Neutrosophic Topp-Leone Extended Exponential distribution modeling with application for bladder cancer patients

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Abstract

The Topp-Leone Extended Exponential distribution is used to simulate human lifetime data patterns in the field of survival analysis. To characterize a variety of uncertain survival data, the neutrosophic Topp-Leone extended exponential distribution (NTLEED) is used. The specified distribution is a great tool for modeling unknown data that is somewhat positively biased. This study covers the primary statistical properties of the constructed NTLEED, including the survival function, hazard rate, and neutrosophic moments. In addition, the neutrosophic parameters are estimated using the popular maximum likelihood estimation technique. To determine whether the predicted neutrosophic parameters were obtained, a simulation study is carried out. Not to mention that actual data has been used to discuss potential real-world applications of NTLEED. Real data were utilized to demonstrate how well the proposed model performed in contrast to the current distributions.

Keywords: Survival analysis; Neutrosophic statistics; hazard function; bladder cancer; Topp-Leone Extended Exponential distribution.

1. Introduction

Statistics come from the domain of mathematics, which starts from probability. Any statistics consists of three major parts: data, the model that governs the data, and methodology embedded to carry out necessary statistical inference. In handling data, many times, probabilities do not support the situation and then possibilities seem to be more appropriate. However, in both probabilities and possibilities, we always think that a certain event occurs or does not occur. But practically, the chances for the occurrence of an event are uncertain and they vary from absolute truth to absolute falsehood. These uncertainties are unsolved by simply adapting the concept of probability or the concept of possibility. To handle this kind of uncertainty, a new theory of probability and possibility called neutrosophic set theory has been introduced by one of the authors of this paper. Even though the domain of neutrosophic set theory starts from data analysis, and the odds of chances for the occurrence of any event are also susceptible to uncertainty.

Statistics is widely used in several fields such as economics, technology, science, and business. Accordingly, these concepts have attracted the attention of many researchers, and several definitions of neutrosophic sets have been made. However, the concept was formally introduced by Smarandache in 1995 [1]. The ideas of are

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analyzed and studied. Statistical analysis was done with the help of neutrosophic sets in 2015. The aim of the paper is to help people understand the importance of neutrosophic numbers and neutrosophic sets in statistics, shedding some light on future research topics. The rest of the paper is organized as follows: the concept of neutrosophic sets and neutrosophic numbers are introduced, and several properties and operations are summarized in chapter two. In chapter three, the neutrosophic ratio, the basics about neighborhood elements neutrosophic ratio, and the neutrosophic statistical operations are examined. In the final chapter, main conclusions and discussions are given. Neutosophic statistics provide an example of how to work with such limitations, such as with ambiguous, scarce, or inconsistent information [2,3,4]. Numerous industries, such as decision-making, pattern identification, data mining, and image processing, use neutrosophic statistics [4,5,6,7].

Neutrophic statistical distributions are a more versatile tool when dealing with imperfect information because attempts have been made to fit data with varying degrees of uncertainty. These distributions are commonly applied in scenarios when the problem in question naturally involves unpredictability, such as when making decisions in the face of uncertainty or when the facts only offer an approximative picture of the actual situation [8,9,10,11,12].

These distributions must accurately reflect the degree of truth, falsity, and uncertainty of the survival probability at each time point as well as the survival data that is currently accessible. Tools and mathematical formalizations related to neutrosophic logic can help achieve this. Neutronosophic probability distribution is the subject of numerous articles [11, 13,14,15,16,17,18,19,20,21,22,23,24].

Because its primary role is the examination of survival statistics, neutrophilic information is very significant. Survival analysis is a statistical technique that focuses on the amount of time until an event occurs [11]. The probability distributions of the temporal data form the foundation of the entire survival analysis. It makes it possible to portray having a distorted or incomplete understanding of what happened.

The Topp-Leone Extended Exponential distribution has numerous applications, one of which is survival analysis. We extended the Topp-Leone Extended Exponential distribution's applicability in neutrosophy when the data in this study is interval form with some degree of indeterminacy. Numerous attributes are examined within the recently suggested distribution, and their uses are explained using both simulated and actual data applications pertaining to bladder cancer.

2. Neutrosophic Topp-Leone Extended Exponential distribution

Topp-Leone distribution is a very general class of distributions because it contains many well-known discrete distributions that are useful in a large number of problems. Many researchers in different application fields, for instance biology, ecology, epidemiology, genetics, and many others obtained their studied models from modeling real issues [25].

