New Concepts in Partner Multineutrosophic Topological Space

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Abstract

We have new introduced two fundamental new concept that characterize partner multineutrosophic sets, the first of which we call the locally partner multineutrosophic interior and the second the locally partner fuzzy exterior, in addition to a concept related to set composition that we call detachable set, with the most important relationships to which these concept are linked and the properties that we have reached with an analytical view of them.

Keyword: Partner multineutrosophic sets; Locally partner multineutrosophic interior and exterior set; detachable set.

1. Introduction

Among the transformations that the scientific and engineering fields witnessed in last century, with the applied sciences, is what zadeh [1] presented through the introduction of fuzzy sets. This illumination has attracted many researchers to transfer their science, mathematical concepts, and other to the X×I(I=[0,1]) space in which the fuzzy sets move, what are the capabilities that characterize this space and the conditions that must be to a chi eve similarity between them. For example, the researcher Raghad [5]studied the local function with the space X×I and obtained different and varied types. But Florentin [7] brought a new and bright idea to the eyes of the world and researcher, and it resonated in the applied fields. He circulated the idea of Zadeh in a uniqe style with in the space X×I×I×I, and it included the foundations for new sets called Neutrosophic sets. But in our paper for new sets called in 2021 [4], both fuzzy sets and soft sets were combined with Neutrosophic crisp sets. But in 2022 [6] we developed a generalized formula to neutrosophic crisp and called Neutrosophic axiol set. In 2023[9], Smarandache, F. generalized the Neutrosophic sets and called then multineutrosiphic sets. Imran et al. [12-14] gave the view of some types of neutrosophic topological groups with respect to neutrosophic alpha open sets, new types of weakly neutrosophic crisp continuity, and new concepts of neutrosophic crisp open sets. Finally, the senses of new types of weakly neutrosophic crisp open mappings and new types of weakly neutrosophic crisp closed functions were informed by Al-Obaidi et al. [15,16].The binary operation $\varepsilon, \cup, =$, the classical operation.

2. Primitive

There are some basic definition of partner sets that serve as an introduction to building the mathematical concepts of our research poth.

Definition 2-1: Let X be nonempty universal set and A be any nonempty subset of X. Then $\{< x, T, I, F >: x \in X\}$ is called neutrosophic set and denoted by briefly by A if T,I,F: $x \to [0,1]$ are degree of truth, indeterminacy, falsehood membership of x corresponding with subset A, respectively. It is clear that $0 \leq \inf\ T + \inf\ I + infF \leq \sup\ T + \sup\ I + \sup\ F \leq 3$

Moreover, if the values of T,I,F are numbers from [0,1], then all of them have single – valued neutrosophic set (SVNS).

Definition 2-2: Let X be universal set and M be any nonempty subset of X, then $\{< x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t >: x \in X\}$ is called multineutrosophic set and denoted briefly by M $T_1, \dots, T_r: X \to [0,1]$ are the degree of truth membership functions of x corresponding with the subset M,$I_1, \dots, I_s: X \to [0,1]$ are the degree of indeterminacy membership function of x corresponding with the subset M, and $F_1, \dots, F_t: X \to [0,1]$

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are the degree of falsehood membership function of x corresponding with the subset M where r+s+t=n. It is obvious that $0 \leq \sum_{i=1}^{r} infT_i + \sum_{j=1}^{t} inf_j I_j + \sum_{k=1}^{t} inf F_k \leq \sum_{i=1}^{r} supT_i + \sum_{j=1}^{s} supI_j + \sum_{k=1}^{t} supF_k \leq n$

Definition 2-3: Let $\wedge_N$,$\wedge_F$ be the neutrosophic intersection and t-norm, respectively. Moreover, let $\vee_N$,$\vee_F$ be the neutrosophic and t-conorm, respectively. Let M_1, M_2 be neutrosophic sets with same(r , s , t)-form. Then $M_1 = \{< x, T_1, \dots, T_r, I_1, \dots, I_s, F_1 \dots, F_t >: x \in X\}$and $M_2\{< x, T_1', \dots, T_r', I_1', \dots, I_s', F_1', \dots, F_t' >: x \in X\}$

(i) The neutrosophic intersection

$$M_1 \wedge_F M_2 = \{< x, T_1 \wedge_F T_1', \dots T_r \wedge_F T_r', I_1 \vee_F I_1', \dots, I_s \vee_F I_s', F_t \vee_F F_t', \dots, F_t \vee_F F_t' >: x \in X\}$$

(ii) The multineutrosophic union

$$M_1 \vee_n M_2 = \{< x, T_1 \vee_N T_1', \dots, T_r \vee_N T_r', I_1 \wedge_N I_1', \dots, I_s \wedge_N I_s', F_t \wedge_N F_t', \dots, F_t \wedge_N F_t' >, x \in X\}$$

3. The Partner Set of Multineutrosophic Set

Definition 3-1: Let X be non-empty universal set and let $M^n = \{< x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t >: x \in X, r + s + t = n\}$ be a multineutrosophic set in X. The fuzzy set M_n is called a **partner set to the multineutrosophic set** $M^n = \{< x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t >: x \in X, r + s + t = n\}$ and it has the following form

$$M_n = \{\langle x, f_{M_n}(x)\rangle: x \in X\}$$

where

$$f_{M_n}(x) = \frac{1}{n}\left(\sum_{i-1}^{r} T_i + \sum_{j=1}^{s} I_j + \sum_{k=1}^{t} F_k\right), \forall x \in X.$$

Definition 3-2[2,8]: Let X by a nonempty universal set. For any multineutrosophic sets A^n and B^m , we defined the union and intersection of them concerning partnerA_p, B_p.

