MAGNIFICATION OF MBJ-NEUTROSOPHIC TRANSLATION ON G-ALGEBRA

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Abstract

In this article, we define the MBJ-neutrosophic magnified translation (MBJNMT) on G-algebra which is the combination of multiplication and translation and study significant results of MBJ-neutrosophic ideal and MBJ-neutrosophic subalgebra by using the notion of MBJ-neutrosophic magnified translation. We investigate the conversion of MBJ-neutrosophic ideal and MBJ-neutrosophic subalgebra with one another and use the idea of intersection and union to produce some important results of MBJ-neutrosophic magnified translation.

Keywords: G-algebra, MBJ-neutrosophic magnified translation.

1.Introduction

Zadeh [1] presented the theory of fuzzy set in 1965. Iseki and Tanaka [2] gave the idea of BCK-algebra in 1978. Iseki [3] introduced the notion of BCI-algebra in 1980 and it is obvious that BCK-algebra is a proper sub class of the class of BCI-algebra. Lee et al. [4] investigated the fuzzy translation, (normalized, maximal) fuzzy extension and fuzzy multiplication of fuzzy subalgebra in BCK/BCI-algebra. Link among fuzzy translation, fuzzy extension and fuzzy multiplication are also studied. Ansari et al. [5] introduced the idea of fuzzy translation, fuzzy extension and fuzzy multiplication of fuzzy β ideal of β-algebra and investigated important characteristic. Priya et al. [6] studied the PS-ideal and its level subsets. Bandaru et al. [7] defined the G-algebra. Lekkoksung [8] focused on fuzzy magnified translation in ternary hemirings, which is a extension of BCI / BCK/Q / KU / d-algebra. Senapati et al. [9] have done extensive research on intuitionistic fuzzy H-ideal in BCK/BCI-algebra. Jana et al. [10] interogated the intuitionistic fuzzy G-subalgebra. Senapati et al. [11] studied fuzzy translations of fuzzy H-ideals in BCK/BCI-algebra. Atanassov [12] introduced the intuitionistic fuzzy sets. Senapati [13] investigated the relationship among intuitionistic fuzzy translation, intuitionistic fuzzy extension and intuitionistic fuzzy multiplication in B-algebra. Kim et al. [14] defined the intuitionistic fuzzy structure of B-algebra. Senapati et al. [15] discussed the fuzzy dot subalgebra and fuzzy dot ideal of B-algebras. Priya et al. [16] has done research on fuzzy translation and fuzzy multiplication in PS-algebra. Chandramouleeswaran et al. [17] investigated fuzzy translation and fuzzy multiplication in BF/BG-algebra. Smarandache [18,19] extended the intuitionistic set to neutrosophic set through Several examples. Jun et al. [20] investigated the commutative falling neutrosophic ideals in BCK-algebra. C. H. Park [21] defined the neutrosophic

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ideal in subtraction algebra and studied it through several properties. Khalid et al. [22] did the research on neutrosophic soft cubic subalgebra through significant characteristic like P-union, R-intersection etc. Khalid et al. [23] interestingly worked on intuitionistic fuzzy translation and multiplication through subalgebra and ideals. Khalid et al. [24] defined the T-neutrosophic cubic set and studied this set through ideals and subalgebras and investigated many results. Takallo et al. [25] defined and studied MBJ-neutrosophic structures and its applications in BCK/BCI-algebras Khalid et al. [26] interpreted the multiplication of neutrosophic cubic set and defined the ϒ-multiplication of neutrosophic cubic set and studied it with neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal and neutrosophic cubic closed normal ideal. He also studied ϒ-multiplication under homomorphism and cartesian product through significant characteristics. Khalid et al. [27] defined and studied the MBJ-neutrosophic T-ideal through union, intersection. further he utilized important characterisitics to interrogate the MBJ-neutrosophic T-ideal under cartesian product.

The purpose of this article is to introduce the idea of MBJ-neutrosophic Magnified translation **(MBJNMT)** on G-algebra. In second section we cite some fundamental definitions which are used to develop the paper. In third section we discussed the MBJ-neutrosophic magnified translation **(MBJNMT)** of MBJ-neutrosophic ideal **(MBJNID)** and MBJ-neutrosophic subalgebra **(MBJNSU).**

2. Preliminaries

First we discuss some definitions which are used to present this article.

