On Two New Algorithms for Solving Mixed Integer Linear Programming Problems and Their Applications

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Abstract:

The objective of this paper is to introduce two new algorithms for dealing with mixed integer linear programming problems, where the first method will be applied to get the efficient cut in the standard cutting plane procedure to obtain the same optimal solution. The second method will be applied with many special conditions to get the global solution instead of the local solution by using cutting-plane and other famous algorithms. On the other hand, we compare our results to the other obtained results by applying other algorithms.

Keywords: Numerical method; Linear programming; Cutting plane; Efficient cut

1. Introduction

The question of correct linear programming is a usual linear programming question on the condition that all or some of the decision variables are constrained and have an integer value, the methods of solving models with integers are based mainly on two principles:

The first principle: is that the correct models cope at the first stage using the simplified method in the solution.

The second principle: is to adopt the final results that represent the optimal solution as the basis for reaching the best solution that makes the values of the basic variables integer values (non-fractional). New additional constraints are then selected that share or provide valid values for the original variables in the model.

This research is focused: first: on our selection of new additional constraints and by adding these new constraints our new model is formed. Secondly: test the optimal solution for linear integer programming on the basis of that positional convergence or overall convergence of the solution.

Here it should be noted that the addition of new entries comes sequentially so that one entry is added to the model and then the calculations are performed as in the (simplified) method. When the solution is incomplete (i.e., the best solution is not found), another new constraint is added and so on until the correct values of the basic variables in the model are reached [2].

2. Mathematical programming:

Mathematical programming (M.P) is one of the methods used in the formulation and resolution of administrative and economic issues by creating a production plan for different actors to increase profits or reduce costs, the general problem of mathematical programming (M.P) is to find the values of variables that maximize or minimize the value of the objective function depending on constraints representing the amount of available resources [1].

2.1. The general formula of the mathematical programming problem is as follows:

$$z = c_1x_1 + c_2x_2 + \dots\dots\dots + c_nx_n \qquad \dots (1)$$

DOI: https://doi.org/10.54216/IJAACI.060106

Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

This objective function is subject to a number of determinants or constraints (i.e. conditions), which take the following form:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{1n}x_n \leq = \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{2n}x_n \leq = \geq b_2 \\ \\ a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{mn}x_n \leq = \geq b_m \end{array}\right\} \quad (2)$$

It is noticeable that the variables x_j are real variables, that is, $(x_1, x_2, \ldots\ldots, x_n \geq 0)$ and that (a_{ij}, b_j, c_j) are constant quantities [3].
With this formula, the problem aims to find a set of positive numbers of variables covered by the system that satisfies the above constraints, the objective function – the set of constraints or conditions-includes all variables with non-negative values [8].

2.2. Integer linear programming problem:

The most scientific applications of linear programming problems (LP) require a solution represented by integers, for example, in production problems It is not possible to complete the production of a car and a half car or a leather bag and a quarter bag, and on this basis the correct linear programming appeared [1] (ILP) Which is a matter of linear programming so that all decision variables (truth) or some of them are constrained to be integer-valued، More precisely, it is a matter of (increasing or decreasing) a linear function (objective function) that is invariant to integer-valued variables and fulfills Equation (2), In general the question of correct programming is more difficult to optimally solve the question of linear programming because we can solve the integer programming problem by solving a sequence of linear programming problems. In the question of integer programming, we will call any integer value of the variables that satisfies (2) the positive solution(feasible solution) And we can call the positive solution Group the positive region (feasible region) for the question of correct programming, so MIN (MAX) the question of correct linear programming is a question of finding the positive solution where the objective function is greater (smaller) or equal to the value of all other existing positive solutions, this solution is called the optimal solution, which is our main goal.
The question of integer linear programming is solved first by ignoring the conditions of variables to take integer values, the problem is then solved as a linear programming problem to find the optimal real solution, then we use the correct linear programming problem algorithms to move from the real solution towards finding the correct positive solutions that is, Solutions in which all variables are true, Correct linear programming algorithms try to find the correct positive solution and then look for another, better solution [4].

2.3. Methods for solving the problem of correct linear programming (ILP)

There are several methods that have developed to solve the problem of integer linear programming (ILP) Most of these methods are based on ignoring the integer constraint to solve the problem and then the fractional values of the variables are processed if they exist the reason is due to the fact that the process of reaching the optimal solution to the problem of correct linear programming (ILP) is carried out through the optimal solution to the problem of linear programming (LP), the optimal values of the optimal solution of the two problems converge and sometimes equal them and that this leads to the reduction of calculations to arrive at the optimal solution of the problem of correct programming[4].

