New algebraic structures approach towards complex interval valued Q-neutrosophic subbisemiring of bisemiring

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Abstract

The notion of complex interval-valued q-neutrosophic subbisemiring (CIVqNSBS) is developed and examined. Additionally, we examine the homomorphic features and significant attributes of CIVqNSBS. We suggest the CIVqNSBS level sets for bisemirings. Consider a complex neutrosophic subset of bisemiring Δ, denoted as $\overrightarrow{\aleph}$, if and only if every non-empty level set $\overrightarrow{Z^{(\partial,\flat)}}$ is a subbisemiring, where $\partial, \flat \in \mathbb{D}[0,1]$, then $\overrightarrow{Z} = (\overrightarrow{Z_{\aleph}^{\dagger}}, \overrightarrow{Z_{\aleph}^{\daleth}}, \overrightarrow{Z_{\aleph}^{\dashv}})$ is a CIVqNSBS of Δ. Let $\overrightarrow{\aleph}$ be the strongest complex neutrosophic relation of bisemiring Δ, and let $\overrightarrow{\Psi}$ be a CIVqNSBS of bisemiring Δ, if and only if $\overrightarrow{\Psi}$ is a CIVqNSBS of $\Delta \times \Delta$, then $\overrightarrow{\aleph}$ is a CIVqNSBS of bisemiring Δ. We show that homomorphic images of all CIVqNSBSs are CIVqNSBSs, and homomorphic pre-images of all CIVqNSBSs are CIVqNSBSs. There are examples given to illustrate our results.

Keywords: CIVqNSBS; CIVqNNSBS; SBS; Homomorphism

1 Introduction

Zadeh[1] invented fuzzy set (FS) theory, which is best suited for handling ambiguity and uncertainty. In a fuzzy set, an element is considered to be member if it has a single value within the interval. However, in practical situations, there can be hesitation, so the degree of non-membership may not always equal 1 minus the degree of membership. The development of FS theory is advancing quickly, leading to the emergence of numerous hybrid fuzzy models. Numerous uncertain theories, including FS,[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS),[3] and spherical FS (SFS),[4] have been developed as a result of the uncertainties. Sets with grades between 0 and 1 referred to as MG make up an FS. According to Atanassov,[2] non-membership grades (NMG) have a value of no more than 1, while IFS is classified as MG. The total of MGs and NMGs when employing a decision-making approach may occasionally be more than 1. The generalized MG and NMG logic, which has a value not exceeding 1 and is determined by the square of the MGs and NMGs, was developed by Yager[3] using PFS logic. These theories are unable to show the neutral condition, which is neither favorable nor unfavorable. Cuong et al.[5] discussed on the image. Three grading points were utilized by FS: positive, neutral, and negative, with the total of these grades not to exceed 1. Additionally, it is more advantageous for some applications than PFS or IFS. It is a three-model independent generalization of FS and IFS that involves falsity, indeterminacy, and truth.

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Neutrosophic set (NS) was developed by Smarandache[6] to deal with ambiguous and inconsistent data. The degree to which an assertion is true, ambiguous or untrue is ascertained using this logic. Complex fuzzy set (CFS) is introduced by.[7] CFSs deals membership functions can have a wide range of values. Unlike a fuzzy membership function with a fixed unit circle, the unit circle of the complex plane is extended to $[0,1]$. A membership function $\mu_X(x)$ that extends to the unit circle in the complex plane rather than only $[0,1]$ characterizes the complex fuzzy set X. Therefore, $\mu_X(x)$ is a complex-valued function that gives any element x in the discourse universe a grade of membership of the type $\rho_X(x) \cdot e^{i\tau_X(x)}$, where $i = \sqrt{-1}$. The two variables, $\rho_X(x)$ and $\tau_X(x)$, which are both real-valued with $\rho_X(x) \in [0,1]$, define the value of $\mu_X(x)$. Semiring logic and its uses were introduced by Golan.[8] Hussian et al.[9] discussed the extension and notion of bisemirings. The bipolar-valued fuzzy sets and related operations are covered by Lee.[10] Ahsan et al. studied fuzzy semirings.[11,12] The notion of bisemirings was first presented by Sen et al.[13] It was recently suggested that intuitionistic fuzzy normal subbisemiring of bisemiring was proposed by Palanikumar et al.[14] The notion of bisemiring was introduced by Palanikumar et al.[15] through the use of bipolar-valued neutrosophic normal sets. The following contributions are made in this paper:

1. The intersection of a every CIVqNSBSs is again a CIVqNSBS of bisemiring Δ.
2. Let $\overrightarrow{\aleph}$ be a CIVqNSBS of Δ and $\overrightarrow{\Psi}$ be a strongest complex interval valued q-neutrosophic relation of Δ. Then $\overrightarrow{\aleph}$ is a CIVqNSBS of bisemiring Δ if and only if $\overrightarrow{\Psi}$ is a CIVqNSBS of $\Delta \times \Delta$.
3. $\overrightarrow{Z} = (\overrightarrow{Z^{\intercal}_{\aleph}} \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}, \overrightarrow{Z^{\daleth}_{\aleph}} \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}, \overrightarrow{Z^{\dashv}_{\aleph}} \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}})$ is a CIVqNSBS of Δ if and only if $\overrightarrow{Z^{(\partial,\flat)}}$ is a subbisemiring of Δ for all $\partial, \flat \in \mathbb{D}[0,1]$.
4. The homomorphic image of every CIVqNSBS is a CIVqNSBS and homomorphic preimage of every CIVqNSBS is a CIVqNSBS.

Aspects of the SBS and CIVqNSBS concept will be examined, and conclusions drawn. The five parts of the article are as follows. An overview of semirings and SBS can be found in Section 1. Information on semiring and SBS preparation is provided in Section 2. Section 3 lists the properties of CIVqNSBS. Numerical examples should be used in the evaluation of CIVqNSBS. The conclusion and future course are indicated in Section 4.

2 Preliminaries

Definition 2.1. [13] An algebraic structure $(\Delta, \uplus, \ominus, \odot)$ is a bisemiring,if $(\Delta, \uplus, \ominus)$ and $(\Delta, \ominus, \odot)$ are semirings, ie.,$(\Delta, \uplus)$, $(\Delta, \ominus)$ and $(\Delta, \odot)$ are semigroups and

1. $z_v \ominus (z_\chi \uplus z_\rho) = (z_v \ominus z_\chi) \uplus (z_v \ominus z_\rho)$,
2. $(z_\chi \uplus z_\rho) \ominus z_v = (z_\chi \ominus z_v) \uplus (z_\rho \ominus z_v)$,
3. $z_v \odot (z_\chi \ominus z_\rho) = (z_v \odot z_\chi) \ominus (z_v \odot z_\rho)$,
4. $(z_\chi \ominus z_\rho) \odot z_v = (z_\chi \odot z_v) \ominus (z_\rho \odot z_v), \ \forall \ z_v, z_\chi, z_\rho \in \Delta$.

Definition 2.2. [6] A NS v in the universe $\mathcal{U}$ is $v = \{\hbar, u^{\intercal}_v(\hbar), u^{\daleth}_v(\hbar), u^{\dashv}_v(\hbar) | \hbar \in \mathcal{U}\}$,where $u^{\intercal}_v(\hbar)$,$u^{\daleth}_v(\hbar)$ $u^{\dashv}_v(\hbar)$ represents the TD,ID and FD of v respectively. Consider the mapping $u^{\intercal}_v : \mathcal{U} \to [0,1], u^{\daleth}_v : \mathcal{U} \to [0,1], u^{\dashv}_v : \mathcal{U} \to [0,1]$ and $0 \preceq u^{\intercal}_v(\hbar) + u^{\daleth}_v(\hbar) + u^{\dashv}_v(\hbar) \preceq 3$.

Definition 2.3. [6] Let $\ell_1 = \langle u^{\intercal}_{\ell_1}, u^{\daleth}_{\ell_1}, u^{\dashv}_{\ell_1}\rangle$,$\ell_2 = \langle u^{\intercal}_{\ell_2}, u^{\daleth}_{\ell_2}, u^{\dashv}_{\ell_2}\rangle$ and $\ell_3 = \langle u^{\intercal}_{\ell_3}, u^{\daleth}_{\ell_3}, u^{\dashv}_{\ell_3}\rangle$ be the three neutrosophic numbers over $\mathcal{U}$. Then

1. $\ell_2 \Cup \ell_3 = \Big\langle \max(\hbar^{\intercal}_{\ell_2}, u^{\intercal}_{\ell_3}), \min(\hbar^{\daleth}_{\ell_2}, u^{\daleth}_{\ell_3}), \min(\hbar^{\dashv}_{\ell_2}, u^{\dashv}_{\ell_3}) \Big\rangle$,
2. $\ell_2 \Cap \ell_3 = \Big\langle \min(\hbar^{\intercal}_{\ell_2}, u^{\intercal}_{\ell_3}), \max(\hbar^{\daleth}_{\ell_2}, u^{\daleth}_{\ell_3}), \max(\hbar^{\dashv}_{\ell_2}, u^{\dashv}_{\ell_3}) \Big\rangle$,

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3. $\ell_2 \succeq \ell_3$ iff $u^{\intercal}_{\ell_2} \succeq u^{\intercal}_{\ell_3}$ and $u^{\daleth}_{\ell_2} \preceq u^{\daleth}_{\ell_3}$ and $u^{\dashv}_{\ell_2} \preceq u^{\dashv}_{\ell_3}$,
4. $\ell_2 = \ell_3$ iff $u^{\intercal}_{\ell_2} = u^{\intercal}_{\ell_3}$ and $u^{\daleth}_{\ell_2} = u^{\daleth}_{\ell_3}$ and $u^{\dashv}_{\ell_2} = u^{\dashv}_{\ell_3}$.

