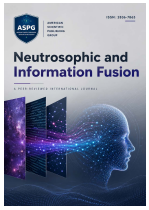
Neutrosophic and Information Fusion

♦ ISSN: 2836-7863 Vol: 04, No. 01, 2024 ♦ PP. 05–11

Article DOI: https://doi.org/10.54216/NIF.040102



FULL LENGTH ARTICLE

On the Computation of Units in Symbolic 4-Plithogenic Ring of Integers

Lee Xu[1,*]

[1] University of Chinese Academy of Sciences, CAS, Mathematics Department, Beijing, China

Email: Leexu1244@yahoo.com

Received: December 09, 2023 Accepted: June 21, 2024 ⋆ Corresponding author

ABSTRACT

In this paper, we study the invertible elements (units) in the symbolic 4-plithogenic ring of integers, where we use a computational algorithm to find all units in the mentioned ring. The all-elements group of 32 symbolic 4-plithogenic ring of integers has been found and listed.

Keywords: Symbolic 4-plithogenic ring ▪ Invertible element ▪ Group of units ▪ Unit

1. INTRODUCTION

The computation of invertible elements (units) in commutative algebraic rings is one of the most important problems in algebra, as well as the classification of the group generated by all units in a ring. The theory of logical rings based on neutrosophic sets and their generalizations has developed through studies of neutrosophic rings, ideals, vector spaces, and matrix representations [14, 15, 16, 20, 21, 22].

Related algebraic foundations include characterizations of neutrosophic rings and ideals, neutrosophic algebras, group rings, subalgebras, and quasigroups [1, 2, 3, 4, 5, 6]. Further developments address rough and fuzzy ideals, modules, fields, hyperrings, tensor products, plithogenic graphs, and matrix algorithms [11, 12, 13, 17, 18, 19, 23, 24, 25, 26, 27, 28]. Studies of units and related invertibility problems in logical rings also provide important motivation for the present work [7, 8, 9, 10].

This has motivated us to study the same problem for 4-plithogenic rings of integers, where we study the invertible elements (units) in the symbolic 4-plithogenic ring of integers, and use a computational algorithm to find all units in the mentioned ring. The all-elements group of 32 symbolic 4-plithogenic ring of integers has been found and listed.

2. MAIN DISCUSSION

Definition 1:

Let

$$4\text{-}sp_z = \left\{ a_0 + \sum_{i=1}^{4} a_i P_i \; ; \; a_i \in \mathbb{Z}, \quad P_i \times P_j = P_{\max(i,j)} \right\} \quad (1)$$

be the symbolic 4-plithogenic ring of integers. Then

$$X = x_0 + \sum_{i=1}^{4} x_i P_i \quad (2)$$

is called a unit if and only if it is invertible in 4-sp_z.

Discussion 2:

Let

$$X = x_0 + \sum_{i=1}^{4} x_i P_i \in 4\text{-}sp_z$$

© 2024 The Author(s). Published by ASPG. This article is licensed under CC BY 4.0.

be a unit. Then

$$\begin{cases} x_0 \in \{-1,1\}, \\ \sum_{i=0}^{1} x_i \in \{-1,1\}, \\ \sum_{i=0}^{2} x_i \in \{-1,1\}, \\ \sum_{i=0}^{3} x_i \in \{-1,1\}, \\ \sum_{i=0}^{4} x_i \in \{-1,1\}. \end{cases} \tag{3}$$

This means that we have 32 units.

Case (1):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Thus $X = 1$.

Case (2):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = -1$.

Case (3):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = 1 - 2P_4$.

Case (4):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Thus $X = 1 - 2P_3 + 2P_4$.

Case (5):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = 1 - 2P_2 + 2P_3$.

Case (6):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_2$.

Case (7):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Thus $X = -1 + 2P_1$.

Case (8):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_3$.

Case (9):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = 1 - 2P_2 + 2P_3 - 2P_4$.

Case (10):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_2 - 2P_4$.

Case (11):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = -1 + 2P_1 - 2P_4$.

Case (12):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = 1 - 2P_2 + 2P_4$.

