The Mathematical Formulas for Inverting Plithogenic Matrices of Special Orders Between 20 and 24

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Abstract

This paper is dedicated to study the Invertibility properties of all plithogenic square matrices of high orders between 20 and 24, where we present the mathematical conditions and formulas for computing the inverses of all plithogenic square matrices with real plithogenic entries. This goal will be completed by proving many theorems that describe the Invertibility of all classic matrix parts of the corresponding symbolic m-plithogenic matrix.

Keywords: 20-plithogenic matrix; invertible plithogenic matrix; plithogenic determinant.

1. Introduction

Plithogenic sets are considered as a new generalization of fuzzy sets presented by Smarandache in [2], and similar to the neutrosophic sets used in the study of commutative and non-commutative algebraic structures [12-16, 20-22], these sets have been used in linear algebra and the theory of spaces [3-4, 23-25], and even in number theory [17-18] and algebraic equations [5-6, 9].

The matrices of order 2 in [7], and of order 3 and order 4 in [8] were studied, where the basic theorems describing the values and special vectors, as well as the determinants associated with these matrices, as well as the necessary and sufficient orthogonality conditions for these matrices, were formulated, through the properties of the partial matrices from which these matrices are formed.

Recently, the studies have been expanded and their results have been generalized by many researchers to include matrices of this type of higher orders starting from order 5 and ending with 19 [19, 26-27].
These generalized studies have motivated us to discuss one of the most important issues related to these matrices, which is the issue of finding algebraic methods, where we find and prove mathematical formulas that express the method of calculating the inverse for the intended high orders.

2. Main Results

Theorem

Let $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$ be a 20-plithogenic matrix of size $n \times n$, hence:

1. $\aleph$ is invertible if and only if $\det \aleph$ is an invertible 20-plithogenic real number.
2. $\aleph^{-1} = {\aleph_0}^{-1} + \left[(\sum_{i=0}^{1} \aleph_i)^{-1} - {\aleph_0}^{-1}\right]P_1 + [(\sum_{i=0}^{2} \aleph_i)^{-1} - (\sum_{i=0}^{1} \aleph_i)^{-1}]P_2 + [(\sum_{i=1}^{3} \aleph_i)^{-1} - (\sum_{i=0}^{2} \aleph_i)^{-1}]P_3 + [(\sum_{i=1}^{4} \aleph_i)^{-1} - (\sum_{i=0}^{3} \aleph_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5} \aleph_i\right)^{-1} - (\sum_{i=0}^{4} \aleph_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} \aleph_i)^{-1} -\right.$

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$\left(\sum_{i=0}^{5} \aleph_i\right)^{-1}\Big] P_6 + [(\sum_{i=1}^{7} \aleph_i)^{-1} - (\sum_{i=0}^{6} \aleph_i)^{-1}]P_7 + [(\sum_{i=1}^{8} \aleph_i)^{-1} - (\sum_{i=0}^{7} \aleph_i)^{-1}]P_8 + [(\sum_{i=1}^{9} \aleph_i)^{-1} - (\sum_{i=0}^{8} \aleph_i)^{-1}]P_9 + [(\sum_{i=1}^{10} \aleph_i)^{-1} - (\sum_{i=0}^{9} \aleph_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} \aleph_i)^{-1} - (\sum_{i=0}^{10} \aleph_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} \aleph_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} \aleph_i)^{-1} - (\sum_{i=0}^{12} \aleph_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} \aleph_i)^{-1} - (\sum_{i=0}^{13} \aleph_i)^{-1}]P_{14} + \Big[\left(\sum_{i=1}^{15} \aleph_i\right)^{-1} - (\sum_{i=0}^{14} \aleph_i)^{-1}\Big] P_{15} + \Big[(\sum_{i=1}^{16} \aleph_i)^{-1} - \left(\sum_{i=0}^{15} \aleph_i\right)^{-1}\Big] P_{16} + [(\sum_{i=1}^{17} \aleph_i)^{-1} - (\sum_{i=0}^{16} \aleph_i)^{-1}]P_{17} + [(\sum_{i=1}^{18} \aleph_i)^{-1} - (\sum_{i=0}^{17} \aleph_i)^{-1}]P_{18} + [(\sum_{i=1}^{19} \aleph_i)^{-1} - (\sum_{i=0}^{18} \aleph_i)^{-1}]P_{19} + [(\sum_{i=1}^{20} \aleph_i)^{-1} - (\sum_{i=0}^{19} \aleph_i)^{-1}]P_{20}$.