2.4. The most commonly used methods are:

A) The Cutting – Planes Method

It is a unique method for solving the problem of integer linear programming, which was published by the scientist [6] RALPH E. GOMORY was named after him GOMORY, CUTTING PLANES METHOD The basic idea of the method of cutting levels is simple for the question of correct linear programming, the question of linear programming is exactly the question of correct programming but without conditions on which variables the solution is valid. If the solution consists of only integer values, it will be the optimal solution for the correct programming problem and no new constraint for the correct programming problem will increase. This new constraint is added so that all correct solutions to the old constraint satisfy the new constraint and the incorrect optimal solution still does not satisfy this constraint. And in the geometric solution, the new constraint added to the linear programming problem will cut off only the incorrect solution and not the correct points that could be positive solutions to the correct programming

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

problem, theoretically (cutting) the incorrect solution by adding one constraint at a time and repeating until the optimal solution consisting of the correct values is reached [4].

The algorithm of cutting levels:

Step 1: (eliminal solution) start by solving the problem given in (1) as a linear programming problem by the odd Simplex Method, ignoring the correct conditions (constraints) on the variables $x_j, j \in I$. If all variables have values, stop, otherwise go to step 2.

Step 2: (constraint selection) choose the line from the last table of solving the linear programming problem in which the base variable $x_{Bi}$is an incorrect value b_i (use the line in which the value of that variable has the largest fractional part, perhaps it will help to reduce the number of iterations and the time it takes to converge) and from it generate the cut-levels constraint.

Step 3: (generate the cut-levels constraint) suppose the chosen line is line i and its equation is:

$$x_{Bi} + \sum_j a_{ij} x_j = b_j,\ j \in I \qquad (3)$$

$$x_{Bi} + \sum_j ([a_{ij}] + f_{ij}) x_j = [b_j] + f_i$$

$$x_{Bi} + \sum_j [a_{ij}] x_j - [b_i] = f_i - \sum_j f_{ij} x_j \leq 0$$

The new entry $$f_i - \sum_j f_{ij} x_j + \delta = 0 \qquad (4)$$

Where $f_{ij} = a_{ij} - [a_{ij}]$is the fractional part of $a_{ij}, 0 \leq f_{ij} \prec 1$

$f_i = b_i - [b_i]$ is the fractional part of $b_i, 0 \leq f_i \prec 1$

δ a new degenerate variable is possible and correct.

Step 4: (testing the most efficient cutter) we test the cutters that we get from Step (3) to get the most efficient cutter in reaching the optimal solution in the fewest steps by compensating for the values of the Slack Variable that we get from the constraints of the original problem in the constraints of cutting the levels that we obtained to obtain straight lines that cut the area of possible solutions and then the cutter that cuts a larger part of that area is selected.

Step 5: (Add constraint) add constraint (4) to the last table of the solution of the odd method and solve it as a question of linear programming, if all the variables $x_j, j \in I$ are correct then the problem is over, otherwise go to step 2 [5] .

The correct programming algorithm:

The philosophy of the correct programming method (Zubaidi) is based on two Tests: the first test is to test the objective function in terms of the fact that the value of the objective function of the correct programming problem (ILP) is less or equal to the value of the objective function in the case of maximization or greater or equal to the value of the objective function in the case of minimization of the linear programming problem (LP) and the second Test is to test the verification of constraints [1].

The steps of this algorithm:

Step 1: finding the optimal solution to the problem without taking into account the integer constraint.

Step 2: we select constraints affecting the model for which the shadow price values are non-zero values.

Step 3: in the case that the optimal solution is represented by higher fractional values of the decision variables, the variable with the lower fractional value is chosen, let it be (x).

Step 4: the first stage: consists in giving x a value represented by the smallest integer greater than its incorrect value, while giving the rest of the variables with the incorrect value values represented by the largest integer less than its incorrect values.

Step 5: calculate the Z_1value for the first stage by substituting the values selected in Step 4 with the target function.

Step 6: calculate $\overline{Z} = Z - Z_1$so that the value of $\overline{Z}$ should be greater or equal to zero in the case of maximization and less or equal to zero in the case of minimization and vice versa we stop and proceed to the second stage.

Step 7: after calculating $\overline{Z}$, we test the verification of the constraints affecting the model by multiplying the coefficients of the left side of the constraints by the increase or decrease of the values of the variables resulting from Step 4.

Step 8: if the constraint signal is smaller or equal, the values calculated in Step 7 must be less or equal to zero and greater or equal to zero if the constraint signal is larger or equal and vice versa, we proceed to the second stage.

Step 9: we choose the constraint with the higher non-zero value resulting from Step 7, if the constraint signal is smaller or equal, but if the constraint signal is larger or equal, we choose the constraint with the lower non-zero value and assume that the value is (K).

Step 10: calculate the value of Q for the constraint, which represents the coefficients of the left side of the constraint

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

less than or equal to $|k|$ in the case that the constraint signal is smaller or equal to and less than or equal to K in the case that the constraint signal is larger or equal to and also the difference between the coefficients of the left-hand side of the constraint that satisfies the condition, we will assume that the values of Q are:

$Q = (\boldsymbol{a_{11}}, \boldsymbol{a_{12}} - \boldsymbol{a_{11}})$

Step 11: in the absence of values for Q, the solution of the first stage represents the optimal solution.