Case (13):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_2 - 2P_3 + 2P_4$.

Case (14):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = -1 + 2P_1 - 2P_3 + 2P_4$.

Case (15):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_3$.

Case (16):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = -1 + 2P_1 - 2P_2 + 2P_3$.

Case (17):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = -1 + 2P_2$.

Case (18):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = -1 + 2P_1 - 2P_3$.

Case (19):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = -1 + 2P_1 - 2P_2 + 2P_4$.

Case (20):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = -1 + 2P_3$.

Case (21):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_4$.

Case (22):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_3 - 2P_4$.

Case (23):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_1 + 2P_2 - 2P_3$.

Case (24):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Then $X = 1 - 2P_2$.

Case (25):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = -1 + 2P_1 - 2P_2 + 2P_3 - 2P_4$.

Case (26):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = -1 + 2P_2 - 2P_4$.

Case (27):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Thus $X = -1 + 2P_2 - 2P_3 + 2P_4$.

Case (28):

For

$$\begin{cases} x_0 = 1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = 1 - 2P_1$.

Case (29):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = 1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = -1 + 2P_1 - 2P_2$.

Case (30):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = 1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = -1 + 2P_2 - 2P_3$.

Case (31):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = 1, \\ \sum_{i=0}^{4} x_i = -1 \end{cases}$$

Thus $X = -1 + 2P_3 - 2P_4$.

Case (32):

For

$$\begin{cases} x_0 = -1, \\ \sum_{i=0}^{1} x_i = -1, \\ \sum_{i=0}^{2} x_i = -1, \\ \sum_{i=0}^{3} x_i = -1, \\ \sum_{i=0}^{4} x_i = 1 \end{cases}$$

Thus $X = -1 + 2P_4$.

3. CONCLUSION

In this paper, we studied the invertible elements (units) in the symbolic 4-plithogenic ring of integers, where we used a computational algorithm to find all units in the mentioned ring. The all-elements group of 32 symbolic 4-plithogenic ring of integers has been found and listed.

REFERENCES

[1] M. S. Hameed, Z. Ahmad, and S. Ali, "Characterization of γ-single valued neutrosophic rings and ideals," *Neutrosophic Sets and Systems*, vol. 50, pp. 47–63, 2022.

[2] T. Nagaiah, L. Bhaskar, P. N. Swamy, and S. Broumi, "A study on neutrosophic algebra," *Neutrosophic Sets and Systems*, vol. 50, pp. 111–118, 2022.

[3] V. Kandasamy and I. Kandasamy, "Neutroalgebra of idempotents in group rings," *Neutrosophic Sets and Systems*, vol. 50, pp. 156–177, 2022.

[4] J. S. Ranjini and V. Mahalakshmi, "Neutrosophic fuzzy x-sub algebra of near-subtraction semigroups," *Neutrosophic Sets and Systems*, vol. 50, pp. 178–187, 2022.

[5] T. Oner, T. Katican, and A. Rezaei, "Neutrosophic $\mathcal{N}$-structures on sheffer stroke BCH-algebras," *Neutrosophic Sets and Systems*, vol. 50, pp. 459–479, 2022.

[6] A. Oyem, T. G. Jaiyeola, J. O. Olaleru, and B. Osoba, "Soft neutrosophic quasigroups," *Neutrosophic Sets and Systems*, vol. 50, pp. 488–503, 2022.

[7] O. Von Shtawzen, "Conjectures for invertible diophantine equations of 3-cyclic and 4-cyclic refined integers," *Journal of Neutrosophic and Fuzzy Systems*, vol. 3, 2022.

[8] O. V. Shtawzen, "On a novel group derived from a generalization of integer exponents and open problems," *Galoitica Journal of Mathematical Structures and Applications*, vol. 1, 2022.

[9] A. Basheer, K. Ahmad, and R. Ali, "On some open problems about *n*-cyclic refined neutrosophic rings and number theory," *Journal of Neutrosophic and Fuzzy Systems*, 2022.

