



Energy of Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Graphs in Decision Making-A Literature Review

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Abstract

This review of the literature delves into the complex interplay between energy measures and decision-making processes in the domains of fuzzy graphs, intuitionistic fuzzy graphs, and neutrosophic graphs. In graph theory, energy is a key quantity that is used to measure structural properties and evaluate decision model dynamics. The research methodically examines the theoretical underpinnings, computational techniques, and practical applications of energy measures in contexts involving decision-making, considering the special features brought forth by fuzzy, intuitionistic fuzzy, and neutrosophic graph models. This review attempts to provide a thorough understanding for researchers and practitioners looking to use energy measures for efficient decision support in the setting of uncertainty contained within these specific graph topologies by synthesizing prior research.

Keywords: Fuzzy graphs; energy; uncertainty; decision-making; intuitionistic fuzzy graphs; neutrosophic graphs; graph theory.

1. Introduction

To tackle the lack of precision and certainty in various practical domains, Zadeh (1965) proposed the concept of fuzzy sets (FSs), which have achieved significant success in diverse fields such as group decision-making, engineering, medical diagnosis, expert systems, and pattern recognition. Atanassov (1986) expanded on the definition of fuzzy sets by introducing a novel element that quantifies the degree of non-membership, leading to the development of intuitionistic fuzzy sets (IFSs) as a generalization of FSs. The shortcomings of both FSs and IFSs, however, are that they do not offer a thorough depiction of all pertinent data pertaining to the issues being investigated. Building upon the foundations laid by Zadeh (1965) and Atanassov (1986), Smarandache (1999) explored the concept of neutrosophic sets (NSs), which are characterized by a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. As a broad variant of fuzzy sets, intuitionistic fuzzy sets, and classical sets, a neutrosophic set captures truthfulness, indeterminacy, and falseness—all of which are real-world complexity in decision-making scenarios. The neutrosophic set holds significant potential within the realm of neutrosophy, and the neutrosophy theory finds extensive application within the domain of graph theory. Subsequently, numerous researchers have delved into the concept of neutrosophic sets in various avenues. Euler, in 1736, pioneered the establishment of graph theory. In the annals of mathematical history, Euler's solution to the famous Königsberg bridge issue is regarded as the founding theorem of graph theory. These days, everyone recognizes this theorem as an area of combinatorial mathematics. Graph theory is widely applied to many fields, including geometry, combinatorics, elliptic curves, topography, decision theory, optimization, and data science, as a powerful tool for comprehending combinatorial problems. A graph serves as a fundamental instrument for illustrating connections between nodes and finds numerous applications in the realm of social networks, making it a suitable foundation for data representation. However, precise graphs fail to capture the essence of any given system due to the prevalence of imprecise data in the contemporary world. The concept of fuzziness was initially employed to define the concept of a fuzzy graph, as introduced by Kaufmann (1973). Subsequently, the field of fuzzy graph theory was developed and refined by Rosenfeld (1975). In the literature, several generalized fuzzy graphs have also been introduced. Among these

developments, Parvathi and Karunambigai (2006) introduced the idea of an intuitionistic fuzzy graph (IFG). The concept of neutrosophic graphs was first presented in the paper "Neutrosophic Graphs" by Kandasamy et al. This study demonstrates that these neutrosophic graphs provide a new insight into the field of graph theory. The presence of more than 200 neutrosophic diagrams in this publication is a noteworthy feature that improves comprehension of these concepts. Fuzzy graphs and intuitionistic fuzzy graphs are generalized by the single valued neutrosophic graph (SVNG).

The notion of graph spectra holds significant importance within the realm of graph theory. In the field of chemistry, Gutman (1978) introduced the concept of graph energy due to its relevance to the overall π -electron energy of specific molecules. Gutman also established both upper and lower bounds for graph energy. Subsequently, Gutman and Zhou (2006) defined the Laplacian energy of a graph as the summation of the absolute differences between the average vertex degree of G and the Laplacian eigenvalues of G . The concept of signless Laplacian energy in a graph was introduced by Gutman in 2009. Anjali and Mathew (2013) conducted an exploration of graph energy within the framework of fuzzy set theory. Building upon this work, Sharbat and Fayazi (2014) proceeded to define the Laplacian energy of a fuzzy graph. Praba et al. (2014) introduced the notion of energy in IFGs (Indeterminate Fuzzy Graphs). Basha and Kartheek (2015) further extended the concept of Laplacian energy from fuzzy graphs to IFGs. However, when faced with scenarios where the descriptions of objects or their relationships, or both, are uncertain and inconsistent, fuzzy graphs and IFGs prove to be inadequate. In order to address this limitation, Akram et al. (2018) expanded the concept of IFGs to single-valued neutrosophic graphs (SVNGs) and proposed multiple novel concepts related to SVNGs and their extensions. Naz et al. (2017) introduced the concept of single-valued neutrosophic digraphs (SVNDGs).

