Possibility Fermatean Neutrosophic Soft Set

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Abstract

In this paper, we introduce the concept of Possibility Fermatean Neutrosophic Soft Set and define some related concepts such as Possibility Fermatean Neutrosophic Soft subset, Possibility Fermatean Neutrosophic Soft null set, and Possibility Fermatean Neutrosophic Soft universal set. Then, we define set-theoretical operations of Possibility Fermatean Neutrosophic Soft Sets such as union, intersection, and complement, and investigate some properties of these operations. We also introduce AND-product and OR-product operations between two Possibility Fermatean Neutrosophic Soft Sets. We propose a decision-making method called the Possibility Fermatean Neutrosophic Soft decision-making method (PFNS-decision-making method) which can be applied to decision-making problems involving uncertainty based on AND-product operation. We finally give a numerical example to display the application of the method that can be successfully applied to the problems.

Keywords: Fuzzy Set, Soft Set; Fuzzy Soft Set; Fermatean Fuzzy Set; Fermatean Fuzzy Soft Set; Intuitionistic Fuzzy Soft Set; Possibility Fuzzy Soft Set; Intuitionistic Possibility Fermatean Fuzzy Soft Set; Possibility Fermatean Neutrosophic Soft Set

1. Introduction

Fuzzy sets were developed by Zadeh [1] to solve problems that contain uncertain information. Some cases cannot deal with a fuzzy set, so Turksen [2] introduced an interval-valued fuzzy set. Atanassove [3] extended the fuzzy set to the Intuitionistic fuzzy set. Which is more general than a fuzzy set. Neutrosophy introduced by Smarandache [4] is a new tool for dealing with problems containing imprecise, indeterminacy, and inconsistent data. Neutrosophic sets which were introduced by Smarandach in 2005 [5] are a generalization of the Intuitionistic fuzzy set. Soft Set is defined by Molodtsov [6] as another commonly used method in handling uncertainties in the data. The concept of a fuzzy soft set was introduced by Maji [7].

Soft Set extended and introduced some of its operations and properties by Maji [8]. Fuzzy soft set extended to Generalized fuzzy soft sets by Majumdar and Samanta in 2010 [9]. Sezgin et al. [10] were proved De Morgan s Law on Soft Set. Neutrosophic Soft Set NSS with basic operation and properties proposed by Maji [11]. The new concept of Generalized neutrosophic soft set GNSS which was introduced by Sahin [12], was an extension of the concept of NSS defined by Maji [8]. NSS was developed by Broumi [13] as a Generalized Neutrosophic Soft Set with basic definitions and operations. He used this concept for solving decision-making problems.

Maji [14] combined intuitionistic fuzzy with soft sets. C¸ Agman and Karatas¸ [15] redefined intuitionistic fuzzy sets and proposed a decision-making method using the intuitionistic fuzzy soft sets. Agarwal et al. [16] defined generalized intuitionistic fuzzy soft sets (GIFSS) and investigated properties of GIFSS. They also put forward similarity measure between two GIFSSs. Das and Kar [17] proposed an algorithmic approach based on intuitionistic fuzzy soft set (IFSS) which explores a particular disease reflecting the agreement of all experts. They also demonstrated effectiveness of the proposed approach using a suitable case study. Deli and C¸ agman [18]

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defined intuitionistic fuzzy parameterized soft sets as an extension of fuzzy parameterized soft sets [19], and suggested a decision-making method based on intuitionistic fuzzy parameterized soft sets. Recently, many researchers published interesting results on intuitionistic fuzzy soft set theory.

Alkhazaleh et al. [20] first introduced concept of the possibility fuzzy soft sets and their operations and gave applications of this theory in a decision-making problem. They also introduced a similarity measure between two possibilities fuzzy soft sets and gave an application of proposed similarity measure method in a medical diagnosis problem. In 2012, Bashir et al. [21] introduced concept of possibility intuitionistic fuzzy soft set and their operations and discussed similarity measure between two possibility intuitionistic fuzzy sets. They also gave an application of this similarity measure. In 2017, Karaasalan.F et al. [22] introduced concept of possibility neutrosophic soft set and their operations and gave application in decision-making.

This paper is structured as follows: In Section 2, covers certain definitions and features of PFNSS.In Section 3, we introduce the concepts of PFNSS, PFN-Soft subset, PFN-Soft null set, Absolute PFNSS, and PFNSS operations. In Section 4 we discuss union and intersection on PFNSSs. In Section 5, we present a MADM problem discussing the selection of the best model for the most attractive laptop based on the computer simulation report and solve it under PFNSSs using a supporting algorithm. Finally, in Section 6, we provide the concluding remarks.

2. Preliminary

In this section, we present some definitions required in this paper.

Definition 1: Fuzzy Set

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(X)$ is interpreted as the degree of membership of the element X in fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples $A = \{(x, \mu_A(x)) : x \in X\}$.

Definition 2: Intuitionistic Fuzzy Set

The intuitionistic fuzzy sets are defined on a non-empty set X as objects having the form $A = \{\langle x, \alpha_A(x), \beta_A(x) : x \in X\}$, where the functions $\alpha_A(x) : X \to [0,1]$ and $\beta_A(x) : X \to [0,1]$, denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively, and $0 \le \alpha_A(x) + \beta_A(x) \le 1$, for all $x \in X$ .Clearly, when $\beta_A(x) = 1 - \alpha_A(x)$, for every $x \in X$, the set A becomes a fuzzy set.

Definition 3: Fermatean Fuzzy Set

Fermatean fuzzy set F in the universe set U is an object with the type $F = \{(u, \alpha F(u), \beta F(u)) : u \in U\}$ where $\alpha F : U \to [0,1]$ and $\beta F : U \to [0,1]$, with the condition $0 \le (\alpha F(u))^3 + (\beta F(u))^3 \le 1$ for all $u \in U$.

Definition 4: Neutrosophic Set

A Neutrosophic set A on the universe of discourse X is defined as $A = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle ; x \in X\}$ where $T; I; X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 5: Fermatean Neutrosophic Set

Let X be a non-empty set (universe). A Fermatean neutrosophic set [FNS] $\{(x, T(x), I(x), F(x) : x \in X\}$, where $T(x), I(x), F(x) \in [0,1], 0 \le (T(x))^3 + (I(x))^3 + (F(x))^3 \le 2$. Then $0 \le (T(x))^3 + (I(x))^3 + (F(x))^3 \le 2$, for all x in X . $T(x)$ is the degree of membership, $I(x)$ is the degree of inderminancy and $F(x)$ is the degree of non-membership. Here $T(x)$ and $F(x)$ are dependent components and $I(x)$ is an independent component.

Definition 6: Soft Set

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Let U be the universal set and E be the set of attributes with respect to U. Let $P(U)$ be the power set of U and $A \subseteq E.A$ pair (F,A) is called a soft set over U and its mapping is given as $F: A \to P(U)$. It is also defined as, $(F,A)=\{F(e)\in P(U): e\in E, F(e)=\varnothing \text{ if } e\notin A\}$.

Definition 7: Fuzzy Soft Set

Let U be the initial universal set and let E be the set of parameters. Let I^U denote the power set of all fuzzy subsets of U .Let $A \subseteq E$. A pair (F,E) is called a fuzzy soft set over U where F is a mapping given by: $F: A \to I^U$.

