Some Results on Single Valued Neutrosophic (Weak) Polygroups

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Abstract

Polygroups are a generalized concept of groups and the concept of single valued neutrosophic set is a generalization of the classical notion of a set. The objective of this paper is to combine the innovative concept of single valued neutrosophic sets and polygroups. In this regard, we introduce the concepts of single valued neutrosophic polygroups and anti- single valued neutrosophic polygroups. Moreover, we investigate their properties and study the relation between level sets of single valued neutrosophic polygroups and (normal) subpolygroups.

Keywords: Polygroup, Weak polygroup, Single valued neutrosophic set, Single valued neutrosophic polygroup, Anti- single valued neutrosophic polygroup

1. Introduction

Florentin Smarandache introduced Neutrosophic sets in 1998 [9], which is the generalization of the fuzzy sets introduced by Lotfi Zadeh in 1965 [13]. Where the latter proposed fuzzy sets as mathematical model of vagueness where elements belong to a given set to some degree that is typically a number that belongs to the unit interval [0,1]. Neutrosophy is a base of Neutrosophic logic which is an extension of fuzzy logic in which indeterminancy is included. In Neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, percentage of indeterminancy in a subset I, and the percentage of falsity in a subset F. A single valued neutrosophic set is an instance of neutrosophic set which can be used in real scientfic and engineering problems. Therefore, the study of single valued neutrosophic sets and their properties have a considerable significance in the sense of applications as well as in understanding the fundamentals of uncertainty.

Algebraic hyperstructures represent a natural generalization of classical algebraic structures and they were introduced by Frederic Marty [8] in 1934 at the eighth Congress of Scandinavian Mathematicians. Where he generalized the notion of a group to that of a hypergroup. In a group, the composition of two elements is an element whereas in a hypergroup, the composition of two elements is a set. Hypergroups have been used in algebra, geometry, convexity, automata theory, combinatorial problems of coloring, lattice theory, Boolean algebras, logic etc., over the years. Comer [5] introduced a special class of hypergroups, using the name of polygroups. He emphasized the importance of polygroups, by analyzing them in connections to graphs, relations, Boolean and cylindric algebras.

The combination between both: fuzzy sets and algebraic hyperstructures, and between neutrosophic sets and algebraic hyperstructures have attracted the attention of many researchers working in these domains and as a result, new branches of research were invented, namely, fuzzy algebraic hyperstructures and neutrosophic algebraic hyperstructures. For more details, we refer to the work done in [1-4, 11, 12].

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As a generalization of (anti-) fuzzy polygroups, this paper combines the notion of single valued neutrosophic set with polygroups to get single valued neutrosophic polygroups and it is constructed as follows: After an Introduction, in Section 2 and Section 3, we present some basic results about single valued neutrosophic sets and about (weak) polygroups. In Section 4, we introduce the notion of single valued nuetrosophic (weak) polygroup and investigate its properties. And finally in Section 5, we study the relation between level sets of single valued neutrosophic polygroups and (normal) subpolygroups.

2. Single valued neutrosophic sets

In this section, we present some basic results about single valued neutrosophic sets.

Single valued neutrosophic sets are generalization of classical sets, fuzzy sets, intuitionistic fuzzy sets and paraconsistent sets, etc.

Definition 2.1. [12] Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$.

Example 2.1. Assume that $X = \{x_1, x_2, x_3\}$, x_1 is importance, x_2 is trustworthiness and x_3 is availability. The values of x_1, x_2 and x_3 are in [0,1]. They are obtained from the questionnaire about flu shot, their option could be a degree of "good effect", a degree of indeterminacy and a degree of "poor effect". A is a single valued neutrosophic set of X defined by $A = \langle 0.4,0.3,0.5\rangle/x_1 + \langle 0.1,0.6,0.3\rangle/x_2 + \langle 0.6,0.2,0.3\rangle/x_3$. And B is a single valued neutrosophic set of X defined by $B = \langle 0.5,0.2,0.4\rangle/x_1 + \langle 0.3,0.6,0.7\rangle/x_2 + \langle 0.5,0.4,0.5\rangle/x_3$.

Definition 2.2. [12] The complement of a single valued neutrosophic set A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - I_A(x)$, $F_{c(A)}(x) = T_A(x)$, for all x in X.