Definition 2.1 [3] An algebra $(Y,*,0)$ of type (2,0) is called a BCI-algebra if it satisfies the following conditions:

i) $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$,

ii) $t_1 * (t_1 * t_2) \leq t_2$,

iii) $t_1 \leq t_1$,

iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,

v) $t_1 \leq 0 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

Definition 2.2 [1] An algebra $(Y,*,0)$ of type (2,0) is called a BCK-algebra if it satisfies the following conditions:

i) $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$,

ii) $t_1 * (t_1 * t_2) \leq t_2$,

iii) $t_1 \leq t_1$,

iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,

v) $0 \leq t_1 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

Definition 2.3 [7] A non-empty set Y with a constant 0 and a binary operation $*$ is said to be G-algebra if it satisfies the following axioms.

G1: $t_1 * t_1 = 0$

G2: $t_1 * (t_1 * t_2) = t_2$, for all $t_1, t_2 \in Y$

A G-algebra is denoted by $(Y,*,0)$.

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Definition 2.4 [7] A non-empty subset S of G-algebra Y is called a G-subalgebra of Y if $t_1 * t_2 \in S\ \forall\ t_1, t_2 \in S$.

Definition 2.5 [15] A non-empty subset I of a G −algebra Y is called an ideal if for any $t_1, t_2 \in Y$,

(i) $0 \in I$,

(ii) $t_1 * t_2 \in I$ and $t_2 \in I \Rightarrow t_1 \in I$.

Definition 2.5 [6] Let Y be a PS-algebra. A fuzzy set B of Y is called a fuzzy PS ideal of Y if it satisfies the following conditions:

i) $\phi(0) \geq \phi(t_1)$,

ii) $\phi(t_1) \geq \min\{\phi(t_2 * t_1), \phi(t_2)\}$, for all $t_1, t_2 \in Y$.

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Let Y be a group of objects denoted generally by t_1. Then a fuzzy set B of Y is defined as $B = \{< t_1, \phi_B(t_1) > | t_1 \in Y\}$, where $\phi_B(t_1)$ is called the membership value of t_1 in B and $\phi_B(t_1) \in [0,1]$.

A fuzzy set B [6] of PS-algebra Y is called a fuzzy PS subalgebra of Y if $\phi(t_1 * t_2) \geq \min\{\phi(t_1), \phi(t_2)\}$, for all $t_1, t_2 \in Y$.

Let a fuzzy subset B [4], [5] of Y and $\alpha \in [0, 1 - \sup\{\phi_B(t_1) | t_1 \in Y\}]$. A mapping $(\phi_B)^T_\alpha | Y \in [0,1]$ is said to be a fuzzy α translation of ϕ_B if it satisfies $(\phi_B)^T_\alpha(t_1) = \phi_B(t_1) + \alpha$, for all $t_1 \in Y$.

Let a fuzzy subset B [4], [5] of Y and $\alpha \in [0,1]$. A mapping $(\phi_B)^M_\alpha | Y \to [0,1]$ is said to be a fuzzy α multiplication of B if it satisfies $(\phi_B)^M_\alpha(t_1) = \alpha.(\phi_B)(t_1)$, for all $t_1 \in Y$.

An intuitionistic fuzzy set **(IFS)** [12] B over Y is an object having the form $B = \{\langle t_1, \phi_B(t_1), \psi_B(t_1)\rangle | t_1 \in Y\}$, where $\phi_B(t_1) | Y \to [0,1]$ and $\psi_B(t_1) | Y \to [0,1]$, with the condition $0 \leq \phi_B(t_1) + \psi_B(t_1) \leq 1$, for all $t_1 \in Y$. $\phi_B(t_1)$ and $\psi_B(t_1)$ represent the degree of existanceship and the degree of non-existanceship of the element t_1 in the set B respectively.

Let $B = \{\langle t_1, \phi_B(t_1), \psi_B(t_1)\rangle | t_1 \in Y\}$ and $B = \{\langle t_1, \phi_B(t_1), \psi_B(t_1)\rangle | t_1 \in Y\}$ be two **IFSs** [12] on Y. Then intersection and union of A and B are indicated by A ∩ B and A ∪ B respectively and are given by

$$A \cap B = \{\langle t_1, \min(\phi_A(t_1), \phi_B(t_1)), \max(\psi_A(t_1), \psi_B(t_1))\rangle | t_1 \in Y\},$$

$$A \cup B = \{\langle t_1, \max(\phi_A(t_1), \phi_B(t_1)), \min(\psi_A(t_1), \psi_B(t_1))\rangle | t_1 \in Y\}.$$

An **IFS** [14] $B = \{\langle t_1, \phi_B(t_1), \psi_B(t_1)\rangle | t_1 \in Y\}$ of Y is called an **IFSU** of Y if it satisfies these two conditions:

(i) $\phi_B(t_1 * t_2) \geq \min\{\phi_B(t_1), \phi_B(t_2)\}$,

(ii) $\psi_B(t_1 * t_2) \leq \max\{\psi_B(t_1), \psi_B(t_2)\}$, for all $t_1, t_2 \in Y$.