1 - $A_p \wedge B_p = \{< x, \min(f_{A_n}(x), g_{B_m}(x) >, x \in X\}$

2- $A_p \vee B_p = \left\{< x, \max\left(f_{A_n}(x), g_{B_m}(x)\right) >, x \in X\right\}$

3- $A_p^c = \{< x, 1 - f_{A_n}(x) >, x \in X\}$, the partner complement of partner set A_n, the partner zero set or partner empty fuzzy set

4- $0_p = \{< x, 0 >, \forall x \in X\}$, the partner universal set or partner universal fuzzy set

5- $1_p = \{< x, 1 >, \forall x \in X\}$,

6- the partner multineutrosophic point PMP is of the form $P_x^\lambda(y) = \begin{cases} \lambda, & y = x \\ 0, & y = 0 \end{cases}, \forall y \in X$

Definition 3-3 [2]: The nonempty collection of partner multineutrosophic sets PMSs, is called partner multineutrosophic topology PMt, if satisfying the following

$1 - 0_p, 1_p \varepsilon \tau_p$

2-if $M_p, N_p \varepsilon \tau_p then M_p \wedge N_p \varepsilon \tau_p$ 3- For any index $\Delta, H_{\lambda P} \varepsilon \tau_p, \forall \in \Delta, then \vee_{\lambda \in \Delta} H_{\lambda p} \varepsilon \tau_p$

Any PMS$M_p \varepsilon \tau_p$, is called partner multineutrosophic open set PMOS, and the complement is called partner multuneutrosophic closed set PMCS where ε is classical belong to.

4. Locally Partner Multineutrosophic Sets LPMIS Interior

The PIM point of PMSs are important tools in diagnosing the form and attributes of the topology derived from it, because through them all topological concepts can be constructed. We will give a special type for partner interior points, which are special point in the universal set with its properties.

Definition 4-1: Let $(1_p, \tau_p)$ be (PMTS) a partner fuzzy neutrosophic point PMP P_x^λ is called locally partner partner multineutrosophic interior of PMTS M^m is defined by

$\ell_i(M_p)_x = \vee_x \left\{G_p \in \tau_p; \lambda \leq f_{G_p}(x) \leq h_{M_p}(x)\right\}$ where $\vee_x H_\lambda^p = sup_{\lambda \in \Pi}\{H_{\lambda_p}(x);$ for any index Π

Example 4-2: Let X={1,2,3} ,

$A^7 = \{(1, (0.3,0.3), (0.3,0.1,0.1), (0.2,0.21), 2, (0.1,0.2), (0.1,0.0,0.1), (0.1,0.1), 3, (0.0,0.1), (0.0,0.1,0.0), (0.2,0.0)\}$

$B^7 = \{(1, (0.1,0.2,0.1), (0.1,0.3), (0.2,0.1), (2, (0.1,0.0,0.4), (0.2,0.2), (0.2,0.3),$
$(3, (0.11,0.4,0.3), (0.1,0.3), (0.1,0.2)\}$, $M^7 = \{(1, (0.0,0.0), (0.1,0.0), (0.0,0.1,0.0),$
$(2, (0.1,0.1), (0.2,0.0), (0.16,0.1,0.1), (3, (0.2,0.1), (0.0,0.3), (0.2,0.0,0.2) \}$

$A_p = \{(1,0.75), (2,0.35), (3,0.2)\}$, $B_p = \{(1,0.4), (2,0.46), (3,0.5)\}$

$A_p \wedge B_p = \{\left(x, \min\left(A_p(x), g_{B_p}(x)\right), \forall x \in X\right\}, A_p \wedge B_p = \{(1,0.4), (2,0.35), (3,0.2)\}$

$A_p \vee B_p = \{(x, \max\left(A_{A_p}(x), g_{B_p}(x), \forall x \in X\right\}, A_p \vee B_p = \{(1,0.75), (2,0.46), (3,0.5)\}$,

$\tau = \{0_p, 1_p, A_p, B_p, A_p \vee B_p, A_p \wedge B_p\}$, $\lambda = 0.2, Let\ M_p = \{(1,0.1), (2,0.38), (30.5)\}$,

$\ell_i(M_p)_x = \vee_x \left\{G_p \in \tau ; \lambda \leq f_{G_p}(x) \leq h_{M_p}(x)\right\}, \ell_i(M_p)_1 = 0 , \ell_i(M_p)_2 = 0.35 , \ell_i(M_p)_3 = 0.5$

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From this we conclude that totally partner set does not necessarily indicate that it is an PMOS set, see example 4-2, also is not PMCS

Definition 4-3: The multineutrosophic interior set is of all LPMIS at PMS $M_r^{\sim}, \forall x \in X$ is called the totally locally fuzzy interior is defined by

$t\ell_i(M_p^{\sim}) = \cup_{x \in X}\, \ell_i(M_p)_x$, where $\cup$ the classical union

it is simply to see that $p_y^\lambda \in t\ell_i(M_p)\, iff\ p_y^\lambda \in \vee_{x\in X}\, \ell_i(M_p)_x$, $\quad iff\ P_y^\lambda \in \ell_i(M_p)_y = \vee\{G_p \in \tau; \lambda \le f_G(y) \le h_{M_p}(y)\}$, $\lambda \le sup f_{G_p^{\sim}}(y)\ ; \forall G_p \varepsilon \tau \ni \lambda \le f_{G_p}(y) \le h_{M_p}(y)$. Also to see that $t\ell_i(1_p) = 1_p$ and $t\ell_i(0_p) = 0_p$

Example : from example 2-3 we have

$t\ell_i(M_p)$ ={(1,0),(2,0.35),(3,0.6)}

From this we conclude that TLPMIS does not necessarily indicate that it is a PMOS, see example2-4 athat $t\ell_i(M_p)_x$ is not PMOS.