Example 2.2. Let A be the SVNS present in Example 2.1. Then

$$c(A) = \langle 0.5,0.7,0.4\rangle/x_1 + \langle 0.3,0.4,0.1\rangle/x_2 + \langle 0.3,0.8,0.6\rangle/x_3.$$

Definition 2.3. [12] Let A and B be single valued neutrosophic sets. Then

- A is contained in B, denoted as $A \subseteq B$, if and only of $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$ for all x in X.
- A and B are equal, written as $A = B$, if and only if A ⊆ B and B ⊆ A.
- The union of A and B is a single valued neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity-membership unctions are related to those of A and B by $T_C(x) = \max(T_A(x), T_B(x))$, $I_C(x) = \max(I_A(x), I_B(x))$, and $F_C(x) = \min(F_A(x), F_B(x))$ for all x in X.
- The intersection of A and B is a single valued neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership unctions are related to those of A and B by $T_C(x) = \min(T_A(x), T_B(x))$, $I_C(x) = \min(I_A(x), I_B(x))$, and $F_C(x) = \max(F_A(x), F_B(x))$ for all x in X.

Example 2.3. Let A and B be the SVNS present in Example 2.1. Then

$$A \cap B = \langle 0.4,0.2,0.5\rangle/x_1 + \langle 0.1,0.6,0.7\rangle/x_2 + \langle 0.5,0.2,0.5\rangle/x_3 \text{ and}$$
$$A \cup B = \langle 0.5,0.3,0.4\rangle/x_1 + \langle 0.3,0.6,0.3\rangle/x_2 + \langle 0.6,0.4,0.3\rangle/x_3.$$

3. (Weak) Polygroups

In this section, we present some definitions and examples related to (weak) polygroups that are used throughout the paper. For more details, we refer to [6, 7].

Let H be a non-empty set and $P^*(H)$ be the collection of all non-empty subsets of H. Ad define " $*$ " as follows:

$$*: H \times H \to P^*(H)$$
$$(x, y) \to x * y$$

Then " $*$ " is called a hyperoperation and $(H,*)$ is called a hypergroupoid.

Definition 3.1.[5] Let $(P,*)$ be a hypergroupoid. Then $(P,*)$ is a polygroup if the following are satisfied for all a, b, c in P.

(1) $a * (b * c) = (a * b) * c$,
(2) There exists e in P with $e * a = a * e = a$ for all a in P,
(3) $x \in y * z$ implies $y \in x * z^{-1}$ and $z \in y^{-1} * x$.

Weak polygroups are generalization of polygroups and they are defined in the same way as polygroups but instead of (1) in Definition 3.1, we have $a * (b * c) \cap (a * b) * c \neq \emptyset$.

In a (weak) polygroup P, $(x^{-1})^{-1} = x$ for all $x \in P$.

Remark 3.1. Every group is a (weak) polygroup.

We present examples on polygroups that are not groups.

Example 3.1. Let $P_1 = \{0, 1\}$. Then $(P_1,*)$ defined in Table 1 is a polygroup with 0 serving as an identity.

Table 1. The polygroup $(P_1,*)$

$*$	0	1
0	0	1
1	1	P_1

Example 3.2. [7] Let $P_2 = \{e, a, b, c\}$. Then $(P_2,.)$ defined in Table 2 is a polygroup with e serving as an identity.

Table 2. The polygroup $(P_2,.)$

.	e	a	b	c
e	e	a	b	c
a	a	e	b	c
b	b	b	{e, a, c}	{b, c}
c	c	c	{b, c}	{e, a, b}

Example 3.3. [7] Let $P_3 = \{e, a, b, c\}$. Then $(P_3,\circ)$ defined in Table 3 is a weak polygroup with e serving as an identity. Moreover, it is not a polygroup.

Table 3. The weak polygroup $(P_3,\circ)$

○	e	a	b	c
e	e	a	b	c
a	a	{e,a}	c	b
b	b	c	{e, b}	a
c	c	b	a	{e, c}

Definition 3.2. [7] Let $(P,*)$ be a polygroup. A subset S of P is subpolygroup of P if and only if $(S,*)$ is a polygroup.

Proposition 3.1. [7] Let $(P,*)$ be a polygroup. A subset S of P is subpolygroup of P if and only $x * y \subseteq S$ and $x^{-1} \in S$ for all $x, y \in S$.

Definition 3.3. [7] Let $(P,*)$ be a polygroup. A subset subpolygroup S of P is a normal subpolygroup of P if $x^{-1} * P * x \subseteq$ P for all $x \in P$.

Example 3.4. Let $(P_2, .)$ be the polygroup in Example 3.2. Then $\{e\}$ and $\{e, a\}$ are subpolygroups of P_2 that are not normal.