Neutrosophic graph theory serves as an extension of classical graph theory, specifically focusing on the notion of indeterminacy. Energy, a crucial concept in graph theory, has been extended to neutrosophic graphs. The energy of neutrosophic graphs finds application in various domains, including decision-making. Laplacian energy and signless Laplacian energy are two other vital energy concepts in neutrosophic graphs. The spectral characteristics of a neutrosophic graph are associated with its Laplacian energy and signless Laplacian energy. A neutrosophic graph's structural qualities can be examined by applying the eigenvalues of its Laplacian matrix and signless Laplacian matrix.

Graph-theoretic principles were merged with fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets to describe uncertain decision-making situations. Pythagorean fuzzy graphs were used in decision-making by Akram et al. (2018) to provide a more adaptable and useful model for managing ambiguity and uncertainty in real-world decision-making situations. They used the development of a satellite communication system and the assessment of reservoir operation systems as examples.

Ye (2013) examined various challenges in multi-criteria decision-making scenarios by utilizing the correlation coefficient in neutrosophic sets. In a similar vein, Ye (2014) introduced a non-stereospecific cross-entropy cross-decision method in the domain of multi-criteria decision-making predicaments. Biswas et al. (2014) proposed a gray-based entropy approach for resolving predicaments involving multiple decisions in neutrosophic single-value sets. Moreover, Biswas and colleagues (2014) proposed a new method for handling multi-criteria decision-making problems involving single-valued neutrosophic sets with particular weights. Using single-valued neutrosophic sets (NSs), Sodenkamp et al. (2018) developed a novel approach to address the influence of independent multi-source uncertainty measures on the dependability of experts' judgments in the context of group multi-criteria decision-making (GMCDM) situations.

The structure of this paper is as follows. A survey of the literature on fuzzy graph energy is given in Section 2. The energy of intuitionistic fuzzy graphs is explored in Section 4. After that, the energy of neutrosophic graphs is thoroughly examined in Section 4. An overview of upcoming studies and a summary round out the paper.

1. Energy of Fuzzy graphs

The techniques dealing with the energy of fuzzy graphs are summarized in Table 1.

Table 1: Categories of fuzzy graphs

Reference	Year	Types of graphs	Significance / Influences
[1]	2013	Fuzzy graphs	Developed a mathematical definition of energy for fuzzy graphs

[2]	2014	Fuzzy graphs	Laplacian energy for fuzzy graphs is defined, and constraints on this energy measure are given.
[3]	2018	Bipolar fuzzy graph	Introduced the concept of energy in bipolar fuzzy graph (BFG) and bipolar fuzzy digraphs (BFDGs).
[4]	2018	Pythagorean fuzzy graphs	The energy and Laplacian energy of Pythagorean fuzzy graphs (PFGs) and Pythagorean fuzzy digraphs (PFDGs) were calculated.
[5]	2018	Bipolar Fuzzy Competition Graphs	Utilizing a novel technique known as bipolar fuzzy extended TOPSIS
[6]	2019	m-polar fuzzy digraphs	Introduced Laplacian and Signless Laplacian energy of m-polar fuzzy digraphs
[7]	2020	Spherical fuzzy graphs	Acquired Bounds on the energy of spherical fuzzy graphs
[8]	2021	Interval-valued fuzzy graphs	Use of max-min operators to find eigenvalues and eigenvectors of interval-valued fuzzy graphs (IVFGs)
[9]	2022	Vague Fuzzy Graph	Explored energy and laplacian energy on Vague Fuzzy Graphs
[10]	2022	Fuzzy Bipolar Pythagorean Graphs	Introduced the concept of fuzzy bipolar pythagorean graphs and explored the energy and laplacian energy

Narayanan & Mathew (2013) undertake an examination of the notion of energy within fuzzy graphs, drawing inspiration from the energy exhibited by p-electrons in the realm of chemistry. They present a precise mathematical definition of energy applicable to all types of graphs, thereby extending the analysis of p-electron energy to molecular graphs. Furthermore, the authors delve into an exploration of the mathematical characteristics of graph energy and ponder the range of possible values it can assume. They also venture into an investigation of various classes of graphs based on their energy levels, including hyperenergetic, hypoenergetic, and equienergetic graphs. The paper also discusses several restrictions on graph energy and looks at real-world uses for it in a variety of disciplines, including computer science, chemistry, physics, and mathematics. The authors stress the importance of graph energy in areas including protecting personal information in databases, predicting the propagation of viruses in computer networks, and pattern recognition. In conclusion, this work significantly advances our understanding of energy in graphs and the applications it has in several fields.

Sharbaf & Fayazi (2014) defined Laplacian energy for fuzzy graphs as the summation of the absolute values of the eigenvalues of the graph and the average degree of the graph. They presented detailed equations and definitions to substantiate their findings and made references to prior research on fuzzy graphs and Laplacian energy. The study contributed to advancing the understanding of Laplacian energy in the context of fuzzy graphs and potentially paved the way for new applications in various fields. Bounds on the Laplacian energy of fuzzy graphs are given in the work. These constraints have a variety of uses and aid in the comprehension of fuzzy graphs' energy characteristics.