Definition 8: Generalized Fuzzy Soft Set

Let $U=\{u_1,u_2,\dots,u_n\}$ be the universal set of elements and $E=\{e_1,e_2,\dots,e_m\}$ be the universal set of parameters. The pair (U,E) is called a soft universe. Let $F: E \to I^U$ and μ be a fuzzy subset of E, *i.e*, $\mu: E \to I=[0,1]$, where I^U is the collection of all Fuzzy subset of U. Let F_μ be the mapping $F: E \to I^U \times I$ be a function defined as follows: $F_\mu(e)=(F(e),\mu(e))$,where $F_e \in I^U$ .Then F_μ is called a Generalized fuzzy soft set (GFSS in short) over the soft universe (U,E). Here for each parameter $e_i, F_\mu(e_i)=(F(e_i),\mu(e_i))$,indicates not only the degree of belongingness of the elements of U in $F(e_i)$,but also the degree of possibility of such belongingness which is represented by μ_i. So, we can write this as follows:

$$F_\mu(e_i)=\left\{\left(\left(\frac{u_1}{F(e_i)(u_1)},\frac{u_2}{F(e_i)(u_2)},\dots\dots,\frac{u_n}{F(e_i)(u_n)}\right),\mu(e_i)\right)\right\}, \forall u\in U, e\in E$$

Definition 9: Fermatean Fuzzy Soft Set

Let E be any set of deferent parameters, and let U be the universe, $A \subseteq E$ a Fermatean fuzzy soft set (FFSS) on U is defined as the pair (F,W) where F is mapping given by $F: W \to FFS(U)$, where $FFS(U)$ is the set of all Fermatean fuzzy sets over U. Here for any parameter $e \in A, F(e)$ is the Fermatean fuzzy set given as $F(e)=\{(u,\alpha F(e)(u), \beta F(e)(u)): u\in U\}$ where $\alpha F(e)(u)$ and $\beta F(e)(u)$ are corresponding degrees of membership and non-membership $0 \le (\alpha F(e)(u))^3 + (\beta F(e)(u))^3 \le 1$. Hence $(F,A)=\{(e,\{(u,\alpha F(e)(u),\beta F(e)(u)\}): e\in A, u\in U\}$.

Definition 10: Neutrosophic Soft Set

Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F: A \to P(U)$.

Definition 11: Intuitionistic Fuzzy Soft Set

Consider U and E as a universe set and a set of parameters, respectively. Let $P(U)$ denotes the set of all Intuitionistic Fuzzy sets of U .Let $A \subseteq E$ .A pair (F,A) is an intuitionistic fuzzy soft set over U. where F is a mapping given by $F: A \to P(U)$.

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Definition 12: Possibility Fuzzy Soft Set

Let $U=\{u_1,u_2,...,u_n\}$ be the universal set of elements and $E=\{e_1,e_2,...,e_m\}$ be the universal set of parameters. The pair (U,E) is called a soft universe. Let $F:E\to I^U$ and μ be a fuzzy subset of E, *i.e*, $\mu:E\to I^U=[0,1]$, where I^U is the collection of all Fuzzy subset of U. Let F_μ be the mapping $F:E\to I^U\times I^U$ be a function defined as follows: $F_\mu(e)=(F(e)(u),\mu(e)(u))$, $\forall u\in U$. Then F_μ is called a Possibility fuzzy soft set (PFSS) over the soft universe (U,E). Here for each parameter $e_i, F_\mu(e_i)=(F(e_i),\mu(e_i))$,indicates not only the degree of belongingness of the elements of U in $F(e_i)$,but also the degree of possibility of such belongingness which is represented by μ_i. So, we can write this as follows:

$$F_\mu=\left\{\left(e,\left(\frac{u}{F(e)(u)},\mu(e)(u)\right)\right)\right\},\forall u\in U,e\in E.$$

Definition 13: Possibility Neutrosophic Soft Set

Let $U=\{u_1,u_2,...,u_n\}$ be the universal set of elements and $E=\{e_1,e_2,...,e_m\}$ be the universal set of parameters, where $N(U)$ be the collection of all neutrosophic set of U and I^U is the collection of all Fuzzy subset of U .A Possibility Neutrosophic Soft Set (PNSS) F_μ over U is a set of ordered pairs defined by:

$$F_\mu=\left\{\left(e_k,\left(\frac{u_j}{F(e_k)(u_j)},\mu(e_k)(u_j)\right)\right)\right\},\forall u_j\in U,e_k\in E\cdot$$

3.Fundamental of Possibility Fermatean Neutrosophic Soft Set

In this section, we introduce the concepts of PFNSS, PFN-Soft subset, PFN-Soft null set, Absolute PFNSS, and PFNSS operations.

Definition 14: Possibility Fermatean Neutrosophic Soft Set

Let $U=\{c_1,c_2,...,c_n\}$ be a universe set of elements and $E=\{e_1,e_2,...,e_m\}$ be the universal set of parameters. Then the pair (U,E) will called soft universe. Let $F:E\to FN(U)$ and μ be a fuzzy subset of E, *i.e*, $\mu:E\to I=[0,1]$, where $FN(U)$ is the collection of all Fermatean Neutrosophic subset of U .

Let $F_\mu:E\to FN(U)\times I^U$ be function defined as follows:

$$F_\mu=\left\{\left(e,\left(\frac{c}{F(e)(c)},\mu(e)(c)\right)\right)\right\},\forall c\in U,e\in E$$

where $F(e)\in FN(U)$, then F_μ is called PFNSS.

Example 1: Let $U=\{c_1,c_2,c_3\}$ to be set of three houses consider some consideration. Assume $E=\{e_1,e_2,e_3\}$ be a set of adjectives where e_1 = cheap, e_2 = colorful, e_3 = location. Let $\mu:E\to I=[1,0]$ .We defined a function.

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$F_\mu : E \to FN(U)\times I^U$ as follows:

$$F_\mu(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle},0.5\right)\right\};$$

$$F_\mu(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.5\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle},0.8\right)\right\};$$

$$F_\mu(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle},0.1\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle},0.7\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.6\right)\right\}.$$

Then F_μ is $(PFNSS)$ over (U,E).

In matrix form this can be expressed as

$$F_\mu=\begin{pmatrix}(\langle 0.2,0.6,0.1\rangle,0.3) & (\langle 0.4,0.2,0.6\rangle,0.4) & (\langle 0.3,0.1,0.7\rangle,0.5)\\ (\langle 0.3,0.4,0.5\rangle,0.2) & (\langle 0.2,0.3,0.4\rangle,0.5) & (\langle 0.7,0.8,0.2\rangle,0.8)\\ (\langle 0.6,0.2,0.1\rangle,0.1) & (\langle 0.3,0.6,0.9\rangle,0.7) & (\langle 0.1,0.2,0.3\rangle,0.6)\end{pmatrix}.$$

Where i^{th} row vector represents $F_\mu(e_i)$, the i^{th} column vector represent c_i, the last column shows the value of μ, and it is said to be called membership matrix of F_μ. It's clearly that $(0.2)^3+(0.1)^3\le 1$ and $(0.2)^3+(0.6)^3+(0.1)^3\le 2$ which satisfy the Fermatean Neutrosophic condition.

Definition 15: Let F_μ and G_δ said to be two PFNSS on (U,E). Then F_μ is said to be a possibility Fermatean Neutrosophic soft subset of G_δ if μ is a fuzzy subset of δ and $F(e)$ is a Fermatean Neutrosophic soft subset of $G(e), \forall e\in E$,where $T_F<T_G, I_F<I_G, F_F>F_G$ *and* $\mu_F<\mu_G$ Here we say that $F_\mu\subseteq G_\delta$.

Example 2: Let $U=\{c_1,c_2,c_3\}$ be a set of three cars, and let $E=\{e_1,e_2,e_3\}$ be a set of parameters were e_1 = cheap, e_2 = expensive and e_3 = yellow. Let F_μ is PFNSS over (U,E) defined as follows:

$$F_\mu(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.2\rangle},0.4\right),\left(\frac{c_2}{\langle 0.4,0.6,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.6,0.1,0.9\rangle},0.1\right)\right\};$$

$$F_\mu(e_2)=\left\{\left(\frac{c_1}{\langle 0.5,0.3,0.8\rangle},0.2\right),\left(\frac{c_2}{\langle 0.7,0.3,0.6\rangle},0.6)\right),\left(\frac{c_3}{\langle 0.1,0.2,0.6\rangle},0.5\right)\right\};$$

$$F_\mu(e_3)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.3\rangle},0.7\right),\left(\frac{c_2}{\langle 0.3,0.5,0.8\rangle},0.8\right),\left(\frac{c_3}{\langle 0.4,0.3,0.7\rangle},0.2\right)\right\}.$$

Let $G_\delta : E\to FN(U)\times I^U$ be another PFNSS over (U,E) defined as follows:

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$$G_\delta(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.1\rangle},0.5\right),\left(\frac{c_2}{\langle 0.6,0.7,0.2\rangle},0.4\right),\left(\frac{c_3}{\langle 0.7,0.2,0.8\rangle},0.2\right)\right\};$$

$$G_\delta(e_2)=\left\{\left(\frac{c_1}{\langle 0.6,0.4,0.7\rangle},0.8\right),\left(\frac{c_2}{\langle 0.9,0.4,0.5\rangle},0.7\right),\left(\frac{c_3}{\langle 0.3,0.5,0.3\rangle},0.6\right)\right\};$$

$$G_\delta(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.7,0.1\rangle},0.8\right),\left(\frac{c_2}{\langle 0.6,0.6,0.7\rangle},0.9\right),\left(\frac{c_3}{\langle 0.5,0.4,0.5\rangle},0.4\right)\right\};$$

It's clear that F_μ is a PFNSS subset of G_δ.