4. Single valued neutrosophic (weak) polygroups, A construction

In this section, we define single valued neutrosophic polygroups and investigate its properties.

Definition 4.1. [6] Let $(P,*)$ be a polygroup and A a fuzzy set over P with a fuzzy membership function μ. Then A is called a fuzzy polygroup over P if for all $x, y \in P$, the following conditions are satisfied.

(1) $\mu(z) \geq \min\{\mu(x), \mu(y)\}$ for all $z \in x * y$,
(2) $\mu(x^{-1}) \geq \mu(x)$.

Remark 4.1. [6] Intersection of fuzzy polygroups over P is a fuzzy polygroup.

Definition 4.2. Let $(P,*)$ be a (weak) polygroup and A a SVNS over P. Then A is called a single valued neutrosophic polygroup (SVNP) over P (single valued neutrosophic weak polygroup (SVNWP) over P) if for all $x, y \in P$, the following conditions are satisfied.

(1) $T_A(z) \geq \min\{T_A(x), T_A(y)\}$, $I_A(z) \geq \min\{I_A(x), I_A(y)\}$, and $F_A(z) \leq max\{F_A(x), F_A(y)\}$ for all $z \in x * y$,
(2) $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \geq I_A(x)$, and $F_A(x^{-1}) \leq F_A(x)$.

Example 4.1. Let $(P_1,*)$ be the polygroup present in Example 3.1 and $A = \langle 0.4,0.5,0.5\rangle/0 + \langle 0.1,0.3,0.7\rangle/1$ is a SVNP over P_1.

Example 4.2. Let $P_3 = \{e, a, b, c\}$ and $(P_3,\circ)$ be the weak polygroup defined in Example 3.3. Then
$A = \langle 0.4,0.5,0.5\rangle/e + \langle 0.1,0.3,0.7\rangle/a + \langle 0.1,0.25,0.9\rangle/b + \langle 0.1,0.25,0.9\rangle/c$ is a SVNWP over P_3.

Remark 4.2. All the theorems and results in this paper that are valid for SVNP are also valid for SVNWP. So, we restrict our results to SVNP.

Proposition 4.1. Let $(P,*)$ be a polygroup and A a SVNP over P. Then the following hold for all $x \in P$.

1. $T_A(x^{-1}) = T_A(x), I_A(x^{-1}) = I_A(x),$ and $F_A(x^{-1}) = F_A(x)$;
2. $T_A(e) \geq T_A(x), I_A(e) \geq I_A(x),$ and $F_A(e) \leq F_A(x)$ where e is the identity in P.

Proof. Let $x \in P$.
Proof of 1.: Definition 4.2 implies that $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \geq I_A(x)$, and $F_A(x^{-1}) \leq F_A(x)$. And having $(x^{-1})^{-1} = x$ implies that $T_A(x) \geq T_A(x^{-1})$, $I_A(x) \geq I_A(x^{-1})$, and $F_A(x) \leq F_A(x^{-1})$. Thus, $T_A(x^{-1}) = T_A(x)$, $I_A(x^{-1}) = I_A(x)$, and $F_A(x^{-1}) = F_A(x)$.
Proof of 2.: Since $e \in x * x^{-1}$, it follows by Definition 4.2 (1) that $T_A(e) \geq \min\,(T_A(x), T_A(x^{-1})) = T_A(x)$, $I_A(e) \geq \min\big(I_A(x), I_A(x^{-1})\big) = I_A(x)$, and $F_A(e) \leq \max\big(F_A(x), F_A(x^{-1})\big) = F_A(x)$.

Example 4.3. Let $(P_2,.)$ be the polygroup present in Example 3.2. Then $A = \langle 0.4,0.5,0.5\rangle/e + \langle 0.5,0.3,0.7\rangle/a + \langle 0.5,0.3,0.7\rangle/b$ is not a SVNP over P_2 as $T_A(e) \geq T_A(a)$ does not hold.

Proposition 4.2. Let $(P,*)$ be a polygroup, A a SVNS over P, and $A^{-1} = \{\langle T_A(x^{-1}), I_A(x^{-1}), F_A(x^{-1})\rangle/x : x \in P\}$. If A is a SVNP over P then $A^{-1} = A$.
Proof. The proof follows from Proposition 4.1.

Proposition 4.3. Let $(P,*)$ be a polygroup and t_1, t_2, t_3 be numbers in the unit interval [0, 1]. If $A = \{\langle t_1, t_2, t_3 \rangle/x : x \in P\}$. Then A is a SVNP over P.