An **IFS** $B = \{\langle t_1, \phi_B(t_1), \psi_B(t_1)\rangle | t_1 \in Y\}$ of Y is said to be an **IFID** of Y if it satisfies these three conditions:

(i) $\phi_B(0) \geq \phi_B(t_1), \psi_B(0) \leq \psi_B(t_1)$,

(ii) $\phi_B(t_1) \geq \min\{\phi_B(t_1 * t_2), \phi_B(t_2)\}$,

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(iii) $\psi_B(t_1) \le \max\{\psi_B(t_1 * t_2), \psi_B(t_2)\}$, for all $t_1, t_2 \in Y$.

Let ϕ be a fuzzy subset [8] of Y, $\alpha \in [0,T]$ and $q \in [0,1]$. A mapping $\phi^{MT}_{\beta\,\alpha}|Y \to [0,1]$ is said to be a fuzzy magnified qα translation of ϕ if it satisfies: $\phi^{MT}_{\beta\,\alpha}(t_1) = \beta.\phi(t_1) + \alpha$ for all $t_1 \in Y$.

Let Y be a non empty set. MBJ-neutrosophic set [25] in Y, is a structure of the form $C = \{\langle M_C t_1, \widehat{B}_C t_1, J_C t_1\rangle | t_1 \in Y\}$ where M_C and J_C are fuzzy sets in Y and M_C is a truth membership function,J_C is a false membership function and $\widehat{B}$ is interval valued fuzzy set in Y and is an Indeterminate Interval Valued membership function.

Let $C = (M_C, \widehat{B}_C, J_C)$ is a MBJ-neutrosphic set of B-algebra Y. Let $t \in [0,1]$, then B is called MBJ-neutrosphic T-ideal (MBJNTID) of Y if it fulfills these assertions:

i) $M_C(0) \ge M_C(t_1)$,$\widehat{B}_C(0) \ge \widehat{B}_C(t_1)$ and $J_C(0) \le J_C(t_1)$.

ii) $M_C(t_1 * t_3) \ge \min\{M_C((t_1 * t_2) * t_3), M_C(t_2)\}$.

iii) $\widehat{B}_C(t_1 * t_3) \ge \mathrm{rmin}\{\widehat{B}_C((t_1 * t_2) * t_3), \widehat{B}_C(t_2)\}$.

iv) $J_C(t_1 * t_3) \le \max\{J_C((t_1 * t_2) * t_3), J_C(t_2)\}$.

3. MBJ-neutrosophic Magnified Translation

In this section, we define MBJ-neutrosophic magnified translation **MBJNMT** and investigate some of its characteristics. we use $Ƕ = \inf\{J_C(t_1) | t_1 \in Y\}$ for any **MBJNS** $C = (M_C, \widehat{B}_C, J_C)$ of Y.

Definition 3.1 Let $C = (M_C, \widehat{B}_C, J_C)$ be a **MBJNS** of Y and $p, q, s \in [0, Ƕ]$, $\lambda \in [0,1]$. An object having the form $C^{MT}_{\lambda\ p,q,s} = \{(M_C)^{MT}_{\lambda\ p}, (\widehat{B}_C)^{MT}_{\lambda\ q}, (J_C)^{MT}_{\lambda\ s}\}$ is said to be a **MBJNMT** of C, when $(M_C)^{MT}_{\lambda\ p}(t_1) = \lambda\,.\,M_C(t_1) + p$, $(\widehat{B}_C)^{MT}_{\lambda\ q}(t_1) = \lambda\,.\,\widehat{B}_C(t_1) + q$, $(J_C)^{MT}_{\lambda\ s}(t_1) = \lambda\,.\,J_C(t_1) - s$ for all $t_1 \in Y$.

