



On Neutrosophic Truncation

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Abstract

Neutrosophic have found their place in neutrosophic studies due to the prevalence of indeterminacy in the world. We present the novel notion of neutrosophic truncated distribution, which is highly significant in analyzing events that involve the exclusion of certain data from the original dataset, particularly where there is a presence of indeterminacy in data. Unsure or ambiguous information, which is disregarded in classical logic, is incorporated within neutrosophic logic due to its focus on both certain and uncertain data. In this paper, the approach of neutrosophic truncation, and truncated distribution of neutrosophic random variable have been introduced, in addition to deriving some of its properties. And other cases discussed neutrosophic truncation depends on the neutrosophic probability function, a classical probability function, and studies neutrosophic probability and neutrosophic interval together. It studies the neutrosophic left truncated and neutrosophic right truncated. Some illustrative examples and statistical properties such as the cumulative function, the moment generating function, the order statistic, and the r th moment are presented.

Keywords: Data truncation; neutrosophic truncated; Left and right neutrosophic truncated; neutrosophic random variable; the moment generating function; the order statistics.

1. Introduction

Researchers are interested in studying distributions in various forms as many problems are difficult to model easily such as engineering, astrological, health, and social. To solve these problems, they should be modeled mathematically. For this reason, many researchers have been introduced in different forms of distribution, including compound, mixture, and truncated distributions. One of these forms is truncated distributions, which is focused on studying the distribution after deleting some of the unimportant or ambiguous data or data that is considered out of study range. Truncation is considered a statistical problem that has wide range of applications, in the survival analysis of patients, the study of the epidemic, astronomy, and the economy. In addition, it is more flexible for modeling data. Truncated statistics goes back to (1898)[1] when Galton analyzed truncated samples which is related to the speed of American trotting horses.

Truncation is defined as a phenomenon in which some samples in the population have a minimum chance of being selected due to their measurement being very short or very long. That is, the probability of an individual is chosen from a population depends on its measurement. Truncated distribution have been studied such as Pareto, left and right gamma distributions, see[2,3,4], and others.

For example, in studies from lifetime, it may be too costly or time-needed to get hold of random sample from a population under study, such as, one cannot identify an individual whose life is shorter or longer than a specific value (truncation limit). Another example, an astronomer cannot determine the luminosity of quasars if they are very dim or very bright. If a set of measurements on quasars, it is double truncated see [5,6]. The number of quasars is required the inclusion of a low and upper on luminosity function or Schechter function, where the low and upper, in case each one or both of them are set or interval, they cannot be represented by classical approach (truncation classical). In the above examples, we find it difficult to calculate the truncated distribution within the classical point or interval

(truncated limit). In addition, ambiguous data or unimportant data is unconsidered in classical approach therefore neutrosophic truncated distribution is considered to include ambiguous or unimportant data with classical data. Florentin first suggested the neutrosophic logic in (1995)[7][15]. Neutrosophic logic introduced many researches on continuous distributions, some method to find neutro- reliability theorem, ... etc.[8,9,10,11,12,18]. Also, researchers introduced the concepts of graph theory and time series in [12][13][14]. Florentin present neutrosophic statistic and neutrosophic measure and neutrosophic integral, see [15],[16]. Sometimes it requires deriving a probability density function (probability mass function) to random variable X and defining its value on the part of the values that are defined in the parameter space for reasons related to the nature of the study. This paper defined the neutrosophic truncation distribution according to neutrosophic logic with different cases depends on neutrosophic random variable and other. The neutrosophic left and right truncated distribution have been introduced. In addition, studied the statistical properties and illustrated examples.

2. Truncation Approach

The truncation approach directly affects one of the properties of the probability density (mass) functions it is that the probability related to space of the random variable after truncation which equal to less than one, $P(\Omega) < 1$ and this is impossible in classical logic, so it must derive a new distribution from the original distribution such that it satisfies the properties of the probability density (mass) function.

Let X be a random variable with probability density function $f(x)$ and the value of X defined on $(-\infty, \infty)$, and suppose we wish restriction probability density function to X defined on a subset Ω^* of sample space Ω , $\Omega^* = \{x: a < x < b\}$, a, b are two numbers defined on Ω .

Let $F(x)$ be the cumulative distribution function of X and it's derived based on function $f(x)$.

$$\text{Let } \xi > 0, \int_a^b \xi f(x) dx = \xi \int_a^b f(x) dx = \xi \left[\int_{-\infty}^b f(x) dx - \int_{-\infty}^a f(x) dx \right] = \xi [F(b) - F(a)]$$

$$\int_a^b (\xi f(x) dx) = 1$$

$$\xi = [F(b) - F(a)]^{-1};$$

the new probability density function after truncation the interval (a, b) is:

$$f(x)^* = [F(b) - F(a)]^{-1} f(x); \quad a < x < b \quad (1)$$

$([b, \infty), (-\infty, a])$, two intervals that have been ignored after truncation.

And we call $f(x)^*$ truncated distribution function from $f(x)$.

3. Neutrosophic Truncated Distribution

In this section, we will define the truncated distribution according to the new logic (neutrosophic logic) which has been introduced in (1995) by Florentin [7].

Neutrosophic logic of any idea is evaluated to have the part of truth in a subset T , the part of indeterminacy in a subset I , and the part of falsity in a subset F such that T, I, F be standard or non-standard real subsets of $] - 0, 1+ [$.