Definition 2.4. [6] For any NS $\ell = \{\hbar, \hbar^{\intercal}_v(\hbar), \hbar^{\daleth}_v(\hbar), \hbar^{\dashv}_v(\hbar)\}$ of $\mathcal{U}$. Then (ζ, τ)-cut is defined as $\{\hbar \in U | \hbar^{\intercal}_v(\hbar) \succeq \zeta, \hbar^{\daleth}_v(\hbar) \succeq \zeta, \hbar^{\dashv}_v(\hbar) \preceq \tau\}$.

Definition 2.5. [6] Let V and Y be two NS s of Δ. Then Cartesian product of V and Y is defined as $V \times Y = \{\hbar^{\mathcal{T}}_{V\times Y}(\hbar, \ell), \hbar^{\daleth}_{V\times Y}(\hbar, \ell), \hbar^{\dashv}_{V\times Y}(\hbar, \ell) | \, for\, all\, \hbar, v \in \Delta\}$,where $\hbar^{\mathcal{T}}_{V\times Y}(\hbar, \ell) = \min\{\hbar^{\mathcal{T}}_V(\hbar), \hbar^{\mathcal{T}}_Y(\ell)\}, \hbar^{\daleth}_{V\times Y}(\hbar, \ell) = \frac{\hbar^{\daleth}_V(\hbar) + \hbar^{\daleth}_Y(\ell)}{2}, \hbar^{\dashv}_{V\times Y}(\hbar, \ell) = \max\{\hbar^{\dashv}_V(\hbar), \hbar^{\dashv}_Y(\ell)\}$.

Definition 2.6. A fuzzy subset v of a bisemiring $(\Delta, \bigcirc_1, \bigcirc_2, \bigcirc_3)$ is represents a fuzzy subbisemiring of Δ if $\hbar_v(\hbar \bigcirc_1 \varepsilon) \succeq \min\{\hbar_v(\hbar), \hbar_v(\varepsilon)\}, \hbar_v(\hbar \bigcirc_2 \varepsilon) \succeq \min\{\hbar_v(\hbar), \hbar_v(\varepsilon)\}, \hbar_v(\hbar \bigcirc_3 \varepsilon) \succeq \min\{\hbar_v(\hbar), \hbar_v(\varepsilon)\}$,for all $\hbar, \varepsilon \in \Delta$.

3 Complex interval valued neutrosophic subbisemiring

Here Δ denotes bisemiring unless other stated, Z stands for real part and $\Im$ stands for imaginary part and $\Theta = 2\pi$.

Definition 3.1. The complex interval valued NS (CIVqNS) $\overrightarrow{\aleph}$ in universal set O,
$\overrightarrow{\aleph} = \left\{\left\langle \hbar, \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}} \right\rangle : \hbar \in O\right\}$,where $\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) = [R^{\intercal L}_{\aleph}, R^{\intercal U}_{\aleph}], \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) = [R^{\daleth L}_{\aleph}, R^{\daleth U}_{\aleph}], \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) = [R^{\dashv L}_{\aleph}, R^{\dashv U}_{\aleph}]$ and $\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}} : O \to D[0, 1]$ represents the truth degree,indeterminacy degree and false degree respectively. For simplicity the symbol $\left\langle \overrightarrow{Z^{\intercal}_{\aleph}}, \overrightarrow{Z^{\daleth}_{\aleph}}, \overrightarrow{Z^{\dashv}_{\aleph}} \right\rangle$ is CIVqNS
$\overrightarrow{\aleph} = \left\{\left\langle \hbar, \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}} \right\rangle : \hbar \in O\right\}$.

Definition 3.2. Let $\overrightarrow{\aleph} = \left\{\hbar, \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}}\right\}$ and $\overrightarrow{\mho} = \left\{\hbar, \overrightarrow{Z^{\intercal}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\mho}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\mho}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\mho}(\hbar,q)}}\right\}$ be two CIVqNSs of O. Then we define the intersection and union operation is defined as

(i) $\overrightarrow{\aleph} \cap \overrightarrow{\mho} = \left\{\left(\hbar, \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\intercal}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\mho}(\hbar,q)}}\}, \min\{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\mho}(\hbar,q)}}\}, \max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\mho}(\hbar,q)}}\}\right) \Big| \hbar \in O\right\}$.

(ii) $\overrightarrow{\aleph} \cup \overrightarrow{\mho} = \left\{\left(\hbar, \max\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\intercal}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\mho}(\hbar,q)}}\}, \max\{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\mho}(\hbar,q)}}\}, \min\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\mho}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\mho}(\hbar,q)}}\}\right) \Big| \hbar \in O\right\}$.

Definition 3.3. For any CIVqNS $\overrightarrow{\aleph} = \left\{\hbar, \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}}, \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}}\right\}$ of a universal set O. Then $(\partial, \flat)$-cut is defined as $\left\{\hbar \in O | \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\hbar,q)}} \succeq \partial, \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\hbar,q)}} \succeq \partial, \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\hbar,q)}} \preceq \flat\right\}$.

Definition 3.4. The Cartesian product of $\overrightarrow{\aleph}$ and $\overrightarrow{\mho}$ is defined as
$\overrightarrow{\aleph \times \mho} = \left\{\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar, \ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal((\hbar,\ell),q)}_{\aleph\times\mho}}}, \overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar, \ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth((\hbar,\ell),q)}_{\aleph\times\mho}}}, \overrightarrow{Z^{\dashv}_{\aleph\times\mho}}(\hbar \cdot e^{i\Theta\overrightarrow{\Im^{\dashv((\hbar,\ell),q)}_{\aleph\times\mho}}}, \ell) | \, for\, all\, \hbar, v \in S\right\}$,where $\overrightarrow{\aleph}$ and $\overrightarrow{\mho}$ be the CIVqNS of O,where

$$\begin{cases} \overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar, \ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal((\hbar,\ell),q)}_{\aleph\times\mho}}} = \min\left\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal(\hbar,q)}_{\aleph}}}, \overrightarrow{Z^{\intercal}_{\mho}}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\intercal(\ell,q)}_{\mho}}}\right\} \\ \overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar, \ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth((\hbar,\ell),q)}_{\aleph\times\mho}}} = \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth(\hbar,q)}_{\aleph}}} + \overrightarrow{Z^{\daleth}_{\mho}}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth(\ell,q)}_{\mho}}}}{2} \\ \overrightarrow{Z^{\dashv}_{\aleph\times\mho}}((\hbar, \ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv((\hbar,\ell),q)}_{\aleph\times\mho}}} = \max\left\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv(\hbar,q)}_{\aleph}}}, \overrightarrow{Z^{\dashv}_{\mho}}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dashv(\ell,q)}_{\mho}}}\right\} \end{cases}$$

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Definition 3.5. For any CFS $\aleph$ of $(\Delta, \heartsuit_1, \heartsuit_2, \heartsuit_3)$ is said to be a CNqSBS of Δ if it satisfies the following conditions:

$$\begin{cases} R_\aleph((\hbar\heartsuit_1\ell), q) \cdot e^{i\Theta\Im_\aleph((\hbar\heartsuit_1\ell),q)} \succeq \min\{Z_\aleph(\hbar, q) \cdot e^{i\Theta\Im_\aleph(\hbar,q)}, R_\aleph(\ell, q) \cdot e^{i\Theta\Im_\aleph(\ell,q)}\} \\ R_\aleph((\hbar\heartsuit_2\ell), q) \cdot e^{i\Theta\Im_\aleph((\hbar\heartsuit_2\ell),q)} \succeq \min\{Z_\aleph(\hbar, q) \cdot e^{i\Theta\Im_\aleph(\hbar,q)}, R_\aleph(\ell, q) \cdot e^{i\Theta\Im_\aleph(\ell,q)}\} \\ R_\aleph((\hbar\heartsuit_3\ell), q) \cdot e^{i\Theta\Im_\aleph((\hbar\heartsuit_3\ell),q)} \succeq \min\{Z_\aleph(\hbar, q) \cdot e^{i\Theta\Im_\aleph(\hbar,q)}, R_\aleph(\ell, q) \cdot e^{i\Theta\Im_\aleph(\ell,q)}\} \end{cases}$$

$\forall\ \hbar, v \in \Delta$.