Example 3.1 Let $Y = \{0,1,2\}$ be a G-algebra. A MBJ-neutrosophic $C = (M_C, \widehat{B}_C, J_C)$ of Y is defined as

$$M_C(t_1) = \begin{cases} [0.2,0.4] & \text{if } t_1 = 0 \\ [0.5,0.7] & \text{if otherwise} \end{cases}$$

$$\widehat{B}_C(t_1) = \begin{cases} [0.4, 0.6] & \text{if } t_1 = 0 \\ [0.5, 0.8] & \text{if otherwise} \end{cases}$$

$$J_C(t_1) = \begin{cases} [0.3,0.5] & \text{if } t_1 = 0 \\ [0.6,0.8] & \text{if otherwise} \end{cases}$$

Then C is a MBJ-neutrosophic subalgebra, choose $\lambda = 0.2, p = 0.04, q = 0.05, s = 0.06$ then the mapping $C^{MT}_{(0.2)\ (0.04,0.05,0.06)}|Y \to [0,1]$ is given by

$$(M_C)^{MT}_{0.2\ 0.04}(t_1) = \begin{cases} [0.08, 0.12] & \text{if } t_1 = 1 \\ [0.14, 0.18] & \text{if otherwise} \end{cases}$$

$$(\widehat{B}_C)^{MT}_{0.2\ 0.05}(t_1) = \begin{cases} [0.13, 0.17] & \text{if } t_1 = 1 \\ [0.15, 0.21] & \text{if otherwise} \end{cases}$$

$$(J_C)^{MT}_{0.2\ 0.06}(t_1) = \begin{cases} [0, 0.04] & \text{if } t_1 = 1 \\ [0.06, 0.1] & \text{if otherwise} \end{cases}$$

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which imply $(M_C)^{M\,T}_{(0.2)(0.04)}(t_1) = (0.2).M_C(t_1) + 0.04$, $(\widehat{B}_C)^{M\,T}_{(0.2)(0.05)}(t_1) = (0.2).B_C(t_1) + 0.05$, $(J_C)^{M\,T}_{(0.2)(0.06)}$

$(t_1) = (0.2).J_C(t_1) - 0.06$ for all $t_1 \in Y$. Hence $C^{M\,T}_{(0.2)\,(0.04,0.05,0.06)}$ is a MBJ-neutrosophic magnified (0.1)(0.02,0.03,0.04) translation.

Theorem 3.1 Let C be a MBJ subset of Y such that $p, q, s \in [0, \text{Ƕ}]$, $\lambda \in [0,1]$ and a mapping $B^{M\,T}_{\lambda\, p,q,s}|Y \to [0,1]$ be a **MBJNMT** of C. If C is **MBJNSU** of Y, then $C^{M\,T}_{\lambda\, p,q,s}$ is a **MBJNSU** of Y.

Proof. Let C be a **MBJNS** of Y, $p, q, s \in [0, \text{Ƕ}]$, $\lambda \in [0,1]$ and a mapping $B^{M\,T}_{\lambda\, p,q,s}|Y \to [0,1]$ be a **MBJNMT** of C. Suppose C is a **MBJNSU** of Y. Then $M_C(t_1 * t_2) \geq \min\{M_C(t_1), M_C(t_2)\}$, $\widehat{B}_C(t_1 * t_2) \geq \text{rmin}\{\widehat{B}_C(t_1), \widehat{B}_C(t_2)\}$, $J_C(t_1 * t_2) \leq \max\{J_C(t_1), J_C(t_2)\}$. Now

$$(M_C)^{M\,T}_{\lambda\, p}(t_1 * t_2) = \lambda\,.\,M_C(t_1 * t_2) + p$$

$$\geq \lambda\,.\,\min\{M_C(t_1), M_C(t_2)\} + p$$

$$= \min\{\lambda\,.\,M_C(t_1) + p, \lambda\,.\,M_C(t_2) + p\}$$

$$(M_C)^{M\,T}_{\lambda\, p}(t_1 * t_2) = \min\{(M_C)^{M\,T}_{\lambda\, p}(t_1), (M_C)^{M\,T}_{\lambda\, p}(t_2)\}$$

$$(M_C)^{M\,T}_{\lambda\, p}(t_1 * t_2) \geq \min\{(M_C)^{M\,T}_{\lambda\, p}(t_1), (M_C)^{M\,T}_{\lambda\, p}(t_2)\},$$

$$(\widehat{B}_C)^{M\,T}_{\lambda\, q}(t_1 * t_2) = \lambda\,.\,\widehat{B}_C(t_1 * t_2) + q$$

$$\geq \lambda\,.\,\text{rmin}\{\widehat{B}_C(t_1), \widehat{B}_C(t_2)\} + q$$

$$= \text{rmin}\{\lambda\,.\,\widehat{B}_C(t_1) + q, \lambda\,.\,\widehat{B}_C(t_2) + q\}$$

$$(\widehat{B}_C)^{M\,T}_{\lambda\, q}(t_1 * t_2) = \text{rmin}\{(\widehat{B}_C)^{M\,T}_{\lambda\, q}(t_1), (\widehat{B}_C)^{M\,T}_{\lambda\, q}(t_2)\}$$