Let X be a random variable that has probability density function $f(x)$, and cumulative function $F(x)$, defined on $(-\infty, \infty)$, if we wish to restrict probability density function to X defined on a subset Ω^* of sample space Ω , $\Omega^* = \{x: a^* < x < b^*\}$, a^*, b^* two neutrosophic numbers defined on Ω .

The neutrosophic truncated distribution define as follow:

$$\begin{aligned} \text{Let } \xi > 0, \int_{a^*}^{b^*} \xi f(x) dx &= \int_{a^*}^{b^*} f(x) dx = \xi \left[\int_{-\infty}^{b^*} f(x) dx - \int_{-\infty}^{a^*} f(x) dx \right] \\ &= \xi [F(b^*) - F(a^*)], \quad \xi = [F(b^*) - F(a^*)]^{-1} \end{aligned}$$

Truncation means delete some values of the domain of the random variables, and the two intervals that have been ignored are $([b^*, \infty), (-\infty, a^*])$;

Where $a^* = a + I$, and $b^* = b + I$ is the neutrosophic number, and I is the indeterminacy part and a, b are the real parts. Where a or b or both may be a set or an interval containing real or number complex. If the coefficients of a and b are real numbers, then a^* , b^* are a real neutrosophic number.

And the new probability function after truncation (a^*, b^*) is:

$$Nf(x)^* = [F(b^*) - F(a^*)]^{-1} f(x); \quad a^* < x < b^* \quad (2)$$

Then we call $Nf(x)^*$ neutrosophic truncated function from $f(x)$, so $Nf(x)^*$ is neutrosophic truncated distribution.

4. Truncated Distribution of Neutrosophic Random Variables

We define the truncated distribution by depended on neutrosophic random variable.

Consider the classical random variable defined on the real number as follow:

$X: \omega \rightarrow \mathfrak{R}$, where ω is the sample space.

If redefine the random variable according to neutrosophic logic as follow [17]:

neutrosophic random variable (NRV) X_{ne} is

$$X_{ne}: \omega \rightarrow \mathfrak{R}(I), \text{ I am indeterminacy. And } X_{ne} = X + I$$

Where the cumulative distribution function of X_{ne} is $F_{X_{ne}}(x) = F_X(x - I)$

And the probability density function of X_{ne} is $f_{X_{ne}}(x) = f_X(x - I)$.

The truncated distribution of neutrosophic random variable (TNRV) is:

$$f_T(X_{ne}) = \frac{f(x_{ne})}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)}, \quad \infty \leq X_{ne} \leq \infty, \quad [\alpha, \beta] \subseteq [-\infty, \infty].$$

$$f_T(X_{ne}) = \frac{f(x_{ne})}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)}, \quad I - \infty \leq X \leq \infty + I,$$

If $X \in [0, 1]$, the probability density function of truncation neutrosophic random variable is:

$$f_T(X_{ne}) = \frac{f(x_{ne})}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)}, \quad 0 \leq X_{ne} \leq 1, \quad [\alpha, \beta] \subseteq [0, 1]$$

$$f_T(X_{ne}) = \frac{f(x_{ne})}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)}, \quad I \leq X \leq I + 1 \quad (3)$$

$$\int_{\alpha}^{\beta} f_T(x_{ne}) dx = \int_{\alpha}^{\beta} \frac{f(x_{ne}) dx}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} = \frac{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} = \frac{F_X(X - I)(\beta) - F_X(X - I)(\alpha)}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} = 1$$

And the cumulative distribution truncated of neutrosophic random variable is:

$$FT_{X_{ne}}(x) = P(X_{ne} \leq x_{ne}) = \frac{F_{X_{ne}}(x_{ne}) - F_{X_{ne}}(\alpha)}{F_{X_{ne}}(x_{ne}) - F_{X_{ne}}(\alpha)} = \frac{F_{X_{ne}}(x_{ne}) - F_{X_{ne}}(\alpha)}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} \quad (4)$$

$$FT_{X_{ne}}(x) = P(X_{ne} \leq x_{ne}) = P(X + I \leq x) = P(X \leq x - I) = FT_X(x - I)$$

We get truncated density function after taking the derivative to x :

$$f_T(X_{ne})(x) = \frac{dF_T(x_{ne})(x)}{dx} = \frac{dF_T(x-I)}{dx} = f_T(X-I)$$

The r^{th} moment about zero of truncated neutrosophic random variable

Consider the neutrosophic random variable $X_{ne} = X + I$, defined on $[-\infty, \infty]$ has probability density function $f_T(X_{ne})$.

The r^{th} moment of neutrosophic random variable after truncation on the interval $[\alpha, \beta]$ where $[\alpha, \beta] \subseteq [-\infty, \infty]$ is:

$$E(X_{Tne}^r) = \int_{-\infty}^{\infty} X_{ne}^r f_T(X_{ne}) dx$$

$$E(X_{Tne}^r) = \left[\frac{\int_{\beta}^{\alpha} X_{ne}^r f(x_{ne}) dx}{F_X(\beta) - F_X(\alpha)} \right] = \frac{\beta^r F_X(\beta) - \alpha^r F_X(\alpha) - r \int_{\beta}^{\alpha} X_{ne}^{r-1} F(x) dx}{F_X(\beta) - F_X(\alpha)} \quad (5)$$