Step 12: calculate the values of $\overline{Q}$ which represent the coefficients of the objective function so that:

The constraint signal is smaller or equal to $\overline{Q} = (c_1, c_2 - c_1)$

The constraint signal is greater or equal to $\overline{Q} = (-c_1, c_1 - c_2)$

Step 13: test the values of $\overline{Q}$ greater than zero, which are smaller or equal to $\overline{Z}$ in the case of maximization, and then choose the higher value, but in the case of minimization, the values of $\overline{Q}$ smaller than zero, which are larger or equal to $\overline{Z}$, are chosen, and vice versa, the solution of the first stage represents the optimal solution.

Step 14: assuming that the value of $\overline{Q}$ chosen in Step 13 is $(c_2 - c_1)$, this means that we move to a later stage that represents an increase in the value of the variable x_2 and a decrease in the value of the variable x_1 by one unit with the values of the other variables constant.

Step 15: the previous steps are recalculated starting from Step 4.

Step 16: the second stage consists in giving X a value representing the largest integer smaller than the incorrect value of X, while giving the rest of the variables values represented by the smallest integer larger than the incorrect values for them, and the previous steps are recalculated.

Step 17: in the event that the first and second stages do not represent the optimal solution, values are given for x and the variables are represented by the largest integer less than their incorrect values.

The previous steps are recalculated.

B) New Mixed Integer Linear Programming:

The method of mixed integer programming requires to formulate it to make some modification in the problem or (algorithm), such as adding some indicative conditions or using some additional variables [6] we would like to point out that the new mixed integer programming results from adding conditions to the level-cutting algorithm and the Zubaidi integer programming algorithm.

The first method is new mixed integer programming:

It is a very successful technique in solving a wide range of correct programming issues and is to provide a guarantee of reaching the optimal solution. We will explain how to perform this technique. A large number of correct linear programming problems can be solved by the new method, the algorithm of which consists exactly of the combination between the method of cutting levels and the Zubaidi correct programming method, and it works as its predecessor in solving a series of (sequential) linear programming problems. The most efficient cutter in the method of cutting levels is tested in two ways:

First: using the line in which the value of that variable has the largest fractional part (of the optimal solution) and from it we generate the constraint of cutting levels.

Secondly, by means of a geometric solution, since the new constraint added to the linear programming problem will (cut off) only the incorrect solution and not the correct points from which it is possible to have positive solutions to the correct programming problem. Theoretically, the incorrect solution is to add one constraint at a time and repeat until the optimal consisting of the correct values is obtained. The new method is equivalent to the first case and better than the second case, because the process of drawing straight lines with two variables is easy, and if it is more than the two variables, it is difficult to draw straight lines, and it may be inaccurate, and this leads to slow convergence and may not lead to the optimal solution.

The first algorithm for new mixed integer programming:

Step (1): (eliminal solution) start by solving the problem given in (1) as a question of linear programming by the individual method Simplex Method, ignoring the correct conditions (constraints) on variables. If all variables $x_j, j \in I$ have integer values, stop, otherwise go to Step2.

Step (2): (choosing a constraint) choose the line from the last table of solving the linear programming problem in

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

which the base variable x_{Bi} has an incorrect value b_i (use the line in which the value of that variable has the largest fractional part, perhaps it will help reduce the number of iterations and the time it takes to converge) and from it generate the constraint of cutting levels.

Step (3): (generate the cut-off constraint of the levels) suppose the chosen line is line i and its equation is:

$$x_{Bi} + \sum_j a_{ij}x_j = b_j, \qquad j \in I$$

$$x_{Bi} + \sum_j ([a_{ij}] + f_{ij})x_j = [b_j] + f_i$$

$$x_{Bi} + \sum_j [a_{ij}]x_j - [b_i] = f_i - \sum_j f_{ij}x_j \leq 0$$

The new entry $$f_i - \sum_j f_{ij}x_j + \delta = 0 \qquad (5)$$

Where $f_{ij} = a_{ij} - [a_{ij}]$is the fractional part of $a_{ij}, 0 \leq f_{ij} \prec 1$

$f_i = b_i - [b_i]$ is the fractional part of $b_i, 0 \leq f_i \prec 1$

δ a new degenerate variable is possible and correct.

Step (4): after generating the cut-level constraints, test the verification of the constraints affecting the model by multiplying the coefficients of the left side of the constraints in increments or decrements for the values of the variables resulting from Step 2.

Step (5): if the constraint signal is smaller or equal, the values calculated in Step 4 must be less or equal to zero and greater or equal to zero if the constraint signal is larger or equal.

Step (6): Select the constraint with the higher non-zero value resulting from Step 4, if the constraint signal is smaller or equal, but if the constraint signal is larger or equal, select the constraint with the lower non-zero value.