Proof. The proof is straightforward.

Remark 4.3. The SVNP present in Proposition 4.2 is called the **constant SNVP**.

Theorem 4.1. Let $(P,*)$ be a polygroup and A a SVNS over P. Then A and $c(A)$ are SVNP over P if and only if A is the constant SVNP.

Proof. If A is the constant SVNP over P then $c(A)$ is also the constant SVNP over P.
Let A and $c(A)$ be SVNP. Then for all x in P, we have:

$$T_A(e) \geq T_A(x), I_A(e) \geq I_A(x), \text{ and } F_A(e) \leq F_A(x), \quad \text{(I)}$$
$$F_A(e) \geq F(x), 1 - I_A(e) \geq 1 - I_A(x), \text{ and } T_A(e) \leq T_A(x) \quad \text{(II)}$$

(I) and (II) implies that $T_A(e) = T_A(x), I_A(e) = I_A(x),$ and $F_A(e) = F_A(x)$. Thus, A is the constant SVNP over P.

Definition 4.3. [6] Let $(P,*)$ be a polygroup and A a fuzzy set over P with membership function μ. Then A is called anti-fuzzy polygroup over P if for all $x, y \in P$, the following conditions are satisfied.

(1) $\mu(z) \leq \max\{\mu(x), \mu(y)\}$ for all $z \in x * y$,
(2) $\mu(x^{-1}) \leq \mu(x)$.

Remark 4.4. [6] Union of anti-fuzzy polygroups over P is an anti-fuzzy polygroup.

Definition 4.4. Let $(P,*)$ be a polygroup and A a SVNS over P. Then A is called an anti-single valued neutrosophic polygroup (ASVNP) over P if for all $x, y \in P$, the following conditions are satisfied.

(1) $T_A(z) \leq \max\{T_A(x), T_A(y)\}$, $I_A(z) \leq \max\{I_A(x), I_A(y)\}$, and $F_A(z) \geq min\{F_A(x), F_A(y)\}$ for all $z \in x * y$,

(2) $T_A(x^{-1}) \leq T_A(x), I_A(x^{-1}) \leq I_A(x)$, and $F_A(x^{-1}) \geq F_A(x)$.

Proposition 4.4. Let $(P,*)$ be a polygroup and A an ASVNP over P. Then the following hold for all $x \in P$.

(1) $T_A(x^{-1}) = T_A(x), I_A(x^{-1}) = I_A(x)$, and $F_A(x^{-1}) = F_A(x)$;

(2) $T_A(e) \leq T_A(x), I_A(e) \leq I_A(x)$, and $F_A(e) \geq F_A(x)$ where e is the identity in P.

Proof. The proof is similar to that of Proposition 4.1.

Example 4.3. Let $(P_1,*)$ be the polygroup present in Example 3.1 and $A = \langle 0.4,0.5,0.9\rangle/0 + \langle 0.5,0.5,0.7\rangle/1$ is an ASVNP over P_1.

Theorem 4.2. Let $(P,*)$ be a polygroup and A be a SVNS over P. Then A is a SVNP over P if and only if T_A and I_A are fuzzy polygroups over P and F_A is an anti-fuzzy polygroup over P.

Proof. The proof follows from the definition of SVNP, fuzzy polygroups, and anti-fuzzy polygroups.

Theorem 4.3. Let $(P,*)$ be a polygroup and A be a SVNS over P. Then A is an ASVNP over P if and only if T_A and I_A are anti-fuzzy polygroups over P and F_A is a fuzzy polygroup over P.

Proof. The proof follows from the definition of ASVNP, fuzzy polygroups, and anti-fuzzy polygroups.

Theorem 4.4. Let $(P,*)$ be a polygroup and A be a SVNS over P. Then A is a SVNP over P if and only if $c(A)$ is an ASVNP over P.

Proof. Let A be a SVNP. Theorem 4.2 asserts that T_A and I_A are fuzzy polygroups over P and F_A is an anti-fuzzy polygroup over P. We get now that $T_{c(A)} = F_A$ and $I_{c(A)} = 1 - I_A$ are anti-fuzzy polygroups over P and $F_{c(A)} = T_A$ is a fuzzy polygroup over P. Theorem 4.3. completes the proof. Similarly, we can prove that if $c(A)$ is an ASVNP over P then A is a SVNP.