$$\begin{aligned} &= \left[\frac{\int_{\beta}^{\alpha} (X+I)^r f(x) dx}{F_X(\beta) - F_X(\alpha)} \right] = \frac{\int_{\beta}^{\alpha} (X)^r f(x) dx + \int_{\beta}^{\alpha} (I)^r f(x) dx}{F_X(\beta) - F_X(\alpha)} = \frac{\beta^r F_X(\beta) - \alpha^r F_X(\alpha) - r \int_{\beta}^{\alpha} X^{r-1} F(x) dx}{F_X(\beta) - F_X(\alpha)} + \frac{[r(I)^r \int_{\beta}^{\alpha} f(x) dx]}{F_X(\beta) - F_X(\alpha)} \\ &= \int \frac{(X)^r f_x(x)}{F_x(\beta) - F_x(\alpha)} dx + I^r \int \frac{f_x(x)}{F_x(\beta) - F_x(\alpha)} dx \\ &= \int_{\beta}^{\alpha} (X)^r f_T(X) dx + I^r \int_{\beta}^{\alpha} f_T(X) dx = E(X_T^r) + I. \quad \text{Since } \int_{\beta}^{\alpha} f_T(X) dx = 1, I^r = I \end{aligned}$$

Properties of expected value for TNRV

1. $E(c X_{Tne} + d + kI) = c E(X_{Tne} + d + kI)$, where $c, d, k \in \mathcal{R}$.
2. $E(X_{Tne} \pm Y_{Tne}) = E(X_{Tne}) \pm E(Y_{Tne})$
3. $E((k + lI) X_{Tne}) = E(k X_{Tne} + l I X_{Tne}) = kE(X_{Tne}) + lIE(X_{Tne})$, where $l, k \in \mathcal{R}$.
4. $|E(X_{Tne})| \leq E(|X_{Tne}|)$.

4.2 The r^{th} Moment about Mean of TNRV

$$\begin{aligned} E(X_{Tne} - E(X_{Tne}))^r &= \int_{-\infty}^{\infty} (X_{Tne} - E(X_{Tne}))^r f_T(X_{ne})(x) dx \\ &= \sum_{\vartheta=0}^r (-1)^{r-\vartheta} \binom{r}{\vartheta} (E(X_{Tne}))^{r-\vartheta} \int_{-\infty}^{\infty} (X_{Tne})^{\vartheta} f_T(X_{ne}) dx \\ &= \sum_{\vartheta=0}^r (-1)^{r-\vartheta} \binom{r}{\vartheta} (E(X_{Tne}))^{(r-\vartheta)} \frac{\int_{\beta}^{\alpha} X_{ne}^{\vartheta} f_T(X_{ne}) dx}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} \\ &= \sum_{\vartheta=0}^r (-1)^{r-\vartheta} \binom{r}{\vartheta} \left(\frac{\beta F_x(\beta) - \alpha F_x(\alpha) - \int_{\beta}^{\alpha} F(x) dx}{F_x(\beta) - F_x(\alpha)} \right)^{(r-\vartheta)} \left(\frac{[\beta^{\vartheta} F_x(\beta) - \alpha^{\vartheta} F_x(\alpha)] - \vartheta \int_{\beta}^{\alpha} X_{ne}^{\vartheta-1} F(x) dx}{F_x(\beta) - F_x(\alpha)} \right) \\ &= \sum_{\vartheta=0}^r (-1)^{r-\vartheta} \binom{r}{\vartheta} \left(\int \frac{X f_x(x)}{F_x(\beta) - F_x(\alpha)} dx + I \int \frac{f_x(x)}{F_x(\beta) - F_x(\alpha)} dx \right)^{(r-\vartheta)} \left(\int \frac{(X)^{\vartheta} f_x(x)}{F_x(\beta) - F_x(\alpha)} dx \right. \\ &\quad \left. + I^{\vartheta} \int \frac{f_x(x)}{F_x(\beta) - F_x(\alpha)} dx \right) \\ &= \sum_{\vartheta=0}^r (-1)^{r-\vartheta} \binom{r}{\vartheta} (E(X_T + I))^{r-\vartheta} \int_{-\infty}^{\infty} (X_T + I)^{\vartheta} f_T(x) dx \end{aligned}$$

$$= \int_{-\infty}^{\infty} [(X_T + I) - E(X_T + I)]^r f_T(x) dx$$

$$E(X_{Tne} - E(X_{Tne}))^r = E[(X_T + I) - E(X_T + I)]^r \tag{6}$$

4.3 Moment Generating Function of TNRV

Consider the neutrosophic random variable $X_{ne} = X + I$, has probability density function $f_T(X_{ne})$

$$\mathcal{M}_{X_{Tne}}(t) = E(e^{tX_{ne}}) = \int_{-\infty}^{\infty} e^{tX_{ne}} f_T(X_{ne}) dx ; -h < t < h, h > 0.$$

$$= \frac{e^{\beta t} F_{X_{ne}}(\beta) - e^{\alpha t} F_{X_{ne}}(\alpha) - t \int_{\alpha}^{\beta} e^{tX_{ne}} F_{X_{ne}}(x) dx}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)}$$

$$\mathcal{M}_{X_{Tne}}(t) = E(e^{tX_{ne}}) = \int_{-\infty}^{\infty} e^{tX_{ne}} f_T(x_{ne}) dx = \int_{-\infty}^{\infty} e^{t(X+I)} f_T(x) dx$$