Step (7): (add the constraint) add the constraint we got from Step 6 to the last table of solving the individual method and solve it as a linear programming problem, if all the variables $x_j, j \in I$ are correct, then the problem is over, otherwise go to step 2.

From Steps (4) to (7), the new cutter is selected for the next iteration and does not depend on the two methods of testing the most efficient cutter used in the planar cutting algorithm.

Second method of new mixed integer programming (NEW 2):

The following are the required iterative steps in the new proposed algorithm as follows:

The second algorithm of new mixed integer programming:

Step (1): Find the optimal solution to the problem without taking into account the integer constraint.

Step (2): we select constraints affecting the model for which the shadow price values are non-zero values.

Step (3): in the case that the optimal solution is represented by higher fractional values of the decision variables, the variable with the lower fractional value is chosen, let it be (x).

Step (4): the first stage: consists in giving x a value represented by the smallest integer greater than its incorrect value, while giving the rest of the variables with the incorrect value values represented by the largest integer less than its incorrect values.

Step (5): calculate the Z_1value for the first stage by substituting the values selected in Step 4 with the target function.

Step (6): calculate $\overline{Z} = Z - Z_1$so that the value of $\overline{Z}$ should be greater or equal to zero in the case of maximization and less or equal to zero in the case of minimization and vice versa we stop and proceed to the second stage.

Step (7): after calculating $\overline{Z}$, we test the verification of the constraints affecting the model by multiplying the coefficients of the left side of the constraints by the increase or decrease of the values of the variables resulting from Step 4.

Step (8): if the constraint signal is smaller or equal, the values calculated in Step 7 must be less or equal to zero and greater or equal to zero if the constraint signal is larger or equal and vice versa, we proceed to the second stage.

Step (9): we choose the constraint with the higher non-zero value resulting from Step 7, if the constraint signal is smaller or equal, but if the constraint signal is larger or equal, we choose the constraint with the lower non-zero value and assume that the value is (K).

Step (10): calculate the value of Q for the constraint, which represents the coefficients of the left side of the constraint less than or equal to $|k|$ in the case that the constraint signal is smaller or equal to and less than or equal to K in the case that the constraint signal is larger or equal to and also the difference between the coefficients of the left-hand side of the constraint that satisfies the condition, we will assume that the values of Q are:

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Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

$Q = (\boldsymbol{a_{11}}, \boldsymbol{a_{12}} - \boldsymbol{a_{11}})$

Step (11): in the absence of values for Q, the solution of the first stage represents the optimal solution.

Step (12): calculate the values of $\overline{Q}$ which represent the coefficients of the objective function so that:

The constraint signal is smaller or equal to $\overline{Q} = (c_1, c_2 - c_1)$

The constraint signal is greater than or equal to $\overline{Q} = (-c_1, c_1 - c_2)$

Step (13): test the values of $\overline{Q}$ greater than zero, which are smaller or equal to $\overline{Z}$ in the case of maximization, and then choose the higher value, but in the case of minimization, the values of $\overline{Q}$ smaller than zero, which are larger or equal to $\overline{Z}$, are chosen, and vice versa, the solution of the first stage represents the optimal solution.

Step (14): assuming that the value of $\overline{Q}$ chosen in Step 13 is $(c_2 - c_1)$, this means that we move to a later stage that represents an increase in the value of the variable x_2 and a decrease in the value of the variable x_1 by one unit with the values of the other variables constant.

Step (15): the previous steps are recalculated starting from Step 4.

Step (16): the second stage consists in giving X a value that represents the largest integer smaller than the incorrect value of X, while giving the rest of the variables values represented by the smallest integer larger than the incorrect values for them, and the previous steps are recalculated.

Step (17): in the event that the first and second stages do not represent the optimal solution, values are given for x and the variables are represented by the largest integer less than their incorrect values. The previous steps are recalculated.

Step (18): the third stage is to give $x_i, i = 1,2,\ldots.,n$ zeros except *i* =1, and then give x_i except *i* =2 and so on. And other values of x are calculated by dividing the value of Z in the optimal solution of the problem by the coefficient x that is in the objective function.

Step (19): calculate the Z_i value of the third stage by substituting the values selected in step 18 with the target function.

Step (20): calculate $\overline{Z} = Z - Z_i$ so that the value of $\overline{Z}$ should be greater or equal to zero in the case of maximization and less or equal to zero in the case of minimization and vice versa the solution obtained in the last stage is the optimal solution.

Step (21): After calculating $\overline{Z}$, we test the verification of the constraints affecting the model by multiplying the coefficients of the left side of the constraints by the increase or decrease of the values of the variables resulting from step 18.

Step (22): if the constraint signal is smaller or equal, the values calculated in step 21 should be less or equal to zero and greater or equal to zero if the constraint signal is larger or equal, if the constraints are met, we compare Z_i with the value of Z in the last step, if $Z_i \prec Z$ is maximized, the values of Z_i is not the optimal solution, but in the case of minimization, $Z_i \prec Z$, the lowest value of Z_i represents the optimal solution. In this algorithm of steps (22)-(18) a comprehensive optimal solution is always obtained, while the Zubaidi algorithm can obtain a local or comprehensive optimal solution.