There are several properties that locally partner pinks have, as show in following theorem.

Theorem 4-4: Let is $(1_p, \tau_p)$ a PMTS, then

1 – $int(M_p) \le t\ell_i(M_p)$

2- $if\ M_r^{\sim} is\ PMS\ then\ t\ell_i(M_p) \le M_p$

3- $if\ M_p \le N_p \to t\ell_i(M_p) \le t\ell_i(N_p)$

4-$t\ell_i(M_p \wedge N_p) = t\ell_i(M_p) t\ell_i(N_p)$

5- $t\ell_i(M_p) \vee t\ell_i(N_p) = t\ell_i(M_p \vee N_p)$

Proof (1): let $P_x^\lambda \in Int(M_p)$

$\Longrightarrow \exists\ G_p \in \tau_p \ni P_x^\lambda \in G_p \le M_p$

$\Longrightarrow \lambda \le f_{G_p}(x) \le sup f_{U_p}(x) \le h_{M_p}(x)$,

$\forall G_p \varepsilon \tau_p \ni \lambda \le f_{U_p}(x) \le h_{M_p}(x)$

$iff\ P_x^\lambda \in \vee_x \{U_p \varepsilon \tau_p; \lambda \le f_{U_p}(x) \le h_{M_p}(x) = \ell_i(M_p) \le \vee_{x\in X}\, \ell_i(M_p)_x = t\ell_i(M_p)$

Proof (2): let $P_x^\lambda \in t\ell_i(M_p)$

$\Longrightarrow P_x^\lambda \in \ell_i(M_p)_x \Longrightarrow P_x^\lambda \in \ell_i(M_p)_x$

$\Longrightarrow \lambda \le f_{G_p}(x) \le h_{M_p}(x)$

$\Longrightarrow P_x^\lambda \in M_p$

$\Longrightarrow t\ell_i(M_p) \le M_p$

$\Longrightarrow P_x^\lambda \in \ell_i(M_p)_x,$

Proof(3): let $P_x^\lambda \in t\ell_i(M_p) \Longrightarrow P_X^\lambda \in \vee\, \ell_i(M_p)_x$

$\Longrightarrow \exists G_p \in \tau_p \ni \lambda \le f_{G_p}(x) \le h_{M_p}(x) \le g_{N_p}(x)$

$\Longrightarrow P_x^\lambda \in \ell_i(N_p)_x \Longrightarrow P_x^\lambda \in t\ell_i(N_p)_x, \Longrightarrow \lambda \le f_{G_p}(x) \le g_{N_p}, \Longrightarrow t\ell_i(M_p) \le t(N_p)$

Proof(4): since

$M_p \wedge N_p \le M_p \Longrightarrow t\ell_i(M_p \wedge N_p) \le t\ell_i(M_p)$

$M_p \wedge N_p \le N_p \Longrightarrow t\ell_i(M_p \wedge N_p) \le t\ell_i(N_p)$

$t\ell_i(M_p \wedge N_p) \le t\ell_i(M_p) \wedge t\ell_i(N_p)$

$\Longleftarrow$ conversely

Let $P_x^\lambda \in t\ell_i(M_p) \wedge t\ell_i(N_p) \Longrightarrow P_x^\lambda \in t\ell_i(M_p) \& P_x^\lambda \in t\ell_i(N_p)$

$\Longrightarrow \exists G_{1p} \varepsilon \tau_p \ni P_x^\lambda \in G_p \le M_p \& G_{2p} \varepsilon \tau_p \ni P_x^\lambda \in G_{2p} \le N_p$

$\Longrightarrow \lambda \le f_{G_{1p}}(x) \le h_{M_p}(x)\ \&\ \lambda \le f_{G_{2p}}(x) \le g_{N_p}(x)$

$\Longrightarrow \lambda \le \min\left(f_{G_{1p}}(x), f_{G_{2p}}(x)\right) \le \min\left(h_{M_p}(x), g_{N_p}(x)\right)$

$\Longrightarrow P_x^\lambda \in \ell_i(M_p \wedge N_p) \Longrightarrow P_x^\lambda \in \ell_i(M_p \wedge N_p) \Longrightarrow P_x^\lambda \in t\ell_i(M_p \wedge N_p)$

$M_p \le M_p \vee N_p \Longrightarrow t\ell_i(M_p) \le t\ell_i(M_p \vee N_p)$ proof(5):

And $N_p \le M_p \vee N_p \Longrightarrow t\ell_i(N_p) \le t\ell_i(M_p \vee N_p)$