Corollary 4.1. Let $(P,*)$ be a polygroup and A_α be a SVNS over P. If A_α is a SVNP over P then $\bigcap_{\alpha\in\Gamma} A_\alpha$ is SVNP over P.

Corollary 4.2. Let $(P,*)$ be a polygroup and A_α is a SVNS over P. If A_α is an ASVNP over P then $\bigcap_{\alpha\in\Gamma} A_\alpha$ is an ASVNP over P.

5. Level sets of single valued neutrosophic (weak) polygroups

In this section, we define level sets of single valued neutrosophic polygroups and relate them to (normal) subpolygroups.

Definition 5.1. Let X be any set, $t = (t_1, t_2, t_3)$ where $0 \leq t_1, t_2 < 1$ and $0 < t_3 \leq 1$, and A be a SVNS over X. Then $A_t = \{x \in X: T_A(x) \geq t_1, I_A(x) \geq t_2, F_A(x) \leq t_3\}$ is called a t-level set of A.

Theorem 5.1. Let $(P,*)$ be a polygroup and A be a SVNS over P. Then A is a SVNP over P if and only if $A_t \neq \emptyset$ is a subpolygroup of P for every $t = (t_1, t_2, t_3)$ where $0 \leq t_1, t_2 < 1$ and $0 < t_3 \leq 1$.

Proof. Let A be a SVNP over P and $x, y \in A_t \neq \emptyset$. For all $z \in x * y$, we have $T_A(z) \geq \min\{T_A(x), T_A(y)\} \geq t_1$, $I_A(z) \geq \min\{I_A(x), I_A(y)\} \geq t_2$, and $F_A(z) \leq max\{F_A(x), F_A(y)\} \leq t_3$. Thus, $x * y \subseteq A_t$. Moreover, having $T_A(x^{-1}) \geq T_A(x) \geq t_1$, $I(x^{-1}) \geq I_A(x) \geq t_2$, and $F_A(x^{-1}) \leq F_A(x) \leq t_3$ implies that $x^{-1} \in A_t$. Thus, A_t is a subpolygroup of P.

Conversely, let $A_t \neq \emptyset$ be a subpolygroup of P and $x, y \in P$. Set $t_1 = \min\{T_A(x), T_A(y)\}$, $t_2 = \min\{I_A(x), I_A(y)\}$, $t_3 = max\{F_A(x), F_A(y)\}$, and $t = (t_1, t_2, t_3)$ Since A_t is a subpolygroup of P, it follows that $x * y \subseteq A_t$ and $x^{-1} \in A_t$. The latter implies that for all $z \in x * y$, $T_A(z) \geq t_1 = \min\{T_A(x), T_A(y)\}$, $I_A(z) \geq t_2 = \min\{I_A(x), I_A(y)$, and $F_A(z) \leq t_3 = max\{F_A(x), F_A(y)\}$. Thus, Condition (1) of Definition 4.1. is satisfied. Moreover, we have $T_A(x^{-1}) \geq t_1 = T_A(x)$, $I(x^{-1}) \geq t_2 = I_A(x)$, and $F_A(x^{-1}) \leq t_3 = F_A(x)$. Thus, Condition (2) of Definition 4.1. is satisfied. Therefore, A is a SVNP over P.

Corollary 5.1. Let $(P,*)$ be a polygroup and A be a SVNP over P. Then P has no non-trivial proper subpolygroups if and only if the constant SNVP and $A = \{\langle t_1, t_2, t_3\rangle/x + \langle t_1', t_2', t_3'\rangle/e : x \neq e \in P\}$ where $t_1 \leq t_1'$, $t_2 \leq t_2'$, and $t_3 \geq t_3'$ are the only SVNP over P.

Example 5.1. Let $P_1 = \{0, 1\}$ and $(P_1,*)$be the polygroup defined in Example 3.1. Then the constant SNVP and $A = \langle t_1, t_2, t_3\rangle/1 + \langle t_1', t_2', t_3'\rangle/0$ where $t_1 \leq t_1'$, $t_2 \leq t_2'$, and $t_3 \geq t_3'$ are the only SVNP over P_1.

Notation 5.1. Let $t = (t_1, t_2, t_3)$ and A a SVNS of P. Then by $A(x) = t$, we mean that $T_A(x) = t_1$, $I_A(x) = t_2$, and $F_A(x) = t_3$. And by $A(x) \leq t$, we mean that $T_A(x) \leq t_1$, $I_A(x) \leq t_2$, and $F_A(x) \geq t_3$.