Naz et al. (2018) examines the notion of energy within the framework of bipolar fuzzy graphs (BFGs) and bipolar fuzzy digraphs (BFDGs), exploring its application in decision-making. The incorporation of bipolarity, vagueness, and imprecision is integral to various aspects of information processing, and bipolar fuzzy sets (BFSs) serve as a suitable means of knowledge representation. The researchers introduce the concept of energy in BFGs and delve into its properties. It is shown that if G is a BFG on n vertices, then $E(G) \leq n^2(1 + \sqrt{n})$ must hold. Additionally, they extend the notion of energy to encompass BFDGs, utilizing the coefficients of $ln - 2$ in the characteristic polynomial to define the energy of BFDGs. Moreover, the study employs the idea of energy in BFDGs to address a decision-making problem pertaining to communication equipment in contemporary warfare. Given the pivotal role of communication equipment in determining the outcome of campaigns, the energy of BFDGs aids in evaluating different alternatives and making the most rational selection. The paper not only introduces the concepts of energy in BFGs and BFDGs within a bipolar fuzzy environment, but also explores

their properties. Furthermore, it illustrates the practical application of energy in decision-making, specifically in the realm of communication equipment in modern warfare.

Akram et al. (2018) focused their attention on the computation of the energy and Laplacian energy of Pythagorean fuzzy graphs (PFGs) and Pythagorean fuzzy digraphs (PFDGs). As an extension of intuitionistic fuzzy sets (IFSs), the authors created Pythagorean fuzzy sets (PFSs) to express obscurity in decision-making tasks. They also obtained lower and upper constraints on PFGs' energy and Laplacian energy. Numerical examples, including the design of a satellite communication system and the assessment of reservoir operation plans, demonstrate the practical significance of these proposed notions in decision making. This work adds new ideas for computing the energy and Laplacian energy of PFGs and PFDGs to the corpus of material already available on fuzzy sets and graphs.

Sarwar et al. (2018) introduced a novel approach called bipolar fuzzy extended TOPSIS, which is designed to handle multi-criteria decision-making problems involving bipolar measurements with both positive and negative values. This method incorporates the ability to incorporate bipolar information into the TOPSIS framework to measure aggregate values on a bipolar scale. In addition, the authors applied their technique to a real-world dental issue and talked about how it may be used to measure the advantages and disadvantages of medical interventions. The study also provides algorithms for calculating the degree of rivalry between species and investigates the application of bipolar fuzzy competition graphs in food webs. In order to examine the degree of competition between species for common prey, the authors show how to construct a bipolar fuzzy competition graph. This technique offers important new information about the interactions between predators and prey in ecosystems.

Akram et al. (2019) conducted a study on fuzzy graphs and their various energy measures, including the energy of m -polar fuzzy digraphs, Laplacian energy of m -polar fuzzy digraphs, and signless Laplacian energy of m -polar fuzzy digraphs. The authors began by providing fundamental definitions of fuzzy graphs, adjacency matrices, eigenvalues, and different types of energies associated with them. The maximum and lower constraints on fuzzy graphs' energy were also covered. We introduced and provided examples for the ideas of energy in m -polar fuzzy digraphs, Laplacian energy, and signless Laplacian energy. Properties of these energy measures in fuzzy digraphs and m -polar fuzzy graphs were also investigated in this article. Fuzzy graphs and their energies are understood better because to this research, which has applications in computer science, physics, chemistry, and mathematics, among other subjects.

Mohamed & Ali (2020) introduced the notion of energy extended to spherical fuzzy graphs and proceed to define the adjacency matrix of such graphs. A spherical fuzzy graph's energy is determined by adding up all of the absolute values of its adjacency matrix's eigenvalues. Additionally, the energy of spherical fuzzy graphs is bounded from below and above, providing insight into the possible range of energy values for these graphs. The authors advance our knowledge of the geometric comparison between several fuzzy set types by contrasting spherical fuzzy sets with intuitionistic fuzzy sets, Pythagorean fuzzy sets, and image fuzzy sets. The study explores several operations on intuitionistic fuzzy graphs and interval-valued Pythagorean fuzzy graphs, building upon earlier efforts on these graph types. The authors utilize the concept of adjacency matrix to effectively compute the energy of spherical fuzzy graphs, providing a practical approach for energy calculation. Furthermore, the paper provides basic definitions that are necessary to understand the topics described, providing a strong basis for further research in the area of spherical fuzzy graphs.

Patra et al. (2021) proposed a novel technique for determining the eigenvalues and eigenvectors of an interval-valued fuzzy graph (IVFG) by employing max-min operators. They also introduced and computed the energy of an IVFG using these operators. Moreover, the paper offers fundamental terminology required to comprehend the subjects discussed, offering a solid foundation for additional study in the field of spherical fuzzy graphs. Specifically, it illustrated how the consumption of food by predators from preys can be represented using interval-valued fuzzy membership values and how eigenvalues and eigenvectors can be utilized to analyze the energy of the ecological system.