The Topp-Leone Extended Exponential distribution (TLEED), proposed by Aidi, et al. [26], considers the extended exponential distribution with a Topp-Leone distribution. More specifically, if a random variable Y has a Gompertz distribution, then the random variable X follows TLEED with density and cumulative distribution functions defined, respectively, by:

$$f(x,\alpha,\beta,\lambda) = 2\alpha\beta\lambda\left(1+\alpha x\right)^{\beta-1} e^{2\left\{1-(1+\alpha x)^{\beta}\right\}} \left\{1-e^{2\left\{1-(\alpha x)^{\beta}\right\}}\right\}^{\lambda-1} \quad (1)$$

$$F(x,\alpha,\beta,\lambda) = \left\{1-e^{2}\left\{1-\left(1+\alpha x\right)^{\beta}\right\}\right\}^{\lambda}, \qquad x>0, \alpha>0, \beta>0, \lambda>0 \quad (2)$$

“The concept of neutrosophic probability as a function $NP :\to [0, 1]^3$ was originally presented by [2], where U is a neutrosophic sample space and defined the probability mapping to take the form $NP(S) = \left(ch(S), ch(neut\ S), ch(anti\ S)\right) = (\eta, \beta, \tau)$ $where\ 0 \le \eta, \beta, \tau \le 1\ and\ 0 \le \eta + \beta + \tau \le 3$. The term Ψ represents the set of sample space, R represents the set of real numbers, and Υ denotes a sample space event, X_N and Y_N denote neutrosophic r.v.

Suppose the neutrosophic variable could be expressed as: $x_N = x_L + x_U I_N$ where $I_N \in \{I_L, I_U\}$ and x_L and $x_U I_N$ denote the determined and indeterminate parts, respectively. Assume that the neutrosophic random variable $x_N \in \{x_L, x_U\}$ follows the TLEED having neutrosophic parameters: $\beta_N \in \{\beta_L, \beta_U\}$, $\alpha_N \in \{\alpha_L, \alpha_U\}$,

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and $\lambda_N \in \{\lambda_L, \lambda_U\}$ where the letters L and U are the lower values and the upper values, respectively". Then, the neutrosophic probability density function of neutrosophic Topp-Leone Extended Exponential distribution (NTLEED) is given by

$$f(x_N, \alpha_N, \beta_N, \lambda_N) = 2\alpha_N \beta_N \lambda_N (1+\alpha_N x_N)^{\beta_N - 1} e^{2\left\{1-(1+\alpha_N x_N)^{\beta_N}\right\}} \left\{1-e^{2\left\{1-(\alpha_N x_N)^{\beta_N}\right\}}\right\}^{\lambda_N - 1} \quad (3)$$

The neutrosophic cumulative density function, the neutrosophic survival, and neutrosophic hazard functions are given below, respectively:

$$F(x_N, \alpha_N, \beta_N, \lambda_N) = \left\{1-e^{2}\left\{1-(1+\alpha_N x_N)^{\beta_N}\right\}\right\}^{\lambda_N}, \quad (4)$$

$$S(x_N, \alpha_N, \beta_N, \lambda_N) = 1-\left\{1-e^{2\left\{1-(1+\alpha_N x_N)^{\beta_N}\right\}}\right\}^{\lambda_N}, \quad (5)$$

$$h(x_N, \alpha_N, \beta_N, \lambda_N) = \frac{2\alpha_N \beta_N \lambda_N (1+\alpha_N x_N)^{\beta_N - 1} e^{2\left\{1-(1+\alpha_N x_N)^{\beta_N}\right\}} \left\{1-e^{2\left\{1-(\alpha_N x_N)^{\beta_N}\right\}}\right\}^{\lambda_N - 1}}{1-\left\{1-e^{2\left\{1-(1+\alpha_N x_N)^{\beta_N}\right\}}\right\}^{\lambda_N}} \quad (6)$$

The moment generating function, $M_X(x_N)$ **of** NTLEED is

$$M_X(x_N) = \sum_{r,j=0}^{\infty} \sum_{i=0}^{r} \sum_{\ell=0}^{[i/\beta]} \frac{x_N^{\,r_N}}{r_N!} \frac{(-1)^{i_N + j_N + r_N} \lambda_N \binom{r_N}{i_N} \binom{\lambda_N - 1}{j_N} [i_N / \beta_N]!}{2^{\frac{i_N}{\beta_N} - \ell_N} \alpha_N^{\,r_N} (j_N + 1)^{\frac{i_N}{\beta_N} - \ell_N + 1} \ell_N!} \quad (7)$$