$$(\widehat{B}_C)^{M\,T}_{\lambda\, q}(t_1 * t_2) \geq \text{rmin}\{(\widehat{B}_C)^{M\,T}_{\lambda\, q}(t_1), (\widehat{B}_C)^{M\,T}_{\lambda\, q}(t_2)\},$$

$$(J_C)^{M\,T}_{\lambda\, s}(t_1 * t_2) = \lambda\,.\,J_C(t_1 * t_2) - s$$

$$\leq \lambda\,.\,\max\{J_C(t_1), J_C(t_2)\} - s$$

$$= \max\{\lambda\,.\,J_C(t_1) - s, \lambda\,.\,J_C(t_2) - s\}$$

$$(J_C)^{M\,T}_{\lambda\, s}(t_1 * t_2) = \max\{(J_C)^{M\,T}_{\lambda\, s}(t_1), (J_C)^{M\,T}_{\lambda\, s}(t_2)\}$$

$$(J_C)^{M\,T}_{\lambda\, s}(t_1 * t_2) \geq \max\{(J_C)^{M\,T}_{\lambda\, s}(t_1), (J_C)^{M\,T}_{\lambda\, s}(t_2)\}.$$

Hence **MBJNMT** $C^{M\,T}_{\lambda\, p,q,s}$ is a **MBJNSU** of Y.

Theorem 3.2 Let C be a MBJ-neutrosophic set of Y such that $p, q, s \in [0, \text{Ƕ}]$, $\lambda \in [0,1]$ and a mapping $C^{M\,T}_{\lambda\, p,q,s}|Y \to [0,1]$ be a **MBJNMT** of C. If $C^{M\,T}_{\lambda\, p,q,s}$ is **MBJNSU** of Y. Then C is a **MBJNSU** of Y.

Proof. Let C be a MBJ-neutrosophic subset of Y, where $p, q, s \in [0, \text{Ƕ}]$, $\lambda \in [0,1]$ and a mapping $C^{M\,T}_{\lambda\, p,q,s}|Y \to [0,1]$ be a **MBJNMT** of C. Let $C^{M\,T}_{\lambda\, p,q,s}$ is a **MBJNSU** of Y, then

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$$\lambda . M_C(t_1 * t_2) + p = (M_C)^{M\,T}_{\lambda\,p}(t_1 * t_2)$$

$$\geq \min\{(M_C)^{M\,T}_{\lambda\,p}(t_1), (M_C)^{M\,T}_{\lambda\,p}(t_2)\}$$

$$= \min\{\lambda . M_C(t_1) + p, \lambda . M_C(t_2) + p\}$$

$$\lambda . M_C(t_1 * t_2) + p = \lambda . \min\{M_C(t_2), M_C(t_1)\} + p,$$

$$\lambda . \widehat{B}_C(t_1 * t_2) + q = (\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1 * t_2)$$

$$\geq \mathrm{rmin}\{(\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1), (\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_2)\}$$

$$= \mathrm{rmin}\{\lambda . \widehat{B}_C(t_1) + q, \lambda . \widehat{B}_C(t_2) + q\}$$

$$\lambda . \widehat{B}_C(t_1 * t_2) + q = \lambda . \mathrm{rmin}\{\widehat{B}_C(t_2), \widehat{B}_C(t_1)\} + q,$$

$$\lambda . J_C(t_1 * t_2) - s = (J_C)^{M\,T}_{\lambda\,s}(t_1 * t_2)$$

$$\leq \max\{(J_C)^{M\,T}_{\lambda\,s}(t_1), (J_C)^{M\,T}_{\lambda\,s}(t_2)\}$$

$$= \max\{\lambda . J_C(t_1) - s, \lambda . J_C(t_2) - s\}$$

$$\lambda . J_C(t_1 * t_2) - s = \lambda . \max\{J_C(t_2), J_C(t_1)\} - s,$$

which imply $M_C(t_1 * t_2) \geq \min\{M_C(t_1), M_C(t_2)\}$,$\widehat{B}_C(t_1 * t_2) \geq \mathrm{rmin}\{\widehat{B}_C(t_1), \widehat{B}_C(t_2)\}$, $J_C(t_1 * t_2) \leq \max\{J_C(t_1)$ $, J_C(t_2)\}$ for all $t_1, t_2 \in Y$. Hence C is a **MBJNSU** of Y.

Theorem 3.3 If C is a **MBJNID** of Y. Then **MBJNMT** $C^{M\,T}_{\lambda\,p,q,s}$ of C is a **MBJNID** of Y for all $p, q, s \in [0, Ƕ]$ and $\lambda \in (0,1]$.