Definition 3.6. For any CFS $\aleph$ of $(\Delta, \heartsuit_1, \heartsuit_2, \heartsuit_3)$ is represents a CNqNSBS of Δ if it satisfies the following conditions:

$$\begin{cases} R_\aleph((\hbar\heartsuit_1\ell), q) \cdot e^{i\Theta\Im_\aleph((\hbar\heartsuit_1\ell),q)} = R_\aleph((\ell\heartsuit_1 u), q) \cdot e^{i\Theta\Im_\aleph((\ell\heartsuit_1 u),q)} \\ R_\aleph((\hbar\heartsuit_2\ell), q) \cdot e^{i\Theta\Im_\aleph((\hbar\heartsuit_2\ell),q)} = R_\aleph((\ell\heartsuit_2 u), q) \cdot e^{i\Theta\Im_\aleph((\ell\heartsuit_2 u),q)} \\ R_\aleph((\hbar\heartsuit_3\ell), q) \cdot e^{i\Theta\Im_\aleph((\hbar\heartsuit_3\ell),q)} = R_\aleph((\ell\heartsuit_3 u), q) \cdot e^{i\Theta\Im_\aleph((\ell\heartsuit_3 u),q)} \end{cases}$$

for all $\hbar, v \in \Delta$.

Definition 3.7. For any CIVqNS $\overrightarrow{\aleph}$ of Δ is said to be a CIVqNSBS of Δ if

$$\begin{cases} \overrightarrow{Z^{\dagger}_\aleph}((\hbar\heartsuit_1\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_1\ell),q)} \succeq \min\{\overrightarrow{Z^{\dagger}_\aleph}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_1\ell),q)}, \overrightarrow{Z^{\dagger}_\aleph}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_1\ell),q)}\} \\ \overrightarrow{Z^{\dagger}_\aleph}((\hbar\heartsuit_2\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_2\ell),q)} \succeq \min\{\overrightarrow{Z^{\dagger}_\aleph}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_2\ell),q)}, \overrightarrow{Z^{\dagger}_\aleph}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_2\ell),q)}\} \\ \overrightarrow{Z^{\dagger}_\aleph}((\hbar\heartsuit_3\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_3\ell),q)} \succeq \min\{\overrightarrow{Z^{\dagger}_\aleph}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_3\ell),q)}, \overrightarrow{Z^{\dagger}_\aleph}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_3\ell),q)}\} \end{cases}$$

$$\begin{cases} \overrightarrow{Z^{\beth}_\aleph}((\hbar\heartsuit_1\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_1\ell),q)} \succeq \frac{\overrightarrow{Z^{\beth}_\aleph}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_1\ell),q)} + \overrightarrow{Z^{\beth}_\aleph}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_1\ell),q)}}{2} \\ \qquad\qquad \text{OR} \\ \overrightarrow{Z^{\beth}_\aleph}((\hbar\heartsuit_2\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\dagger}_\aleph}((\hbar\heartsuit_2\ell),q)} \succeq \frac{\overrightarrow{Z^{\beth}_\aleph}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_2\ell),q)} + \overrightarrow{Z^{\beth}_\aleph}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_2\ell),q)}}{2} \\ \qquad\qquad \text{OR} \\ \overrightarrow{Z^{\beth}_\aleph}((\hbar\heartsuit_3\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_3\ell),q)} \succeq \frac{\overrightarrow{Z^{\beth}_\aleph}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_3\ell),q)} + \overrightarrow{Z^{\beth}_\aleph}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_\aleph}((\hbar\heartsuit_3\ell),q)}}{2} \end{cases}$$

$$\begin{cases} \overrightarrow{Z^{\daleth}_\aleph}((\hbar\heartsuit_1\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_1\ell),q)} \preceq \max\{\overrightarrow{Z^{\daleth}_\aleph}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_1\ell),q)}, \overrightarrow{Z^{\daleth}_\aleph}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_1\ell),q)}\} \\ \overrightarrow{Z^{\daleth}_\aleph}((\hbar\heartsuit_2\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_2\ell),q)} \preceq \max\{\overrightarrow{Z^{\daleth}_\aleph}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_2\ell),q)}, \overrightarrow{Z^{\daleth}_\aleph}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_2\ell),q)}\} \\ \overrightarrow{Z^{\daleth}_\aleph}((\hbar\heartsuit_3\ell), q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_3\ell),q)} \preceq \max\{\overrightarrow{Z^{\daleth}_\aleph}(\hbar, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_3\ell),q)}, \overrightarrow{Z^{\daleth}_\aleph}(\ell, q) \cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_\aleph}((\hbar\heartsuit_3\ell),q)}\} \end{cases}$$

for all $\hbar, v \in \Delta$.

Example 3.8. Consider the bisemiring $\Delta = \{\alpha, \gamma, \chi, \rho\}$ with the Cayley table:

$\heartsuit_1$	α	γ	χ	ρ
α	α	α	α	α
γ	α	γ	α	γ
χ	α	α	χ	χ
ρ	α	γ	χ	ρ

$\heartsuit_2$	α	γ	χ	ρ
α	α	γ	χ	ρ
γ	γ	γ	ρ	ρ
χ	χ	ρ	χ	ρ
ρ	ρ	ρ	ρ	ρ

$\heartsuit_3$	α	γ	χ	ρ
α	α	α	α	α
γ	α	γ	χ	ρ
χ	ρ	ρ	ρ	ρ
ρ	ρ	ρ	ρ	ρ

	$(w, q) = \alpha$	$(w, q) = \gamma$
$\overrightarrow{Z^{\dagger}_\aleph}(w, q)$	$[0.85e^{i2\pi(0.7)}, 0.95e^{i2\pi(0.8)}]$	$[0.75e^{i2\pi(0.6)}, 0.85e^{i2\pi(0.7)}]$
$\overrightarrow{Z^{\beth}_\aleph}(w, q)$	$[0.55e^{i2\pi(0.4)}, 0.65e^{i2\pi(0.5)}]$	$[0.45e^{i2\pi(0.3)}, 0.55e^{i2\pi(0.4)}]$
$\overrightarrow{Z^{\daleth}_\aleph}(w, q)$	$[0.65e^{i2\pi(0.5)}, 0.75e^{i2\pi(0.6)}]$	$[0.75e^{i2\pi(0.6)}, 0.85e^{i2\pi(0.65)}]$

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	$(w,q)=\chi$	$(w,q)=\rho$
$\overrightarrow{Z^{\intercal}_{\aleph}}(w,q)$	$[0.45e^{i2\pi(0.3)}, 0.55e^{i2\pi(0.4)}]$	$[0.65e^{i2\pi(0.5)}, 0.75e^{i2\pi(0.6)}]$
$\overrightarrow{Z^{\daleth}_{\aleph}}(w,q)$	$[0.25e^{i2\pi(0.2)}, 0.35e^{i2\pi(0.3)}]$	$[0.35e^{i2\pi(0.2)}, 0.45e^{i2\pi(0.3)}]$
$\overrightarrow{Z^{\gimel}_{\aleph}}(w,q)$	$[0.95e^{i2\pi(0.8)}, 0.85e^{i2\pi(0.7)}]$	$[0.9e^{i2\pi(0.75)}, 0.95e^{i2\pi(0.8)}]$

Hence, $\overrightarrow{\aleph}$ is a CIVqNSBS of Δ.

Theorem 3.9. *The intersection of a every CIVqNSBSs is again a CIVqNSBS of* Δ.

Proof. Let $\{\overrightarrow{\Psi_i} : i \in I\}$ be the family of CIVqNSBSs of Δ and $\overrightarrow{\aleph} = \bigcap_{i\in I}\overrightarrow{\Psi_i}$. Let $\hbar, v \in \Delta$.

Now,

$$\begin{aligned}
\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)} &= \inf_{i\in I}\ \overrightarrow{Z^{\intercal}_{\Psi_i}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)} \\
&\succeq \inf_{i\in I}\ \min\{\overrightarrow{Z^{\intercal}_{\Psi_i}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)}, \overrightarrow{Z^{\intercal}_{\Psi_i}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)}\} \\
&= \min\left\{\inf_{i\in I}\ \overrightarrow{Z^{\intercal}_{\Psi_i}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)}, \inf_{i\in I}\ \overrightarrow{Z^{\intercal}_{\Psi_i}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)}\right\} \\
&= \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)}, \overrightarrow{Z^{\intercal}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_1\ell),q)}\}
\end{aligned}$$

Similarly,
$\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar\heartsuit_2\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_2\ell),q)} \succeq \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_2\ell),q)}, \overrightarrow{Z^{\intercal}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_2\ell),q)}\}$,
$\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar\heartsuit_3\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_3\ell),q)} \succeq \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_3\ell),q)}, \overrightarrow{Z^{\intercal}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\heartsuit_3\ell),q)}\}$.
Now,

$$\begin{aligned}
\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)} &= \inf_{i\in I}\ \overrightarrow{Z^{\daleth}_{\Psi_i}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)} \\
&\succeq \inf_{i\in I}\ \frac{\overrightarrow{Z^{\daleth}_{\Psi_i}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)} + \overrightarrow{Z^{\daleth}_{\Psi_i}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)}}{2} \\
&= \frac{\inf_{i\in I}\ \overrightarrow{Z^{\daleth}_{\Psi_i}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)} + \inf_{i\in I}\ \overrightarrow{Z^{\daleth}_{\Psi_i}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)}}{2} \\
&= \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)} + \overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_1\ell),q)}}{2}
\end{aligned}$$