$\Longrightarrow t\ell_i(M_p) \vee t\ell_i(N_p) \le t\ell_i(M_p \vee N_p)$

$\Longleftarrow let\ P_x^\lambda \in t\ell_i(M_p \vee N_p) \Longrightarrow P_x^\lambda \in \vee_x\, \ell_i(M_p \vee N_p) \Longrightarrow P_x^\lambda \in \ell_i(M_p \vee N_p) = \vee_x \{G_p \varepsilon \tau_p \ni \lambda \le f_{G_p}(x)$
$\le h_{(M_p \vee N_p)}(x) = \max(g_{M_p}(x), k_{N_p}(x)\}$

$= \vee_x \{G_p \varepsilon \tau_p \ni \lambda \le f_{G_p}(x) \le g_{M_p}(x) or\ \lambda \le f_{G_p}(x) \le k_{N_p}(x)$

$\Longrightarrow either P_x^\lambda \in \vee_x \left\{G_p \varepsilon \tau_p \ni \lambda \le f_{G_p}(x) \le g_{M_p}(x)\right\} or\ P_x^\lambda \in \vee_x \left\{G_p \varepsilon \tau_p \ni \lambda \le f_{G_p}(x) \le k_{N_p}(x)\right\}$

$\Longrightarrow either\ P_x^\lambda \in \ell_i(M_p) or P_x^\lambda \in \ell_i(N_p) either\ P_x^\lambda \in t\ell_i(M_p)\ or\ P_x^\lambda \in t\ell_i(N_p) \Longrightarrow P_x^\lambda \in t\ell_i(M_p) \vee t\ell_i(N_p)$

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The converse of part (i), (ii)and (iii) are incorrect, as show in examples 4-5

Example 4-5: Let X={1,2,3,4,5}

A^8
$= \{(1,(0.1,0.0,0.2),(0.0,0.2,0.1),(0.3,0.1),(2(,0.0,0.0,0.1),(0.1,0.2,0.0),(0.2,0.1),(3,(0.08,0.2,0.1),(0.2,0.0,0.1)$
$(0.1,0.1),(4,(0.11,0.0,0.1),(0.1,0.2,0.1),(0.0,0.1),(5,(0.0,0.1,0.0),(0.0,0.2,0.1),(0.1,0.0)\}$

$B^8, = \{(1,(0.2,0.0,0.3),(0.1,0.4,0.1),(0.2,0.2),(2,(0.13,0.2,0.5),(0.1,0.0,0.0),(0.1,0.2),$
$(3,(0.1,0.18,0.2),(0.0,0.2,0.0),(0.1,0.1),(4,(0.1,0.1,0.0),(0.2,0.11,0.0),(0.2,0.0),(5(0.08,0.01,0.0),(0.03,0.07,0.1$
$1),(0.19,0.0)\}$

$M^8 = \{(1,(0.1,0.1,0.2),(0.0,0.2,0.0),(0.3,0.1),(2(0.2,0.11,0.0),(0.1,0.1,0.2),(0.1,0.2),$
$(3,(0.15,0.2,0.1),(0.09,0.11,0.03),(0.17,0.1),(4,(0.19,0.18,0.02),(0.1,0.2,0.1),(0.28,0.02),$
$(5,(0.2,0.14,0.09),(0.11,0.1,0.07),(0.13,0.2)\}$

$N^5 = \{(1,(0.1,0.3),(0.2,0.1),(0.3),(2,(0.025,0.2),(0.1,0.01),(0.09),(3,(0.072,0.1),(0.3,0.4),(0.2),$
$(4,0.06),(0.19,0.01),(0.2),(5,(0.05,0.03),(0.07,0.05),(0.05)\}$

$S^4 = \{(1,(0.1,0.2),(0.08),(0.22),(2,(0.03,0.08),(0.12),(0.2),(3,(0.51,,0.2),(0.1),(0.2),$
$(4,(0.05,0.06),(0.07),(0.17),(5,(0.04,0.1),(0.1),(0.1)\}$

$A_p = \{(1,0.5,0.5),(2,0.5.0.2),(3,0.33,0.1),(4,0.25,0.05),(5,0.2,0.3)\}$

$B_p = \{(1,o.5,0.5,0.5),(2,0.33,0.4,0.4),(3,0.25,0.33,0.3),(4,0.2,0.28,0.23),(5,0.16,0.25,0.19)\}$

$A_p = \{(1,0.5),(2,0.35),(3,0.215),(4,0.15),(5,0.25)\}$

$B_p = \{(1,0.5),(2,0.37),(3,0.19),(4,0.23),(5,0.2)\}$

$B_p \wedge B_p = \{(x, \min(f_A(x), g_b(x), \quad \forall x \in X\} = \{(1,0.5),(2,0.35),(3,0.19),(4,0.15),(5,0.2)\}$

$A_p \vee B_p = \{(x, \max(f_A(x), g_B(x), \quad \forall x \in X\}\} = \{(1,0.5),(2,0.37),(3,0.215),(4,0.23),(5,0.25)\}$

$\tau_p = \{0_p, 1_p, A_p, B_p, A_p \wedge B_p, A_p \vee B_p\}$

$M_p = \{(1,0.5),(2,0.505),(3,0.525),(4,0.545),(5,0.75)$

$N_p = \{(1,0.415),(2,0.2125),(3,0.161),(4,0.1405),(5,0.125)\}$

$\ell_i(M_p)_1 = 0.5, \quad \ell_i(M_p)_2 = 0.37, \quad \ell_i(M_p)_3 = 0.215, \quad \ell_i(M_p)_4 = 0.23, \quad \ell_i(M_p)_5 = 0.25$