Li et al. (2022) presented the notion of vague fuzzy graph structures (VFGS), which is a generalization of vague graphs (VGs) and fuzzy graphs (FGs) to explain ambiguity and uncertainty in decision-making situations. They define the adjacency matrix (AM), energy, and Laplacian energy (LE) of a VFGS and extend the concept of energy from ambiguous graphs to VFGSs. The paper explored the application of energy in decision-making problems and presents three specific applications of energy in this context. The authors emphasized the significance of energy in graph theory, particularly in the fields of biological and chemical sciences, and proposed the utilization of energy and Laplacian energy in modeling problems related to VFGSs.

Rajeshwari (2022) introduced the notion of Fuzzy Bipolar Pythagorean Graphs (FBPGs) and investigates their energy and Laplacian energy. After defining FBPGs as a combination of a fuzzy graph and a Pythagorean fuzzy

set, the authors went on to define energy and Laplacian energy in terms of FBPGs. The paper studied the Laplacian energy and the lower and upper constraints on the energy of FBPGs. The authors provide a mathematical characterization of FBPGs by employing Laplacian matrices and connection matrices. The spectra of FBPGs are analyzed by looking at the eigenvalues of the Laplacian matrix and connection matrix. The paper incorporates illustrative examples to showcase the computations and characteristics of FBPGs. The discoveries presented in the paper contribute to the comprehension and examination of FBPGs, which hold potential for applications in decision-making processes and various domains such as clinical analysis and corporate security. In the next section, some of the available energies of intuitionistic fuzzy graphs are discussed.

3. Energy of Intuitionistic fuzzy graphs

Table 2 presents an overview of various methodologies employed in the energy of Intuitionistic- FGs.

Table 2: Categories of Intuitionistic fuzzy graphs

Reference	Year	Type of graphs	Significance/ Influences
[11]	2014	Intuitionistic Fuzzy Graph	Explored the concept of energy in intuitionistic fuzzy graphs
[12]	2015	Intuitionistic Fuzzy Graph	Introduced the concept of Laplacian energy of intuitionistic fuzzy graphs
[13]	2016	Intuitionistic fuzzy graph	Analysed the spreading rate of a virus on the extreme energy of an intuitionistic fuzzy graph
[14]	2016	Intuitionistic Fuzzy Directed Graph	Energy of an intuitionistic fuzzy directed graph
[15]	2018	Intuitionistic fuzzy directed graph	Defined the degree matrix and Laplacian energy of intuitionistic fuzzy directed graphs
[16]	2019	Dominating Intuitionistic Fuzzy Graph	Upper and lower bounds for the energy of dominating intuitionistic fuzzy graphs
[17]	2020	Intuitionistic Fuzzy Graphs with TOPSIS method	Developed MATLAB software for computing signless Laplacian energy of an intuitionistic fuzzy graph with TOPSIS method.
[18]	2020	Bipolar Intuitionistic Fuzzy Graphs	Notions of subdivisions in intuitionistic bipolar fuzzy graphs
[19]	2023	Intuitionistic Fuzzy Matrix	Developed a MAGDM model using the IFM energy method
[20]	2023	Intuitionistic Fuzzy Graphs	Discussed the use of Laplacian energy and regression coefficient measurements

Chandrasekaran et al. (2014) focused on the concept of energy in intuitionistic fuzzy graphs, which incorporates uncertainty and hesitancy in their edges and vertices. The authors proposed a method for calculating the energy of an intuitionistic fuzzy graph based on the determinant of its adjacency matrix. They defined the energy of an intuitionistic fuzzy graph as the sum of the absolute values of the eigenvalues of its adjacent matrix. This paper presents a mathematical formulation and algorithm for computing the energy of an intuitionistic fuzzy graph. The authors also discussed the properties and characteristics of the energy of intuitionistic fuzzy graphs, including their relationship to the number of vertices and edges in the graph. This study provides insights into the structural properties of intuitionistic fuzzy graphs and makes a valuable contribution to the field of fuzzy graph theory.

Basha and Kartheek (2015) introduced the concept of Laplacian energy in the domain of intuitionistic fuzzy graphs. The authors delved into the topic of labeling Laplacian eigenvalues in a graph with n components, where $\mu_{n-i} = 0$ for $i = 0, 1, 2, \dots, p - 1$ and $\mu_{n-p} > 0$. The Laplacian energy of a graph G is precisely defined as the summation of the distances of the Laplacian eigenvalues of G and the average degree $d(G)$ of G . Furthermore,

the authors presented the Laplacian energy of an intuitionistic fuzzy graph, which is equal to the summation of the distances of the Laplacian eigenvalues and the average degree of the graph. The Laplacian energy of an intuitionistic fuzzy graph can be effectively computed by employing the intuitionistic fuzzy adjacency and intuitionistic fuzzy Laplacian matrices of the graph. The authors proposed a theorem that provides a mathematical representation of the Laplacian energy of an intuitionistic fuzzy graph.

Deepa et al. (2016a) analyzed the propagation rate of a virus on the extreme energy of an intuitionistic fuzzy graph, utilizing a website as a real-time example. The authors modelled the website as an intuitionistic fuzzy graph, explored the correlation between the number of visitors and the energy of the graph, and offered fundamental definitions pertaining to the energy of an intuitionistic fuzzy graph. Additionally, the authors constructed max-min intuitionistic fuzzy matrices for various time periods and calculated extreme energy, thereby presenting a comparison of energy and the spreading rate of the virus during these periods.