Similarly,
$\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar\heartsuit_2\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_2\ell),q)} \succeq \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_2\ell),q)} + \overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_2\ell),q)}}{2}$ and
$\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar\heartsuit_3\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_3\ell),q)} \succeq \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_3\ell),q)} + \overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\heartsuit_3\ell),q)}}{2}$.
Now,

$$\begin{aligned}
\overrightarrow{Z^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)} &= \sup_{i\in I}\ \overrightarrow{Z^{\gimel}_{\Psi_i}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)} \\
&\preceq \sup_{i\in I}\ \max\{\overrightarrow{Z^{\gimel}_{\Psi_i}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)}, \overrightarrow{Z^{\gimel}_{\Psi_i}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)}\} \\
&= \max\left\{\sup_{i\in I}\ \overrightarrow{Z^{\gimel}_{\Psi_i}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)}, \sup_{i\in I}\ \overrightarrow{Z^{\gimel}_{\Psi_i}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)}\right\} \\
&= \max\{\overrightarrow{Z^{\gimel}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)}, \overrightarrow{Z^{\gimel}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_1\ell),q)}\}
\end{aligned}$$

Similarly,
$\overrightarrow{Z^{\gimel}_{\aleph}}((\hbar\heartsuit_2\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_2\ell),q)} \preceq \max\{\overrightarrow{Z^{\gimel}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_2\ell),q)}, \overrightarrow{Z^{\gimel}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_2\ell),q)}\}$,and
$\overrightarrow{Z^{\gimel}_{\aleph}}((\hbar\heartsuit_3\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_3\ell),q)} \preceq \max\{\overrightarrow{Z^{\gimel}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_3\ell),q)}, \overrightarrow{Z^{\gimel}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\heartsuit_3\ell),q)}\}$.
Thus, $\overrightarrow{\aleph}$ is a CIVqNSBS of Δ.

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Theorem 3.10. *If $\overrightarrow{\aleph}$ and $\overrightarrow{\mho}$ be the CIVqNSBSs of Δ_1 and Δ_2 respectively,then $\overrightarrow{\aleph\times\mho}$is a CIVqNSBS of $\Delta_1\times\Delta_2$.*

Proof. Let $\hbar_1,\hbar_2\in\Delta_1$ and $\ell_1,\ell_2\in\Delta_2$. Then $(\hbar_1,\ell_1)$ and $(\hbar_2,\ell_2)$ are in $\Delta_1\times\Delta_2$. Now

$$\begin{aligned}
&\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_1(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_1(\hbar_2,\ell_2),q)]}\\
=\;&\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2,\ell_1\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2,\ell_1\heartsuit_1\ell_2),q)}\\
=\;&\min\{\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2),q)},\overrightarrow{Z^{\intercal}_{\mho}}((\ell_1\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\mho}}((\ell_1\heartsuit_1\ell_2),q)}\}\\
\succeq\;&\min\{\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1)},\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)}\},\min\{\overrightarrow{Z^{\intercal}_{\mho}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_1,q)},\overrightarrow{Z^{\intercal}_{\mho}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_2,q)}\}\}\\
=\;&\min\{\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1)},\overrightarrow{Z^{\intercal}_{\mho}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_1)}\},\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)},\overrightarrow{Z^{\intercal}_{\mho}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_2,q)}\}\}\\
=\;&\min\{\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)},\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\}
\end{aligned}$$

Also $\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_2(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_2(\hbar_2,\ell_2),q)]}$
$\succeq\min\{\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)},\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\}$

and $\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_3(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_3(\hbar_2,\ell_2),q)]}$
$\succeq\min\{\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)},\overrightarrow{Z^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\}$.
Now,

$$\begin{aligned}
&\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_1(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_1(\hbar_2,\ell_2),q)]}\\
=\;&\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2,\ell_1\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2,\ell_1\heartsuit_1\ell_2),q)}\\
=\;&\frac{\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2),q)}+\overrightarrow{Z^{\daleth}_{\mho}}((\ell_1\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\ell_1\heartsuit_1\ell_2),q)}}{2}\\
\succeq\;&\frac{1}{2}\left[\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_2,q)}}{2}+\frac{\overrightarrow{Z^{\daleth}_{\mho}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\mho}}(\ell_1,q)}+\overrightarrow{Z^{\daleth}_{\mho}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\mho}}(\ell_2,q)}}{2}\right]\\
=\;&\frac{1}{2}\left[\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\mho}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\mho}}(\ell_1,q)}}{2}+\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_2,q)}+\overrightarrow{Z^{\daleth}_{\mho}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\mho}}(\ell_2,q)}}{2}\right]\\
=\;&\frac{1}{2}\left[\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)}+\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\right]
\end{aligned}$$

Also
$\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_2(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_2(\hbar_2,\ell_2),q)]}\succeq\frac{1}{2}\Big[\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)}+$
$\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\Big]$
and
$\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_3(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_3(\hbar_2,\ell_2),q)]}\succeq\frac{1}{2}\Big[\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)}+$
$\overrightarrow{Z^{\daleth}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\Big]$.
Now,

$$\begin{aligned}
&\overrightarrow{Z^{\Finv}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_1(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_1(\hbar_2,\ell_2),q)]}\\
=\;&\overrightarrow{Z^{\Finv}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2,\ell_1\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2,\ell_1\heartsuit_1\ell_2),q)}\\
=\;&\max\{\overrightarrow{Z^{\Finv}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph\times\mho}}((\hbar_1\heartsuit_1\hbar_2),q)},\overrightarrow{Z^{\Finv}_{\mho}}((\ell_1\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph\times\mho}}((\ell_1\heartsuit_1\ell_2),q)}\}\\
\preceq\;&\max\{\max\{\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar_2,q)}\},\max\{\overrightarrow{Z^{\Finv}_{\mho}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\mho}}(\ell_1,q)},\overrightarrow{Z^{\Finv}_{\mho}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\mho}}(\ell_2,q)}\}\}\\
=\;&\max\{\max\{\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\Finv}_{\mho}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\mho}}(\ell_1,q)}\},\max\{\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar_2,q)},\overrightarrow{Z^{\Finv}_{\mho}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\mho}}(\ell_2,q)}\}\}\\
=\;&\max\{\overrightarrow{Z^{\Finv}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)},\overrightarrow{Z^{\Finv}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\}
\end{aligned}$$

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Also $\overrightarrow{Z^{\dashv}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_2(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_2(\hbar_2,\ell_2),q)]}$
$\preceq \max\{\overrightarrow{Z^{\dashv}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)},\overrightarrow{Z^{\dashv}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}}((\hbar_2,\ell_2),q)\}$,
$\overrightarrow{Z^{\dashv}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_3(\hbar_2,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}[((\hbar_1,\ell_1)\heartsuit_3(\hbar_2,\ell_2),q)]}$
$\preceq \max\{\overrightarrow{Z^{\dashv}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_1,\ell_1),q)},\overrightarrow{Z^{\dashv}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph\times\mho}}((\hbar_2,\ell_2),q)}\}$.
Thus, $\overrightarrow{\aleph\times\mho}$ is a CIVqNSBS of Δ.

Corollary 3.11. *If $\overrightarrow{\aleph_1},\overrightarrow{\aleph_2},...,\overrightarrow{\aleph_n}$ be the finite collection of CIVqNSBSs of $\Delta_1,\Delta_2,...,\Delta_n$ respectively. Then $\overrightarrow{\aleph_1\times\aleph_2\times...\times\aleph_n}$ is a CIVqNSBS of $\Delta_1\times\Delta_2\times...\times\Delta_n$.*

Definition 3.12. Let $\overrightarrow{\aleph}\subseteq\Delta$,the strongest CIVqN relation on Δ is

$$\begin{cases}\overrightarrow{Z^{\intercal}_{\Psi}}((\hbar,\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}((\hbar,\ell),q)}=min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell,q)}\}\\ \overrightarrow{Z^{\ddagger}_{\Psi}}((\hbar,\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\ddagger}_{\Psi}}((\hbar,\ell),q)}=\frac{\overrightarrow{Z^{\ddagger}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\ddagger}_{\aleph}}(\hbar,q)}+\overrightarrow{Z^{\ddagger}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\ddagger}_{\aleph}}(\ell,q)}}{2}\\ \overrightarrow{Z^{\dashv}_{\Psi}}((\hbar,\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\Psi}}((\hbar,\ell),q)}=max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\dashv}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\ell,q)}\}\end{cases}$$

Theorem 3.13. *Let $\overrightarrow{\aleph}$ be a CIVqNSBS of Δ and $\overrightarrow{\Psi}$ be a strongest complex interval valued neutrosophic relation of Δ. Then $\overrightarrow{\aleph}$ is a CIVqNSBS of $\Delta\times\Delta$ if and only if $\overrightarrow{\Psi}$ is a CIVqNSBS of $\Delta\times\Delta$.*

