On the Computation of Units in Symbolic 4- Plithogenic Ring of Integers

Lee Xu
University of Chinese Academy of Sciences, CAS, Mathematics Department, Beijing, China
Email: Leexu1244@yahoo.com

Abstract

In this paper, we study the invertible elements (units) in the symbolic 4-plithogenic ring of integers, where we use a computational algorithm to find all units in the mentioned ring. The all-elements group of 32 symbolic 4-plithogenic ring of integers has been found and listed.

Keywords: Symbolic 4-plithogenic ring; Invertible element; Group of units; Unit

1. Introduction

The computation of invertible elements (units) in commutative algebraic rings is one of the most important problems in algebra, as well as the classification of the group generated by all units in a ring. The theory of logical rings based on neutrosophic sets and their generalizations began with many works, see [14-16, 20-22], where we can find the concept of neutrosophic ring, neutrosophic ideals, and homeomorphisms. In [1], symbolic 2-plithogenic rings were defined for the first time, and then they were generalized for higher orders [23-27] with many related algebraic substructures based on them, such as spaces, matrices, and special elements [2-6, 17-19]. In [7-13], the group of units of several logical rings was studied, such as n-cyclic refined neutrosophic units, and plithogenic units.
This has motivated us to study the same problem for 4-plithogenic rings of integers, where we study the invertible elements (units) in the symbolic 4-plithogenic ring of integers, where we use a computational algorithm to find all units in the mentioned ring. The all-elements group of 32 symbolic 4-plithogenic ring of integers has been found and listed.

2. Main Discussion

Definition 1:

Let $4-sp_z=\{a_0+\sum_{i=1}^{4}a_iP_i\,;\ \ a_i\in Z\,,P_i\times P_j=P_{\max(i,j)}\}$ be the symbolic 4-plithogenic ring of integers, then $X=x_0+\sum_{i=1}^{4}x_iP_i$ is called a unit if and only if it is invertible in $4-sp_z$.

Discussion 2:

Let $X=x_0+\sum_{i=1}^{4}x_iP_i\in\ 4-sp_z$ be a unit, then:

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

$$\begin{cases} x_0 \in \{-1,1\} \\ \sum_{i=0}^{1} x_i \in \{-1,1\} \\ \sum_{i=0}^{2} x_i \in \{-1,1\} \\ \sum_{i=0}^{3} x_i \in \{-1,1\} \\ \sum_{i=0}^{4} x_i \in \{-1,1\} \end{cases}$$

This means that we have 32 units.

Case (1):

$$\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Thus $X = 1$.

Case (2):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$

Thus $X = -1$.

Case (3):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$

Thus $X = 1 - 2P_4$.

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

Case (4):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Thus $X = 1 - 2P_3 + 2P_4$.

Case (5):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = +1 \\ \sum_{i=0}^{4} x_i = +1 \end{cases}$

Then $X = 1 - 2P_2 + 2P_3$.

Case (6):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = +1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = 1 - 2P_1 + 2P_2$.

Case (7):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = +1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$ $, X = -1 + 2P_1$

Case (8):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = +1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$

Then $X = 1 - 2P_3$.

Case (9):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = +1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$

$X = 1 - 2P_2 + 2P_3 - 2P_4$.

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

Case (10):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$

Then $X = 1 - 2P_1 + 2P_2 - 2P_4$.

Case (11):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$

Then $X = -1 + 2P_1 - 2P_4$.

Case (12):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = 1 - 2P_2 + 2P_4$.

Case (13):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = 1 - 2P_1 + 2P_2 - 2P_3 + 2P_4$.

Case (14):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = -1 + 2P_1 - 2P_3 + 2P_4$.

Case (15):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = 1 - 2P_1 + 2P_3$.

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

Case (16):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = -1 + 2P_1 - 2P_2 + 2P_3$.

Case (17):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = -1 + 2P_2$.

Case (18):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$

Then $X = -1 + 2P_1 - 2P_3$.

Case (19):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = -1 + 2P_1 - 2P_2 + 2P_4$.

Case (20):

For $\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = -1 + 2P_3$.

Case (21):

For $\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$

Then $X = 1 - 2P_1 + 2P_4$.

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

Case (22):

$$\text{For}\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_3 - 2P_4$.

Case (23):

$$\text{For}\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_2 - 2P_3$.

Case (24):

$$\text{For}\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_2$.

Case (25):

$$\text{For}\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

$X = -1 + 2P_1 - 2P_2 + 2P_3 - 2P_4$.

Case (26):

$$\text{For}\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = +1 \\ \sum_{i=0}^{3} x_i = +1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

$X = -1 + 2P_2 - 2P_4$.

Case (27):

$$\text{For}\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

$X = -1 + 2P_2 - 2P_3 + 2P_4$.

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

Case (28):

$$\text{For}\begin{cases} x_0 = 1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

$X = 1 - 2P_1$.

Case (29):

$$\text{For}\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = 1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

$X = -1 + 2P_1 - 2P_2$.

Case (30):

$$\text{For}\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = 1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

$X = -1 + 2P_2 - 2P_3$.

Case (31):

$$\text{For}\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = 1 \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

$X = -1 + 2P_3 - 2P_4$.