$t\ell_i(M_p)_x = \vee_x \ell_i(M_p)_x$

$t\ell_i(M_p)_x = \{(1,5),(2,0.37),(3,0.215),(4,0.23),(5,0.25)\}$

$$int(M_p) = \vee \{G_p \varepsilon \tau_p;\ G_p \le M_p\} = N_p \vee (N_p \wedge M_p) = \{(1,0.5),(2,0.35),(3,0.215),(4,0.23),(5,0.2)\} \le t\ell_i(M_p)_x$$

$\Rightarrow t\ell_i(M_p) \nleq int(M_p)$ and $M_p \nleq t\ell_i(M_p)$

The converse of the part (iii) is not true from the example

$S_p = \{(1,0.25),(2,0.215),(3,0.19),(4,0.175),(5,0.17)\}$

$\ell_i(S_p)_1 = 0, \quad \ell_i(S_p)_2 = 0, \quad \ell_i(S_p)_3 = 0.19, \quad \ell_i(S_p)_4 = 0.15, \quad \ell_i(S_p)_5 = 0$

$t\ell_i(S_p) = \{(1,0),(2,0),(3,0.19),(4,0.15),(5,0)\}$

$S_p \le M_p \quad but\ t\ell_i(M_p)_x \nleq t\ell_i(S_p)_x.$

5. Locally Partner Multineutrosophic Exterior Set LPMES

Now in this section, we will give the form of the description of point outside the partner set, its relationship with the points inside the partner set, and the relationships that connect them together, while examining its properties.

Definition 5-1: A fuzzy point P_x^λ is called locally fuzzy exterior of a partner set $M^{\sim p}$ at the point $x \in X$ is defined by

$\ell_{ex}(M_p)_x = \vee_x \{G_p \in \tau_p;\ \lambda \le f_{G_p}(x) \le h_{1-M_p}(x)$ and $t\ell_e(M_p) = \vee_{x \in X} \ell_e(M_p)_x$ is called totally partner exterior of partner set M_p

Example 5-2: $A^6 = \{(1,(0.5,0.3),(0.1,0.2),(0.2,0.2),(2,(0.05,0.1),(0.2,0.2),(0.3,0.1)$
$(3,(0.03,0.08),(0.12,0.03),(0.17,0.1),(4,(0.1,0.2),(0.1,0.1),(0.08,0.12),(5,(0.4,0.2),(0.3,0.2),(0.28,0.12))\}$

B^6
$= \{(1,(0.1,0.2),(0.02,0.08,0.1),(0.1,0.1),(2(0.2,0.2),(0.3,0.1,0.1),(0.4,0.2),(3,(0.6,0.3),0.3,0.3,0.1),(0.5,0.4),$
$(4,(0.1,0.3),(0.20,0.27,0.1),(0.1,0.1),(5,(0.05,0.05),(0.2,0.2,0.08),90.2,0.02)\}$

$A^6 = \{(1,1,0.5),(2,0.5,0.25),(3,0.33,0.2),(4,0,0.7),(5,0.9,0.5)\}$

$B^7 = \{(1,0,0.7),(2,0.5,1),(3,1,0.15),(4,0.9,0.2),(5,0.2,0.6)\}$

$A_p = \left\{\left(x, \frac{\sum f(x)}{n}\right), \forall x \in X\right\}, A_p = \{(1,0.75),(2,0.375),(3,0.265),(4,0.35),(5,0.7)\}$

$B_p = \{(1,0.35),(2,0.75),(3,0.31),(4,0.55),(5,0.4)\}$

$A_p \wedge B_p = \{(x, \min(f(x), g(x)), \forall x \in X\} = \{(1,0.35),(2,0.75),(3,0.31),(4,0.35),(5,0.4)\}$

$A_p \vee B_p = \{(x, \max(f(x), g(x)), \forall x \in X\} = \{(1,0.75),(2,0.357),(3,0.265),(4,0.55),(5,0.7)\}$

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$\tau_p = \{0_p, 1_p, A_p, B_p, A_p \vee B_p, A_p \wedge B_p\}$

Let $M_p = \{(1,0.52), (2,0.81), (3,0.43), (4,0.7), (5,0.33)\}$

$M_p^{\sim\#c} = \{(x, 1 - h_{M^{\sim p}}(x), \forall x \in X\ = \{(1, o.48), (2,0.643), (3,0.735), (4,0.45), (5,0.67)\}$

$\ell_e(M_p)_1 = 0.35, \ell_e(M_p\ )_2 = 0.375, \ell_e(M_p)_3 = 0.265, \ell_e(M_p)_4 = 0.35, \ell_e(M_p)_5 = 0.7$

$t\ell_e(M_p) = \{(1,0.35), (2,0.375), (3,0.265), (4,0.35), (5, (0.7)\}$

1- By carefully analysing the above definition logically, we can conclude the following, $t\ell_e(0_p) = 1_p$ and $t\ell_e(1_p) = 0_p$

2- we can also notice $t\ell_e(M_p) \wedge M_p = 0_p$ for any PMSs M_p

Theorem 5-3: Let $(1_p, \tau_p)$ be partner fuzzy topological space and let $A_p and\ B_p$ are PMSs then