Proof. Suppose $\overrightarrow{\aleph}$ is a CIVqNSBS of $\Delta\times\Delta$ and $\overrightarrow{\Psi}$ be the strongest complex interval valued neutrosophic relation of Δ.
For any $\hbar=(\hbar_1,\hbar_2),\ell=(\ell_1,\ell_2)\in\Delta\times\Delta$. Now,

$$\begin{aligned}&\overrightarrow{Z^{\intercal}_{\Psi}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}((\hbar\heartsuit_1\ell),q)}\\ =\ &\overrightarrow{Z^{\intercal}_{\Psi}}[((((\hbar_1,\hbar_2),q)\heartsuit_1((\ell_1,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}[((((\hbar_1,\hbar_2),q)\heartsuit_1((\ell_1,\ell_2),q)]}\\ =\ &\overrightarrow{Z^{\intercal}_{\Psi}}(\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}(\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2)}\\ =\ &\min\{\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)},\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)}\}\\ \succeq\ &\min\{\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_1,q)}\},\\ &\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_2,q)}\}\}\\ =\ &\min\{\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)}\},\\ &\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_2,q)}\}\}\\ =\ &\min\{\overrightarrow{Z^{\intercal}_{\Psi}}((\hbar_1,\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}((\hbar_1,\hbar_2),q)},\overrightarrow{Z^{\intercal}_{\Psi}}((\ell_1,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}((\ell_1,\ell_2),q)}\}\\ =\ &\min\{\overrightarrow{Z^{\intercal}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}(\hbar,q)},\overrightarrow{Z^{\intercal}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}(\ell,q)}\}\end{aligned}$$

Also $\overrightarrow{Z^{\intercal}_{\Psi}}((\hbar\heartsuit_2\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}((\hbar\heartsuit_2\ell),q)}\succeq\min\{\overrightarrow{Z^{\intercal}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}(\hbar,q)},\overrightarrow{Z^{\intercal}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}(\ell,q)}\}$,
$\overrightarrow{Z^{\intercal}_{\Psi}}((\hbar\heartsuit_3\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}((\hbar\heartsuit_3\ell),q)}\succeq\min\{\overrightarrow{Z^{\intercal}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}(\hbar,q)},\overrightarrow{Z^{\intercal}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}(\ell,q)}\}$.

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Now, $\overrightarrow{Z^{\daleth}_{\Psi}}(\hbar\heartsuit_1\ell)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\hbar\heartsuit_1\ell)}$

$$= \overrightarrow{Z^{\daleth}_{\Psi}}[((((\hbar_1,\hbar_2),q)\heartsuit_1(\ell_1,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}[((((\hbar_1,\hbar_2),q)\heartsuit_1((\ell_1,\ell_2),q),q)]}$$
$$= \overrightarrow{Z^{\daleth}_{\Psi}}((\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2,q))}$$
$$= \frac{\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)}+\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}((\hbar_2\heartsuit_1\ell_2),q)}{2}$$
$$\succeq \frac{1}{2}\left[\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell_1,q)}}{2}+\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_2,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell_2,q)}}{2}\right]$$
$$= \frac{1}{2}\left[\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_2,q)}}{2}+\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\ell_1,q)}}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell_2,q)}}{2}\right]$$
$$= \frac{\overrightarrow{Z^{\daleth}_{\Psi}}((\hbar_1,\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\hbar_1,\hbar_2),q)}+\overrightarrow{Z^{\daleth}_{\Psi}}((\ell_1,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\ell_1,\ell_2),q)}}{2}$$
$$= \frac{\overrightarrow{Z^{\daleth}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}(\hbar,q)}}+\overrightarrow{Z^{\daleth}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\hbar,q)}}{2}$$

Also $\overrightarrow{Z^{\daleth}_{\Psi}}((\hbar\heartsuit_2\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\hbar\heartsuit_2\ell),q)}\succeq\frac{\overrightarrow{Z^{\daleth}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}(\hbar,q)}}+\overrightarrow{Z^{\daleth}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\ell,q)}}{2}$
and $\overrightarrow{Z^{\daleth}_{\Psi}}((\hbar\heartsuit_3\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\hbar\heartsuit_3\ell),q)}\succeq\frac{\overrightarrow{Z^{\daleth}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}(\hbar,q)}}+\overrightarrow{Z^{\daleth}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\ell,q)}}{2}$.
Similarly, $\overrightarrow{Z^{\beth}_{\Psi}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}((\hbar\heartsuit_1\ell),q)}\preceq\max\{\overrightarrow{Z^{\beth}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}(\hbar,q)\cdot}},\overrightarrow{Z^{\beth}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}(\ell,q)}}\}$,
$\overrightarrow{Z^{\beth}_{\Psi}}((\hbar\heartsuit_2\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}((\hbar\heartsuit_2\ell),q)}\preceq\max\{\overrightarrow{Z^{\beth}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}(\hbar,q)}},\overrightarrow{Z^{\beth}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}(\ell,q)}}\}$ and
$\overrightarrow{Z^{\beth}_{\Psi}}((\hbar\heartsuit_3\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}((\hbar\heartsuit_3\ell),q)}\preceq\max\{\overrightarrow{Z^{\beth}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}(\hbar,q)}},\overrightarrow{Z^{\beth}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}(\ell}}\}$.
Therefore, $\overrightarrow{\Psi}$ is a CIVqNSBS of $\Delta\times\Delta$.

Conversely,suppose that $\overrightarrow{\Psi}$ is a CIVqNSBS of $\Delta\times\Delta$. Let $\hbar=((\hbar_1,\hbar_2),q),\ell=((\ell_1,\ell_2),q)\in\Delta\times\Delta$. Now,

$$\min\{\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}},\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_2\heartsuit_1\ell_2),q)}}\}$$
$$= \overrightarrow{Z^{\intercal}_{\Psi}}(\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}(\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2)}}$$
$$= \overrightarrow{Z^{\intercal}_{\Psi}}[((\hbar_1,\hbar_2),q)\heartsuit_1((\ell_1,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{v}[((\hbar_1,\hbar_2),q)\heartsuit_1((\ell_1,\ell_2),q)]}}$$
$$= \overrightarrow{Z^{\intercal}_{\Psi}}(\hbar\heartsuit_1\ell)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}(\hbar\heartsuit_1\ell)}}$$
$$\succeq\min\{\overrightarrow{Z^{\intercal}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}(\hbar,q)}},\overrightarrow{Z^{\intercal}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}(\ell,q)}}\}$$
$$= \min\{\overrightarrow{Z^{\intercal}_{\Psi}}((\hbar_1,\hbar_2),q)\}\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}((\hbar_1,\hbar_2),q)\}}},\overrightarrow{Z^{\intercal}_{\Psi}}((\ell_1,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}((\ell_1,\ell_2),q)}\}$$
$$= \min\{\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)}\},\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_1,q)}},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_2,q)}}\}\}$$

If $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}}\preceq\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_2\heartsuit_1\ell_2),q)}}$, then $\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)}\preceq\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)}$ and $\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_1,q)}}\preceq\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_2,q)}}$. We get $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}}\succeq\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_1,q)}}\}$ for all $\hbar_1,\ell_1\in\Delta$,and
$\min\{\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_2\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_1\heartsuit_2\ell_1),q)}},\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_2\heartsuit_2\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_2\heartsuit_2\ell_2),q)}}\}\succeq\min\{\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)}\},\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_1,q)}},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_2,q)}}\}\}$
If $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_2\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_1\heartsuit_2\ell_1),q)}}\preceq\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_2\heartsuit_2\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_2\heartsuit_2\ell_2),q)}}$,then $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_2\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}((\hbar_1\heartsuit_2\ell_1),q)}}\succeq\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}(\ell_1,q)}}\}$.
$\min\{\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)},\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_2\heartsuit_3\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_2\heartsuit_3\ell_2),q)}\}\succeq\min\{\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_2,q)}\},\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_2,q)}\}\}$
If $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)}\preceq\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_2\heartsuit_3\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_2\heartsuit_3\ell_2),q)}$,then $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)}\succeq\min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\intercal}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell_1,q)}\}$.
Now,

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$$\frac{1}{2}\Big[\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}}+\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth((\hbar_2\heartsuit_1\ell_2),q)}_{\aleph}}}\Big]$$
$$=\overrightarrow{Z^{\daleth}_{\Psi}}(\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}(\hbar_1\heartsuit_1\ell_1,\hbar_2\heartsuit_1\ell_2)}}$$
$$=\overrightarrow{Z^{\daleth}_{\Psi}}[((\hbar_1,\hbar_2),q)\heartsuit_1((\ell_1,\ell_2),q)]\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}[((\hbar_1,\hbar_2),q)\heartsuit_1((\ell_1,\ell_2),q)]}}$$
$$=\overrightarrow{Z^{\daleth}_{\Psi}}((\hbar\heartsuit_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}((\hbar\heartsuit_1\ell),q)}}$$
$$\succeq\frac{\overrightarrow{Z^{\daleth}_{\Psi}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth(\hbar,q)}_{\Psi}}}+\overrightarrow{Z^{\daleth}_{\Psi}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}(\ell,q)}}}{2}$$
$$=\frac{\overrightarrow{Z^{\daleth}_{\Psi}}((\hbar_1,\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}((\hbar_1,\hbar_2),q)}}+\overrightarrow{Z^{\daleth}_{\Psi}}((\ell_1,\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}}}{2}$$
$$=\frac{1}{2}\left[\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_2,q)}}{2}+\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\ell_1,q)}}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\ell_2,q)}}}{2}\right]$$