Proof. Suppose $C = (M_C, \widehat{B}_C, J_C)$ is a **MBJNID** of Y. Then

$$(M_C)^{M\,T}_{\lambda\,p}(0) = \lambda . M_C(0) + p \geq \lambda . M_C(t_1) + p$$

$$(M_C)^{M\,T}_{\lambda\,p}(0) = (M_C)^{M\,T}_{\lambda\,p}(t_1),$$

$$(\widehat{B}_C)^{M\,T}_{\lambda\,q}(0) = \lambda . \widehat{B}_C(0) + q \geq \lambda . \widehat{B}_C(t_1) + q$$

$$(\widehat{B}_C)^{M\,T}_{\lambda\,q}(0) = (\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1),$$

$$(J_C)^{M\,T}_{\lambda\,s}(0) = \lambda . J_C(0) - s \leq \lambda . J_C(t_1) - s$$

$$(J_C)^{M\,T}_{\lambda\,s}(0) = (J_C)^{M\,T}_{\lambda\,s}(t_1)$$

Now

$$(M_C)^{M\,T}_{\lambda\,p}(t_1) = \lambda . M_C(t_1) + p$$

$$\geq \lambda . \min\{M_C(t_1 * t_2), M_C(t_2)\} + p$$

$$= \min\{\lambda . M_C(t_1 * t_2) + p, \lambda . M_C(t_2) + p\}$$

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$(M_C)^{MT}_{\lambda p}(t_1) = \min\{(M_C)^{MT}_{\lambda p}(t_1 * t_2), (M_C)^{MT}_{\lambda p}(t_2)\}$

$\Rightarrow (M_C)^{MT}_{\lambda p}(t_1) \geq \min\{(M_C)^{MT}_{\lambda p}(t_1 * t_2), (M_C)^{MT}_{\lambda p}(t_2)\},$

$(\widehat{B}_C)^{MT}_{\lambda q}(t_1) = \lambda . \widehat{B}_C(t_1) + q$

$\geq \lambda . r\min\{\widehat{B}_C(t_1 * t_2), \widehat{B}_C(t_2)\} + q$

$= r\min\{\lambda . \widehat{B}_C(t_1 * t_2) + q, \lambda . \widehat{B}_C(t_2) + q\}$

$(\widehat{B}_C)^{MT}_{\lambda q}(t_1) = r\min\{(\widehat{B}_C)^{MT}_{\lambda q}(t_1 * t_2), (\widehat{B}_C)^{MT}_{\lambda q}(t_2)\}$

$\Rightarrow (\widehat{B}_C)^{MT}_{\lambda q}(t_1) \geq r\min\{(\widehat{B}_C)^{MT}_{\lambda q}(t_1 * t_2), (\widehat{B}_C)^{MT}_{\lambda q}(t_2)\},$

$(J_C)^{MT}_{\lambda s}(t_1) = \lambda . J_C(t_1) - s$

$\leq \lambda . \max\{J_C(t_1 * t_2), J_C(t_2)\} - s$

$= \max\{\lambda . J_C(t_1 * t_2) - s, \lambda . J_C(t_2) - s\}$

$(J_C)^{MT}_{\lambda s}(t_1) = \max\{(J_C)^{MT}_{\lambda s}(t_1 * t_2), (J_C)^{MT}_{\lambda s}(t_2)\}$

$\Rightarrow (J_C)^{MT}_{\lambda s}(t_1) \leq \max\{(J_C)^{MT}_{\lambda s}(t_1 * t_2), (J_C)^{MT}_{\lambda s}(t_2)\}$

for all $t_1, t_2 \in Y$ and all $p, q, s \in [0, Ƕ]$, $\lambda \in (0,1]$. Hence $C^{MT}_{\lambda\, p,q,s}$ of C is a **MBJNID** of Y.

Theorem 3.4 If C is a MBJ-neutrosophic set of Y such that **MBJNMT** $C^{MT}_{\lambda\, p,q,s}$ of C is a **MBJNID** of Y for all $p, q, s \in [0, Ƕ]$ and $\lambda \in (0,1]$, then C is a **MBJNID** of Y.

