,1,2,3), e_{89} = (4,1,3,2), e_{90} = (2,3,1,4), \\
e_{91} &= (2,3,4,1), e_{92} = (3,2,1,4), e_{93} = (3,2,4,1), \\
e_{94} &= (2,4,1,3), e_{95} = (2,4,3,1), e_{96} = (4,2,1,3), \\
e_{97} &= (4,2,3,1), e_{98} = (3,4,1,2), e_{99} = (3,4,2,1),
\end{aligned}$$

$$e_{100} = (4,3,1,2), e_{101} = (4,3,2,1).$$

The corresponding units of $U(3 - sp_{Z_3})$ are:

$$\begin{aligned}
f^{-1}(e_1) &= 1, f^{-1}(e_2) = 1 + p_3, f^{-1}(e_3) = 1 + p_2 + 4p_3, \\
f^{-1}(e_4) &= 1 + p_1 + 4p_2, f^{-1}(e_5) = 2 + 4p_1, f^{-1}(e_6) = 1 + 2p_3, \\
f^{-1}(e_7) &= 1 + 2p_2 + 3p_3, f^{-1}(e_8) = 1 + 2p_1 + 3p_2, f^{-1}(e_9) = 3 + 3p_1, f^{-1}(e_{10}) = 3, f^{-1}(e_{11}) = 1 + \\
&+ 3p_3, f^{-1}(e_{12}) = 1 + 3p_2 + 2p_3, f^{-1}(e_{13}) = 1 + 3p_1 + 2p_2, f^{-1}(e_{14}) = 4 + 2p_1, f^{-1}(e_{15}) = 4, f^{-1}(e_{16}) = 2 + \\
&+ 4p_3, f^{-1}(e_{17}) = 2 + 4p_2 + p_3, f^{-1}(e_{18}) = 2 + 4p_1 + p_2, f^{-1}(e_{19}) = 1 + p_1, f^{-1}(e_{20}) = 3 + 3p_3, f^{-1}(e_{21}) = 3 + \\
&+ 3p_2 + 2p_3, f^{-1}(e_{22}) = 3 + 3p_1 + 2p_2, f^{-1}(e_{23}) = 1 + 2p_1, f^{-1}(e_{24}) = 4 + 2p_3, f^{-1}(e_{25}) = 4 + 2p_2 + \\
&+ 3p_3, f^{-1}(e_{26}) = 4 + 2p_1 + 3p_2, f^{-1}(e_{27}) = 1 + 3p_1, f^{-1}(e_{28}) = 2 + p_3, f^{-1}(e_{29}) = 2 + p_2 + 4p_3, f^{-1}(e_{30}) = \\
&+ 2 + p_1 + 4p_2, f^{-1}(e_{31}) = 3 + 4p_1, f^{-1}(e_{32}) = 2 + 2p_3, f^{-1}(e_{33}) = 2 + 2p_2 + 3p_3, f^{-1}(e_{34}) = 2 + 2p_1 + \\
&+ 3p_2, f^{-1}(e_{35}) = 4 + 3p_1, f^{-1}(e_{36}) = 3 + p_3, f^{-1}(e_{37}) = 3 + p_2 + 4p_3, f^{-1}(e_{38}) = 3 + p_1 + 4p_2, f^{-1}(e_{39}) = \\
&+ 4 + 4p_1, f^{-1}(e_{40}) = 1 + p_2, f^{-1}(e_{41}) = 2 + 4p_2, f^{-1}(e_{42}) = 1 + p_1 + 4p_3, f^{-1}(e_{43}) = 2 + 4p_1 + p_3, f^{-1}(e_{44}) = \\
&+ 1 + p_1 + 4p_2 + p_3, f^{-1}(e_{45}) = 2 + 4p_1 + p_2 + 4p_3, f^{-1}(e_{46}) = 3 + 3p_2, f^{-1}(e_{47}) = 1 + 2p_2, f^{-1}(e_{48}) = 3 + \\
&+ 3p_1 + 2p_3, f^{-1}(e_{49}) = 1 + 2p_1 + 3p_3, f^{-1}(e_{50}) = 3 + 3p_1 + 2p_2 + 3p_3, f^{-1}(e_{51}) = 1 + 2p_1 + 3p_2 + \\