$$\int_{-\infty}^{\infty} e^{tx+tl} f_T(x) dx = \frac{e^{\beta t+tl} F_X(\beta) - e^{\alpha t+tl} F_X(\alpha) - t \int_{\alpha}^{\beta} e^{tx+tl} F_X(x) dx}{F_X(\beta) - F_X(\alpha)}$$

$$= \frac{e^{tI} (e^{\beta t} F_X(\beta) - e^{\alpha t} F_X(\alpha)) - t \int_{\alpha}^{\beta} e^{tx} F_X(x) dx}{F_X(\beta) - F_X(\alpha)}$$

$$= (e^{tI}) \frac{(e^{\beta t} F_X(\beta) - e^{\alpha t} F_X(\alpha)) - t \int_{\alpha}^{\beta} e^{tx} F_X(x) dx}{F_X(\beta) - F_X(\alpha)}$$

$$\mathcal{M}_{X_{Tne}}(t) = (e^{tI}) \int_{-\infty}^{\infty} e^{tx} f_T(x) dx = (e^{tI}) E(e^{tx}) = (e^{tI}) \mathcal{M}_{X_T}(t) \tag{7}$$

Properties of MGF for TNRV

- 1- $\mathcal{M}_{TX_{ne}}(0) = 1;$
 - 2- $\frac{d\mathcal{M}_{TX_{ne}}(t)}{dt} |_{t=0} = E(X_{Tne});$
- $$\frac{d\mathcal{M}_{TX_{ne}}(t)}{dt} = \frac{d e^{tI}}{dt} \mathcal{M}_{TX}(t=0) + \frac{d\mathcal{M}_{TX}}{dt} e^{tI} (t=0)$$
- $$= I e^{tI} \mathcal{M}_{TX}|_{t=0} + \mathcal{M}_{TX} e^{tI} |_{t=0}$$
- $$= I \mathcal{M}_{TX}(0) + \mathcal{M}_{TX}$$
- $$= E(X_T) + I = E(X_T)$$
- 3- $\frac{d^n \mathcal{M}_{TX_{ne}}(t)}{dt^n} |_{t=0} = E(X_{Tne}^n)$
 - 4- If $Y_{Tne} = (c + Id)X_{Tne} + (l + kI)$, then $\mathcal{M}_{Y_{Tne}} = e^{c+Id} e^{t(l+k)I} \mathcal{M}_{X_{Tne}}(c + Id)t$ where $c, d, k, l \in \mathcal{R}$

$$\mathcal{M}_{Y_{Tne}} = E(e^{tY_{Tne}}) = E(e^{t[(c+Id)X_{Tne}+(l+kI)]}) = E(e^{t[(c+Id)(X_{Tne}+I)]} e^{t(l+kI)})$$

$$= e^{t(l+kI)} E(e^{t[(cX_{Tne}+cl+dIX_{Tne}+dl^2)]})$$

$$= e^{t(l+kI)} e^{t(c+dI)} \mathcal{M}_{X_{Tne}}(c + Id)t \tag{8}$$

4.4 The Order Statistic of Truncated Neutrosophic Random Variables

Let $x_{ne_1}, x_{ne_2}, x_{ne_3}, \dots, x_{ne_m}$ be the independent and identical distribution of neutrosophic random variables with truncated probability density function and neutrosophic cumulative distribution. Then $x_{ne_{[1]}} \leq x_{ne_{[2]}} \leq x_{ne_{[3]}} \dots \leq x_{ne_{[m]}}$, be the order statistic, where $x_{ne_{[i]}}$ are the arranged in order of increasing. If neutrosophic random variable has probability density function and cumulative function as equation (3), and (4) respectively.

The neutrosophic truncated density function of order statistics is:

$$\begin{aligned} \varphi_{ne[\ell]}(x_{ne}) &= \frac{m!}{(\ell-1)!(m-\ell)!} f_T(X_{ne}) \left(FT_{X_{ne}}(x) \right)^{\ell-1} \left(1 - FT_{X_{ne}}(x) \right)^{m-\ell} \\ &= \frac{m! f(x_{ne}) / (\ell-1)!(m-\ell)!}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} \left(\frac{F_{X_{ne}}(x_{ne}) - F_{X_{ne}}(\alpha)}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} \right)^{\ell-1} \left(1 - \frac{F_{X_{ne}}(x_{ne}) - F_{X_{ne}}(\alpha)}{F_{X_{ne}}(\beta) - F_{X_{ne}}(\alpha)} \right)^{m-\ell} \end{aligned}$$

5. Cases of Neutrosophic Truncated Distribution

Since the new logic (neutrosophic logic) includes extremes and ambiguity. So In this section, we will discuss cases that are related to the neutrosophic truncation according to neutrosophic logic.

5.1 Neutrosophic Truncated Distributions Depended on Neutrosophic Probability Distributions

Let X be a random variable with neutrosophic probability density function $Nf(x)$ and the value of X defined on $(-\infty, \infty)$, and suppose the restriction probability density function to X define on a subset Ω^* of sample space Ω , $\Omega^* = \{x: a < x < b\}$, a, b two numbers defined on Ω . The neutrosophic truncated distribution is:

$$Nf(x)^* = [F(b) - F(a)]^{-1} Nf(x); \quad a < x < b$$

That is, the two intervals that have been ignored are $([b, \infty), (-\infty, a])$.

$$Nf(x)^* = [F(b^*) - F(a^*)]^{-1} f(x); \quad a^* < x < b^* \tag{9}$$

Where $Nf(x)$ is the probability function of neutrosophic continuous distribution which contains some indeterminacy in the function or in it parameter(s) for the probability distribution.