Theorem 5.2. Let $(P,*)$ be a polygroup. Then every subpolygroup of P is a level set of a SVNP over P.

Proof. Let S be a subpolygroup of P and $t = (t_1, t_2, t_3)$ where $0 < t_1, t_2 < 1$ and $0 < t_3 < 1$. Define the SVNS over P as follows:

$$A(x) = \begin{cases} (t_1, t_2, t_3) & if\ x \in S, \\ (0,0,1) & otherwise. \end{cases}$$

Let $t' = (t_1', t_2', t_3')$. Then $A_{t'} = \begin{cases} S & if\ t_1 \geq t_1', t_2 \geq t_2', and\ t_3 \leq t_3' \\ P & if\ t_1' = 0, t_2' = 0, and\ t_3' = 1, \\ \emptyset & otherwise. \end{cases}$ is either $\emptyset$ or a subpolygroup of P.

Using Theorem 5.1, we get that A is a SVNP over P.

Definition 5.2. Let $(P,*)$ be a polygroup and A be a SVNP over P. Then A is said to be a normal SVNP over P if $A(z) = A(z')$ for all $z \in x \star y, z' \in y \star x$.

Example 5.2. Let $(P,*)$ be a polygroup and A be a SVNP over P. Then the constant SNVP is a normal SNVP over P.

Theorem 5.3. Let $(P,*)$ be a polygroup and A is a SVNS over P. Then A is a normal SVNP over P if and only if $A_t \neq \emptyset$ is a normal subpolygroup of P for every $t = (t_1, t_2, t_3)$ where $0 \leq t_1, t_2 < 1$ and $0 < t_3 \leq 1$.

Proof. Let A be a normal SVNP over P and $x, y \in A_t \neq \emptyset$. Theorem 5.1 asserts that $A_t \neq \emptyset$ is a subpolygroup of P. Let $x \in P$. We need to show that $x^{-1} * A_t * x \subseteq A_t$. Let $z \in x^{-1} * A_t * x$. Then there exist y in A_t such that $z \in x^{-1} * y * x$ and hence $z \in x^{-1} * p$ where $p \in y * x$. The latter implies that $y \in p * x^{-1}$. And since A is a normal SVNP over P, it follows that $A(z) = A(y)$. Thus, $z \in A_t$.

Conversely, let $A_t \neq \emptyset$ be a normal subpolygroup of P. Theorem 5.1 asserts that A is a SVNP over P. To show that A is a normal SVNP over P, it suffices to show that $A(z) = A(z')$ for all $z \in x \star y, z' \in y \star x$. Let $z \in x \star y, z' \in y \star x$ with $A(z') = t$. Having $z' \in y \star x$ implies that $y \in z' \star x^{-1}$. The latter implies that Let $z \in x \star z' * x^{-1}$. Since $z' \in A_t$ and $A_t \neq \emptyset$ is a normal subpolygroup of P, it follows that $z \in A_t$ and hence, $A(z) \geq A(z') = t$. Similarly, we get that $A(z') \geq A(z)$.

Corollary 5.2. Let $(P,*)$ be a polygroup and A be a SVNP over P. Then P has no proper normal subpolygroups if and only if the constant SNVP is the only normal SNVP over P.

Example 5.3. Let $P_2 = \{e, a, b, c\}$ and $(P_2, .)$ be the polygroup defined in Example 3.2. Then the constant SNVP is the only normal SVNP over P_2.

Theorem 5.4. Let $(P,*)$ be a polygroup. Then every normal subpolygroup of P is a level set of a normal SVNP over P.

Proof. The proof is the same as that of Theorem 5.2.

Corollary 5.5. Let $(P,*)$ be a polygroup and A be a SVNP over P. Then $A^* = \{x \in P: A(x) = A(e)\}$ is a subpolygroup of P. Moreover, if A is a normal SVNP over P then A^* is a normal subpolygroup of P.

Proof. Let $t = A(e)$. Then $A_t = \{x \in P: T_A(x) \geq T_A(e), I_A(x) \geq I_A(e), F_A(x) \leq F_A(e)\}$. Proposition 4.1, 2. asserts that $A_t = \{x \in P: T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)\} = A^*$. Theorem 5.1 and Theorem 5.3 complete the proof.

6. Conclusion

This paper has introduced an algebraic hyperstructure of single valued neutrosophic sets in the form of single valued neutrosophic polygroups and anti- single valued neutrosophic polygroups. Several interesting properties of the new defined notions were discussed. The results of this paper can be considered as a generalization for the work related to fuzzy polygroups.

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