3- Parameter Estimation of NTLEED

Maximum likelihood estimation (MLE) method is mostly used in estimating NTLEED parameters. Let $x_{N1}, x_{N2}, \ldots, x_{Nn}$ be a random sample of size n from the NTLEED. The log-likelihood function is then given by

$$\log L = n\ln(2) + n\ln(\alpha_N \lambda_N \beta_N) + (\beta_N - 1)\sum_{i=1}^{n} \ln(1+\alpha_N x_{N i_N}) + 2n - 2\sum_{i=1}^{n} (1+\alpha_N x_{N i_N})^{\beta_N} + (\lambda_N - 1)\sum_{i=1}^{n} \ln\left(1-e^{2\left\{1-(1+\alpha_N x_N)^{\beta_N}\right\}}\right) \quad (8)$$

Thus, the MLE of $\hat{\alpha}_N, \hat{\beta}_N$ and $\hat{\lambda}_N$ for α_N, β_N and λ_N are the solutions to the nonlinear equations:

$$\frac{\partial L}{\partial \alpha_N} = \frac{n}{\alpha_N} - 2\beta_N \sum_{i=1}^{n} x_{Ni} u_{Ni}^{\,\beta_N - 1} + (\beta_N - 1)\sum_{i=1}^{n} \frac{x_{Ni}}{1+\alpha_N x_{Ni}} + 2(\lambda_N - 1)\beta_N \sum_{i=1}^{n} \frac{x_{Ni} u_{Ni}^{\,\beta_N - 1} e^{2(1-u_{Ni}^{\,\beta_N})}}{1-e^{2(1-u_{Ni}^{\,\beta_N})}} = 0 \quad (9)$$

$$\frac{\partial L}{\partial \beta_N} = \frac{n}{\beta_N} + \sum_{i=1}^{n} \ln u_{Ni} - 2\sum_{i=1}^{n} u_{Ni}^{\,\beta_N} \ln u_{Ni} + 2(\lambda_N - 1)\sum_{i=1}^{n} \frac{u_{Ni}^{\,\beta_N} \ln(u_{Ni}) e^{2(1-u_{Ni}^{\,\beta_N})}}{1-e^{2(1-u_{Ni}^{\,\beta_N})}} = 0 \quad (10)$$

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$$\frac{\partial L}{\partial \lambda_N} = \frac{n}{\lambda_N} + \sum_{i=1}^{n} \ln\left(1 - e^{2\left\{1-(1+\alpha_N x_{Ni})^{\beta_N}\right\}}\right) = 0 \quad (11)$$

where $u_{Ni}(\alpha_N, x_{Ni}) \equiv u_{Ni} = 1 + \alpha_N x_{Ni}$.

5. Simulation results

A Monte Carlo simulation is run in R software with several sample sizes, $n = 20, 50, 150, 300$ and neutrosophic parameters in three cases: (1) $\beta_N \in [1, 1.8]$, $\alpha_N \in [1.3, 2]$, $\lambda_N \in [0.5, 1]$, (2) $\beta_N \in [1.5, 2.5]$, $\alpha_N \in [2, 3]$, $\lambda_N \in [1, 2]$, and (3) $\beta_N \in [2.5, 4]$, $\alpha_N \in [3, 4.5]$, $\lambda_N \in [1.8, 3]$. The simulation is replicated for 1000 times. For every number of n, performance metrics such as the neutrosophic mean square error (NMSE), the neutrosophic average bias (NAB). The results are given in Tables 1 -3. These Tables show that, for both neutrosophic parameters, the NAB and NMSE decrease with increasing sample sizes, as would be predicted. Moreover, the results of the study indicate that the neutrosophic MLE for the NTLEED provides precise estimation with a larger sample size.

Table 1: Average performance for case 1

N	NAB			NMSE		
	β_N	α_N	λ_N	β_N	α_N	λ_N
20	[0.018, 0.018]	[0.021, 0.025]	[0.020, 0.022]	[0.031, 0.033]	[0.041, 0.043]	[0.039, 0.044]
50	[0.015, 0.017]	[0.017, 0.020]	[0.015, 0.018]	[0.029, 0.028]	[0.035, 0.038]	[0.035, 0.041]
150	[0.010, 0.013]	[0.013, 0.018]	[0.010, 0.015]	[0.022, 0.021]	[0.030, 0.035]	[0.030, 0.036]
300	[0.005, 0.008]	[0.010, 0.015]	[0.005, 0.010]	[0.018, 0.017]	[0.025, 0.028]	[0.022, 0.027]