Definition 16: A PFNSS is said to be a null PFNSS, denoted by φ_0 such that:
$\varphi_0(e)=\left(F(e)(c),\mu(e)(c)\right),\forall e\in E$, where $F(e)=(0,0,1)$ and $\mu(e)=0,\forall e\in E$.

Example 3: Let $U=\{c_1,c_2,c_3\}$ be a set of three blouses, and let $E=\{e_1,e_2,e_3\}$ be a set of adjectives where e_1 = bright, e_2 = colorful and e_3 = cheap. Let $\mu:E\to I^U$. We defined a function $F_\mu:E\to FN(U)\times I^U$ which is a PFNSS over (U,E) defined as follows:

$$F_\mu(e_1)=\left\{\left(\frac{c_1}{\langle 0,0,1\rangle},0\right),\left(\frac{c_2}{\langle 0,0,1\rangle},0\right),\left(\frac{c_3}{\langle 0,0,1\rangle},0\right)\right\};$$

$$F_\mu(e_2)=\left\{\left(\frac{c_1}{\langle 0,0,1\rangle},0\right),\left(\frac{c_2}{\langle 0,0,1\rangle},0\right),\left(\frac{c_3}{\langle 0,0,1\rangle},0\right)\right\};$$

$$F_\mu(e_3)=\left\{\left(\frac{c_1}{\langle 0,0,1\rangle},0\right),\left(\frac{c_2}{\langle 0,0,1\rangle},0\right),\left(\frac{c_3}{\langle 0,0,1\rangle},0\right)\right\};$$

$$F_\mu=\begin{pmatrix}(\langle 0,0,1\rangle,0) & (\langle 0,0,1\rangle,0) & (\langle 0,0,1\rangle,0)\\ (\langle 0,0,1\rangle,0) & (\langle 0,0,1\rangle,0) & (\langle 0,0,1\rangle,0)\\ (\langle 0,0,1\rangle,0) & (\langle 0,0,1\rangle,0) & (\langle 0,0,1\rangle,0)\end{pmatrix}.$$

Then F_μ is a null PFNSS.

Definition 17: A PFNSS is said to be a Possibility absolute Fermatean Neutrosophic Soft Set denoted by A_1, such that $A_1(e)=\left(F(e)(c),\mu(e)(c)\right),\forall e\in E$, where $F(e)=(1,0,0)$ and $\mu(e)=1,\forall e\in E$.

Example 4: Let $U=\{c_1,c_2,c_3\}$ be a set of three buses. Let $E=\{e_1,e_2,e_3\}$ be a set of parameters were e_1 = yellow, e_2 = long and e_3 = Mercedes. Let F_μ be a PFNSS over (U,E).

$$F_\mu(e_1)=\left\{\left(\frac{c_1}{\langle 1,0,0\rangle},1\right),\left(\frac{c_2}{\langle 1,0,0\rangle},1\right),\left(\frac{c_3}{\langle 1,0,0\rangle},1\right)\right\};$$

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$$F_{\mu}(e_2)=\left\{\left(\frac{c_1}{\langle 1,0,0\rangle},1\right),\left(\frac{c_2}{\langle 1,0,0\rangle},1\right),\left(\frac{c_3}{\langle 1,0,0\rangle},1\right)\right\};$$

$$F_{\mu}(e_3)=\left\{\left(\frac{c_1}{\langle 1,0,0\rangle},1\right),\left(\frac{c_2}{\langle 1,0,0\rangle},1\right),\left(\frac{c_3}{\langle 1,0,0\rangle},1\right)\right\};$$

$$F_{\mu}=\begin{pmatrix}(\langle 1,0,0\rangle,1) & (\langle 1,0,0\rangle,1) & (\langle 1,0,0\rangle,1)\\(\langle 1,0,0\rangle,1) & (\langle 1,0,0\rangle,1) & (\langle 1,0,0\rangle,1)\\(\langle 1,0,0\rangle,1) & (\langle 1,0,0\rangle,1) & (\langle 1,0,0\rangle,1)\end{pmatrix}.$$

Then F_{μ} is called Possibility Absolute Fermatean Neutrosophic Soft Set.

Definition 18: Let F_{μ} be a PFNSS over (U,E). Then the complement of F_{μ}, denoted by $F_{\mu}^{\ c}$ is defined by $F_{\mu}^{c}=G_{\delta}$ such that: $F_{\mu}^{c}(e)=D_{\delta}(e)=(D(e),\delta(e)),\forall e\in E$ where $D(e)$ denoted the Fermatean Neutrosophic complement, where $\mu^{c}(e)=1-\mu, T^{c}(e)=F(e), I^{c}(e)=1-I(e), F^{c}(e)=T(e)$ and $\delta(e)$ is a fuzzy complement of $\mu(e)$.

Example 5: Consider the matrix notation in Example 1

$$F_{\mu}=\begin{pmatrix}(\langle 0.2,0.6,0.1\rangle,0.3) & (\langle 0.4,0.2,0.6\rangle,0.4) & (\langle 0.3,0.1,0.7\rangle,0.5)\\(\langle 0.3,0.4,0.5\rangle,0.2) & (\langle 0.2,0.3,0.4\rangle,0.5) & (\langle 0.7,0.8,0.2\rangle,0.8)\\(\langle 0.6,0.2,0.1\rangle,0.1) & (\langle 0.3,0.6,0.9\rangle,0.7) & (\langle 0.1,0.2,0.3\rangle,0.6)\end{pmatrix}.$$

By using the basic fuzzy complement and Fermatean Neutrosophic complement, we have $F_{\mu}^{c}=G_{\delta}$, where G_{δ} is:

$$G_{\delta}=\begin{pmatrix}(\langle 0.1,0.4,0.2\rangle,0.7) & (\langle 0.6,0.8,0.4\rangle,0.6) & (\langle 0.7,0.9,0.3\rangle,0.5)\\(\langle 0.5,0.6,0.3\rangle,0.8) & (\langle 0.4,0.7,0.2\rangle,0.5) & (\langle 0.2,0.2,0.7\rangle,0.2)\\(\langle 0.1,0.8,0.6\rangle,0.9) & (\langle 0.9,0.4,0.3\rangle,0.3) & (\langle 0.3,0.8,0.1\rangle,0.4)\end{pmatrix}.$$

4.Union and Intersection of PFNSSs

Definition 19: The Union of two PFNSSs F_{μ} and G_{δ}, denoted by $F_{\mu}\tilde{\bigcup}G_{\delta}$, is a PFNSS $H_{\upsilon}:E\to FN(U)\times I^{U}$ defined by $H_{\upsilon}=\left(H(e)(c),\upsilon(e)(c)\right),\forall e\in E$, such that $H(e)=\left(F(e)\tilde{\bigcup}G(e)\right)$ and $\upsilon(e)=s\left(\mu(e),\delta(e)\right)$, where

$$F\bigcup G=\left\{\left(x,\max\left(T_{F}(x),T_{G}(x)\right),\min\left(I_{F}(x),I_{G}(x)\right),\min\left(F_{F}(x),F_{G}(x),\max\left(\mu_{G}(x),\mu_{F}(x)\right)\right)\right\}$$

, s is an s-norm and $\tilde{\bigcup}$ is a Fermatean Neutrosophic soft union.