[10] A. Basheer, K. D. Ahmad, and R. Ali, "Examples on some novel diophantine equations derived from the group of units problem in *n*-cyclic refined neutrosophic rings of integers," *Galoitica Journal of Mathematical Structures and Applications*, vol. 3, 2022.

[11] V. S. Subha, G. Rajasekar, and S. Soundaravalli, "Rough neutrosophic ideals in a ring," *Neutrosophic Sets and Systems*, vol. 50, pp. 504–514, 2022.

[12] B. Elavarasan, K. Porselvi, Y. B. Jun, and G. Muhiuddin, "Neutrosophic $\aleph$-filters in semigroups," *Neutrosophic Sets and Systems*, vol. 50, pp. 515–531, 2022.

[13] M. S. Hameed, Z. Ahmad, S. Broumi, and S. Ali, "On characterizations of $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic hyperrings and hyperideals," *Neutrosophic Sets and Systems*, vol. 50, pp. 614–628, 2022.

[14] M. Abobala, M. B. Ziena, R. I. Doewes, and Z. Hussein, "The representation of refined neutrosophic matrices by refined neutrosophic linear functions," *International Journal of Neutrosophic Science*, vol. 19, no. 1, pp. 342–349, 2022.

[15] M. Abobala, "On refined neutrosophic matrices and their applications in refined neutrosophic algebraic equations," *Journal of Mathematics*, 2021.

[16] S. Broumi, M. B. Zeina, M. Lathamaheswari, A. Bakali, and M. Talea, "A maple code to perform operations on single valued neutrosophic matrices," *Neutrosophic Sets and Systems*, vol. 49, pp. 485–508, 2022.

[17] K. Hemabala and B. S. Kumar, "Neutrosophic multi fuzzy ideals of near ring," *Neutrosophic Sets and Systems*, vol. 49, pp. 184–197, 2022.

[18] M. M. Abed, N. Hassan, and F. Al-Sharqi, "On neutrosophic multiplication module," *Neutrosophic Sets and Systems*, vol. 49, pp. 198–208, 2022.

[19] T. Chalapathi, K. K. Naidu, and H. D. Babu, "Algebraic properties of finite neutrosophic fields," *Neutrosophic Sets and Systems*, vol. 49, pp. 253–261, 2022.

[20] F. Smarandache and M. Abobala, "*n*-refined neutrosophic vector spaces," *International Journal of Neutrosophic Science*, vol. 7, pp. 47–54, 2020.

[21] M. Abobala, "A study of AH-substructures in *n*-refined neutrosophic vector spaces," *International Journal of Neutrosophic Science*, vol. 9, pp. 74–85, 2020.

[22] M. Abobala, "On the characterization of maximal and minimal ideals in several neutrosophic rings," *Neutrosophic Sets and Systems*, vol. 45, 2021.

[23] B. R. and U. Paul, "Tensor product of neutrosophic submodules of an *r*-module," *Neutrosophic Sets and Systems*, vol. 49, pp. 458–470, 2022.

[24] P. K. Singh, "Intuitionistic plithogenic graph and its cut for knowledge processing tasks," *Neutrosophic Sets and Systems*, vol. 49, pp. 70–91, 2022.

[25] M. Abobala, "Neutrosophic real inner product spaces," *Neutrosophic Sets and Systems*, vol. 43, 2021.

[26] I. Silambarasan, R. Udhayakumar, F. Smarandache, and S. Broumi, "Some algebraic structures of neutrosophic fuzzy sets," *International Journal of Neutrosophic Science*, vol. 19, no. 2, pp. 30–41, 2022.

[27] M. F. Alaswa, "On the neutrosophic characteristic polynomials and neutrosophic cayley–hamilton theorem," *International Journal of Neutrosophic Science*, vol. 18, no. 3, pp. 59–71, 2022.

[28] K. D. Ahmad, N. N. Thjeel, M. M. Abdul Zahra, and R. A. Jaleel, "On the classification of *n*-refined neutrosophic rings and its applications in matrix computing algorithms and linear systems," *International Journal of Neutrosophic Science*, vol. 18, no. 4, pp. 08–15, 2022.