Deepa et al. (2016b) discussed the concept of energy in graphs and fuzzy graphs and presented established results for these subjects. The authors examined the energy of an intuitionistic fuzzy directed graph using a real-life example, considering its relevance in various real-world networks. This study enhances the understanding of directed graphs and their energy, thereby expanding the knowledge in the fields of graph theory and fuzzy graph theory. In addition, the authors explored the characteristic polynomial and determinant of the graph, which were utilized in the analysis of its energy. The findings of this study have potential applications in network analysis, optimization, and modeling, where a comprehensive understanding of the energy of directed graphs is crucial.

Praba et al. (2018) extended the concept of Laplacian energy to intuitionistic fuzzy directed graphs by defining the degree matrix and Laplacian energy for such graphs. They also derived the upper and lower bounds for the Laplacian energy of intuitionistic fuzzy directed graphs. Challenging the existing conjecture that the inequality $E(G) \leq LE(G)$ holds true for all graphs, the authors propose that it does not hold for intuitionistic fuzzy directed graphs. Numerical examples are provided to illustrate the defined principles and analyze the outcomes.

Deepa et al. (2019) introduced the concept of dominating energy for intuitionistic fuzzy directed graphs, and established upper and lower bounds for the energy of dominating intuitionistic fuzzy graphs, considering both real and complex eigenvalues. The study of domination in graphs originated from the game of chess and is well-established. The authors defined the least number of queens necessary to cover a chessboard, and introduced the concept of edge domination. Furthermore, a numerical example is presented to illustrate the discussed ideas, showing a dominating intuitionistic fuzzy graph. The paper also provides proofs and corollaries related to dominating intuitionistic fuzzy graphs, including the substitution of equations and the relationship between dominating sets and common eigenvectors. The significance of this study lies in its contribution to the field of domination of graphs, specifically in the context of intuitionistic fuzzy graphs. It expands on existing concepts and provides new insights into the energy of dominating intuitionistic fuzzy graphs.

Ramesh and Basha (2020) focused on the development of MATLAB software for various computations related to the signless Laplacian energy of intuitionistic fuzzy matrices and graphs, as well as intuitionistic fluffy weighted averaging (IFWA) using the TOPSIS method. This study utilized theoretical perspectives from previous studies and provided appropriate illustrations to demonstrate the functionality of the developed software. Specifically, the MATLAB program was applied to select the simplest alliance partner for an automobile company, showcasing its practical relevance in decision-making processes. This study contributes to the field of group decision making by providing a computational tool and methodology for analyzing and selecting partners based on the signless Laplacian energy of intuitionistic fuzzy graphs. In addition, the research paper included the representation of alternatives, rules, and weights in a table for better visibility. Overall, this paper presents a comprehensive study on the utilization of signless Laplacian energy and the TOPSIS method for partner selection, making a valuable contribution to the field of decision-making in the context of intuitionistic fuzzy graphs.

Alnaser et al. (2020) introduced and examined the concepts of IFG subdivisions, a matrix of bipolar intuitionistic fuzzy graphs, and line intuitionistic bipolar fuzzy graphs in their research paper. The authors provided definitions and analyses of these concepts, and established mathematical frameworks and principles for the representation and examination of intricate systems. The utilization of intuitionistic bipolar fuzzy graphs of subdivisions permits a more intricate depiction of uncertainty and ambiguity within graph structures, thereby enhancing the comprehension of complex relationships. The matrix of a bipolar intuitionistic fuzzy graph enables the investigation of the relationships and interactions between elements in a graph, offering insights into the connectivity and interdependencies within a system. The line intuitionistic bipolar fuzzy graph provides a means to model and analyze the connectivity and strength of different paths within a graph, thus contributing to the understanding of network structures and flow of information. The work conducted in this study makes valuable contributions to the field of applied mathematics and information sciences by introducing novel tools and

concepts for the analysis and representation of complex systems, with potential applications in decision-making, optimization algorithms, and data analysis.

Li and Ye (2023) presented the development of a multi-attribute group decision-making (MAGDM) model using the Intuitionistic Fuzzy Measure (IFM) energy method. This model effectively manages MAGDM problems that encompass all IFM information on attribute weights, decision-maker weights, and attribute values. The developed MAGDM model was applied to the problem of selecting hospital locations in Shaoxing City, China. The practicality and validity of the model were demonstrated through this application. The results indicate that the developed MAGDM model using the IFM energy method outperforms the existing intuitionistic fuzzy MAGDM methods in terms of comprehensive IFM information expressions and the ability to handle MAGDM problems with comprehensive IFM information. The superiority and novelty of the developed model lies in its capacity to fully represent all IFM information and provide effective solutions for intuitionistic fuzzy MAGDM problems.