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Example 6: Let $U=\{u_1,u_2,u_3\}$ and $E=\{e_1,e_2,e_3\}$. Let F_μ be a PFNSS defined as follows:

$$F_\mu(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle},0.5\right)\right\};$$

$$F_\mu(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.5\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle},0.8\right)\right\};$$

$$F_\mu(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle},0.1\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle},0.7\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.6\right)\right\};$$

Let $G_\delta : E \to FN(U)\times I^U$ be another PFNSS over (U,E) defined as follows:

$$G_\delta(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle},0.4\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle},0.5\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle},0.3\right)\right\};$$

$$G_\delta(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.7\right)\right\};$$

$$G_\delta(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.8\right)\right\}.$$

To find the union, we use the Fermatean Neutrosophic soft union as fuzzy union.

method:

$$H_\upsilon(e_1)=\left\{\left(\frac{c_1}{\max\langle 0.2,0.3\rangle,\min\langle 0.6,0.2\rangle,\min\langle 0.1,0.1\rangle},\max\langle 0.3,0.4\rangle\right),\right.$$
$$\left(\frac{c_2}{\max\langle 0.4,0.2\rangle,\min\langle 0.2,0.4\rangle,\min\langle 0.6,0.3\rangle},\max\langle 0.4,0.5\rangle\right),$$
$$\left.\left(\frac{c_3}{\max\langle 0.3,0.1\rangle,\min\langle 0.1,0.7\rangle,\min\langle 0.7,0.2\rangle},\max\langle 0.5,0.3\rangle\right)\right\}$$
$$=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle},0.4\right),\left(\frac{c_2}{\langle 0.4,0.2,0.3\rangle},0.5\right),\left(\frac{c_3}{\langle 0.3,0.1,0.2\rangle},0.5\right)\right\}.$$

The Similarly, we get.

$$H_\upsilon(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.3,0.4\rangle},0.2\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle},0.5\right),\left(\frac{c_3}{\langle 0.7,0.2,0.2\rangle},0.8\right)\right\};$$

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$$H_{\upsilon}(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle},0.2\right),\left(\frac{c_2}{\langle 0.3,0.6,0.4\rangle},0.7\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.8\right)\right\}.$$

In matrix notation, we get.

$$H_{\upsilon}(e)=\begin{pmatrix}(\langle 0.3,0.2,0.1\rangle,0.4) & (\langle 0.4,0.2,0.3\rangle,0.5) & (\langle 0.3,0.1,0.2\rangle,0.5)\\ (\langle 0.3,0.3,0.4\rangle,0.2) & (\langle 0.7,0.1,0.2\rangle,0.5) & (\langle 0.7,0.2,0.2\rangle,0.8)\\ (\langle 0.6,0.2,0.1\rangle,0.2) & (\langle 0.3,0.6,0.4\rangle,0.7) & (\langle 0.1,0.2,0.3\rangle,0.8)\end{pmatrix}.$$

Definition 20: The Intersection of two PFNSSs F_{μ} and G_{δ}, denoted by $F_{\mu}\tilde{\bigcap}G_{\delta}$, is a PFNSSs $H_{\upsilon}:E\rightarrow FN(U)\times I^{U}$ defined by $H_{\upsilon}=\left(H(e)(c),\upsilon(e)(c)\right),\forall e\in E$, such that $H(e)=\left(F(e)\tilde{\bigcap}G(e)\right)$ and $\upsilon(e)=t\left(\mu(e),\delta(e)\right)$, where $F\bigcap G=\left\{\left(x,\min\left(T_F(x),T_G(x)\right),\max\left(I_F(x),I_G(x)\right),\max\left(F_F(x),F_G(x),\min\left(\mu_G(x),\mu_F(x)\right)\right)\right\}$, t is a fuzzy t-norm and $\tilde{\bigcap}$ is a Fermatean Neutrosophic soft intersection.

Example 7: Refer to Example 6

$$F_{\mu}(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle},0.5\right)\right\};$$

$$F_{\mu}(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.5\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle},0.8\right)\right\};$$

$$F_{\mu}(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle},0.1\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle},0.7\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.6\right)\right\};$$

Let $G_{\delta}:E\rightarrow FN(U)\times I^{U}$ be another PFNSS over (U,E) defined as follows:

$$G_{\delta}(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle},0.4\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle},0.5\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle},0.3\right)\right\};$$

$$G_{\delta}(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.7\right)\right\};$$

$$G_{\delta}(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.8\right)\right\}.$$

To find the intersection, we use the Fermatean Neutrosophic soft intersection as fuzzy intersection method:

$$H_{\upsilon}(e_1)=\left\{\left(\frac{c_1}{\min\langle 0.2,0.3\rangle,\max\langle 0.6,0.2\rangle,\max\langle 0.1,0.1\rangle},\min\langle 0.3,0.4\rangle\right),\right.$$

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$$\left(\frac{c_2}{\min\langle 0.4,0.2\rangle,\max\langle 0.2,0.4\rangle,\max\langle 0.6,0.3\rangle},\min\langle 0.4,0.5\rangle\right),$$

$$\left.\left(\frac{c_3}{\min\langle 0.3,0.1\rangle,\max\langle 0.1,0.7\rangle,\max\langle 0.7,0.2\rangle},\min\langle 0.5,0.3\rangle\right)\right\}$$

$$=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.2,0.4,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.1,0.7,0.7\rangle},0.3\right)\right\}.$$

Similarly, we get.

$$H_\upsilon(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.4,0.5\rangle},0.1\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.8,0.3\rangle},0.7\right)\right\};$$

$$H_\upsilon(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle},0.1\right),\left(\frac{c_2}{\langle 0.1,0.7,0.9\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.6\right)\right\};$$

$$H_\upsilon(e)=\begin{pmatrix}(\langle 0.2,0.6,0.1\rangle,0.3) & (\langle 0.2,0.4,0.6\rangle,0.4) & (\langle 0.1,0.7,0.7\rangle,0.3)\\ (\langle 0.2,0.4,0.5\rangle,0.1) & (\langle 0.2,0.3,0.4\rangle,0.3) & (\langle 0.1,0.8,0.3\rangle,0.7)\\ (\langle 0.4,0.3,0.5\rangle,0.1) & (\langle 0.1,0.7,0.9\rangle,0.3) & (\langle 0.1,0.9,0.3\rangle,0.6)\end{pmatrix}.$$

Proposition 1: Let F_μ , G_δ and H_υ any three PFNSS over (U,E) .Then the following holds:

I. $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$

II. $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$

III. $F_\mu \tilde{\cup}\left(G_\delta \tilde{\cup} H_\upsilon\right)=\left(F_\mu \tilde{\cup} G_\delta\right)\tilde{\cup} H_\upsilon$

IV. $F_\mu \tilde{\cap}\left(G_\delta \tilde{\cap} H_\upsilon\right)=\left(F_\mu \tilde{\cap} G_\delta\right)\tilde{\cap} H_\upsilon$

Proof (I)

$$F_\mu \tilde{\cup} G_\delta = \left(\left(F\tilde{\cup} G\right), s(\mu,\delta)\right)$$

$=\left(\left(G\tilde{\cup} F\right), s(\delta,\mu)\right)$, since $\tilde{\cup}$ is commutative and s is commutative.

$$=G_\delta \tilde{\cup} F_\mu$$

Proof (II)

$$F_\mu \tilde{\cap} G_\delta = \left(\left(F\tilde{\cap} G\right), t(\mu,\delta)\right)$$

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$=\left(\left(G\tilde{\cap}F\right),t\left(\delta,\mu\right)\right)$, since $\tilde{\cap}$ is commutative and t is commutative.

$=G_{\delta}\tilde{\cap}F_{\mu}$

Proof (III)

$$F_{\mu}\tilde{\cup}\left(G_{\delta}\tilde{\cup}H_{\upsilon}\right)=\left(\left(F\tilde{\cup}\left(G\tilde{\cup}H\right),s\left(\mu,s\left(\delta,\upsilon\right)\right)\right)$$
$$=\left(\left(F\tilde{\cup}G\right)\tilde{\cup}H\right),s\left(\mu,\delta,s\left(\upsilon\right)\right)$$
$$=\left(F_{\mu}\tilde{\cup}G_{\delta}\right)\tilde{\cup}H_{\upsilon}$$

since $\tilde{\cup}$ is verify Demorgan laws and associative and s is associative.