5.2 Neutrosophic Truncated Distributions Depended on Classical Probability Distributions

Let X is be a random variable with probability density function $f(x)$ and the value of X defined on $(-\infty, \infty)$, and suppose the restriction probability density function to X defined on a subset Ω^* of sample space Ω , $\Omega^* = \{x: a^* < x < b^*\}$, a^*, b^* are two neutrosophic numbers defined on Ω . The neutrosophic distribution after truncation (a^*, b^*) is:

$$Nf(x)^* = [F(b^*) - F(a^*)]^{-1} f(x); \quad a^* < x < b^*$$

The two intervals that have been ignored are $([b^*, \infty), (-\infty, a^*])$. two intervals $[b^*, \infty)$ and $(-\infty, a^*]$ are neutrosophic interval. $(-\infty, a^*]$ may be $(-\infty, a + I]$ or $(-\infty, a + Iy]$

$$Nf(x)^* = (P[a^* < x < b^*])^{-1} f(x); \quad x \in (a^*, b^*) \tag{10}$$

In this case, the truncated probability density function is defined on a neutrosophic truncation interval. That is, the interval that will be truncated contains some indeterminacy.

5.3 Neutrosophic Truncated Distribution Depended on Neutrosophic Probability Distributions And Neutrosophic Truncation Interval.

Let X be a random variable with neutrosophic probability density function $Nf(x)$ and the value of X defined on $(-\infty, \infty)$, and suppose the restriction probability density function to X define on a subset Ω^* to sample space, $\Omega^* = \{x: a^* < x < b^*\}$, a^*, b^* two neutrosophic numbers defined on Ω . The neutrosophic truncated distribution is:

$$Nf(x)^* = (P[a^* < x < b^*])^{-1} (Nf(x)); x \in (a^*, b^*).$$

The two intervals that have been ignored are $([b^*, \infty), (-\infty, a^*])$. two intervals $[b^*, \infty)$ and $(-\infty, a^*]$ are neutrosophic interval. $(-\infty, a^*]$ may be $(-\infty, a + I]$ or $(-\infty, a + Iy]$, and the neutrosophic density function $Nf(x)$ of x .

$$Nf(x)^* = (P[a^* < x < b^*])^{-1} (Nf(x)); x \in (a^*, b^*). \quad (11)$$

The interval of truncation and the probability density function contain some indeterminacy. And therefore we are dealing with neutrosophic probability density function and neutrosophic the interval truncation.

6. Types of Neutrosophic Truncation

6.1 Neutrosophic Left Truncation

Truncated from below, low values of X are cut off, and the range is from some minimum value to positive infinity $\{X \min, \infty\}$.

The probability density function of X defined on the subset of Ω , $\Omega^* = \{x: a < x < \infty\}$, $a \in \Omega$ is

the neutrosophic left truncated distribution defined as:

$$NLf(x)^* = \frac{Nf(x)}{P(a < x < \infty)} = \frac{Nf(x)}{1-F(a)}, x \in (a, \infty).$$

But, if $\Omega^* = \{x: a^* < x < \infty\}$, a^* be a neutrosophic number defined on Ω .

The neutrosophic left truncated distribution after truncation (a^*, ∞) is:

$$NLf(x)^* = \frac{f(x)}{P(a^* < x < \infty)} = \frac{f(x)}{1-F(a^*)}, x \in (a^*, \infty). \text{ Or, } NLf(x)^* = \frac{Nf(x)}{P(a^* < x < \infty)} = \frac{Nf(x)}{1-F(a^*)}, x \in (a^*, \infty).$$

6.2 Neutrosophic Right Truncation

Truncated from upper, upper values of X are cut off so your range is from some negative infinity to maximum value of X $\{-\infty, X \max\}$

the probability density function of X defined on the subset of Ω is:

$$\Omega^* = \{x: -\infty < x < b\}, b \in \Omega.$$

$$NRf(x)^* = \frac{Nf(x)}{P(-\infty < X < b)} = \frac{Nf(x)}{F(b)}, x \in (-\infty, b).$$

If $\Omega^* = \{-\infty < x < b^*\}$, b^* be a neutrosophic number defined on Ω .

The neutrosophic right truncated distribution after truncation $(-\infty, b^*)$ is:

$$NRf(x)^* = \frac{f(x)}{P(-\infty < X < b^*)} = \frac{f(x)}{F(b^*)}, x \in (-\infty, b^*). \text{ Or, } NRf(x)^* = \frac{Nf(x)}{P(-\infty < X < b^*)} = \frac{Nf(x)}{F(b^*)}, x \in (-\infty, b^*).$$

7. Applications

Example (1)

Let X be a random variable with probability density function $f(x) = \alpha e^{-\alpha x}$; $x \geq 0$, find the probability density function, and cumulative distribution function of X on $[2, \infty]$.