Akula and Basha (2023) focused on intuitionistic fuzzy graphs (IFGs), which serve as extensions of fuzzy graphs and offer a better description of ambiguity in decision-making issues than fuzzy graphs. The authors computed the Laplacian energy and correlation coefficient of IFGs to address group decision-making problems supported by intuitionistic fuzzy preference relationships (IFPR). This study presents a methodology for calculating the comparative position loads of establishments by utilizing the undecided corroboration of IFPR and the correlation coefficient of one personality IFPR with respect to other items. Furthermore, the authors encompassed a substantial number of establishments in the comprehensive IFPR and devised a correlation coefficient technique to examine the significance of alternatives and determine the optimal choice. Finally, we introduce a collaborative decision-making technique into a money-investing scheme that can be adapted to various beneficial investing schemes. The classification of various energies in different aspects of neutrosophic graphs is addressed in the next section.

4. Energy of Neutrosophic fuzzy graphs

Table 3 presents a concise overview of the several methodologies employed to address the energy properties of neutrosophic graphs.

Table 3: Classification of Neutrosophic graphs

Reference	Year	Type of graphs	Significance/Influences
[21]	2018	Single-Valued Neutrosophic Graphs	Energy, Laplacian energy, and signless Laplacian energy in Single-Valued Neutrosophic Graph
[22]	2018	Analytic hierarchy process (AHP) and SWOT analysis	Evaluation of both quantitative and qualitative factors through comparison matrices.
[23]	2019	Interval-valued neutrosophic graphs	Computations of spectrum and energy of interval-valued neutrosophic graphs using MATLAB
[24]	2019	Interval-valued neutrosophic numbers	Developed the algorithm for neutrosophic shortest path problem between each pair of nodes in the network.
[25]	2019	Neutrosophic graph-based multicriteria decision-making model	Derived cartesian product, composition, union and join of two single valued neutrosophic sets
[26]	2019	Complex neutrosophic graph	Laplacian energy of a complex neutrosophic graph is computed
[27]	2020	Single-valued neutrosophic graphs	Introduced dominating sets and dominating numbers for energy graphs in single-valued neutrosophic graphs.
[28]	2020	Bipolar single-valued neutrosophic graph	Residue product, rejection, maximal product, and symmetric difference of BSVNGs had been explored with examples.
[29]	2020	Sum distance in a neutrosophic graph	Derived degree, eccentricity, radius, and diameter based on this metric

[30]	2020	ELECTRE approach	Multi-attribute decision-making in a refined neutrosophic environment
[31]	2021	Single-valued neutrosophic graphs (SVNG)	Definitions and operations in SVNG, such as rejection, maximal product, symmetric difference, and residue product are derived.
[32]	2021	Bipolar single-valued neutrosophic graphs	Concepts of energy, Laplacian energy, and signless Laplacian energy under bipolar single-valued neutrosophic graphs derived
[33]	2022	Balanced Neutrosophic Graphs	Properties of complementary and self-complementary balanced neutrosophic graphs derived
[34]	2022	Fermatean neutrosophic graphs	Derived regular Fermatean neutrosophic graphs, strong Fermatean neutrosophic graphs, cartesian, composition, and lexicographic product of Fermatean neutrosophic graphs
[35]	2023	Complex fermatean neutrosophic graph	Set theoretical operations such as complement, union, join, ring-sum, and cartesian product of CFNG derived.
[36]	2024	Interval-Valued Fermatean Neutrosophic Domain	Novel optimization approach for the Minimum Spanning Tree Problem within the interval-valued fermatean neutrosophic domain, addressing uncertainty in edge weights

Naz et al. (2018) proposed the concepts of energy, Laplacian energy, and signless Laplacian energy in single-valued neutrosophic graphs (SVNGs). The authors created these energy concept' attributes in SVNGs and constructed the connections between them. Real-world examples are provided to demonstrate how the suggested concepts may be used. The Laplacian and signless Laplacian spectra of SVNGs are also covered in this paper. Additionally, the number of non-zero SVNG eigenvalues was ascertained by the authors, and a correlation with the Laplacian energy was found. The signless Laplacian energy of SVNGs was defined, and its computation was explained by the authors.

Abdel-Basset et al. (2018) In order to address the shortcomings of the conventional SWOT analysis in terms of factor quantification and alternative ranking, the authors sought to improve it through integration with the analytic hierarchy process (AHP). The suggested model was implemented by the authors in a neutrosophic setting, taking into consideration hazy and contradictory real-world data. A genuine case study of Starbucks was conducted to validate the suggested model and show how it may be used in strategic planning and decision-making.

Broumi et al. (2019a) suggested utilizing MATLAB to compute the energy and spectrum of interval-valued neutrosophic graphs using mathematical notations and computations. This work addressed the difficulties in creating interval-valued neutrosophic graphs and assessing uncertainty and indeterminacy in graphical analytics. It also expands the mathematical algebra of neutrosophic graphs to include these graphs. The authors provided a thorough description of the fundamental ideas behind interval-valued neutrosophic graphs as well as their notions and attributes. This study established a practical framework for interpreting and evaluating interval-valued neutrosophic graphs in real-world applications, as well as a mathematical foundation for handling uncertainty in graph data. MATLAB was used for the computational analysis, method creation, and mathematical modeling in this study.