If $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}}\preceq\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}((\hbar_2\heartsuit_1\ell_2),q)}}$, then $\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}\preceq$ $\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_2,q)}$ and $\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\ell_1,q)}}\preceq\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\ell_2,q)}}$.
We get $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)}\succeq\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}(\ell_1,q)}}}{2}$.
Similarly, $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_2\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}((\hbar_1\heartsuit_2\ell_1),q)}}\succeq\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell_1,q)}}{2}$
and $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}((\hbar_1\heartsuit_3\ell_1),q)}}\succeq\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar_1,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}(\ell_1,q)}}}{2}$.
Similarly to prove that
$\max\{\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}},\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_2\heartsuit_1\ell_2),q)}}\}\preceq\max\{\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot$ $e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_2,q)}\},\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_1,q)}},\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_2,q)}}\}\}$
If $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}}\succeq\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_2\heartsuit_1\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_2\heartsuit_1\ell_2),q)}}$, then $\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_1,q)}\succeq$ $\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_2,q)}$ and $\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_1,q)}}\succeq\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_2,q)}}$.
We get $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_1\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_1\heartsuit_1\ell_1),q)}}\preceq\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_1,q)}}\}$.
$\max\{\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_2\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_1\heartsuit_2\ell_1),q)}},\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_2\heartsuit_2\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_2\heartsuit_2\ell_2),q)}}\}$
$\preceq\max\{\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_2,q)}\},\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_1,q)}},\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}\}\}$
If $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_2\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_1\heartsuit_2\ell_1),q)}}\succeq\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_2\heartsuit_2\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_2\heartsuit_2\ell_2),q)}}$, then $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_2\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_1\heartsuit_2\ell_1),q)}}\preceq$ $\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_1,q)}}\}$.
$\max\{\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_1\heartsuit_3\ell_1),q)}},\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_2\heartsuit_3\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}((\hbar_2\heartsuit_3\ell_2),q)}}\}\preceq\max\{\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot$ $e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_2,q)}\},\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_1,q)}},\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}(\ell_2,q)}}\}\}$
If $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}((\hbar_1\heartsuit_3\ell_1),q)}}\succeq\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_2\heartsuit_3\ell_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}((\hbar_2\heartsuit_3\ell_2),q)}}$, then $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_3\ell_1),q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}((\hbar_1\heartsuit_3\ell_1),q)}}\preceq$ $\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}(\hbar_1,q)},\overrightarrow{Z^{\dashv}_{\aleph}}(\ell_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}(\ell_1,q)}}\}$.
Therefore, $\overrightarrow{\aleph}$ is a CIVqNSBS of Δ.

Theorem 3.14. *Suppose that $\overrightarrow{\aleph}$ is a subset of Δ. Then $\overrightarrow{Z}=(\overrightarrow{Z^{\top}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}}},\overrightarrow{Z^{\daleth}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}},\overrightarrow{Z^{\dashv}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}})$ is a CIVqNSBS of Δ if and only if $\overrightarrow{Z^{(\partial,\flat)}}$ is a subbisemiring of Δ for all $\partial,\flat\in\mathbb{D}[0,1]$.*

Proof. Assume that $\overrightarrow{Z}$ is a CIVqNSBS of Δ. For each $\partial,\flat\in\mathbb{D}[0,1]$ and $\hbar_1,\hbar_2\in\overrightarrow{Z^{(\partial,\flat)}}$. Now, $\overrightarrow{Z^{\top}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}}}(\hbar_1,q)\succeq\partial$, $\overrightarrow{Z^{\top}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}}}(\hbar_2,q)\succeq\partial$ and $\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_1,q)\succeq\partial$, $\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_2,q)\succeq\partial$ and $\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_1,q)\preceq\flat$, $\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_2,q)\preceq\flat$. Now, $\overrightarrow{Z^{\top}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2),q)\succeq$ $\min\{\overrightarrow{Z^{\top}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}}}(\hbar_1,q),\overrightarrow{Z^{\top}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\top}_{\aleph}}}(\hbar_2,q)\}\succeq\partial$ and $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2),q)\succeq$ $\frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_1,q)+\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_2,q)}{2}\succeq\frac{\partial+\partial}{2}=\partial$ and $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2),q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2),q)\preceq\max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1,q)\cdot$ $e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_1,q),\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_2,q)\}\preceq\flat$. This implies that $\hbar_1\heartsuit_1\hbar_2\in\overrightarrow{Z^{(\partial,\flat)}}$. Similarly, $\hbar_1\heartsuit_2\hbar_2\in\overrightarrow{Z^{(\partial,\flat)}}$

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and $\hbar_1 \heartsuit_3 \hbar_2 \in \overrightarrow{Z^{(\partial,\flat)}}$. Hence,$\overrightarrow{Z^{(\partial,\flat)}}$ is a subbisemiring of Δ,for all $\partial, \flat \in \mathbb{D}[0,1]$.

Conversely,assume that $\overrightarrow{Z^{(\partial,\flat)}}$ is a subbisemiring of Δ and $\partial, \flat \in \mathbb{D}[0,1]$. Suppose if there exist $\hbar_1, \hbar_2 \in \Delta$ such that $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) < \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\hbar_1, q), \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\hbar_2, q)\}$,$\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) < \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_1,q)+\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_2,q)}{2}$ and $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) > \max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_1, q), \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_2, q)\}$. For $\partial, \flat \in \mathbb{D}[0,1]$ such that $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) < \partial \preceq \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\hbar_1, q), \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\hbar_2, q)\}$ and $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) < \partial \preceq \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_1,q)+\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_2,q)}{2}$ and $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) > \flat \succeq \max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_1, q), \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_2, q)\}$. Thus,$\hbar_1, \hbar_2 \in \overrightarrow{Z^{(\partial,\flat)}}$,but $\hbar_1\heartsuit_1\hbar_2 \notin \overrightarrow{Z^{(\partial,\flat)}}$. This contradicts,$\overrightarrow{Z^{(\partial,\flat)}}$ is a SBS of Δ. Therefore $\overrightarrow{Z^{\intercal}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) \succeq \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_1, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\hbar_1, q), \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar_2, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\hbar_2)\}$, $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) \succeq \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_1,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_1,q)+\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar_2,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar_2,q)}{2}$ and $\overrightarrow{Z^{\dashv}_{\aleph}}((\hbar_1\heartsuit_1\hbar_2), q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}((\hbar_1\heartsuit_1\hbar_2), q) \preceq \max\{\overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_1, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_1, q), \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar_2, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar_2, q)\}$. Similarly,$\heartsuit_2$ and $\heartsuit_3$ cases. Hence $\overrightarrow{Z} = (\overrightarrow{Z^{\intercal}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}, \overrightarrow{Z^{\daleth}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}, \overrightarrow{Z^{\dashv}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}})$ is a CIVqNSBS of Δ.

Definition 3.15. Let $(\Delta_1, \oslash_1, \oslash_2, \oslash_3)$ and $(\Delta_2, \sqcap_1, \sqcap_2, \sqcap_3)$ be any two bisemirings. The mapping $\Xi : \Delta_1 \to \Delta_2$ and $\overrightarrow{\aleph}$ be any CIVqNSBS in Δ_1, $\overrightarrow{\Psi}$ be any CIVqNSBS in $\Xi(\Delta_1) = \Delta_2$. If $\overrightarrow{Z_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im_{\aleph}}} = [\overrightarrow{Z^{\intercal}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}, \overrightarrow{Z^{\daleth}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}, \overrightarrow{Z^{\dashv}_{\aleph}}\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}]$ is a CIVqNS in Δ_1,then $\overrightarrow{Z_{\Psi}}$ is a CIVqNS in Δ_2,defined by

$$\overrightarrow{Z^{\intercal}_{\Psi}}(\ell, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\ell, q) = \begin{cases} \sup \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}(\hbar, q) & if \ \hbar \in \Xi^{-1}\ell \\ 0 & otherwise \end{cases}$$

$$\overrightarrow{Z^{\daleth}_{\Psi}}(\ell, q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\ell, q) = \begin{cases} \sup \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}}(\hbar, q) & if \ \hbar \in \Xi^{-1}\ell \\ 0 & otherwise \end{cases}$$

$$\overrightarrow{Z^{\dashv}_{\Psi}}(\ell, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\ell, q) = \begin{cases} \inf \overrightarrow{Z^{\dashv}_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\aleph}}}(\hbar, q) & if \ \hbar \in \Xi^{-1}\ell \\ 1 & otherwise \end{cases}$$

for all $\hbar \in \Delta_1$ and $v \in \Delta_2$ is represents the image of $R_{\aleph}$ under Ξ.
Similarly,If $\overrightarrow{Z_{\Psi}}\cdot e^{i\Theta\overrightarrow{\Im_{\Psi}}} = [\overrightarrow{Z^{\intercal}_{\Psi}}\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}}, \overrightarrow{Z^{\daleth}_{\Psi}}\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}}, \overrightarrow{Z^{\dashv}_{\Psi}}\cdot e^{i\Theta\overrightarrow{\Im^{\dashv}_{\Psi}}}]$ is a CIVqNS in Δ_2,then CIVqNS $\overrightarrow{Z_{\aleph}} = \Xi \circ \overrightarrow{Z_{\Psi}}$ in Δ_1 [ie,the CIVqNS defined by $\overrightarrow{Z_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im_{\aleph}}(\hbar,q)} = \overrightarrow{Z_{\Psi}}(\Xi(\hbar, q))\cdot e^{i\Theta\overrightarrow{\Im_{\aleph}}(\Xi(\hbar,q))}$] is represents the preimage of $\overrightarrow{Z_{\Psi}}$ under Ξ.