&+ 2p_3, f^{-1}(e_{52}) = 4 + 2p_2, f^{-1}(e_{53}) = 1 + 3p_2, f^{-1}(e_{54}) = 1 + 3p_1 + 2p_3, f^{-1}(e_{55}) = 4 + 2p_1 + 3p_3, f^{-1}(e_{56}) = \\
&+ 1 + 3p_1 + 2p_2 + 3p_3, f^{-1}(e_{57}) = 4 + 2p_1 + 3p_2 + 2p_3, f^{-1}(e_{58}) = 2 + p_2, f^{-1}(e_{59}) = 3 + 4p_2, f^{-1}(e_{60}) = 2 + \\
&+ p_1 + 4p_3, f^{-1}(e_{61}) = 3 + 4p_1 + p_3, f^{-1}(e_{62}) = 3 + p_2 + 4p_3, f^{-1}(e_{63}) = 2 + p_1 + 4p_2 + p_3, f^{-1}(e_{64}) = 2 + \\
&+ 2p_2, f^{-1}(e_{65}) = 4 + 3p_2, f^{-1}(e_{66}) = 2 + 2p_1 + 3p_3, f^{-1}(e_{67}) = 4 + 3p_1 + 2p_3, f^{-1}(e_{68}) = 2 + 2p_1 + 3p_2 + \\
&+ 2p_1 + 2p_3, f^{-1}(e_{69}) = 4 + 3p_1 + 2p_2 + 3p_3, f^{-1}(e_{70}) = 3 + p_2, f^{-1}(e_{71}) = 4 + 4p_2, f^{-1}(e_{72}) = 4 + 4p_1 + \\
&+ p_3, f^{-1}(e_{73}) = 3 + p_1 + 4p_3, f^{-1}(e_{74}) = 3 + p_1 + 4p_2 + p_3, f^{-1}(e_{75}) = 4 + 4p_1 + p_2 + 4p_3, f^{-1}(e_{76}) = \\
&+ 2, f^{-1}(e_{77}) = 4, f^{-1}(e_{78}) = 1 + p_1 + p_2 + p_3, f^{-1}(e_{79}) = 2 + 4p_1 + 2p_2 + p_3, f^{-1}(e_{80}) = 1 + p_1 + 2p_2 + \\
&+ 4p_3, f^{-1}(e_{81}) = 2 + 4p_1 + 3p_2 + 4p_3, f^{-1}(e_{82}) = 1 + 2p_1 + 4p_2 + 2p_3, f^{-1}(e_{83}) = 1 + 2p_1 + p_2 + \\
&+ 3p_3, f^{-1}(e_{84}) = 3 + 3p_1 + p_2 + 2p_3, f^{-1}(e_{85}) = 3 + 3p_1 + 3p_2 + 3p_3, f^{-1}(e_{86}) = 1 + 3p_1 + 3p_2 + \\
&+ p_3, f^{-1}(e_{87}) = 1 + 3p_1 + 4p_2 + 4p_3, f^{-1}(e_{88}) = 4 + 2p_1 + p_2 + p_3, f^{-1}(e_{89}) = 4 + 2p_1 + 2p_2 + \\
&+ 4p_3, f^{-1}(e_{90}) = 2 + p_1 + 3p_2 + 3p_3, f^{-1}(e_{91}) = 2 + p_1 + p_2 + 2p_3, f^{-1}(e_{92}) = 3 + 4p_1 + 4p_2 + \\
&+ 3p_3, f^{-1}(e_{93}) = 3 + 4p_1 + 2p_2 + 2p_3, f^{-1}(e_{94}) = 2 + 2p_1 + 2p_2 + 2p_3, f^{-1}(e_{95}) = 2 + 2p_1 + 4p_2 + \\
&+ 3p_3, f^{-1}(e_{96}) = 4 + 3p_1 + 4p_2 + 2p_3, f^{-1}(e_{97}) = 4 + 3p_1 + p_2 + 3p_3, f^{-1}(e_{98}) = 3 + p_1 + 2p_2 + \\
&+ p_3, f^{-1}(e_{99}) = 3 + p_1 + 3p_2 + 4p_3, f^{-1}(e_{100}) = 4 + 4p_1 + 3p_2 + p_3, f^{-1}(e_{101}) = 4 + 4p_1 + 4p_2 + 4p_3.
\end{aligned}$$

By continuing the same argument, we get the rest of the units.