Solution:

$$f(x)^* = \frac{f(x)}{P[2 < x < \infty]} = \alpha e^{\alpha(2-x)}; \quad x \geq 2$$

The neutrosophic probability density function $Nf(x) = \alpha_N e^{-\alpha_N x}$; $x \geq 0$, $\alpha_N = \alpha + I$

$$Nf(x)^* = \frac{Nf(x)}{P[a < x < b]}, \quad x \in [2, \infty]$$

$$Nf(x)^* = \frac{\alpha_N e^{-\alpha_N x}}{P[2 < x < \infty]} = \frac{\alpha_N e^{-\alpha_N x}}{P[2 < x < \infty]} = \frac{\alpha_N e^{-\alpha_N x}}{(e^{-\alpha_N a} - e^{-\alpha_N b})}$$

$$\alpha_N = \left[\frac{1}{4}I, 0.2\right], \quad \frac{1}{4}I = \left[\frac{1}{2}, \frac{1}{4}\right], \quad \alpha_N = \left[\frac{1}{2}, 0.2\right] \text{ and } \left[\frac{1}{4}, 0.2\right]$$

When $\alpha_N = \left[\frac{1}{2}, 0.2\right]$;

$$Nf(x)^* = \frac{\left[\frac{1}{2}, 0.2\right] e^{-\left[\frac{1}{2}, 0.2\right]x}}{(e^{-\left[\frac{1}{2}, 0.2\right]a} - e^{-\left[\frac{1}{2}, 0.2\right]b})}$$

$$\text{either } Nf(x)^* = \frac{\frac{1}{2} e^{-\frac{1}{2}x}}{(e^{-\frac{1}{2}a})} = \frac{1}{2} e^{(1-\frac{1}{2})x}, \quad \text{or } Nf(x)^* = \frac{0.2 e^{-0.2x}}{(e^{-(2)(0.2)})} = 0.2 e^{(0.4-0.2x)}.$$

$$\alpha_N = \left[\frac{1}{4}, 0.2\right], \quad Nf(x)^* = \frac{\left[\frac{1}{4}, 0.2\right] e^{-\left[\frac{1}{4}, 0.2\right]x}}{(e^{-\left[\frac{1}{4}, 0.2\right]a} - e^{-\left[\frac{1}{4}, 0.2\right]b})}$$

$$\text{either } Nf(x)^* = \frac{\frac{1}{4} e^{-\frac{1}{4}x}}{(e^{-\frac{1}{4}a})} = \frac{1}{4} e^{\frac{1}{2}(1-\frac{1}{2})x}, \quad \text{or } Nf(x)^* = \frac{0.2 e^{-0.2x}}{(e^{-(2)(0.2)})} = 0.2 e^{(0.4-0.2x)}$$

If the lower limit interval truncated (a) is not clear, its ranges between 1.9 and 2.1, $a^* = [1.9, 2.1]$

The neutrosophic truncated distribution is:

$$Nf(x)^* = \frac{f(x)}{P\{1.9 < x < \infty\}} = \frac{e^{-x}}{F(\infty) - F(1.9)} = e^{1.9-x};$$

$$\text{And } Nf(x)^* = \frac{f(x)}{P\{2.1 < x < \infty\}} = \frac{e^{-x}}{F(\infty) - F(2.1)} = e^{2.1-x}, \quad \text{When } \{x: a^* < x < \infty\}, \quad a^* = [1.9, 2.1]$$

The cumulative distribution when $x = 3$ is; $F(x = 3)^* = P(X \leq 3) = 1 - e^{2-3} = 1 - \frac{1}{e} = 0.63$; $x \geq 2$

And neutrosophic cumulative distribution of the truncated distribution is:

$$NF(x)^* = \int_{1.9}^{\infty} e^{1.9-u} du = 1 - e^{1.9-x}, \quad x \geq 1.9. \quad NF(x)^* = \int_{2.1}^{\infty} e^{2.1-u} du = 1 - e^{2.1-x}, \quad x \geq 2.1.$$

$$NF(x = 3)^* = \int_{1.9}^{\infty} e^{1.9-u} du = 1 - e^{-1.1} = 0.6671, \quad x \geq 1.9.$$

$$NF(x = 3)^* = \int_{2.1}^{\infty} e^{2.1-u} du = 1 - e^{2.1-3} = 0.593, x \geq 2.1.$$

$$F(x = 3) = P(X \leq 3) = 1 - e^{2-3} = 1 - \frac{1}{e} = 0.63 ; x \geq 0$$

Neutrosophic cumulative function is [0.593, 0.6671], and the cumulative classical value (0.63) belongs to the interval [0.593, 0.6671].