1- if A_p nonzero PMCS then $t\ell_e(A_p)\ \neq 0$

2- $t\ell_i(M_p) \leq\ t\ell_e(t\ell_e(M_p)$

3- $\ell_{ex}(B_p \wedge A_p)_x = \ell_{ex}(A_p)_x \vee \ell_{ex}(B_p)_x$

4- $\ell_{ex}(A_p \vee B_p)_x = \ell_{ex}(A_p)_x \wedge \ell_{ex}(B_p)_x$

5- $t\ell_{ex}(A_p \wedge B_p)_x = t\ell_{ex}(A_p)_x \vee t\ell_{ex}(B_p)_x$

6- $t\ell_{ex}(A_p \vee B_p)_x \leq t\ell_{ex}(A_p)_x \wedge t\ell_{ex}(B_p)_x$

7- $\ell_{ex}(1_p - A_p) = \ell_i(A_p) and\ \ell_i(1_p - A_p) = \ell_{ex}(A_p)$

Proof:

3- $\ell_e(A_p \wedge B_p)_x = \vee_x \left\{V_p \varepsilon \tau_p, f_{V_p}(x) \leq 1 - \min\left(g_{A_p^\sim}(x), h_{B_p^\sim}(x)\right)\right\} = \max(1 - f_{a_p^\sim}(x), 1 - g_{B_p^\sim}(x))$

$=(\vee_x | V_p^\sim \varepsilon \tau_p^\sim, f_{A_p^\sim}(x) \leq 1 - h_{B_p^\sim}(x)\} = \ell_e(A_p^\sim)_x \vee \ell_e(B_p^\sim)_x$

4- $\ell_{ex}(A_p^\sim \vee B_p^\sim)_x = \vee \left\{V_p^\sim \varepsilon \tau_p^\sim, f_{V_p^\sim}(x) \leq 1 - \max\left(g_{A_p^\sim}(x), h_{B_p^\sim}(x)\right)\right\} = \min(1 - g_{A^{\sim p}}(x), 1 - h_{B^{\sim p}}(x))$

$=\left[\vee \left\{V_p^\sim \varepsilon \tau_p^\sim, f_{V_p^\sim}(x) \leq 1 - g_{A_p^\sim}(x)\right\}\right] \wedge \left[\vee \left\{V_p^\sim \varepsilon \tau_p^\sim, f_{V_p^\sim}(x) \leq 1 - h_{B_p^\sim}(x)\right\}\right] = \ell_{ex}(B_p^\sim)_x \wedge \ell_{ex}(B_p^\sim)_x$

5- $A_p^\sim \leq A_p^\sim \vee B_p^\sim\ \ \&\ B_p^\sim \leq A_p^\sim \vee B_p^\sim$

$\Longrightarrow t\ell_e(A_p^\sim \vee B_p^\sim) \leq t\ell_e(A_p^\sim)\ \ \&\ t\ell_e(A_p^\sim \vee B_p^\sim) \leq t\ell_e(B_p^\sim)$

$t\ell_e(A_p^\sim \vee B_p^\sim)\ \leq t\ell_{ex}(A_p^\sim)\ \wedge t\ell_{ex}(B_p^\sim)$

6- $\ell_e(1_p^\sim - A_p^\sim)_x = \vee_x \left\{V_p^\sim \varepsilon \tau_p^\sim, f_{V_p^\sim}(x) \leq 1 - g_{1_p^\sim - A_p^\sim}(x) = g_{1_p^\sim - (1_p^\sim - A_p^\sim)}(x) = g_{A_p^\sim}(x)\right\} = \ell_i(A_p^\sim)$

The converse of part (6) is not true

Example 5-4: Let X={1,2,3,4}

$A^5 = \{(1, (0.2,0.3), (0.3), (0.23,0.27), (2, (0.05,0.1), (0.3), (0.2,0.2), (3, (0.06,0.07), (0.03), (0.27,0.13), (4, (0.05,0.025), (0.05), (0.8,0.12)\}$

$B^5 = \{(1,0.03,0.05), (0.06), (0.04,0.15), (2, (0.08,0.1), (0.13), (0.07,0.2), (3, (0.021,0.1,), (0.2), (0.2,0.1)$ (4,(0.0760.2),(0.2),(0.3,0.1)}

$M^5 = \{(1, (0.05,0.1), (0.2), (0.3,0.1), (2, (0.04,0.1), (0.1), (0.23,0.17), (3, (0.014,0.2), (0.1), (0.23,0.17)$ (4,(0.01,0.1),(0.08),(0.02,0.2)}, $N^5\{(1, (0.03,0.08), (0.02), (0.1,0.2), (2, (0.03,0.08), (0.1), (0.1,0.2)$ (3,(0.2,0.3),(0.2),(0.1,0.1),(4(0.06,0.3),(0.2),(0.4,0.1)}

$A_p^\sim = \{(1,0.65), (2,0.425), (3,0.56), (4,0.1625)\}, B_p^\sim = \{(1,0.34), (2,0.29), (3,0.31), (4,0.348)\}$

$A_p^\sim \vee B_p^\sim = \{(1,0.65), (2,0.425), (3,0.56), (4,0.348)\}, A_p^\sim \wedge B_p^\sim = \{(1,0.34), (2,0.29), (3,0.31), (4,0.1625)\}$