Case (32):

$$\text{For}\begin{cases} x_0 = -1 \\ \sum_{i=0}^{1} x_i = -1 \\ \sum_{i=0}^{2} x_i = -1 \\ \sum_{i=0}^{3} x_i = -1 \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

$X = -1 + 2P_4$.

3. Conclusion

In this paper, we studied the invertible elements (units) in the symbolic 4-plithogenic ring of integers, where we used a computational algorithm to find all units in the mentioned ring. The all-elements group of 32 symbolic 4-plithogenic ring of integers has been found and listed.

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

References

[1] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[2] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.

[3] Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.

[4] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and TheirAlgebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.

[5] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[6] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.

[7] Von Shtawzen, O., " Conjectures For Invertible Diophantine Equations Of 3-Cyclic and 4-Cyclic Refined Integers", Journal Of Neutrosophic And Fuzzy Systems, Vol.3, 2022.

[8] Von Shtawzen, O., " On A Novel Group Derived From A Generalization Of Integer Exponents and Open Problems", Galoitica journal Of Mathematical Structures and Applications, Vol 1, 2022.

[9] Basheer, A., Ahmad, K., and Ali, R., " On Some Open Problems About n-Cyclic Refined Neutrosophic Rings and Number Theory", Journal Of Neutrosophic And Fuzzy Systems,, 2022.

[10] Alrida Basheer , Katy D. Ahmad , Rozina Ali., "Examples on Some Novel Diophantine Equations Derived from the Group of Units Problem in n-Cyclic Refined Neutrosophic Rings of Integers", Galoitica Journal Of Mathematical Structures And Applications, Vol.3, 2022.

[11] Sankari, H., and Abobala, M., " On The Group of Units Classification In 3-Cyclic and 4-cyclic Refined Rings of Integers And The Proof of Von Shtawzens' Conjectures", International Journal of Neutrosophic Science, 2023.

[12] Sankari, H., and Abobala, M., " On The Classification of The Group of Units of Rational and Real 2-Cyclic Refined Neutrosophic Rings", Neutrosophic Sets and Systems, 2023.

[13] Sankari, H., and Abobala, M., " On The Algebraic Homomorphisms Between Symbolic 2-Plithogenic Rings And 2-cyclic Refined Rings", Neutrosophic Sets and Systems, 2023.

[14] Abobala, M., Ziena, M.B., Doewes, R.I., Hussein, Z. The Representation of Refined Neutrosophic Matrices By Refined Neutrosophic Linear Functions, *International Journal of Neutrosophic Science*, 2022, 19(1), pp. 342–349

[15] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021

[16] Sankari, H. Abobala, M. (2024). On The Computational Properties of 3-Cyclic and 4-Cyclic Refined Matrices and the Diagonalization Algorithm. *International Journal of Advances in Applied Computational Intelligence*, 6(2), 37-45. DOI: https://doi.org/10.54216/IJAACI.060204

[17] Soueycatt, M., Charchekhandra, B., Abu Hakmeh, R., " On The Foundations Of Symbolic 5-Plithogenic Number Theory", Neutrosophic Sets and Systems, vol. 59, 2023.

[18] Merkepci, H., and Rawashdeh, A., " On The Symbolic 2-Plithogenic Number Theory and Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.

[19] avier Gamboa-Cruzado, Renatto Oyague Guerra, Enrique Condor Tinoco, Guillermo Paucar-Carlos, Juan Gamarra Moreno, Warshine Barry. (2024). A Study of the 16-Plithogenic and 17-Plithogenic Square Real Matrices and Their Properties. Journal of International Journal of Neutrosophic Science, 23 (3), 63-76 (Doi : https://doi.org/10.54216/IJNS.230306).

[20] Smarandache F., and Abobala, M., "n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.

[21] Abobala, M.,. "A Study of AH-Substructures in n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[22] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.

[23] Nader Mahmoud Taffach , Ahmed Hatip., "A Review on Symbolic 2-Plithogenic Algebraic Structures " Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.

[24] Nader Mahmoud Taffach , Ahmed Hatip.," A Brief Review on The Symbolic 2-Plithogenic Number Theory and Algebraic Equations ", Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024

[25] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.

[26] P. Prabakaran, Bilal Abdallah, Turayeva Dinara Tulkunovna. (2024). On Certain Algebraic Properties of Symbolic 3-Plithogenic Real Square Matrices. Journal of International Journal of Neutrosophic Science, 23 (4), 350-357 (Doi : https://doi.org/10.54216/IJNS.230427).

[27] Necati Olgun. (2023). On The 15-Plithogenic Square Real Matrices. Galoitica: Journal of Mathematical Structures and Applications, 9 (2), 41-48 (Doi : https://doi.org/10.54216/GJMSA.090205).

[28] Noor Edin Rabeh, Othman Al-Basheer, Sara Sawalmeh, Rozina Ali. (2023). An Algebraic Approach to n-Plithogenic Square Matrices For $18 \leq n \leq 19$. Journal of Neutrosophic and Fuzzy Systems, 7 (2), 08-23 (Doi : https://doi.org/10.54216/JNFS.070201)

DOI: https://doi.org/10.54216/NIF.040102
Received: December 09, 2023 Accepted: June 21, 2024