Illustrative examples:

The new proposed methods were applied to many examples in order to demonstrate their efficiency. In this paragraph we will review some of these examples to clarify:

Example (1): solve the following correct linear programming problem:

$$\boldsymbol{Max\ Z = 5x_1 + 8x_2}$$
$$\boldsymbol{s.t\ \ x_1 + x_2 \leq 6}$$
$$\boldsymbol{5\,x_1 + 9x_2 \leq 45}$$
$$\boldsymbol{x_1, x_2} \geq 0 \qquad \textit{and integers}$$

Solution: Simplex Method

$$\boldsymbol{Min\ U = -5x_1 - 8x_2}$$
$$\boldsymbol{s.t\ \ x_1 + x_2} + \delta_1 \boldsymbol{= 6}$$
$$\boldsymbol{5\,x_1 + 9x_2} + \delta_2 \boldsymbol{= 45}$$
$$\boldsymbol{x_1, x_2}, \delta_1, \delta_2 \geq 0$$

Table 1

Basic	$\boldsymbol{b_i}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	δ_1	δ_2
x_1	9/4	1	0	9/4	-1/4
x_2	15/4	0	1	-5/4	1/4
- *U*	165/7	0	0	5/4	3/4

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

The optimal solution to the problem of linear programming without taking into account the integer constraint is:

$x_1 = \mathbf{9/4},\qquad x_2 = 15/4,\qquad Z = 41\frac{1}{4}$

Using the first method for the new mixed integer programming:

From table 1 we get:

$$-Z - \frac{5}{4}\delta_1 - \frac{3}{4}\delta_2 = -41\frac{1}{4}$$
$$x_1 + \frac{9}{4}\delta_1 - \frac{1}{4}\delta_2 = \frac{9}{4}$$
$$x_2 - \frac{5}{4}\delta_1 + \frac{1}{4}\delta_2 = \frac{15}{4}$$
$$x_1, x_2, \delta_1, \delta_2 \geq 0$$

The first cutter: $-Z - 2\delta_1 - \delta_2 + 42 = \frac{3}{4} - \frac{3}{4}\delta_1 - \frac{1}{4}\delta_2$

The second cutter: $x_1 + 2\delta_1 - \delta_2 - 2 = \frac{1}{4} - \frac{1}{4}\delta_1 - \frac{3}{4}\delta_2$

The third cutter: $x_1 - 2\delta_1 - 3 = \frac{3}{4} - \frac{3}{4}\delta_1 - \frac{1}{4}\delta_2$

Cutters first = third $\frac{3}{4} - \frac{3}{4}\delta_1 - \frac{1}{4}\delta_2 \leq 0$

Second $\frac{1}{4} - \frac{1}{4}\delta_1 - \frac{3}{4}\delta_2 \leq 0$

Testing the most efficient cutter among the cutters extracted from the issue:

The first cutter is $\frac{3}{4} - \frac{3}{4}\delta_1 - \frac{1}{4}\delta_2 + \delta_3 = 0$, $\delta_3 \geq 0$

The second cutter is $\frac{1}{4} - \frac{1}{4}\delta_1 - \frac{3}{4}\delta_2 + \delta_4 = 0$, $\delta_4 \geq 0$

from the original problem we extract the values δ_1, δ_2 in the following form:

$\delta_1 = 6 - x_1 - x_2$ (6)

$\delta_2 = 45 - 5x_1 - 9x_2$ (7)

We substitute equations (6) and (7) in the first and second cutouts to find the basic cutouts in the matter, which are denoted by the variables in the matter After compensation we get:

$$2x_1 + 3x_2 \leq 15 \ldots\ldots\ldots\ldots\ldots\ldots\ldots c_1$$

$$4x_1 + 7x_2 \leq 35 \ldots\ldots\ldots\ldots\ldots\ldots\ldots c_2$$

We test the most efficient cutter by compensating the amount of increase or decrease in the values of x_1, x_2 in the cutters as follows:

$$(\frac{-1}{4})2 + (-\frac{3}{4})3 = -2\frac{3}{4}$$
$$(\frac{-1}{4})4 + (-\frac{3}{4})7 = -6\frac{1}{4}$$

Since the most efficient cutter is the first cutter, so we use the first cutter to solve the problem:

$$\frac{3}{4} - \frac{3}{4}\delta_1 - \frac{1}{4}\delta_2 \leq 0$$
$$\frac{3}{4} - \frac{3}{4}\delta_1 - \frac{1}{4}\delta_2 + \delta_3 = 0 \ , \qquad \delta_3 \geq 0$$
$$-\frac{3}{4}\delta_1 - \frac{1}{4}\delta_2 + \delta_3 = -\frac{3}{4}$$