Proof

1). Let $\aleph = \aleph_0 + \sum_{i=1}^{20} \aleph_i P_i$, then $\aleph$ is invertible if and only if there exists $G = G_0 + \sum_{i=1}^{20} G_i P_i$ such that: $\aleph \times G = U_{n\times n}$, hence:

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$$
\begin{cases}
\aleph_0 G_0 = U_{n\times n} \\
\sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} I_i - \aleph_0 G_0 = O_{n\times n} \\
\sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i - \sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} G_i = O_{n\times n} \\
\sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} G_i - \sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i = O_{n\times n} \\
\sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} G_i - \sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} I_i = O_{n\times n} \\
\sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i - \sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} I_i = O_{n\times n} \\
\sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i - \sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i = O_{n\times n} \\
\sum_{i=0}^{7} \tau_i \sum_{i=0}^{7} G_i - \sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i = O_{n\times n} \\
\sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i - \sum_{i=0}^{7} \aleph_i \sum_{i=0}^{7} G_i = O_{n\times n} \\
\sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i - \sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i = O_{n\times n} \\
\sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i - \sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i = O_{n\times n} \\
\sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i - \sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i = O_{n\times n} \\
\sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i - \sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i = O_{n\times n} \\
\sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i - \sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i = O_{n\times n} \\
\sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i - \sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i = O_{n\times n} \\
\sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i - \sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i = O_{n\times n} \\
\sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i - \sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i = O_{n\times n} \\
\sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i - \sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i = O_{n\times n} \\
\sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i - \sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i = O_{n\times n} \\
\sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i - \sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i = O_{n\times n} \\
\sum_{i=0}^{20} \aleph_i \sum_{i=0}^{20} G_i - \sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i = O_{n\times n}
\end{cases}
$$

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This implies that:

$$\begin{cases} \aleph_0 G_0 = U_{n\times n} \\ \sum_{i=0}^{q} \aleph_i \sum_{i=0}^{q} G_i = U_{n\times n} \end{cases} ;\ 1 \le q \le 20$$

Hence $\det\left(\sum_{i=0}^{q} \aleph_i\right) \neq 0$ for all $1 \le q \le 20$, so that $\det(\tau)$ is invertible in $20 - SP_R$.

2). $\sum_{i=0}^{q} G_i = \left(\sum_{i=0}^{q} \aleph_i\right)^{-1}$for $1 \le q \le 20$, therefor

$\aleph^{-1} = {\aleph_0}^{-1} + \left[(\sum_{i=0}^{1} \aleph_i)^{-1} - {\aleph_0}^{-1}\right]P_1 + [(\sum_{i=0}^{2} \aleph_i)^{-1} - (\sum_{i=0}^{1} \aleph_i)^{-1}]P_2 + [(\sum_{i=0}^{3} \aleph_i)^{-1} - (\sum_{i=0}^{2} \aleph_i)^{-1}]P_3 + [(\sum_{i=0}^{4} \aleph_i)^{-1} - (\sum_{i=0}^{3} \aleph_i)^{-1}]P_4 + \left[\left(\sum_{i=0}^{5} \aleph_i\right)^{-1} - (\sum_{i=0}^{4} \aleph_i)^{-1}\right] P_5 + \left[(\sum_{i=0}^{6} \aleph_i)^{-1} - \left(\sum_{i=0}^{5} \aleph_i\right)^{-1}\right] P_6 + [(\sum_{i=0}^{7} \aleph_i)^{-1} - (\sum_{i=0}^{6} \aleph_i)^{-1}]P_7 + [(\sum_{i=0}^{8} \aleph_i)^{-1} - (\sum_{i=0}^{7} \aleph_i)^{-1}]P_8 + [(\sum_{i=0}^{9} \aleph_i)^{-1} - (\sum_{i=0}^{8} \aleph_i)^{-1}]P_9 + [(\sum_{i=0}^{10} \aleph_i)^{-1} - (\sum_{i=0}^{9} \aleph_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \aleph_i)^{-1} - (\sum_{i=0}^{10} \aleph_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \aleph_i)^{-1} - (\sum_{i=0}^{11} \aleph_i)^{-1}]P_{12} + [(\sum_{i=0}^{13} \aleph_i)^{-1} - (\sum_{i=0}^{12} \aleph_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \aleph_i)^{-1} - (\sum_{i=0}^{13} \aleph_i)^{-1}]P_{14} + \left[\left(\sum_{i=0}^{15} \aleph_i\right)^{-1} - (\sum_{i=0}^{14} \aleph_i)^{-1}\right] P_{15} + \left[(\sum_{i=0}^{16} \aleph_i)^{-1} - \left(\sum_{i=0}^{15} \aleph_i\right)^{-1}\right] P_{16} + [(\sum_{i=0}^{17} \aleph_i)^{-1} - (\sum_{i=0}^{16} \aleph_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \aleph_i)^{-1} - (\sum_{i=0}^{17} \aleph_i)^{-1}]P_{18} + [(\sum_{i=0}^{19} \aleph_i)^{-1} - (\sum_{i=0}^{18} \aleph_i)^{-1}]P_{19} + [(\sum_{i=0}^{20} \aleph_i)^{-1} - (\sum_{i=0}^{19} \aleph_i)^{-1}]P_{20}.$