Proof. Suppose **MBJNMT** $C^{MT}_{\lambda\, p,q,s}$ is a **MBJNID** of Y for some $p, q, s \in [0, Ƕ]$, $\lambda \in (0,1]$ and $t_1, t_2 \in Y$. Then

$\lambda . M_C(0) + p = (M_C)^{MT}_{\lambda p}(0)$

$\geq (M_C)^{MT}_{\lambda p}(t_1)$

$\lambda . M_C(0) + p = \lambda . M_C(t_1) + p,$

$\lambda . \widehat{B}_C(0) + q = (\widehat{B}_C)^{MT}_{\lambda q}(0)$

$\geq (\widehat{B}_C)^{MT}_{\lambda q}(t_1)$

$\lambda . \widehat{B}_C(0) + q = \lambda . \widehat{B}_C(t_1) + q,$

$\lambda . J_C(0) - s = (J_C)^{MT}_{\lambda s}(0)$

$\leq (J_C)^{MT}_{\lambda s}(t_1)$

$\lambda . J_C(0) - s = \lambda . J_C(t_1) - s,$

which imply $M_C(0) \geq M_C(t_1)$,$\widehat{B}_C(0) \geq \widehat{B}_C(t_1)$,$J_C(0) \leq J_C(t_1)$. Now, we have

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$$\lambda . M_C(t_1) + p = (M_C)^{M\,T}_{\lambda\,p}(t_1)$$

$$\geq \min\{(M_C)^{M\,T}_{\lambda\,p}(t_1 * t_2), (M_C)^{M\,T}_{\lambda\,p}(t_2)\}$$

$$= \min\{\lambda . M_C(t_1 * t_2) + p, \lambda . M_C(t_2) + p\}$$

$$\lambda . M_C(t_1) + p = \lambda . \min\{M_C(t_1 * t_2), M_C(t_2)\} + p,$$

$$\lambda . \widehat{B}_C(t_1) + q = (\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1)$$

$$\geq \mathrm{rmin}\{(\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1 * t_2), (\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_2)\}$$

$$= \mathrm{rmin}\{\lambda . \widehat{B}_C(t_1 * t_2) + q, \lambda . \widehat{B}_C(t_2) + q\}$$

$$\lambda . \widehat{B}_C(t_1) + q = \lambda . \mathrm{rmin}\{\widehat{B}_C(t_1 * t_2), \widehat{B}_C(t_2)\} + q,$$

$$\lambda . J_C(t_1) - s = (J_C)^{M\,T}_{\lambda\,s}(t_1)$$

$$\leq \max\{(J_C)^{M\,T}_{\lambda\,s}(t_1 * t_2), (J_C)^{M\,T}_{\lambda\,s}(t_2)\}$$

$$= \max\{\lambda . J_C(t_1 * t_2) - s, \lambda . J_C(t_2) - s\}$$

$$\lambda . J_C(t_1) - s = \lambda . \max\{J_C(t_1 * t_2), J_C(t_2)\} - s$$

which imply $M_C(t_1) \geq \min\{M_C(t_1 * t_2), M_C(t_2)\}$, $\widehat{B}_C(t_1) \geq \mathrm{rmin}\{\widehat{B}_C(t_1 * t_2), \widehat{B}_C(t_2)\}$, $J_C(t_1) \leq \max\{J_C(t_1 * t_2), J_C(t_2)\}$ for all $t_1, t_2 \in Y$. Hence C is a **MBJNID** of Y.

Theorem 3.5 Intersection of any two **MBJNMT** $C^{M\,T}_{\lambda\,p,q,s}$ of a **MBJNID** C of Y is a **MBJNID** of Y.

Proof. Suppose $C^{M\,T}_{\lambda\,p,q,s}$ and $C^{M\,T}_{\lambda'\,p',q',s'}$ are two **MBJNMTs** of **MBJNID** C of Y, where $p, q, s, p', q', s' \in [0, Ʋ]$ and $\lambda, \lambda' \in (0,1]$. Assume $p \leq p', q \leq q', s \leq s'$ and $\lambda = \lambda'$. Since $C^{M\,T}_{\lambda\,p,q,s}$ and $C^{M\,T}_{\lambda'\,p',q',s'}$ are **MBJNIDs** of Y. So

$$((M_C)^{M\,T}_{\lambda\,p} \cap (M_C)^{M\,T}_{\lambda'\,p'})(t_1) = \min\{(M_C)^{M\,T}_{\lambda\,p}(t_1), (M_C)^{M\,T}_{\lambda'\,p'}(t_1)\}$$

$$= \min\{\lambda . M_C(t_1) + p, \lambda' . M_C(t_1) + p'\}$$

$$= \lambda . M_C(t_1) + p$$

$$((M_C)^{M\,T}_{\lambda\,p} \cap (M_C)^{M\,T}_{\lambda'\,p'})(t_1) = (M_C)^{M\,T}_{\lambda\,p}(t_1),$$

$$((\widehat{B}_C)^{M\,T}_{\lambda\,q} \cap (\widehat{B}_C)^{M\,T}_{\lambda'\,q'})(t_1) = \mathrm{rmin}\{(\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1), (\widehat{B}_C)^{M\,T}_{\lambda'\,q'}(t_1)\}$$