Table 2

Basic	b_i	x_1	x_2	δ_1	δ_2	δ_3
x_1	9/4	1	0	9/4	-1/4	0
x_2	15/4	0	1	-5/4	1/4	0
- *U*	165/7	0	0	5/4	3/4	0
δ_3	-3/4	0	0	-3/4	-1/4	1

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

Table 3

Basic	b_i	x_1	x_2	δ_1	δ_2	δ_3
x_1	0	1	0	0	-1	3
x_2	5	0	1	0	2/3	-5/3
- U	40	0	1	0	-1/3	-5/3
δ_1	1	0	0	1	1/3	-4/3

Hence $\boldsymbol{x_1} = 0, \quad \boldsymbol{x_2} = \mathbf{5}, \quad \boldsymbol{Z} = \mathbf{40}$

Using the second method for new mixed integer programming:

Table 1 represents the optimal solution to the problem without taking into account the integer constraint, which is:

$$\boldsymbol{x_1} = 2\frac{1}{4}, \quad \boldsymbol{x_2} = \mathbf{3}\frac{3}{4}, \quad \boldsymbol{Z} = \mathbf{41}\frac{1}{4}$$

The values of the shadow prices are $\delta_2 = \frac{3}{4}\, \delta_1 = 1\frac{1}{4}$, which means that the two constraints affect the model.

The first stage: We choose the variable $\boldsymbol{x_2}$ because it has the highest fraction, that is, that:

$$\boldsymbol{x_1} = 2, \quad \boldsymbol{x_2} = \mathbf{4}, \quad \boldsymbol{Z_1} = 42$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_1} = 41\frac{1}{4} - 42 = -\frac{3}{4}$$

Since the value is negative, we proceed directly to the second stage.

The second stage:

$$\boldsymbol{x_1} = 3, \quad \boldsymbol{x_2} = \mathbf{3}, \quad \boldsymbol{Z_2} = 39$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_2} = 41\frac{1}{4} - 39 = 2\frac{1}{4}$$

We test the verification of the first and second entries as follows:

1) $(\frac{3}{4})1 - (\frac{3}{4})1 = 0$
2) $(\frac{3}{4})5 - (\frac{3}{4})9 = -3$

Since the values (0,-3) are smaller or equal to zero, this means that the constraints are checked and from the values (0,-3)The second constraint is tested to calculate the values of Q_2:

$Q_2 =$…………..

The solution:

$$\boldsymbol{x_1} = 3, \quad \boldsymbol{x_2} = \mathbf{3}, \quad \boldsymbol{Z_2} = 39$$

The third stage:

$$\boldsymbol{x_1} = 0, \quad \boldsymbol{x_2} = \mathbf{5}, \quad \boldsymbol{Z_3} = 40$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_3} = 41\frac{1}{4} - 40 = 1\frac{1}{4}$$

We test check constraints:

1) $-1(\frac{9}{4}) + 1(\frac{5}{4}) = -1$
2) $-5(\frac{9}{4}) + 9(\frac{5}{4}) = 0$

Since the values are less than or equal to zero, this means that the constraints are checked:

$$Q_1 = (a_{11}, a_{12} - a_{11}, a_{11} - a_{12})$$

$$\overline{Q}_1 = \ldots\ldots\ldots\ldots..$$

The fourth stage:

$$\boldsymbol{x_1} = 8, \quad \boldsymbol{x_2} = \mathbf{0}, \quad \boldsymbol{Z_4} = 40$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_4} = 41\frac{1}{4} - 40 = 1\frac{1}{4}$$

We test check constraints:

1) $1(\frac{23}{4}) - 1(\frac{15}{4}) = 2$
2) $5(\frac{23}{4}) - 9(\frac{15}{4}) = -\frac{30}{4}$

Since not all values are less than or equal to zero, this means that the constraints are not fulfilled so the point does not represent a possible solution to the issue. So the optimal solution is: $\boldsymbol{x_1} = 0, \quad \boldsymbol{x_2} = \mathbf{5}, \quad \boldsymbol{Z} = 40$

Example (2): Solve the following correct linear programming problem:

$$MinZ = 2x_1 - 3x_2 - 3x_3$$

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

$$s.t \quad -x_1 + x_2 + 3x_3 \le 8$$
$$3x_1 + 2x_2 - x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

The solution: Simplex Method

Table 4

Basic	b_i	x_1	x_2	x_3	δ_1	δ_2
x_3	6/7	-5/7	0	1	2/7	-1/7
x_2	38/7	8/7	1	0	1/7	3/7
Z	132/7	23/7	0	0	9/7	6/7

For an optimal solution to the problem of linear programming without taking into account the integer constraint is:

$$x_3 = 6/7, \quad x_2 = 38/7, \quad Z = -132/7$$

Using the first method of new mixed integer programming:

We get from table 4

$$Z + \frac{9}{7}\delta_1 + \frac{6}{7}\delta_2 + \frac{23}{7}x_1 = \frac{132}{7}$$
$$x_1 + \frac{1}{7}\delta_1 + \frac{3}{7}\delta_2 + \frac{8}{7}x_1 = \frac{38}{7}$$
$$x_3 + \frac{2}{7}\delta_1 - \frac{1}{7}\delta_2 - \frac{5}{7}x_1 = \frac{6}{7}$$
$$x_1, x_2, x_3, \delta_1, \delta_2 \ge 0$$

the first cutter $\quad Z + 3x_1 + \delta_1 - 18 = \frac{6}{7} - \frac{2}{7}x_1 - \frac{2}{7}\delta_1 - \frac{6}{7}\delta_2 \le 0$

the second cutter $\quad x_2 + x_1 - 5 = \frac{3}{7} - \frac{1}{7}x_1 - \frac{1}{7}\delta_1 - \frac{3}{7}\delta_2 \le 0$

the third cutter $\quad x_3 - x_1 - \delta_2 = \frac{6}{7} - \frac{2}{7}x_1 - \frac{2}{7}\delta_1 - \frac{6}{7}\delta_2 \le 0$

Cutters

The first = the third $\quad \frac{6}{7} - \frac{2}{7}x_1 - \frac{2}{7}\delta_1 - \frac{6}{7}\delta_2 \le 0$

The second $\quad \frac{3}{7} - \frac{1}{7}x_1 - \frac{1}{7}\delta_1 - \frac{3}{7}\delta_2 \le 0$

Testing the most efficient cutter among the cutters extracted from the issue:

the first cutter $\quad \frac{6}{7} - \frac{2}{7}x_1 - \frac{2}{7}\delta_1 - \frac{6}{7}\delta_2 + \delta_3 = 0, \quad \delta_3 \ge 0$

the second cutter $\quad \frac{3}{7} - \frac{1}{7}x_1 - \frac{1}{7}\delta_1 - \frac{3}{7}\delta_2 + \delta_4 = 0, \quad \delta_4 \ge 0$

hence:

$$\delta_1 = 8 + x_1 - x_2 - 3x \quad \ldots(8)$$
$$\delta_9 = 10 - 3x_1 - 2x_2 + x_3 \quad \ldots(9)$$

We substitute equations (9),(8) in the first and second cutouts to find the basic cutouts in the matter, which are denoted by the variables in the matter

$$2x_1 + 2x_2 \le 10 \ldots\ldots\ldots\ldots\ldots\ldots c_1$$

$$x_1 + x_2 \le -5 \ldots\ldots\ldots\ldots\ldots\ldots c_2$$

We get:

To find the most efficient cutter, we compensate for the decrease in the values of x_1, x_2, x_3, in cutters:

$$2(0) + 2(-\frac{3}{7}) + 0 = -\frac{6}{7}$$
$$1(0) + 1(-\frac{3}{7}) + 0 = -\frac{3}{7}$$

If the most efficient cutter is the second cutter, then we solve the question using the second cutter:

$$\frac{3}{7} - \frac{1}{7}x_1 - \frac{1}{7}\delta_1 - \frac{3}{7}\delta_2 \le 0$$
$$\frac{3}{7} - \frac{1}{7}x_1 - \frac{1}{7}\delta_1 - \frac{3}{7}\delta_2 + \delta_3 = 0, \quad \delta_3 \ge 0$$
$$-\frac{1}{7}x_1 - \frac{1}{7}\delta_1 - \frac{3}{7}\delta_2 + \delta_3 = -\frac{3}{7}$$

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

Table 5

Basic	b_i	x_1	x_2	x_3	δ_1	δ_2	δ_3
x_3	6/7	-5/7	0	1	2/7	-1/7	0
x_2	38/7	8/7	1	0	1/7	3/7	0
Z	132/7	23/7	1	0	9/7	6/7	0
δ_3	3/7	-1/7	0	0	-1/7	-3/7	1

Table 6

Basic	b_i	x_1	x_2	x_3	δ_1	δ_2	δ_3
x_3	1	-3/5	0	1	2/5	0	-5
x_2	5	4/5	1	0	-1/5	0	3/5
Z	18	3	0	0	1	0	2
δ_2	1	1/3	0	0	1/3	1	-7/3

then $\boldsymbol{x_1} = 0,\ \boldsymbol{x_2} = 5,\ \boldsymbol{x_3} = \mathbf{1},\ \boldsymbol{Z} = -18$

Using the second method of new mixed integer programming:

Table 4 represents the optimal solution to the problem without taking into account the integer constraint, which is:

$$\boldsymbol{x_1} = 0, \quad \boldsymbol{x_2} = \frac{38}{7}, \quad \boldsymbol{x_3} = \frac{6}{7}, \quad \boldsymbol{Z} = \frac{-132}{7}$$

The values of the shadow prices are

$$\delta_2 = \frac{5}{7} ، \delta_1 = \frac{11}{7}$$

which means that the two constraints affect the model.