Theorem

Let $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$ be a 21-plithogenic matrix of size $n \times n$, hence:

1. $\aleph$ is invertible if and only if $\det \aleph$ is an invertible 21-plithogenic real number.
2. $\aleph^{-1} = {\aleph_0}^{-1} + \left[(\sum_{i=0}^{1} \aleph_i)^{-1} - {\aleph_0}^{-1}\right]P_1 + [(\sum_{i=0}^{2} \aleph_i)^{-1} - (\sum_{i=0}^{1} \aleph_i)^{-1}]P_2 + [(\sum_{i=1}^{3} \aleph_i)^{-1} - (\sum_{i=0}^{2} \aleph_i)^{-1}]P_3 + [(\sum_{i=1}^{4} \aleph_i)^{-1} - (\sum_{i=0}^{3} \aleph_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5} \aleph_i\right)^{-1} - (\sum_{i=0}^{4} \aleph_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} \aleph_i)^{-1} - \left(\sum_{i=0}^{5} \aleph_i\right)^{-1}\right] P_6 + [(\sum_{i=1}^{7} \aleph_i)^{-1} - (\sum_{i=0}^{6} \aleph_i)^{-1}]P_7 + [(\sum_{i=1}^{8} \aleph_i)^{-1} - (\sum_{i=0}^{7} \aleph_i)^{-1}]P_8 + [(\sum_{i=1}^{9} \aleph_i)^{-1} - (\sum_{i=0}^{8} \aleph_i)^{-1}]P_9 + [(\sum_{i=1}^{10} \aleph_i)^{-1} - (\sum_{i=0}^{9} \aleph_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} \aleph_i)^{-1} - (\sum_{i=0}^{10} \aleph_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} \aleph_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} \aleph_i)^{-1} - (\sum_{i=0}^{12} \aleph_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} \aleph_i)^{-1} - (\sum_{i=0}^{13} \aleph_i)^{-1}]P_{14} + \left[\left(\sum_{i=1}^{15} \aleph_i\right)^{-1} - (\sum_{i=0}^{14} \aleph_i)^{-1}\right] P_{15} + \left[(\sum_{i=1}^{16} \aleph_i)^{-1} - \left(\sum_{i=0}^{15} \aleph_i\right)^{-1}\right] P_{16} + [(\sum_{i=1}^{17} \aleph_i)^{-1} - (\sum_{i=0}^{16} \aleph_i)^{-1}]P_{17} + [(\sum_{i=1}^{18} \aleph_i)^{-1} - (\sum_{i=0}^{17} \aleph_i)^{-1}]P_{18} + [(\sum_{i=1}^{19} \aleph_i)^{-1} - (\sum_{i=0}^{18} \aleph_i)^{-1}]P_{19} + [(\sum_{i=1}^{20} \aleph_i)^{-1} - (\sum_{i=0}^{19} \aleph_i)^{-1}]P_{20} + [(\sum_{i=1}^{21} \aleph_i)^{-1} - (\sum_{i=0}^{20} \aleph_i)^{-1}]P_{21}.$

Proof

1). Let $\aleph = \aleph_0 + \sum_{i=1}^{21} \aleph_i P_i$, then $\aleph$ is invertible if and only if there exists $G = G_0 + \sum_{i=1}^{21} G_i P_i$ such that: $\aleph \times G = U_{n\times n}$, hence:

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$$\begin{cases} \aleph_0 G_0 = U_{n\times n} \\ \sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} I_i - \aleph_0 G_0 = O_{n\times n} \\ \sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i - \sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} G_i = O_{n\times n} \\ \sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} G_i - \sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i = O_{n\times n} \\ \sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} G_i - \sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} I_i = O_{n\times n} \\ \sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i - \sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} I_i = O_{n\times n} \\ \sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i - \sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i = O_{n\times n} \\ \sum_{i=0}^{7} \tau_i \sum_{i=0}^{7} G_i - \sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i = O_{n\times n} \\ \sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i - \sum_{i=0}^{7} \aleph_i \sum_{i=0}^{7} G_i = O_{n\times n} \\ \sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i - \sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i = O_{n\times n} \\ \sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i - \sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i = O_{n\times n} \\ \sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i - \sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i = O_{n\times n} \\ \sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i - \sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i = O_{n\times n} \\ \sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i - \sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i = O_{n\times n} \\ \sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i - \sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i = O_{n\times n} \\ \sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i - \sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i = O_{n\times n} \\ \sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i - \sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i = O_{n\times n} \\ \sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i - \sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i = O_{n\times n} \\ \sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i - \sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i = O_{n\times n} \\ \sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i - \sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i = O_{n\times n} \\ \sum_{i=0}^{20} \aleph_i \sum_{i=0}^{20} G_i - \sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i = O_{n\times n} \end{cases}$$