Proof (IV)

$$F_{\mu}\tilde{\cap}\left(G_{\delta}\tilde{\cap}H_{\upsilon}\right)=\left(\left(F\tilde{\cap}\left(G\tilde{\cap}H\right),t\left(\mu,t\left(\delta,\upsilon\right)\right)\right)$$
$$=\left(\left(F\tilde{\cap}G\right)\tilde{\cap}H\right),t\left(\mu,\delta,t\left(\upsilon\right)\right)$$
$$=\left(F_{\mu}\tilde{\cap}G_{\delta}\right)\tilde{\cap}H_{\upsilon}$$

since $\tilde{\cap}$ is verify Demorgan laws and associative and t is associative.

Proposition 2: Let F_{μ} and G_{δ} are two PFNSS over (U,E) .Then the following hold:

I. $\left(F_{\mu}\tilde{\cap}G_{\delta}\right)^{c}=\left(F_{\mu}^{c}\tilde{\cup}G_{\delta}^{c}\right)$

II. $\left(F_{\mu}\tilde{\cup}G_{\delta}\right)^{c}=\left(F_{\mu}^{c}\tilde{\cap}G_{\delta}^{c}\right)$

Proof (I)

$$\left(F_{\mu}\tilde{\cap}G_{\delta}\right)^{c}=\left(\left(F\tilde{\cap}G\right)^{c},t\left(\mu,\delta\right)^{c}\right)$$

$=\left(\left(F^{c}\tilde{\cup}G^{c}\right),s\left(\mu^{c},\delta^{c}\right)\right)$, since $\tilde{\cup}$ is soft complement and s is complementing.

$$=\left(F_{\mu}{}^{c}\tilde{\cup}G_{\delta}{}^{c}\right)$$

Proof (II)

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$$\left(F_\mu \tilde{\bigcup} G_\delta\right)^c = \left(\left(F \tilde{\bigcup} G\right)^c, s\left(\mu,\delta\right)^c\right)$$

$$= \left(\left(F^c \tilde{\bigcap} G^c\right), t\left(\mu^c,\delta^c\right)\right)$$, since $\tilde{\bigcap}$ is soft complement and t is soft complement.

$$= \left(F_\mu{}^c \tilde{\bigcap} G_\delta{}^c\right)$$

5. AND and OR Operations on FNSSs with Applications in Decision Making

In this section, we introduce the definitions of AND and OR operations on Possibility Fermatean Neutrosophic Soft Set, Applications of Possibility Fermatean Neutrosophic Soft Set in decision-making problem are given.

Definition 21: If $\left(F_\mu, A\right)$ and $\left(G_\delta, B\right)$ are two PFNSSs then "$\left(F_\mu, A\right)$ AND $\left(G_\delta, B\right)$", denoted by $\left(F_\mu, A\right)\wedge\left(G_\delta, B\right)$ is defined by $\left(F_\mu, A\right)\wedge\left(G_\delta, B\right) = \left(H_\lambda, A\times B\right)$, where

$$H_\lambda\left(\alpha,\beta\right) = \left(H\left(\alpha,\beta\right)(c), \lambda\left(\alpha,\beta\right)(c)\right),\ \forall\left(\alpha,\beta\right)\in A\times B,$$

such that $H\left(\alpha,\beta\right) = \left(F\left(\alpha\right)\bigcap G\left(\beta\right)\right)$ and $\lambda\left(\alpha,\beta\right) = t\left(\mu\left(\alpha\right),\delta\left(\beta\right)\right)$, for all $\left(\alpha,\beta\right)\in A\times B$, where

$$F\wedge G = \left\{\left(x, \min\left(T_F\left(x\right),T_G\left(x\right)\right), \max\left(I_F\left(x\right),I_G\left(x\right)\right), \max\left(F_F\left(x\right),F_G\left(x\right), \min\left(\mu_G\left(x\right),\mu_F\left(x\right)\right)\right)\right)\right\}$$

and $\tilde{\bigcap}$ is Fermatean Neutrosophic soft intersection and t is a t-norm.

Example 8: Assume the universal consists of three cars c_1, c_2, c_3 that is, $U = \left\{c_1, c_2, c_3\right\}$ and there are three parameters $E = \left\{e_1, e_2, e_3\right\}$ which describe their performances according to certain specific tasks. Suppose Mr.X wants to buy one such car depending on either of the parameters only. Let there be two observations F_μ and G_δ by two experts defined as follows:

$$F_\mu(e_1) = \left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle},0.5\right)\right\};$$

$$F_\mu(e_2) = \left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.5\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle},0.8\right)\right\};$$

$$F_\mu(e_3) = \left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle},0.1\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle},0.7\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.6\right)\right\};$$

$$G_\delta(e_1) = \left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle},0.4\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle},0.5\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle},0.3\right)\right\};$$

$$G_\delta(e_2) = \left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.7\right)\right\};$$

$$G_\delta(e_3) = \left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.8\right)\right\};$$

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$$H_\lambda(e_1,e_1)=\left\{\left(\frac{c_1}{\min\langle 0.2,0.3\rangle,\max\langle 0.6,0.2\rangle,\max\langle 0.1,0.1\rangle},\min\langle 0.3,0.4\rangle\right),\right.$$

$$\left(\frac{c_2}{\min\langle 0.4,0.2\rangle,\max\langle 0.2,0.4\rangle,\max\langle 0.6,0.3\rangle},\min\langle 0.4,0.5\rangle\right),$$

$$\left.\left(\frac{c_3}{\min\langle 0.3,0.1\rangle,\max\langle 0.1,0.7\rangle,\max\langle 0.7,0.2\rangle},\min\langle 0.5,0.3\rangle\right)\right\}.$$

$$H_\lambda(e_1,e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.2,0.4,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.1,0.7,0.7\rangle},0.3\right)\right\}.$$

The Similarly, we get.

$$H_\lambda(e_1,e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.7\rangle},0.5\right)\right\};$$

$$H_\lambda(e_1,e_3)=\left\{\left(\frac{c_1}{(0.2,0.6,0.5)},0.2\right),\left(\frac{c_2}{(0.1,0.7,0.6)},0.3\right),\left(\frac{c_3}{(0.1,0.9,0.7)},0.5\right)\right\};$$

$$H_\lambda(e_2,e_1)=\left\{\left(\frac{c_1}{(0.3,0.4,0.5)},0.2\right),\left(\frac{c_2}{(0.2,0.4,0.4)},0.5\right),\left(\frac{c_3}{(0.1,0.8,0.2)},0.3\right)\right\};$$

$$H_\lambda(e_2,e_2)=\left\{\left(\frac{c_1}{(0.2,0.4,0.5)},0.1\right),\left(\frac{c_2}{(0.2,0.3,0.4)},0.3\right),\left(\frac{c_3}{(0.1,0.8,0.3)},0.7\right)\right\};$$

$$H_\lambda(e_2,e_3)=\left\{\left(\frac{c_1}{(0.3,0.4,0.5)},0.2\right),\left(\frac{c_2}{(0.1,0.7,0.4)},0.3\right),\left(\frac{c_3}{(0.1,0.9,0.3)},0.8\right)\right\};$$

$$H_\lambda(e_3,e_1)=\left\{\left(\frac{c_1}{(0.3,0.2,0.1)},0.1\right),\left(\frac{c_2}{(0.2,0.6,0.9)},0.5\right),\left(\frac{c_3}{(0.1,0.7,0.3)},0.3\right)\right\};$$

$$H_\lambda(e_3,e_2)=\left\{\left(\frac{c_1}{(0.2,0.3,0.4)},0.1\right),\left(\frac{c_2}{(0.3,0.6,0.9)},0.3\right),\left(\frac{c_3}{(0.1,0.2,0.3)},0.6\right)\right\};$$

$$H_\lambda(e_3,e_3)=\left\{\left(\frac{c_1}{(0.4,0.3,0.5)},0.1\right),\left(\frac{c_2}{(0.1,0.7,0.9)},0.3\right),\left(\frac{c_3}{(0.1,0.9,0.3)},0.6\right)\right\}.$$