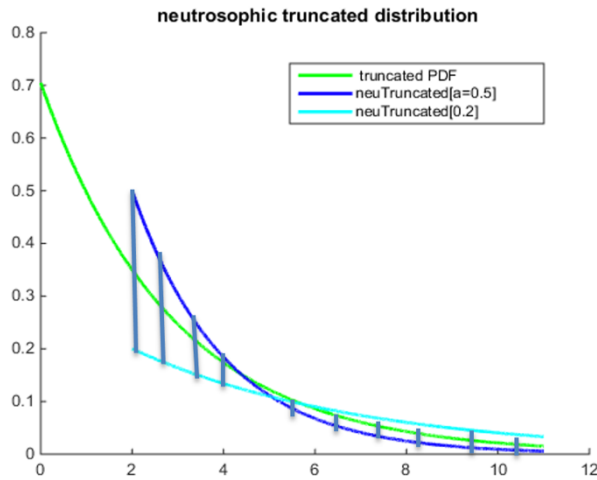


Figure1: Neutrosophic truncated on $\alpha_N = [0.5, 0.2]$

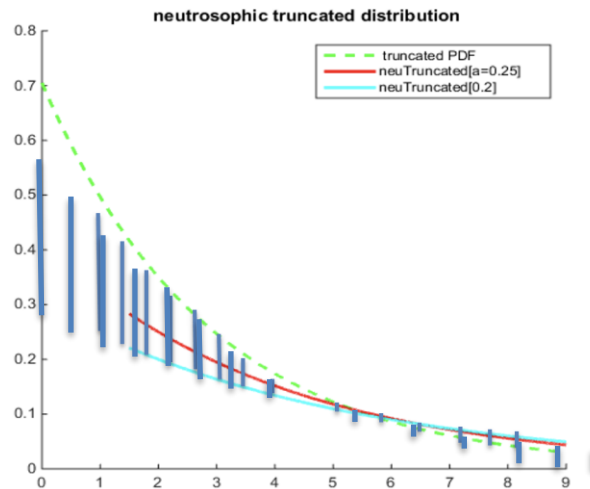


Figure 2: Neutrosophic truncated on $\alpha_N = [0.25, 0.2]$

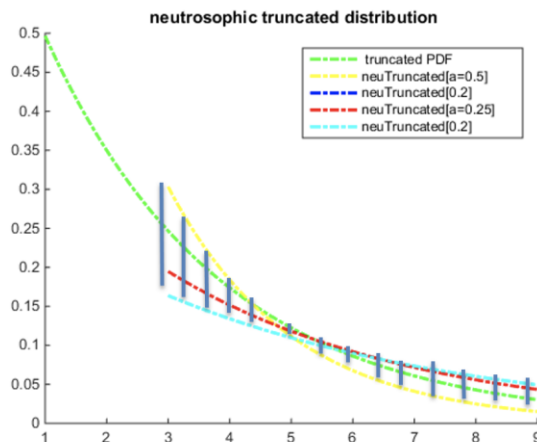


Figure 3: Neutrosophic truncated when $\alpha_N=[0.25 I, 0.2]$

In figure(1) the probability density function of neutrosophic truncated defined when , $\alpha_N=[0.5, 0.2]$, and figure(2) the probability density function of neutrosophic truncated defined when $\alpha_N=[0.25, 0.2]$. While in figure(3), the probability density function of neutrosophic truncated defined when $\alpha_N=[0.25 I, 0.2]$.

Example (2)

Let $f(x) = 6x(1 - x)$; $0 < x < 1$, the probability density function to random variable X. Find the probability density function on the interval $[1/4, 1]$, and the r^{th} moment about zero.

$$f(x)^* = \frac{6x(1-x)}{P\left[\frac{1}{4} < x < 1\right]} = \frac{6x(1-x)}{[F(1) - F(\frac{1}{4})]}$$

$$F(x) = 3x^2 - 2x^3, F(1) = 1; F\left(\frac{1}{4}\right) = \frac{5}{32}$$

$$\text{The truncated density function on } [a, b] = \left[\frac{1}{4}, 1\right]. f(x)^* = \frac{6x(1-x)}{\frac{27}{32}} = \frac{64}{9} x(1-x); \frac{1}{4} < x < 1$$

The left truncated $(a, \infty)=(1/4, \infty)$ is;

$$f(x)^* = \frac{f(x)}{1-F(a)} = \frac{6x(1-x)}{1-F(1/4)} = \frac{6x(1-x)}{1-5/32} = \frac{6x(1-x)}{0.84375}, x \in \left(\frac{1}{4}, \infty\right).$$

The r^{th} moment of truncated density function on interval $[1/4, 1]$ is:

$$E(X^r) = \int_{\frac{1}{4}}^1 X^r f(x)^* dx = \int_{\frac{1}{4}}^1 x^{r+1} \frac{64}{9} (1-x) dx = \frac{64}{9} \int_{\frac{1}{4}}^1 (x^{r+1} - x^{r+2}) dx$$

$$\text{If } r = 1; E(X) = \frac{64}{9} \int_{\frac{1}{4}}^1 (x^2 - x^3) dx = 0.5617$$

Find the probability density function of truncated distribution on the interval $[1/4 I, 1]$ $a = \frac{1}{4} I = \left[\frac{1}{2}, \frac{1}{5}\right]$

When the interval that truncation has contains some indeterminacy. Neutrosophic Left truncated distribution is:

$$NLf(x)^* = \frac{f(x)}{P(a^* < x < \infty)} = \frac{f(x)}{1-F(a^*)}, x \in (a^*, \infty).$$

$$\frac{f(x)}{1-F(a^*)} = \frac{6x(1-x)}{1-(1/2)} = \frac{6x(1-x)}{0.5}, (\text{when } a = 1/2)$$

$$Nlf(x)^* = \frac{f(x)}{P(a^* < x < \infty)} = \frac{f(x)}{1-F(a^*)}, x \in (a^*, \infty).$$

$$\frac{f(x)}{1-F(a^*)} = \frac{6x(1-x)}{1-(0.056)} = \frac{6x(1-x)}{0.944}, \text{ (when } a = 1/5)$$

Then neutrosophic left truncated is the range between $(\frac{6x(1-x)}{0.5}, \frac{6x(1-x)}{0.944})$

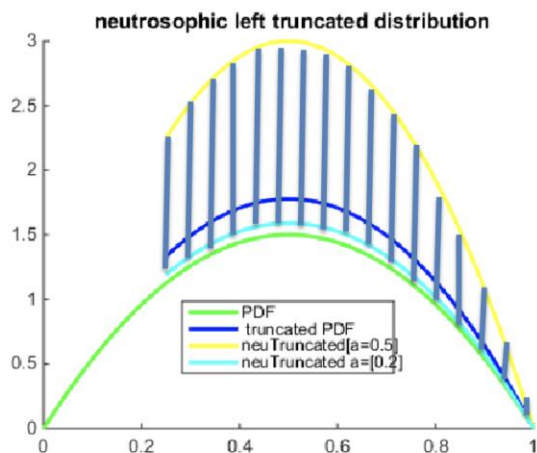


Figure 4: left truncated, PDF, and neutrosophic left truncated $\alpha_N=[0.25 I, 0.2]$.