Broumi et al. (2019b) In a network operating in a neutrosophic environment, a unique technique for solving the shortest route problem (SPP) utilizing Bellman's algorithm is presented. This approach represents the network's edge lengths using interval-valued neutrosophic numbers (IVNNs). The effectiveness of the proposed algorithm was validated using a numerical example. To demonstrate the benefits of the new algorithm over the current approaches, a comparative analysis was performed. To compare IVNNs, this research presented a rating method based on a scoring function. This study used interval-valued neutrosophic sets to address indeterminacy in real-

world problems. The proposed technique provides a straightforward and efficient method for resolving SPP in an unpredictable setting.

Şahin (2019) introduced the innovative notion of single-valued neutrosophic graphs, delving into their characteristics, such as Cartesian product, composition, union, and join. The authors then formulate a multicriteria decision-making model based on neutrosophic graphs by employing two distinct methodologies to address single-valued neutrosophic decision-making problems. By utilizing two illustrative examples, the authors convincingly demonstrated the applicability, feasibility, effectiveness, and advantages of the proposed approaches. Compared with traditional and fuzzy models, single-valued neutrosophic models offer enhanced precision, flexibility, and compatibility. The incorporation of graph theory in various disciplines, including engineering, computer science, and medical diagnosis, highlights the wide-ranging applicability of the proposed single-valued neutrosophic graph-based model. Furthermore, the study expands upon the findings regarding crisp graphs, fuzzy graphs, and intuitionistic fuzzy graphs in the realm of single-valued neutrosophic graphs. This contribution represents a significant advancement in the field of neutrosophic set theory by introducing a novel tool to handle indeterminate, inconsistent, and uncertain information in real-life applications.

Khan et al. (2019) included complex neutrosophic graphs in their introduction and advancement of neutrosophic set theory. In addition to proposing the definition of energy for a complex neutrosophic graph in terms of its adjacency matrix, the authors also defined the adjacency matrix of a complex neutrosophic graph. Furthermore, they derived the lower and upper bounds of energy. The concept of Laplacian energy for a complex neutrosophic graph is introduced, and the findings on the Laplacian spectra of complex neutrosophic graphs are presented. The notions of energy and Laplacian energy for complex neutrosophic graphs were successfully illustrated using a numerical example. According to the study, more investigation into the energy and spectrum of intricate neutrosophic graphs could reveal new findings in subsequent research.

Mullai and Broumi (2020) The notion of dominating energy in neutrosophic graphs was first presented by the authors, who also defined a number of related terminologies, including the complement of neutrosophic graphs, eigenvalues for the dominating energy, and dominating the neutrosophic adjacency matrix. The authors stress the importance of dominating energy in the field of energy applications and contend that using neutrosophic graph theory yields more effective results than current techniques. Additionally, this study explores the union and join operations of neutrosophic graphs with respect to dominant energy and provides a number of theorems in this regard. Basic definitions and results from the domination theory of graphs are included in this paper; these are crucial for understanding the planned study. The dominant energy of neutrosophic graphs and the dominant neutrosophic adjacency matrix have mathematical representations. There are also formulas and equations for the trace of the dominating neutrosophic adjacency matrix and the sum of squares of the eigenvalues. The paper ends by providing a brief summary of the findings and highlighting the significance of the research project.

Malik et al. (2020) conducted a study to characterize and explore the properties of bipolar single-valued neutrosophic graphs (BSVNGs) and their applications in different domains. As an extension of single-valued neutrosophic graphs, intuitionistic fuzzy graphs, bipolar intuitionistic fuzzy graphs, BSVNGs have been introduced. They studied the properties of BSVNGs by defining four operations (residue product, rejection, maximal product, and symmetric difference) and provided examples. In a BSVNG, the researchers also calculated and talked about the vertex's degree as well as its total degree. They also created an algorithm to put a useful technique pertaining to BSVNGs into practice. The results of this study advance the knowledge of BSVNGs and their use in a variety of sectors, including neural networks, energy systems, and coding.

Expanding on the idea of a strong sum distance in fuzzy graphs, Das et al. (2020) proposed the notion of sum distance in neutrosophic graphs. The authors established a sum distance metric by defining the weights of the edges in a neutrosophic graph. They examined the degree, eccentricity, radius, and diameter of neutrosophic graphs among other features. The study also demonstrated the practical use of sum distance in a neutrosophic graph to resolve the travelling salesman issue. The authors suggested more studies in this field and discussed the potential of neutrosophic graphs to handle inaccurate data.

Sayyadi et al. (2020) presented a model of the ELECTRE approach, employing neutrosophic sets to handle uncertain, imprecise, incomplete, and inconsistent information in real-life systems. The model was demonstrated through an example involving the evaluation of three alternatives by four experts, based on five neutrosophic linguistic variables. These variables were converted into neutrosophic numbers and the integrated matrix, criteria weights, and expert weights were computed. Concordance and discordance matrices were then calculated based on the description provided. Finally, the alternatives were ranked, with A3, A2, and A1 being classified as first, second, and third, respectively.