In matrix notation we get

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$$H_\lambda = \begin{pmatrix} (\langle 0.2,0.6,0.1\rangle,0.3) & (\langle 0.2,0.4,0.6\rangle,0.4) & (\langle 0.1,0.7,0.7\rangle,0.3) \\ (\langle 0.2,0.4,0.6\rangle,0.1) & (\langle 0.4,0.2,0.6\rangle,0.3) & (\langle 0.1,0.2,0.7\rangle,0.5) \\ (\langle 0.2,0.6,0.5\rangle,0.2) & (\langle 0.1,0.7,0.6\rangle,0.3) & (\langle 0.1,0.9,0.7\rangle,0.5) \\ (\langle 0.3,0.4,0.5\rangle,0.2) & (\langle 0.2,0.4,0.4\rangle,0.5) & (\langle 0.1,0.8,0.2\rangle,0.3) \\ (\langle 0.2,0.4,0.5\rangle,0.1) & (\langle 0.2,0.3,0.4\rangle,0.3) & (\langle 0.1,0.8,0.3\rangle,0.7) \\ (\langle 0.3,0.4,0.5\rangle,0.2) & (\langle 0.1,0.7,0.4\rangle,0.3) & (\langle 0.1,0.9,0.3\rangle,0.8) \\ (\langle 0.3,0.2,0.1\rangle,0.1) & (\langle 0.2,0.6,0.9\rangle,0.3) & (\langle 0.1,0.7,0.3\rangle,0.3) \\ (\langle 0.2,0.3,0.4\rangle,0.1) & (\langle 0.1,0.7,0.9\rangle,0.3) & (\langle 0.1,0.2,0.3\rangle,0.6) \\ (\langle 0.4,0.3,0.5\rangle,0.1) & (\langle 0.1,0.9,0.3\rangle,0.6) & (\langle 0.1,0.9,0.3\rangle,0.6) \end{pmatrix}$$

Matrix representation of AND

Definition 22 :If (F_μ, A) and (G_δ, B) are two PFNSSs then "(F_μ, A) OR (G_δ, B)", denoted by $(F_\mu, A)\vee(G_\delta, B)$ is defined by $(F_\mu, A)\vee(G_\delta, B) = (H_\lambda, A\times B)$, Where $H_\lambda(\alpha,\beta) = (H(\alpha,\beta)(c), \lambda(\alpha,\beta)(c))$, $\forall(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta) = (F(\alpha)\tilde{\bigcup} G(\beta))$ and $\lambda(\alpha,\beta) = s(\mu(\alpha),\delta(\beta))$, for all $(\alpha,\beta)\in A\times B$, where

$$F\vee G = \{(x, \max(T_F(x), T_G(x)), \min(I_F(x), I_G(x)), \min(F_F(x), F_G(x), \max(\mu_G(x), \mu_F(x)))\}$$

$\tilde{\bigcup}$ is Fermatean Neutrosophic Soft union and s is an s-norm.

Example 9: Consider Example 8, where :

$$F_\mu(e_1) = \left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle},0.5\right)\right\};$$

$$F_\mu(e_2) = \left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.5\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle},0.8\right)\right\};$$

$$F_\mu(e_3) = \left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle},0.1\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle},0.7\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.6\right)\right\};$$

$$G_\delta(e_1) = \left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle},0.4\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle},0.5\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle},0.3\right)\right\};$$

$$G_\delta(e_2) = \left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.7\right)\right\};$$

$$G_\delta(e_3) = \left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.8\right)\right\}.$$

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$$H_\lambda(e_1,e_1)=\left\{\left(\frac{c_1}{(\max(0.2,0.3),\min(0.6,0.2),\min(0.1,0.1))},\max(0.3,0.4)\right),\right.$$

$$\left(\frac{c_2}{(\max(0.4,0.2),\min(0.2,0.4),\min(0.6,0.3))},\max(0.4,0.5)\right),$$

$$\left.\left(\frac{c_3}{(\max(0.3,0.1),\min(0.1,0.7),\min(0.7,0.2))},\max(0.5,0.3)\right)\right\}.$$

$$H_\lambda(e_1,e_1)=\left\{\left(\frac{c_1}{(0.3,0.2,0.1)},0.4\right),\left(\frac{c_2}{(0.4,0.2,0.3)},0.5\right),\left(\frac{c_3}{(0.3,0.1,0.2)},0.5\right)\right\}.$$

The Similarly, we get.

$$H_\lambda(e_1,e_2)=\left\{\left(\frac{c_1}{(0.2,0.3,0.1)},0.3\right),\left(\frac{c_2}{(0.7,0.1,0.2)},0.4\right),\left(\frac{c_3}{(0.3,0.1,0.3)},0.7\right)\right\};$$

$$H_\lambda(e_1,e_3)=\left\{\left(\frac{c_1}{(0.4,0.3,0.1)},0.3\right),\left(\frac{c_2}{(0.4,0.2,0.4)},0.4\right),\left(\frac{c_3}{(0.3,0.1,0.3)},0.8\right)\right\};$$

$$H_\lambda(e_2,e_1)=\left\{\left(\frac{c_1}{(0.3,0.2,0.1)},0.4\right),\left(\frac{c_2}{(0.2,0.3,0.3)},0.5\right),\left(\frac{c_3}{(0.7,0.7,0.2)},0.8\right)\right\};$$

$$H_\lambda(e_2,e_2)=\left\{\left(\frac{c_1}{(0.3,0.3,0.4)},0.2\right),\left(\frac{c_2}{(0.7,0.1,0.2)},0.5\right),\left(\frac{c_3}{(0.7,0.2,0.2)},0.8\right)\right\};$$

$$H_\lambda(e_2,e_3)=\left\{\left(\frac{c_1}{(0.4,0.3,0.5)},0.2\right),\left(\frac{c_2}{(0.2,0.3,0.4)},0.5\right),\left(\frac{c_3}{(0.7,0.8,0.2)},0.8\right)\right\};$$

$$H_\lambda(e_3,e_1)=\left\{\left(\frac{c_1}{(0.6,0.2,0.1)},0.4\right),\left(\frac{c_2}{(0.3,0.4,0.3)},0.7\right),\left(\frac{c_3}{(0.1,0.2,0.2)},0.6\right)\right\};$$

$$H_\lambda(e_3,e_2)=\left\{\left(\frac{c_1}{(0.6,0.2,0.1)},0.1\right),\left(\frac{c_2}{(0.7,0.1,0.2)},0.7\right),\left(\frac{c_3}{(0.1,0.2,0.3)},0.7\right)\right\};$$

$$H_\lambda(e_3,e_3)=\left\{\left(\frac{c_1}{(0.5,0.2,0.1)},0.2\right),\left(\frac{c_2}{(0.3,0.6,0.4)},0.7\right),\left(\frac{c_3}{(0.1,0.2,0.3)},0.8\right)\right\}.$$

In matrix notation we get

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$$H_{\lambda}=\begin{pmatrix} (\langle 0.3,0.2,0.1\rangle,0.4) & (\langle 0.4,0.2,0.3\rangle,0.5) & (\langle 0.3,0.1,0.2\rangle,0.5) \\ (\langle 0.2,0.3,0.1\rangle,0.4) & (\langle 0.7,0.1,0.2\rangle,0.4) & (\langle 0.3,0.1,0.3\rangle,0.7) \\ (\langle 0.4,0.3,0.1\rangle,0.3) & (\langle 0.4,0.2,0.4\rangle,0.4) & (\langle 0.3,0.1,0.3\rangle,0.8) \\ (\langle 0.3,0.2,0.1\rangle,0.4) & (\langle 0.2,0.3,0.3\rangle,0.5) & (\langle 0.7,0.7,0.2\rangle,0.8) \\ (\langle 0.3,0.3,0.4\rangle,0.2) & (\langle 0.7,0.1,0.2\rangle,0.5) & (\langle 0.7,0.2,0.2\rangle,0.8) \\ (\langle 0.4,0.3,0.5\rangle,0.2) & (\langle 0.2,0.3,0.4\rangle,0.5) & (\langle 0.7,0.8,0.2\rangle,0.8) \\ (\langle 0.6,0.2,0.1\rangle,0.4) & (\langle 0.3,0.4,0.3\rangle,0.7) & (\langle 0.1,0.1,0.2\rangle,0.6) \\ (\langle 0.6,0.2,0.1\rangle,0.1) & (\langle 0.7,0.1,0.2\rangle,0.7) & (\langle 0.1,0.2,0.3\rangle,0.7) \\ (\langle 0.5,0.2,0.1\rangle,0.2) & (\langle 0.3,0.6,0.4\rangle,0.7) & (\langle 0.1,0.2,0.3\rangle,0.8) \end{pmatrix}$$