In Figure 4, the curves of the probability density function and left truncated on the interval [0.25,1] are illustrated, such that a blue curve represents the left truncated, While the neutrosophic left truncated is represented by a region between two curves that colored yellow when a=1/2, and turquoise when a=1/5.

From the above examples, when truncation limit is an interval (lower, upper or both), the area of truncation is wider than when truncation limit is a point, where the truncation point is a classical number. The interval truncation could contain indeterminacy number. Through the neutrosophic truncated distribution will allow unimportant or ambiguous data to be included in random variable or probability density function or truncated limit.

8. Conclusions

In this paper, the neutrosophic truncated continuous distributions have been studied for the first time. neutrosophic truncated continuous distributions are defined with different cases, such as neutrosophic truncated neutrosophic random variable, neutrosophic truncated of neutrosophic probability function, neutrosophic truncated of neutrosophic interval truncation with classical probability function, and neutrosophic truncated of the neutrosophic probability function with the neutrosophic interval truncation. That is, Through the neutrosophic truncated distribution will allow unimportant or ambiguous data to be included in random variable or probability density function or truncated limit. the neutrosophic left truncated and neutrosophic right truncated distributions have been introduced.in addition, some statistical properties such as expectation, moment generating function, the rth moment, and order statistics are derived and proved. some examples are suggested.

References

[1] F.Galton, "Examination into the registered speeds of American trotting House". with remarks on their value as Hereditary data, November 29,1889.
 [2] L. Zaninetti, M. Ferraro, "On the truncated Pareto Distribution with Applications" Central European Journal of Physics, Vol.6, No.1, 2002.
 [3] L. Zaninetti, "A right and left truncated gamma distribution with application to the stars" Advanced Studies Theorem Physics, Vol. 7, No.23, 1139–1147, 2013.
 [4] M. Ali, S. Nadarajah, "Truncated Pareto Distribution". Computer Communication, vol. 30,1–4, 2006.

- [5] Lorenzo Zaninetti, "probability distribution in Astrophysics", international journal of astronomy and astrophysics, value 9, 2019.
- [6] Efron B, Petrosian, "Nonparametric methods for doubly truncated data". journal of the American Statistical Association, Vol. 94 ,824–834, 1999.
- [7] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math. pp. 287-297, 2005.
- [8] R. Alhabib, M. M. Ranna, H. Farah and A. Salama, "Some Neutrosophic Probability Distributions," Neutrosophic Sets and Systems, vol. 22, pp. 30-38, 2018.
- [9] Kawther F. Alhasan and F. Smarandache, "Neutrosophic Weibull distribution and Neutrosophic Family Weibull Distribution," Neutrosophic Sets and Systems, vol. 28, pp. 191-199, 2019.
- [10] Kawther F. Alhasan, A.A. Salama, Florentin Smarandache. "Introduction to Neutrosophic Reliability Theory." International Journal of Neutrosophic Science, Vol. 15, No. 1, 2021 ,PP. 52-61.
- [11] S. Broumi, M. Talea. A. Bakkali and F. Smarandache, Single Valued Neutrosophic Graph, Journal of New theory, N 10, 2016.
- [12] S. Broumi, Mohamed T, Assis B, and Florentin, Single valued neutrosophic graphs, journal of new theory. No 10, 2016.
- [13] Alhabib, Rafif, Salama A. A., "The Neutrosophic Time Series-Study Its Models Linear –Logarithmic and test the Coefficients Significance of Its linear model" Neutrosophic Sets and Systems, Vol. 33, pp. 105-115, 2020.
- [14] A. Manshath, E. Kungumaraj, E. Lathanayagam, M. C. Joe Anand, Nivetha Martin, Elangovan Muniyandy, S. Indrakumar, Neutrosophic Integrals by Reduction Formula and Partial Fraction Methods for Indefinite Integrals, International Journal of Neutrosophic Science , Vol. 23 , No. 1 , (2024) : 08-16 (Doi : <https://doi.org/10.54216/IJNS.230101>).
- [15] Smarandache F., "Neutrosophical statistics". Sitech & Education publishing, 2014.
- [16] Srishti Kumari, Azarudheen S., Synergizing Neutrosophy and Randomized Blocks Design: Development and Analytical Insights, International Journal of Neutrosophic Science , Vol. 23 , No. 1 , (2024) : 125-133 (Doi : <https://doi.org/10.54216/IJNS.230111>)
- [17] Smarandache F., "A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press, Rehoboth, NM, 1999.
- [18] Kawther F Alhasan . study some methods to measure reliability neutrosophically, International Journal of Neutrosophic Science, Vol. 24, no 2, 2024 .