Table 2: Average performance for case 2

N	NAB			NMSE		
	β_N	α_N	λ_N	β_N	α_N	λ_N
20	[0.025, 0.028]	[0.031, 0.035]	[0.032, 0.038]	[0.047, 0.051]	[0.044, 0.050]	[0.038, 0.043]
50	[0.021, 0.022]	[0.027, 0.030]	[0.027, 0.031]	[0.043, 0.047]	[0.041, 0.047]	[0.034, 0.039]
150	[0.017, 0.019]	[0.022, 0.025]	[0.022, 0.027]	[0.038, 0.042]	[0.036, 0.041]	[0.029, 0.035]
300	[0.013, 0.016]	[0.015, 0.018]	[0.016, 0.021]	[0.032, 0.037]	[0.030, 0.035]	[0.021, 0.026]

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Table 3: Average performance for case 3

N	NAB			NMSE		
	β_N	α_N	λ_N	β_N	α_N	λ_N
20	[0.033, 0.040]	[0.032, 0.038]	[0.035, 0.042]	[0.048, 0.052]	[0.044, 0.051]	[0.041, 0.046]
50	[0.028, 0.034]	[0.027, 0.032]	[0.029, 0.037]	[0.042, 0.048]	[0.040, 0.046]	[0.034, 0.040]
150	[0.022, 0.029]	[0.022, 0.027]	[0.022, 0.030]	[0.037, 0.041]	[0.035, 0.040]	[0.028, 0.035]
300	[0.017, 0.022]	[0.014, 0.021]	[0.014, 0.022]	[0.031, 0.036]	[0.031, 0.036]	[0.022, 0.026]

5. Applications

In this section, the practical application was utilized in terms of sample data to determine potential interest towards the distribution of NTLEED. We are talking about the data set that is a aggregation of 128 patients with cancer who's months-long remission periods. The remission times adopted in the analysis reported here apply to a selected sample of patients with bladder cancer and are mostly meant for illustration [27]. Consequently, the findings on the goodness of fit of four probability distributions, which includes the KS statistic, reveal that the NTLEED is one of the probable distributions for the remission times. According to [16], the information presented here demonstrates that the remission times for a number of cancer patients, including those listed in [7.26, 8.2], [12, 14.77], [15, 17.2], [5.3, 7.1], [75.02, 81], and [1.5, 3.2], are not exact but rather given at irregular intervals.

We evaluate the model suitability of the proposed NTLEED with the neutrosophic exponential distribution (NED) applications for complicated data processing studied by [16]. The log-likelihood value (LogL), Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and Kolmogorov-Smirnov (KS) test are the techniques used to determine which model best fits the data. This is the model that has highest LogL values and lowest AIC, BIC & KS statistic values. Also, it establishes that when p-value increases, the model which forms the best fit for the neutrosophic results is achieved. The model sufficiency indicators and the Neutrosophic Maximum Likelihood Estimators for each group are presented in detail in Table 4. Therefore, arising from the outcomes presented above is evidence that the NTLEED has an advantage over the NED in data. This representation of the proposed model's performance is highlighted with bold values in the table below.

Table 4: The cancer patients' data neutrosophic distributions

	NED	NTLEED
Parameter	$\alpha_N = [0.1081, 0.10822]$	$\beta_N = [1.053, 1.122]$
		$\alpha_N = [0.517,\ 0.831]$
		$\lambda_N = [2.031,\ 2.154]$
LogL	[10.352, 13.241]	**[77.631, 79.567]**
AIC	[63.508, 65.334]	**[148.381, 150.674]**

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BIC	[60.218, 61.229]	**[147.348, 148.106]**
KS-value	[0.752, 0.774]	**[0.227, 0.284]**
KS- p-value	[1.135×10^{-6}, 1.188×10^{-6}]	**[0.966, 0.979]**

6. Conclusions

Here, a new distribution, known as neutrosophic Topp Leone extended exponential distribution (NTLEED) is introduced. About the given application data, the works above examined can be used for survival and dependability uncertainty by utilizing the well-known distribution. Further, the moments of neutrosophy, neutrosophic hazard rate, and the neutrosophic survival function have been investigated as the primary statistical modules of the proposed NTLEED. The illustration of the neutrosophic MLEs after construction present the MSEs and neutrosophic average bias for several instances in terms of sample size. To naturally reach the computed values of neutrosophic parameters, simulation research was conducted. The analysis of the simulation data reveals that the two factors that played a crucial role in getting a far better estimate of an unknown parameter are the sample size and the neutrosophic parametric value. For example, the collecting of remission times from one hundred twenty-eight cancer patients is the other compelling evidence supporting the use of NTLEED where neutrosophic condition is present.

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