Matrix representation of OR

An Application of PFNSS in Decision-Making

Definition 23: [22] Let $g_{\upsilon},h_{\rho}\in PN(U,E)$, $f_{\mu}=g_{\upsilon}\wedge h_{\rho}$, and f_{μ}^{t}, f_{μ}^{i} and f_{μ}^{f} be the truth, indeterminacy and falsity matrices of $\wedge$ product matrix, respectively. Then, weighted matrices of f_{μ}^{t}, f_{μ}^{i} and f_{μ}^{f}, denoted by $\wedge^{t}$, $\wedge^{i}$ and $\wedge^{f}$, are defined as follows:

$$\wedge^{t}(e_{kj},u_{r})=t_{(g_{\upsilon}\wedge h_{\rho})(e_{kj})}(u_{r})+(\upsilon_{kr}(e_{k})\wedge p_{jr}(e_{j}))-t_{(g_{\upsilon}\wedge h_{\rho})(e_{kj})}(u_{r})\times(\mu_{kr}(e_{k})\wedge\upsilon_{jr}(e_{j}))$$

$$\wedge^{i}(e_{kj},u_{r})=i_{(g_{\upsilon}\wedge h_{\rho})(e_{kj})}(u_{r})\times(\upsilon_{kr}(e_{k})\wedge p_{jr}(e_{j}))$$

$$\wedge^{f}(e_{kj},u_{r})=f_{(g_{\upsilon}\wedge h_{\rho})(e_{kj})}(u_{r})\times(\upsilon_{kr}(e_{k})\wedge p_{jr}(e_{j}))$$

for j, k, r $\in\Lambda$.

Definition 24 :[22] Let $g_{\upsilon},h_{\rho}\in PN(U,E)$, $f_{\mu}=g_{\upsilon}\wedge h_{\rho}$, and let $\wedge^{t}$, $\wedge^{i}$ and $\wedge^{f}$ be the weighted matrices of f_{μ}^{t}, f_{μ}^{i} and f_{μ}^{f}, respectively. Then, in the weighted matrices $\wedge^{t}$, $\wedge^{i}$ and $\wedge^{f}$ scores of $u^{n}\in U$, denoted by $s^{t}(u_{n}),s^{i}(u_{n})$ and $s^{f}(u_{n})$ are defined as follows:

$$s^{t}(u_{n})=\sum_{kj\in\Lambda}\delta_{kj}^{t}(u_{n})$$

$$s^{i}(u_{n})=\sum_{kj\in\Lambda}\delta_{kj}^{i}(u_{n})$$

$$s^{f}(u_{n})=\sum_{kj\in\Lambda}\delta_{kj}^{f}(u_{n})$$

Where

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$$s^t(u_n)=\begin{cases}\wedge^t(e_{kj},u_n), & \wedge^t(e_{kj},u_n)=\max\{\wedge^t(e_{kj},u_m):u_m\in U\\ 0, & otherwise\end{cases}$$

$$s^i(u_n)=\begin{cases}\wedge^i(e_{kj},u_n), & \wedge^i(e_{kj},u_n)=\max\{\wedge^i(e_{kj},u_m):u_m\in U\\ 0, & otherwise\end{cases}$$

$$s^f(u_n)=\begin{cases}\wedge^f(e_{kj},u_n), & \wedge^f(e_{kj},u_n)=\max\{\wedge^f(e_{kj},u_m):u_m\in U\\ 0, & otherwise\end{cases}.$$

Definition 25: [22] Let $s^t(u_n), s^i(u_n)$ *and* $s^f(u_n)$ be scored of $u_n\in U$ in the weighted matrices $\wedge^t, \wedge^i$ *and* $\wedge^f$ .Then, decision score of $u_n\in U$, denoted by $ds(u_n)$, is defined by

$$ds(u_n)=s^t(u_n)-s^i(u_n)-s^f(u_n).$$

Now, we construct a PFNS-decision making method by the following algorithm:

Algorithm:

Step 1: Input the Possibility Fermatean Neutrosophic soft sets,

Step 2: Construct the AND matrix,

Step 3: Construct the truth, indeterminacy, and falsity matrices of the AND matrix,

Step 4: Construct the weighted matrices $\wedge^t, \wedge^i$ *and* $\wedge^f$,

Step 5: Compute the score of $u_t\in U$, for each of the weighted matrices,

Step 6: Compute the decision score, for all $u_t\in U$,

Step 7: The optimal decision is to select $u_t=\max ds(u_i)$.

Example 10: Assume that $U=\{c_1,c_2,c_3\}$ is a set of three laptops and $E=\{e_1,e_2,e_3\}$ = {Price, Battery, CPU} is a set of parameters which is attractiveness of laptops. Suppose that Mr.X wants to buy the most suitable laptop.

Step 1: Based on the choice parameters of Mr. X, PFNSS F_μ *and* G_δ constructed by two experts are as follows:

$$F_\mu(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle},0.5\right)\right\};$$

$$F_\mu(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.5\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle},0.8\right)\right\};$$

$$F_\mu(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle},0.1\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle},0.7\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.6\right)\right\};$$

$$G_\delta(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle},0.4\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle},0.5\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle},0.3\right)\right\};$$

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$$G_{\delta}(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.7\right)\right\};$$

$$G_{\delta}(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.8\right)\right\}.$$

$$H_{\lambda}(e_1,e_1)=\left\{\left(\frac{c_1}{\min\langle 0.2,0.3\rangle,\max\langle 0.6,0.2\rangle,\max\langle 0.1,0.1\rangle},\min\langle 0.3,0.4\rangle\right),\right.$$

$$\left(\frac{c_2}{\min\langle 0.4,0.2\rangle,\max\langle 0.2,0.4\rangle,\max\langle 0.6,0.3\rangle},\min\langle 0.4,0.5\rangle\right),$$

$$\left.\left(\frac{c_3}{\min\langle 0.3,0.1\rangle,\max\langle 0.1,0.7\rangle,\max\langle 0.7,0.2\rangle},\min\langle 0.5,0.3\rangle\right)\right\}.$$

$$H_{\lambda}(e_1,e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle},0.3\right),\left(\frac{c_2}{\langle 0.2,0.4,0.6\rangle},0.4\right),\left(\frac{c_3}{\langle 0.1,0.7,0.7\rangle},0.3\right)\right\}.$$

Similarly, we get.

$$H_{\lambda}(e_1,e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.7\rangle},0.5\right)\right\};$$

$$H_{\lambda}(e_1,e_3)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.1,0.7,0.6\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.7\rangle},0.5\right)\right\};$$

$$H_{\lambda}(e_2,e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.2,0.4,0.4\rangle},0.5\right),\left(\frac{c_3}{\langle 0.1,0.8,0.2\rangle},0.3\right)\right\};$$

$$H_{\lambda}(e_2,e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.4,0.5\rangle},0.1\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.8,0.3\rangle},0.7\right)\right\};$$

$$H_{\lambda}(e_2,e_3)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle},0.2\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.8\right)\right\};$$

$$H_{\lambda}(e_3,e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle},0.1\right),\left(\frac{c_2}{\langle 0.2,0.6,0.9\rangle},0.5\right),\left(\frac{c_3}{\langle 0.1,0.7,0.3\rangle},0.3\right)\right\};$$

$$H_{\lambda}(e_3,e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle},0.1\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle},0.6\right)\right\};$$

$$H_{\lambda}(e_3,e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle},0.1\right),\left(\frac{c_2}{\langle 0.1,0.7,0.9\rangle},0.3\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle},0.6\right)\right\}.$$