Theorem 3.16. *The homomorphic image of every CIVqNSBS is a CIVqNSBS.*

Proof. The mapping $\Xi : \Delta_1 \to \Delta_2$ be any homomorphism. Now,$\Xi((\hbar \oslash_1 \ell), q) = \Xi(\hbar, q) \sqcap_1 \Xi(\ell, q)$, $\Xi((\hbar \oslash_2 \ell), q) = \Xi(\hbar, q) \sqcap_2 \Xi(\ell, q)$ and $\Xi((\hbar \oslash_3 \ell), q) = \Xi(\hbar, q) \sqcap_3 \Xi(\ell, q)$ for all $\hbar, v \in \Delta_1$. Let $\overrightarrow{\Psi} = \Xi(\overrightarrow{\aleph})$, $\overrightarrow{\aleph}$ is any CIVqNSBS of Δ_1. Let $\Xi(\hbar, q), \Xi(\ell, q) \in \Delta_2$. Let $\hbar \in u\Xi^{-1}(\Xi(\hbar, q))$ and $v \in \Xi^{-1}(\Xi(\ell, q))$ be such that $\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar,q)} = \sup_{\hbar \in \Xi^{-1}(\Xi(\hbar,q))} \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar,q)}$ and $\overrightarrow{Z^{\intercal}_{\aleph}}(\ell, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell,q)} = \sup_{\hbar \in \Xi^{-1}(\Xi(\ell,q))} \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar,q)}$. Now,

$$\begin{aligned}
\overrightarrow{Z^{\intercal}_{\Psi}}((\Xi(\hbar) \sqcap_1 \Xi(\ell)), q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} &= \sup_{(\hbar',q) \in \Xi^{-1}(\Xi(\hbar,q)\sqcap_1\Xi(\ell,q))} \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar', q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar',q)} \\
&= \sup_{(\hbar',q) \in \Xi^{-1}(\Xi((\hbar\oslash_1\ell),q)} \overrightarrow{Z^{\intercal}_{\aleph}}(\hbar', q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar',q)} \\
&= \overrightarrow{Z^{\intercal}_{\aleph}}((\hbar \oslash_1 \ell), q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}((\hbar\oslash_1\ell),q)} \\
&\succeq \min\{\overrightarrow{Z^{\intercal}_{\aleph}}(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\hbar,q)}, \overrightarrow{Z^{\intercal}_{\aleph}}(\ell, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\aleph}}(\ell,q)}\} \\
&= \min\{\overrightarrow{Z^{\intercal}_{\Psi}}\Xi(\hbar, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}\Xi(\hbar,q)}, \overrightarrow{Z^{\intercal}_{\Psi}}\Xi(\ell, q)\cdot e^{i\Theta\overrightarrow{\Im^{\intercal}_{\Psi}}\Xi(\ell,q)}\}.
\end{aligned}$$

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Thus,$\overrightarrow{Z^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \succeq \min\{\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)}\}$.
Similarly,$\overrightarrow{Z^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_2\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_2\Xi(\ell)),q)} \succeq \min\{\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)}\}$
and
$\overrightarrow{Z^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_3\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_3\Xi(\ell)),q)} \succeq \min\{\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)}\}$.
Let $\hbar\in\Xi^{-1}(\Xi(\hbar,q))$ and $v\in\Xi^{-1}(\Xi(\ell,q))$ be such that $\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}(\hbar,q)} = \sup_{\hbar\in\Xi^{-1}(\Xi(\hbar,q))}\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}(\hbar,q)}$ and $\overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}(\ell,q)} = \sup_{\hbar\in\Xi^{-1}(\Xi(\ell,q))}\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)}$. Now,

$$\begin{aligned}
\overrightarrow{Z^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} &= \sup_{(\hbar',q)\in\Xi^{-1}(\Xi(\hbar,q)\sqcap_1\Xi(\ell,q))}\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar',q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar',q)}\\
&= \sup_{(\hbar',q)\in\Xi^{-1}(\Xi((\hbar\oslash_1\ell),q)}\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar',q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar',q)}\\
&= \overrightarrow{Z^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)}\\
&\succeq \frac{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar,q)}+\overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell,q)}}{2}\\
&= \frac{\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\hbar,q)}+\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\ell,q)}}{2}.
\end{aligned}$$

Thus,$\overrightarrow{Z^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \succeq \frac{\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\hbar,q)}+\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\ell,q)}}{2}$.
Similarly,$\overrightarrow{Z^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_2\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_2\Xi(\ell)),q)} \succeq \frac{\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\hbar,q)}+\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\ell,q)}}{2}$ and
$\overrightarrow{Z^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_3\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_3\Xi(\ell)),q)} \succeq \frac{\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\hbar,q)}+\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\ell,q)}}{2}$.
Let $\Xi(\hbar,q),\Xi(\ell,q)\in\Delta_2$. Let $\hbar\in\Xi^{-1}(\Xi(\hbar,q))$ and $v\in\Xi^{-1}(\Xi(\ell,q))$ be such that $\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar,q)} = \inf_{\hbar\in\Xi^{-1}(\Xi(\hbar,q))}\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar,q)}$ and $\overrightarrow{Z^{\Finv}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\ell,q)} = \inf_{\hbar\in\Xi^{-1}(\Xi(\ell,q))}\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar,q)}$. Now,

$$\begin{aligned}
\overrightarrow{Z^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} &= \inf_{(\hbar',q)\in\Xi^{-1}(\Xi(\hbar,q)\sqcap_1\Xi(\ell,q))}\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar',q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar',q)}\\
&= \inf_{(\hbar',q)\in\Xi^{-1}(\Xi((\hbar\oslash_1\ell),q)}\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar',q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar',q)}\\
&= \overrightarrow{Z^{\Finv}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}((\hbar\oslash_1\ell),q)}\\
&\preceq \max\{\overrightarrow{Z^{\Finv}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\Finv}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\aleph}}(\ell,q)}\}\\
&= \max\{\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\ell,q)}\}.
\end{aligned}$$

Thus,$\overrightarrow{Z^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \preceq \max\{\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\ell,q)}\}$.
Similarly,$\overrightarrow{Z^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_2\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_2\Xi(\ell)),q)} \preceq \max\{\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\ell,q)}\}$
and $\overrightarrow{Z^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_3\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \preceq \max\{\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\ell,q)},\overrightarrow{Z^{\Finv}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\Finv}_{\Psi}}\Xi(\ell,q)}\}$.
Thus, $\overrightarrow{\Psi}$ is a CIVqNSBS of Δ_2.

Theorem 3.17. *The homomorphic preimage of every CIVqNSBS is a CIVqNSBS.*

Proof. The mapping $\Xi:\Delta_1\to\Delta_2$ be a homomorphism. Now,$\Xi((\hbar\oslash_1\ell),q)=\Xi(\hbar,q)\sqcap_1\Xi(\ell,q),\Xi((\hbar\oslash_2\ell),q)=\Xi(\hbar,q)\sqcap_2\Xi(\ell,q)$ and $\Xi((\hbar\oslash_3\ell),q)=\Xi(\hbar,q)\sqcap_3\Xi(\ell,q)$ for all $\hbar,v\in\Delta_1$. Let $\overrightarrow{\Psi}=\Xi(\overrightarrow{\aleph}),\overrightarrow{\Psi}$ is a CIVqNSBS of Δ_2. Let $\hbar,v\in\Delta_1$. Now,$\overrightarrow{Z^{\mathsf{T}}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\aleph}}((\hbar\oslash_1\ell),q)} = \overrightarrow{Z^{\mathsf{T}}_{\Psi}}(\Xi((\hbar\oslash_1\ell),q))\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}(\Xi((\hbar\oslash_1\ell),q))} = \overrightarrow{Z^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \succeq \min\{\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\Psi}}\Xi(\ell,q)}\} = \min\{\overrightarrow{Z^{\mathsf{T}}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\mathsf{T}}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\aleph}}(\ell,q)}\}$. Thus,$\overrightarrow{Z^{\mathsf{T}}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\aleph}}((\hbar\oslash_1\ell),q)} \succeq \min\{\overrightarrow{Z^{\mathsf{T}}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\aleph}}(\hbar)},\overrightarrow{Z^{\mathsf{T}}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\mathsf{T}}_{\aleph}}(\ell,q)}\}$. Now,$\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)} = \overrightarrow{Z^{\daleth}_{\Psi}}(\Xi((\hbar\oslash_1\ell),q))\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\hbar\oslash_1\ell),q)} = \overrightarrow{Z^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\hbar\oslash_1\ell),q)} \succeq \frac{\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\hbar,q)}+\overrightarrow{Z^{\daleth}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}\Xi(\ell,q)}}{2} =$

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$\frac{\overrightarrow{Z^{\beth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\hbar,q)}+\overrightarrow{Z^{\beth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\ell,q)}}{2}$. Thus, $\overrightarrow{Z^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)} \succeq \frac{\overrightarrow{Z^{\beth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\hbar,q)}+\overrightarrow{Z^{\beth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\ell,q)}}{2}$. Now, $\overrightarrow{Z^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)} = \overrightarrow{Z^{\beth}_{\Psi}}(\Xi((\hbar\oslash_1\ell),q))\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}(\Xi((\hbar\oslash_1\ell),q))} = \overrightarrow{Z^{\beth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \preceq \max\{\overrightarrow{Z^{\beth}_{\Psi}}\Xi(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}\Xi(\hbar,q)},\overrightarrow{Z^{\beth}_{\Psi}}\Xi(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}\Xi(\ell,q)}\} = \max\{\overrightarrow{Z^{\beth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\beth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\ell,q)}\}$. Thus, $\overrightarrow{Z^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)} \preceq \max\{\overrightarrow{Z^{\beth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\beth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\ell,q)}\}$.