Zeng et al. (2021) examined the basic properties of novel operations, such as rejection, maximal product, symmetric difference, and residue product, in single-valued neutrosophic graphs (SVNG). The notions of a vertex's degree and the total degree in SVNG are presented in. Vertex degrees under the maximal product, symmetric difference, residue product, and rejection operations were assigned specific requirements. The practical achievements of the research were highlighted by an algorithm described in the publication, coupled with the application of SVNG in the field of food and agriculture organization. The study also demonstrated that an SVNG is produced when two SVNGs are rejected. It provides a mathematical expression for computing the rejection of two SVNGs and characterizes the rejection operation in terms of crisp graphs. Additionally, this study demonstrated that one SVNG is the residue product of two SVNGs.

Mohamad et al. (2021) The authors of this study suggest using bipolar single-valued neutrosophic graphs (BSVNGs) in conjunction with energy, Laplacian energy, and signless Laplacian energy to improve decision-making in scenarios with both positive and negative uncertainty. Using examples to clarify its features, they provided the idea of a bipolar single-valued neutrosophic graph (BSVNG) for an energy graph. In-depth explanations are provided for the properties of eigenvalues, energy upper and lower bounds, Laplacian energy, and signless Laplacian energy in the BSVNGs. To further emphasize the importance of the former, a numerical comparison between the use of bipolar and non-bipolar concepts was presented. The applicability and effectiveness of the proposed technique in real-life scenarios were illustrated by applying energy, Laplacian energy, and signless Laplacian energy in BSVNGs to the selection of renewable energy sources.

Sivakumar et al. (2022) presented the concept of balanced neutrosophic graphs based on density functions and their properties were examined by the authors in this research article. They list the prerequisites—self-complementary, regular, complete, and strong—that a neutrosophic graph must meet to be categorized as a balanced neutrosophic graph. This study explores the basic definitions and theorems needed in graph theory and provides several features of complementary neutrosophic graphs. In addition, the elements of self-complementarity, regularity, completeness, and strong neutrosophic graphs are considered when discussing the requirements for classifying a neutrosophic graph as a balanced neutrosophic graph. Furthermore, the characteristics of balanced neutrosophic graphs that are complementary and self-complementary are studied. This study also discusses the weight of a single-valued neutrosophic graph (SVNG) in terms of the true membership value, indeterminacy membership value, and falsity membership value.

Broumi et al. (2022) The notion of Fermatean neutrosophic graphs and operations on Fermatean neutrosophic graphs are introduced by the writers in their academic work. Regular Fermatean neutrosophic graphs and strong Fermatean neutrosophic graphs are defined, and their features are discussed. This work investigates Fermatean neutrosophic graphs' Cartesian, compositional, and lexicographic products, providing techniques for merging and contrasting various graphs. The authors show how Fermatean neutrosophic graphs can be used in decision-making situations such as choosing a power plant. Furthermore, for fuzzy and neutrosophic sets, this research offers a geometric interpretation of membership, non-membership, and indeterminacy grades.

Broumi et al. (2023) In this study, the authors define a complex fermatean neutrosophic graph (CFNG) and discuss some basic graphical concepts, such as the order, size, degree, and total degree of a vertex in CFNG. They research set theory operations on CFNG, including cartesian product, complement, union, join, and ring-sum. Additionally, the idea of a regular graph in a complex Fermatean neutrosophic environment is presented and investigated. To solve a multi-criteria decision-making problem, including an unknown graphical structure of characteristics, the authors employed CFNG. Furthermore, it shows how CFNG can be used in the educational system to assess a lecturer's productivity in research. The application of complex neutrosophic sets (CNS) to the management of uncertainty in complex-valued physical variables was also discussed in this study.

Dhouib et al. (2024) The Dhouib-Matrix-MSTP (DM-MSTP) method is extended for interval-valued fermatean fuzzy graphs, showcasing its efficiency and effectiveness in solving the Minimum Spanning Tree Problem. A structured presentation of fundamental concepts, method outlines, numerical examples, and conclusions provides a comprehensive insight into the proposed solution. The study highlights the successful application of the DM-MSTP method in a case study, demonstrating its functionality and efficacy in solving the Minimum Spanning Tree Problem under interval-valued fermatean neutrosophic domain. The paper lays the groundwork for future research to explore the application of DM-MSTP to other variants of the Minimum Spanning Tree Problem and its extension to other neutrosophic domains, indicating promising directions for further advancements in graph theory.

5. Conclusion

The examination of fuzzy graphs has shown their adaptability in managing imprecise and unclear information, offering a sturdy framework for the decision-making process. Meanwhile, intuitionistic fuzzy graphs have

broadened this concept by integrating the concept of hesitation, enabling a more detailed portrayal of uncertainty in systems supporting decision-making. Furthermore, the analysis has also illuminated the emerging realm of neutrosophic graphs, which consider the existence of neutral or indeterminate elements, thereby expanding the range of graph theory to capture the complexities of phenomena observed in the real world. As we navigate through the constantly evolving landscape of uncertainty in various domains, such as finance, healthcare, and artificial intelligence, the insights gained from this study serve as a foundation for continued research and exploration. Future studies in this field may focus on refining existing models, devising innovative methodologies, and exploring practical applications that leverage the potential of these graph structures to address intricate and dynamic challenges.

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