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Step 2: consider the PFNSS AND matrix,

$$H_\lambda = \begin{pmatrix} c_{1,\lambda} & c_{2,\lambda} & c_{3,\lambda} \\ (\langle 0.2,0.6,0.1\rangle,0.3) & (\langle 0.2,0.4,0.6\rangle,0.4) & (\langle 0.1,0.7,0.7\rangle,0.3) \\ (\langle 0.2,0.4,0.6\rangle,0.1) & (\langle 0.4,0.2,0.6\rangle,0.3) & (\langle 0.1,0.2,0.7\rangle,0.5) \\ (\langle 0.2,0.6,0.5\rangle,0.2) & (\langle 0.1,0.7,0.6\rangle,0.3) & (\langle 0.1,0.9,0.7\rangle,0.5) \\ (\langle 0.3,0.4,0.5\rangle,0.2) & (\langle 0.2,0.4,0.4\rangle,0.5) & (\langle 0.1,0.8,0.2\rangle,0.3) \\ (\langle 0.2,0.4,0.5\rangle,0.1) & (\langle 0.2,0.3,0.4\rangle,0.3) & (\langle 0.1,0.8,0.3\rangle,0.7) \\ (\langle 0.3,0.4,0.5\rangle,0.2) & (\langle 0.1,0.7,0.4\rangle,0.3) & (\langle 0.1,0.9,0.3\rangle,0.8) \\ (\langle 0.3,0.2,0.1\rangle,0.1) & (\langle 0.2,0.6,0.9\rangle,0.3) & (\langle 0.1,0.7,0.3\rangle,0.3) \\ (\langle 0.2,0.3,0.4\rangle,0.1) & (\langle 0.1,0.7,0.9\rangle,0.3) & (\langle 0.1,0.2,0.3\rangle,0.6) \\ (\langle 0.4,0.3,0.5\rangle,0.1) & (\langle 0.1,0.9,0.3\rangle,0.6) & (\langle 0.1,0.9,0.3\rangle,0.6) \end{pmatrix}$$

Matrix representation of AND

Step 3: We Construct the T, I, and F matrices of the AND matrix,

$$\begin{pmatrix} \langle 0.2,0.3\rangle & \langle 0.2,0.4\rangle & \langle 0.1,0.3\rangle \\ \langle 0.2,0.1\rangle & \langle 0.4,0.3\rangle & \langle 0.1,0.5\rangle \\ \langle 0.2,0.2\rangle & \langle 0.1,0.3\rangle & \langle 0.1,0.5\rangle \\ \langle 0.3,0.2\rangle & \langle 0.2,0.5\rangle & \langle 0.1,0.3\rangle \\ \langle 0.2,0.1\rangle & \langle 0.2,0.3\rangle & \langle 0.1,0.7\rangle \\ \langle 0.3,0.2\rangle & \langle 0.1,0.3\rangle & \langle 0.1,0.8\rangle \\ \langle 0.3,0.1\rangle & \langle 0.2,0.3\rangle & \langle 0.1,0.3\rangle \\ \langle 0.2,0.1\rangle & \langle 0.1,0.3\rangle & \langle 0.1,0.6\rangle \\ \langle 0.4,0.1\rangle & \langle 0.1,0.6\rangle & \langle 0.1,0.6\rangle \end{pmatrix} \begin{pmatrix} \langle 0.6,0.3\rangle & \langle 0.4,0.4\rangle & \langle 0.7,0.3\rangle \\ \langle 0.4,0.1\rangle & \langle 0.2,0.3\rangle & \langle 0.2,0.5\rangle \\ \langle 0.6,0.2\rangle & \langle 0.7,0.3\rangle & \langle 0.9,0.5\rangle \\ \langle 0.4,0.2\rangle & \langle 0.4,0.5\rangle & \langle 0.8,0.3\rangle \\ \langle 0.4,0.1\rangle & \langle 0.3,0.3\rangle & \langle 0.8,0.7\rangle \\ \langle 0.4,0.2\rangle & \langle 0.7,0.3\rangle & \langle 0.9,0.8\rangle \\ \langle 0.2,0.1\rangle & \langle 0.6,0.3\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.1\rangle & \langle 0.7,0.3\rangle & \langle 0.9,0.6\rangle \\ \langle 0.3,0.1\rangle & \langle 0.9,0.6\rangle & \langle 0.1,0.6\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.1,0.3\rangle & \langle 0.6,0.4\rangle & \langle 0.7,0.3\rangle \\ \langle 0.6,0.1\rangle & \langle 0.6,0.3\rangle & \langle 0.7,0.5\rangle \\ \langle 0.5,0.2\rangle & \langle 0.6,0.3\rangle & \langle 0.7,0.5\rangle \\ \langle 0.5,0.2\rangle & \langle 0.4,0.5\rangle & \langle 0.2,0.3\rangle \\ \langle 0.5,0.1\rangle & \langle 0.4,0.3\rangle & \langle 0.3,0.7\rangle \\ \langle 0.5,0.2\rangle & \langle 0.4,0.3\rangle & \langle 0.3,0.8\rangle \\ \langle 0.1,0.1\rangle & \langle 0.9,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.4,0.1\rangle & \langle 0.9,0.3\rangle & \langle 0.3,0.6\rangle \\ \langle 0.5,0.1\rangle & \langle 0.3,0.6\rangle & \langle 0.3,0.6\rangle \end{pmatrix}$$

Matrix F_μ^t *of* AND *Matrix* F_μ^i *of* AND *Matrix* F_μ^f *of* AND

Step 4: We obtain weighted matrices $\wedge^t, \wedge^i$ *and* $\wedge^f$ by using Definition 23

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$\wedge^t$	c_1	c_2	c_3
e_{11}	0.44	<u>0.52</u>	0.37
e_{12}	0.28	<u>0.58</u>	0.55
e_{13}	0.37	0.37	<u>0.55</u>
e_{21}	0.44	<u>0.60</u>	0.37
e_{22}	0.28	0.44	<u>0.73</u>
e_{23}	0.44	0.37	<u>0.82</u>
e_{31}	0.37	<u>0.44</u>	0.37
e_{32}	0.28	0.37	<u>0.64</u>
e_{33}	0.46	<u>0.64</u>	<u>0.64</u>

$\wedge^i$	c_1	c_2	c_3
e_{11}	0.18	0.16	<u>0.21</u>
e_{12}	0.04	0.06	<u>0.10</u>
e_{13}	0.12	0.21	<u>0.45</u>
e_{21}	0.08	0.20	<u>0.24</u>
e_{22}	0.04	0.09	<u>0.56</u>
e_{23}	0.08	0.21	<u>0.72</u>
e_{31}	0.02	<u>0.18</u>	0.06
e_{32}	0.03	0.21	<u>0.54</u>
e_{33}	0.03	<u>0.54</u>	0.06

$\wedge^f$	c_1	c_2	c_3
e_{11}	0.03	<u>0.24</u>	0.21
e_{12}	0.06	0.18	<u>0.35</u>
e_{13}	0.10	0.18	<u>0.35</u>
e_{21}	0.10	<u>0.20</u>	0.06
e_{22}	0.05	0.12	<u>0.21</u>
e_{23}	0.10	0.20	<u>0.24</u>
e_{31}	0.01	<u>0.27</u>	0.09
e_{32}	0.04	<u>0.27</u>	0.18
e_{33}	0.05	<u>0.18</u>	<u>0.18</u>

Matrix F_μ^t *of* Truth *Matrix* F_μ^i *of* Indeterminacy *Matrix* F_μ^f *of* Falsity

Step 5: For all $c \in U$,we find scores by using Definition 24 as following:

$$s^t(c_1)=0, s^t(c_2)=3.05, s^t(c_3)=3.38$$

$$s^i(c_1)=0 \quad , s^i(c_2)=0.72 \quad , s^i(c_3)=2.82$$

$$s^f(c_1)=0 \quad , s^f(c_2)=1.16 \quad , s^f(c_3)=1.33$$

Step 6: For all $c \in U$,we find scores by using Definition 25 as following:

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$$ds(c_1) = 0 - 0 - 0 = 0$$
$$ds(c_2) = 3.05 - 0.72 - 1.16 = 1.17$$
$$ds(c_3) = 3.38 - 2.82 - 1.33 = -0.77$$

Step 7: The optimal condition selection for Mr. X is c_2.

6. Conclusion

In this paper, we have introduced the concept of Possibility Fermatean Neutrosophic Soft Set and studied some of its properties as complement, union, intersection, AND and OR. Applications of this theory has been given to solve a decision-making problem.

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