The first stage: We choose the variable $\boldsymbol{x_3}$ because it has the highest fraction, that is:

$$\boldsymbol{x_1} = 0, \boldsymbol{x_2} = 5, \boldsymbol{x_3} = 1, \boldsymbol{Z_1} = \mathbf{-18}$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_1} = -18\frac{6}{7} - (-18) = -\frac{6}{7}$$

We test check constraints:

$$-(0) - (\frac{3}{7}) + 3(\frac{1}{7}) = 0$$

$$3(0) - 2(\frac{3}{7}) + (\frac{1}{7}) = -1$$

Since the values (0,-1) are smaller or equal to zero, this means that the constraints are checked and from the values (0,-1)The second constraint is tested to calculate the values of Q_2:

$$Q_2 = (a_{21} - a_{22}) = 1$$

$$\overline{Q}_2 = (c_1 - c_2) = 5$$

solution: $\boldsymbol{x_1} = 0,\ \boldsymbol{x_2} = 5,\ \boldsymbol{x_3} = \mathbf{1},\ \boldsymbol{Z} = -18$

The second stage:

$$\boldsymbol{x_1} = 9, \boldsymbol{x_2} = 0, \boldsymbol{x_3} = \mathbf{0}, \boldsymbol{Z_2} = \mathbf{18}$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_2} = -18\frac{6}{7} - 18 = -36\frac{6}{7}$$

We test the verification of the first and second entries as follows:

1) $-(9) + (\frac{-38}{7}) + 3(\frac{-6}{7}) = \frac{-119}{7}$
2) $3\frac{3}{4} + 2(\frac{-38}{7}) - (\frac{-6}{7}) = \frac{119}{7}$

The third stage:

$$\boldsymbol{x_1} = 0, \boldsymbol{x_2} = 6, \boldsymbol{x_3} = \mathbf{0}, \boldsymbol{Z_3} = \mathbf{-18}$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_2} = -18\frac{6}{7} - (-18) = -\frac{6}{7}$$

We test check constraints:

1) $-(0) + (\frac{4}{7}) + 3(\frac{-6}{7}) = -2$
2) $3(0) + 2(\frac{4}{7}) - (\frac{-6}{7}) = 2$

The fourth stage:

$$\boldsymbol{x_1} = 0, \boldsymbol{x_2} = 0, \boldsymbol{x_3} = \mathbf{6}, \boldsymbol{Z_4} = \mathbf{-18}$$

$$\overline{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{Z_4} = -18\frac{6}{7} - (-18) = -\frac{6}{7}$$

DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

We test check constraints:

1) $-(0)+(\frac{-38}{7})+3(\frac{36}{7})=10$
2) $3(0)+2(\frac{-38}{7})-(\frac{36}{7})=\frac{-112}{7}$

the optimal solution is: $x_1=0, x_2=5, x_3=1, Z_3=-18$

3. Results:

Based on the planar cutting algorithm and the Zubaidi algorithm, the two new algorithms NEW2, NEW1, we include the results of a number of nonlinear functions that have been solved by the aforementioned algorithms, and these algorithms all depend on the optimal solution.

Table 7: The results of a number of nonlinear functions

	PROBLEM	CUTTING PLANES	ALIH.	NEW 1	NEW 2
1.	$MaxZ=5x_1+2x_2$ $s.t\ \ 5x_1+4x_2\leq 20$ $x_1,x_2\geq 0$	20	20	20	20
2.	$MaxZ=5x_1+8x_2$ $s.t\ \ x_1+x_2\leq 6$ $5x_1+9x_2\leq 45$ $x_1,x_2\geq 0$	40	39	40	40
3.	$MaxZ=3x_1+4x_2$ $s.t\ \ 2x_1+x_2\leq 6$ $2x_1+3x_2\leq 9$ $x_1,x_2\geq 0$	12	10	12	12
4.	$MaxZ=5x_1+4x_2$ $s.t\ \ x_1+x_2\leq 5$ $10x_1+6x_2\leq 45$ $x_1,x_2\geq 0$	23	23	23	23
5.	$MinZ=2x_1-3x_2-3x_3$ $s.t\ \ -x_1+x_2+3x_3\leq 8$ $x_1+2x_2-3x_3\leq 18$ $x_1,x_2,x_3\geq 0$	-18	-18	-18	-18
6.	$MaxZ=4x_1+5x_2+7x_3$ $s.t\ \ 3x_1+2x_2+3x_3\leq 20$ $3x_1+5x_2+7x_3\leq 30$ $2x_1+3x_2+4x_3\leq 0$ $x_1,x_2,x_3\geq 0$	33	33	33	33

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DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024

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DOI: https://doi.org/10.54216/IJAACI.060106
Received: November 11, 2023 Revised: January 27, 2023 Accepted: June 19, 2024