$\tau_p^\sim = \{0_p^\sim, 1_p^\sim, A_p^\sim, B_p^\sim, A_p^\sim \vee B_p^\sim, A_p^\sim \wedge B_p^\sim\}$

$, M_p^\sim = \{(1,0.325), (2,0.31), (3,0.257), (4,0.205)\}, N_p^\sim = \{(1,0.215), (2,0.35), (3,0.45), (0.53)\}$

$(M_p^\sim)^c = \{(1,0.675), (2,0.69), (3,0.743), (4,0.795)\}, (N_p^\sim)^c = \{(1,0.785), (2,0.65), (3,0.55), (4,0.46)\}$

$\ell_{ex}(M_p^\sim)_1 = 0.65,\ \ \ell_{ex}(M_p^\sim)_2 = 0.425,\ \ \ell_{ex}(M_p^\sim)_3 = 0.56,\ \ \ell_{ex}(M_p^\sim)_4 = 0.348$

$t\ell_{ex}(M_p^\sim)_x = \{(1,0.65), (2,0.425), (3,0.56), (4,0.348)\}, t\ell_{ex}(N_p^\sim)_x = \{(1,0.65), (2,0.425), (3,0.31), (4\}$

$t\ell_{ex}(M_p^\sim)_x \wedge\ t\ell_{ex}(N_p^\sim)_x = \{(1,0.65), (2,0.425), (3, (0.31)\}, M_p^\sim \vee N_p^\sim = \{(1,0.325), (2,0.35), (3,0.45)\}$

$M_p^\sim \vee N_p^\sim = \{(1,0.675), (2,0.65), (3,0.55)\}$

Theorem5-5: Each of the following properties are equivalent, for any PMS in PMTS $(1_p^\sim, \tau_p^\sim)$

a – $t\ell_{ex}(A_p^\sim)\ = 0_p^\sim$

b- the only PMOS set contain in $1_p^\sim - A_p^\sim$ is PM empty set

c- the only PMCS set contain in $A_p^\sim$ is the PM universal set

proof: $a \leftrightarrow b$

$let\ \forall x \in X, 0_p^\sim = \ell_e(A_p^\sim)_x = \vee\ \ell_e(A_p^\sim)_x$

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$iff\ \forall x \in X, 0_p^{\sim} = \ell_e(A_p^{\sim})_x = \wedge_x \{G_p^{\sim} \varepsilon \tau_p^{\sim} \ni f_{G_p^{\sim}}(x) = 1_p^{\sim} - g_{A_p^{\sim}}(x)\}$
$iff \forall x \in X, G_p^{\sim} \varepsilon \tau_p^{\sim} \ni f_{G_p^{\sim}}(x) = 0_p^{\sim}$
Iff the only PMOS contain in $1_p^{\sim} - A_p^{\sim}$ is PM empty set

Corollary5-6: If $t\ell_{ex}(A_p^{\sim}) = 0_p^{\sim}\ then\ \forall U_p^{\sim} \varepsilon \tau_p^{\sim} \exists x \in X\ sch\ that\ \min(f_{U_p^{\sim}}(x), g_{A_p^{\sim}}(x)) \neq 0_p^{\sim}$
Proof: if possible $\exists 0_p^{\sim} \neq U_p^{\sim} \varepsilon \tau_p^{\sim}\ and\ \forall y \in X \min(f_{u^{\sim p}}(y), g_{A^{\sim p}}(y)) = 0^{\sim}$
$U_p^{\sim} \wedge A_p^{\sim} = 0_p^{\sim}\ then\ U_p^{\sim} < 1_p^{\sim} - A_p^{\sim}$
Then the PMOS $U_p^{\sim}$ contain in`$1_p^{\sim} - A_p^{\sim}$ which contradiction by theorem 5-5
For any a PMTS $(1_p^{\sim}, \tau_p^{\sim})$ and PMS $M_p^{\sim}$, we get $t\ell_i(M_p^{\sim}) = t\ell_i\left(t\ell_i(M_p^{\sim})\right)$, from this characteristic we conclude that $t\ell_e(M_p^{\sim}) = t\ell_e\left(1_p^{\sim} - t\ell_e(M_p^{\sim})\right)$
Through definition 2-1 and 3-1, we conclude that is not necessarily $t\ell_i(M_p^{\sim}) \wedge t\ell_e(M_p^{\sim}) = M_p^{\sim}$, and as in example 2-1, where $t\ell_i(M_p^{\sim}) = \{(1,0), (2,0.55), (3,0.5)\} and\ t\ell_e(M_p^{\sim}) = \{(1,0.75), (2,0.46). (3,0.5)\}, so\ t\ell_i(M^{\sim p}) \wedge t\ell_e(M_p^{\sim}) = t\ell_i(M_p^{\sim}) \neq 0_p^{\sim}$
Definition 5-7: A PMS $M_p^{\sim}$ is detachable set if $t\ell_i(M_p^{\sim}) \wedge t\ell_e(M_p^{\sim}) = 0_p^{\sim}$
Example 5-8: Let X={1,2,3},
$A^6 = \{(0.3,0.2), (0.3), (0.1,0.08,0.02), (2, (0.1,0.06), (0.2), (0.3,0.1,0.1), (3, (0.05,0.1), (0.1), (0.1,0.02,0.42)\}$
$B^6 = \{(1, (0.1,0.1), (0.1,0.2), (0.1), (2,)0.09,0.18), (0.12,0.3), (0.3), (3, (0.05,0.33), (0.27,0.3), (0.2)\}$
$M^5 = \{(1, (0.2), (0.1,0.2), (0.3,0.1), (2, (0.1), (0.1,0.08), (0.1,0.2), (3(0.17), (0.2,0.2), (0.21,0.19)\}$
$A_p^{\sim} = \{(1,0.5), (2,0.43), (3,0.425)\}, B^{\sim p}\{(1,0.75), (2,0.495), (3,0.375)\},$
$A_p^{\sim} \wedge B_p^{\sim} = \{(1,0.5), (2,0.43), (3,0.375)\}, A_p^{\sim} \vee B_p^{\sim}\{(1,0.75), (2,0.495), (3,0.425)\},$