3. Results

We will write all the computed units in table called the units table of the symbolic plithogenic ring.

Table 1: Units table of $3 - sp_{Z_3}$

Unit in $(Z_3)^4$	Corresponding Unit in $3 - sp_{Z_3}$	
(1,1,1,1)	1	1
(1,1,1,2)	$1 + p_3$	2
(1,1,2,1)	$1 + p_2 + 2p_3$	3
(1,2,1,1)	$1 + p_1 + 2p_2$	4
(2,1,1,1)	$2 + 2p_1$	5
(2,2,2,2)	2	6
(2,2,2,1)	$2 + 2p_3$	7
(2,2,1,2)	$2 + 2p_2 + p_3$	8
(2,1,2,2)	$2 + 2p_1 + p_2$	9
(1,2,2,2)	$1 + p_1$	10
(1,1,2,2)	$1 + p_2$	11

(1,2,1,2)	$1 + p_1 + 2p_2 + p_3$	12
(2,2,1,1)	$2 + 2p_2$	13
(2,1,2,1)	$2 + 2p_1 + p_2 + 2p_3$	14
(2,1,1,2)	$2 + 2p_1 + p_3$	15
(1,2,2,1)	$1 + p_1 + 2p_3$	16

Table 2: Units table of $3 - sp_{Z_4}$

Unit in $(Z_4)^4$	Corresponding Unit in $3 - sp_{Z_4}$	
(1,1,1,1)	1	1
(1,1,1,3)	$1 + 2p_3$	2
(1,1,3,1)	$1 + 2p_2 + 2p_4$	3
(1,3,1,1)	$1 + 2p_1 + 2p_2$	4
(3,1,1,1)	$3 + 2p_1$	5
(3,3,3,3)	3	6
(3,3,3,1)	$3 + 2p_4$	7
(3,3,1,3)	$3 + 2p_2 + 2p_4$	8
(3,1,3,3)	$3 + 2p_1 + 2p_2$	9
(1,3,3,3)	$1 + 2p_1$	10
(1,1,3,3)	$1 + 2p_2$	11
(3,3,1,1)	$3 + 2p_2$	12
(1,3,3,1)	$1 + 2p_1 + 2p_4$	13
(3,1,1,3)	$3 + 2p_1 + 2p_4$	14
(3,1,3,1)	$3 + 2p_1 + 2p_2 + 2p_4$	15
(1,3,1,3)	$1 + 2p_1 + 2p_2 + 2p_4$	16

Table 3: Units table of $3 - sp_Z$

Unit in $(Z)^4$	Corresponding Unit in $3 - sp_Z$	
(1,1,1,1)	1	1
(1,1,1,-1)	$1 - 2p_3$	2
(1,1,-1,1)	$1 - 2p_2 + 2p_3$	3
(1,-1,1,1)	$1 - 2p_1 + 2p_2$	4
(-1,1,1,1)	$-1 + 2p_1$	5
(-1,-1,-1,-1)	-1	6
(-1,-1,-1,1)	$-1 + 2p_3$	7
(-1,-1,1,-1)	$-1 + 2p_2 - 2p_3$	8
(-1,1,-1,-1)	$-1 + 2p_1 - 2p_2$	9
(1,-1,-1,-1)	$1 - 2p_1$	10
(1,1,-1,-1)	$1 - 2p_2$	11
(-1,-1,1,1)	$-1 + 2p_2$	12
(1,-1,-1,1)	$1 - 2p_1 + 2p_3$	13
(-1,1,1,-1)	$-1 + 2p_1 - 2p_3$	14
(-1,1,-1,1)	$-1 + 2p_1 - 2p_2 + 2p_3$	15
(1,-1,1,-1)	$1 - 2p_1 + 2p_2 - 2p_3$	16

4. Conclusion

In this paper, we study and find the symbolic m-plithogenic units in many symbolic plithogenic rings for some special values of m, where we presented a full classification of many different symbolic n-plithogenic group of units as direct products of well-known groups by building suitable and well-defined algebraic isomorphisms.

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