$$= \mathrm{rmin}\{\lambda . \widehat{B}_C(t_1) + q, \lambda' . \widehat{B}_C(t_1) + q'\}$$

$$= \lambda . \widehat{B}_C(t_1) + q$$

$$((\widehat{B}_C)^{M\,T}_{\lambda\,q} \cap (\widehat{B}_C)^{M\,T}_{\lambda'\,q'})(t_1) = (\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1),$$

$$((J_C)^{M\,T}_{\lambda\,s} \cap (J_C)^{M\,T}_{\lambda'\,s'})(t_1) = \max\{(J_C)^{M\,T}_{\lambda\,s}(t_1), (J_C)^{M\,T}_{\lambda'\,s'}(t_1)\}$$

$$= \max\{\lambda . J_C(t_1) - s, \lambda' . J_C(t_1) - s'\}$$

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$= \lambda . J_C(t_1) - s$

$((J_C)^{M\,T}_{\lambda\,s} \cap (J_C)^{M\,T}_{\lambda'\,s'})(t_1) = (J_C)^{M\,T}_{\lambda\,s}(t_1).$

Hence $C^{M\,T}_{\lambda\,p,q,s} \cap C^{M\,T}_{\lambda'\,p',q',s'}$ is **MBJNID** of Y.

Theorem 3.6 Union of any two **MBJNMT** $C^{M\,T}_{\lambda\,p,q,s}$ of a **MBJNID** C of Y is a **MBJNID** of Y.

Proof. Suppose $C^{M\,T}_{\lambda\,p,q,s}$ and $C^{M\,T}_{\lambda'\,p',q',s'}$ are two **MBJNMTs** of **MBJNID** C of Y, where $p, q, s, p', q', s' \in [0, Ƀ]$ and $\lambda, \lambda' \in (0,1]$. Assume $p \leq p', q \leq q', s \leq s'$ and $\lambda = \lambda'$. Since $C^{M\,T}_{\lambda\,p,q,s}$ and $C^{M\,T}_{\lambda'\,p',q',s'}$ are **MBJNIDs** of Y. Then

$((M_C)^{M\,T}_{\lambda\,p} \cup (M_C)^{M\,T}_{\lambda'\,p'})(t_1) = \max\{(M_C)^{M\,T}_{\lambda\,p}(t_1), (M_C)^{M\,T}_{\lambda'\,p'}(t_1)\}$

$= \max\{\lambda . M_C(t_1) + p, \lambda' . M_C(t_1) + p'\}$

$= \lambda . M_C(t_1) + p$

$((M_C)^{M\,T}_{\lambda\,p} \cup (M_C)^{M\,T}_{\lambda'\,p'})(t_1) = (M_C)^{M\,T}_{\lambda\,p}(t_1),$

$((\widehat{B}_C)^{M\,T}_{\lambda\,q} \cup (\widehat{B}_C)^{M\,T}_{\lambda'\,q'})(t_1) = \mathrm{rmax}\{(\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1), (\widehat{B}_C)^{M\,T}_{\lambda'\,q'}(t_1)\}$

$= \mathrm{rmax}\{\lambda . \widehat{B}_C(t_1) + q, \lambda' . \widehat{B}_C(t_1) + q'\}$

$= \lambda . \widehat{B}_C(t_1) + q$

$((\widehat{B}_C)^{M\,T}_{\lambda\,q} \cup (\widehat{B}_C)^{M\,T}_{\lambda'\,q'})(t_1) = (\widehat{B}_C)^{M\,T}_{\lambda\,q}(t_1),$

$((J_C)^{M\,T}_{\lambda\,s} \cup (J_C)^{M\,T}_{\lambda'\,s'})(t_1) = \min\{(J_C)^{M\,T}_{\lambda\,s}(t_1), (J_C)^{M\,T}_{\lambda'\,s'}(t_1)\}$

$= \min\{\lambda . J_C(t_1) - s, \lambda' . J_C(t_1) - s'\}$

$= \lambda . J_C(t_1) - s$

$((J_C)^{M\,T}_{\lambda\,s} \cup (J_C)^{M\,T}_{\lambda'\,s'})(t_1) = (J_C)^{M\,T}_{\lambda\,s}(t_1)$

4. Conclusion

In this article, we defined **MBJNMT** of MBJ-neutrosophic set on G-algebra. Moreover, deep study of **MBJNMT** will lead us to study the neutrosophic theory on some other framework. For future work, magnification can be applied on MBJ-neutrosophic soft set and T-MBJ-neutrosophic set.

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