$\tau_p^{\sim} = \{0_p^{\sim}, 1_p^{\sim}, A_p^{\sim}, B_p^{\sim}, A_p^{\sim} \wedge B_p^{\sim}, A_p^{\sim} \vee B_p^{\sim}\}, , M_p^{\sim} = \{(1,0.45), (2,0.25), (3,0.17)\},$
$(M_p^{\sim})^c = \{(1,0.55), (2,0.75), (3,0.83)\}$
$\ell_i(M_P^{\sim})_1 = 0^{\sim p}, \ell_i(M_p^{\sim})_2 = 0_p^{\sim}, \ell_i(M_p^{\sim})_3 = 0_p^{\sim}, t\ell_i(M_p^{\sim}) = \{(1, 0_p^{\sim}), (2, 0_p^{\sim}), (3, 0_p^{\sim})\}$
$\ell_e(M_p^{\sim})_1 = 0.5, \ell_e(M_p^{\sim})_2 = 0.495, \ell_e(M_p^{\sim})_3 = 0.425\ , t\ell_e(M_p^{\sim}) = \{(1,0.5), (2,0.495), (3,0.425)\}$
$t\ell_i(M_p^{\sim}) \wedge t\ell_e(M_p^{\sim}) = 0_p^{\sim}$

Though above definition, we ,conclude that $\forall x \in X, \ell_i(M_p^{\sim})_x \wedge \ell_e(M_p^{\sim})_x = 0_p^{\sim}$
Definition 5-9: For any PMS $M_p^{\sim}$ in PMTS $(1_p^{\sim}, \tau_p^{\sim})$ put ther partner multineutrosophic conduced PMCOS of is defined by form$\gamma_p(M_p)_x = \ell_i(M_p)_x \vee \ell_e(M_p)_x$ and the totally partner multineutrosophic conduced YPMCOS of is defined by $\gamma^{\sim p} = \cup_{x \in X} \gamma_x^{\sim p}$
Example 5-10: Let X={1,2,3}
$A^6 = \{(1, (0.03,0.3), (0.17,0.13), (0.2,0.2), (2, (0.11,0.09), (0.23,0.07), (0.22,0.08),$
$(3, (0.01,0.1), (0.1,0.1), (0.1,0.1)\}, B^6 = \{(1, (0.01,0.09), (0.03,0.17), (0.1,0.2), (2, (0.05,0.04), (0.06,0.1),$
(0.2,0.2),(3,(0.06,0.3),(0.3,0.2),(0.1,0.1)},
$M^5\{(1, (0.05,0.05), (0.06,0.04), (0.05), (2, (0.07,0.08), (0.2,0.2), (0.2),$
(3,(0.034,0.2),(0.3,0.1),(0.2)}
$A_p^{\sim} = \{(1,0.65), (2,0.4), (3,0.255)\}, B_p^{\sim} = \{(1,0.3), (2,0.325), (3,0.53)\}, A_p^{\sim} \vee B_p^{\sim}$
$= \{(1,0.65), (2,0.325), (3,0.255)\}$
$A_p^{\sim} \wedge B_p^{\sim} = \{(1,0.3), (2,0.325), (3,0.255)\}, \tau_p^{\sim} = \{1_p^{\sim}, 0^{\sim}{}_p, A_p^{\sim}, B_p^{\sim}, A_p^{\sim} \wedge B_p^{\sim}, A_p^{\sim} \vee B_p^{\sim}\}$
Let M={1,2,3},$M_2^{\sim} = \{(1,0.125), (2,0.36), (3,0.437)\}, \ell_i(M_2^{\sim})_1 = 0_n^{\sim}, \ell_i(M_2^{\sim})_2 = 0.325, \ell_i(M_2^{\sim})_3 = 0.255$
$M_p^{\sim c} = \{(1,0.875), (2,0.64), (3,0.563)\}, \ell_i(M_p^{\sim})_1 = 0.65, \ell_i(M_p^{\sim})_2 = 0.4, \ell_i(M_p^{\sim})_3 = 0.53$
$\gamma_p(M_p)_x = \ell_e(M_p^{\sim})_x \vee \ell_i(M_p^{\sim})_x, \gamma_p^1 = 0.65, \gamma_p^2 = 0.4, \gamma_p^3 = 0.53$

5. Conclusion

Throughout definition of LPMIS and LPMES, we have made clear that they are not necessarily TLPMISs, and in the same context we have also shown that TLPMIS and TLPMES are not necessarily PMOSs.
It is possiple to generalize the concepts presented by both Abdulsada [10] and Al-Swidi[11] and study them on the PMSspresented.

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