Theorem 3.18. *If $\Xi:\Delta_1\to\Delta_2$ is a homomorphism,then $\Xi(\overrightarrow{\aleph_{(\partial,\flat)}})$ is a subbisemiring of CIVqNSBS $\overrightarrow{\Psi}$ of Δ_2.*

Proof. The mapping $\Xi:\Delta_1\to\Delta_2$ be a homomorphism. Now, $\Xi((\hbar\oslash_1\ell),q)=\Xi(\hbar,q)\sqcap_1\Xi(\ell,q),\Xi((\hbar\oslash_2\ell),q)=\Xi(\hbar,q)\sqcap_2\Xi(\ell,q)$ and $\Xi((\hbar\oslash_3\ell),q)=\Xi(\hbar,q)\sqcap_3\Xi(\ell,q)$ for all $\hbar,v\in\Delta_1$. Let $\overrightarrow{\Psi}=\Xi(\overrightarrow{\aleph}),\overrightarrow{\aleph}$ is a CIVqNSBS of Δ_1. By Theorem 3.16, $\overrightarrow{\Psi}$ is a CIVqNSBS of Δ_2. Let $\overrightarrow{\aleph_{(\partial,\flat)}}$ be any subbisemiring of $\overrightarrow{\aleph}$. Suppose that $\hbar,v\in\overrightarrow{\aleph_{(\partial,\flat)}}$. Then $u\oslash_1 v,u\oslash_2 v$ and $u\oslash_3 v\in\overrightarrow{\aleph_{(\partial,\flat)}}$. Now, $\overrightarrow{Z^{\daleth}_{\Psi}}(\Xi(\hbar,q))\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\Xi(\hbar,q))} = \overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar,q)} \succeq \partial, \overrightarrow{Z^{\daleth}_{\Psi}}(\Xi(\ell,q))\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\Xi(\ell,q))} = \overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell,q)} \succeq \partial$. Thus, $\overrightarrow{Z^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \succeq \overrightarrow{Z^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)} \succeq \partial$. Now, $\overrightarrow{Z^{\gimel}_{\Psi}}(\Xi(\hbar,q))\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\Psi}}(\Xi(\hbar,q))} = \overrightarrow{Z^{\gimel}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}(\hbar,q)} \succeq \partial, \overrightarrow{Z^{\gimel}_{\Psi}}(\Xi(\ell,q))\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\Psi}}(\Xi(\ell,q))} = \overrightarrow{Z^{\gimel}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}(\ell,q)} \succeq \partial$. Thus, $\overrightarrow{Z^{\gimel}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \succeq \overrightarrow{Z^{\gimel}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\oslash_1\ell),q)} \succeq \partial$. Now, $\overrightarrow{Z^{\beth}_{\Psi}}(\Xi(\hbar,q))\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}(\Xi(\hbar,q))} = \overrightarrow{Z^{\beth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\hbar,q)} \preceq \flat, \overrightarrow{Z^{\beth}_{\Psi}}(\Xi(\ell,q))\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}(\Xi(\ell,q))} = \overrightarrow{Z^{\beth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\ell,q)} \preceq \flat$. Thus, $\overrightarrow{Z^{\beth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \preceq \overrightarrow{Z^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)} \preceq \flat$, for all $\Xi(\hbar,q),\Xi(\ell,q)\in\Delta_2$. Similarly other operations, $\Xi(\overrightarrow{\aleph_{(\partial,\flat)}})$ is a subbisemiring of CIVqNSBS $\overrightarrow{\Psi}$ of Δ_2.

Theorem 3.19. *If $\Xi:\Delta_1\to\Delta_2$ is any homomorphism,then $\overrightarrow{\aleph_{(\partial,\flat)}}$ is a subbisemiring of CIVqNSBS $\overrightarrow{\aleph}$ of Δ_1.*

Proof. The mapping $\Xi:\Delta_1\to\Delta_2$ be any homomorphism. We have $\Xi((\hbar\oslash_1\ell),q)=\Xi(\hbar,q)\sqcap_1\Xi(\ell,q),\Xi((\hbar\oslash_2\ell),q)=\Xi(\hbar,q)\sqcap_2\Xi(\ell,q)$ and $\Xi((\hbar\oslash_3\ell),q)=\Xi(\hbar,q)\sqcap_3\Xi(\ell,q)$ for all $\hbar,v\in\Delta_1$. Let $\overrightarrow{\Psi}=\Xi(\overrightarrow{\aleph}),\overrightarrow{\Psi}$ is a CIVqNSBS of Δ_2. By Theorem 3.17, $\overrightarrow{\aleph}$ is a CIVqNSBS of Δ_1. Let $\Xi(\overrightarrow{\aleph_{(\partial,\flat)}})$ be a subbisemiring of $\overrightarrow{\Psi}$. Suppose that $\Xi(\hbar,q),\Xi(\ell,q)\in\Xi(\overrightarrow{\aleph_{(\partial,\flat)}})$. Now, $\Xi((\hbar\oslash_1\ell),q),\Xi((\hbar\oslash_2\ell),q)$ and $\Xi((\hbar\oslash_3\ell),q)\in\Xi(\overrightarrow{\aleph_{(\partial,\flat)}})$. Now, $\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar,q)} = \overrightarrow{Z^{\daleth}_{\Psi}}(\Xi(\hbar,q))\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\Xi(\hbar,q))} \succeq \partial, \overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell,q)} = \overrightarrow{Z^{\daleth}_{\Psi}}(\Xi(\ell,q))\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\Psi}}(\Xi(\ell,q))} \succeq \partial$. Thus, $\overrightarrow{Z^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}((\hbar\oslash_1\ell),q)} \succeq \min\{\overrightarrow{Z^{\daleth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\daleth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\daleth}_{\aleph}}(\ell,q)}\} \succeq \partial$. Now, $\overrightarrow{Z^{\gimel}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}(\hbar,q)} = \overrightarrow{Z^{\gimel}_{\Psi}}(\Xi(\hbar,q))\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\Psi}}(\Xi(\hbar,q))} \succeq \partial, \overrightarrow{Z^{\gimel}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}(\ell,q)} = \overrightarrow{Z^{\gimel}_{\Psi}}(\Xi(\ell,q))\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\Psi}}(\Xi(\ell,q))} \succeq \partial$. Thus, $\overrightarrow{Z^{\gimel}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}((\hbar\oslash_1\ell),q)} \succeq \frac{\overrightarrow{Z^{\gimel}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}(\hbar,q)}+\overrightarrow{Z^{\gimel}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\gimel}_{\aleph}}(\ell,q)}}{2} \succeq \partial$. Now, $\overrightarrow{Z^{\beth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\hbar,q)} = \overrightarrow{Z^{\beth}_{\Psi}}(\Xi(\hbar,q))\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}(\Xi(\hbar,q))} \preceq \flat, \overrightarrow{Z^{\beth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\ell,q)} = \overrightarrow{Z^{\beth}_{\Psi}}(\Xi(\ell,q))\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}(\Xi(\ell,q))} \preceq \flat$. Thus, $\overrightarrow{Z^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}((\hbar\oslash_1\ell),q)} = \overrightarrow{Z^{\beth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\Psi}}((\Xi(\hbar)\sqcap_1\Xi(\ell)),q)} \preceq \max\{\overrightarrow{Z^{\beth}_{\aleph}}(\hbar,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\hbar,q)},\overrightarrow{Z^{\beth}_{\aleph}}(\ell,q)\cdot e^{i\Theta\overrightarrow{\Im^{\beth}_{\aleph}}(\ell,q)}\} \preceq \flat$, for all $\hbar,v\in\Delta_1$. Similarly other operations, $\overrightarrow{\aleph_{(\partial,\flat)}}$ is a subbisemiring of CIVqNSBS $\overrightarrow{\aleph}$ of Δ_1.

4 Conclusion and future direction

A novel kind of neutrosophic SBSis presented in this work. The concept of three grades is approached in a novel way by the complex neutrosophic subbisemiring, which describes three grades in terms of a complex number. A complicated form of three grades signifies a paradigm change since it converts three grades into a two dimensional parameter. Neutrosophic SBSwith complicated interval values was defined. The concepts of CIVqNNSBS and CIVqNSBS level sets were established. Applying the Q-cubic set and anti Q-cubic set to bisemiring is our aim. Furthermore, an analysis of the properties of various transformations is conducted. We are attempting to handle images and signals in addition to data analysis, thus it makes sense to employ F-transforms to widen the use of new fuzzy structures. Therefore, in the future, we should think about utilizing cubic subbisemiring, soft set CIVqNSBS, and soft set CIVqNNSBS.

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