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And $\sum_{i=0}^{21} \aleph_i \sum_{i=0}^{21} G_i - \sum_{i=0}^{20} \aleph_i \sum_{i=0}^{20} G_i = O_{n\times n}$

$$\begin{cases} \aleph_0 G_0 = U_{n\times n} \\ \sum_{i=0}^{q} \aleph_i \sum_{i=0}^{q} G_i = U_{n\times n} \end{cases} ;\ 1 \le q \le 21$$

Hence $\det\left(\sum_{i=0}^{q} \aleph_i\right) \neq 0$ for all $1 \le q \le 21$, so that $\det(\aleph)$ is invertible in $21 - SP_R$.

2). $\sum_{i=0}^{q} G_i = \left(\sum_{i=0}^{q} \aleph_i\right)^{-1}$for $1 \le q \le 21$, therefor

$\aleph^{-1} = {\aleph_0}^{-1} + \left[(\sum_{i=0}^{1} \aleph_i)^{-1} - {\aleph_0}^{-1}\right]P_1 + [(\sum_{i=0}^{2} \aleph_i)^{-1} - (\sum_{i=0}^{1} \aleph_i)^{-1}]P_2 + [(\sum_{i=0}^{3} \aleph_i)^{-1} - (\sum_{i=0}^{2} \aleph_i)^{-1}]P_3 + [(\sum_{i=0}^{4} \aleph_i)^{-1} - (\sum_{i=0}^{3} \aleph_i)^{-1}]P_4 + \left[\left(\sum_{i=0}^{5} \aleph_i\right)^{-1} - (\sum_{i=0}^{4} \aleph_i)^{-1}\right]P_5 + \left[(\sum_{i=0}^{6} \aleph_i)^{-1} - \left(\sum_{i=0}^{5} \aleph_i\right)^{-1}\right]P_6 + [(\sum_{i=0}^{7} \aleph_i)^{-1} - (\sum_{i=0}^{6} \aleph_i)^{-1}]P_7 + [(\sum_{i=0}^{8} \aleph_i)^{-1} - (\sum_{i=0}^{7} \aleph_i)^{-1}]P_8 + [(\sum_{i=0}^{9} \aleph_i)^{-1} - (\sum_{i=0}^{8} \aleph_i)^{-1}]P_9 + [(\sum_{i=0}^{10} \aleph_i)^{-1} - (\sum_{i=0}^{9} \aleph_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \aleph_i)^{-1} - (\sum_{i=0}^{10} \aleph_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \aleph_i)^{-1} - (\sum_{i=0}^{11} \aleph_i)^{-1}]P_{12} + [(\sum_{i=0}^{13} \aleph_i)^{-1} - (\sum_{i=0}^{12} \aleph_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \aleph_i)^{-1} - (\sum_{i=0}^{13} \aleph_i)^{-1}]P_{14} + \left[\left(\sum_{i=0}^{15} \aleph_i\right)^{-1} - (\sum_{i=0}^{14} \aleph_i)^{-1}\right]P_{15} + \left[(\sum_{i=0}^{16} \aleph_i)^{-1} - \left(\sum_{i=0}^{15} \aleph_i\right)^{-1}\right]P_{16} + [(\sum_{i=0}^{17} \aleph_i)^{-1} - (\sum_{i=0}^{16} \aleph_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \aleph_i)^{-1} - (\sum_{i=0}^{17} \aleph_i)^{-1}]P_{18} + [(\sum_{i=0}^{19} \aleph_i)^{-1} - (\sum_{i=0}^{18} \aleph_i)^{-1}]P_{19} + [(\sum_{i=0}^{20} \aleph_i)^{-1} - (\sum_{i=0}^{19} \aleph_i)^{-1}]P_{20} + [(\sum_{i=0}^{21} \aleph_i)^{-1} - (\sum_{i=0}^{20} \aleph_i)^{-1}]P_{21}$.

Theorem

Let $\aleph = \aleph_0 + \sum_{i=1}^{22} \aleph_i P_i$ be a 22-plithogenic matrix of size $n \times n$, hence:

1. $\aleph$ is invertible if and only if $\det \aleph$ is an invertible 22-plithogenic real number.
2. $\aleph^{-1} = {\aleph_0}^{-1} + \left[(\sum_{i=0}^{1} \aleph_i)^{-1} - {\aleph_0}^{-1}\right]P_1 + [(\sum_{i=0}^{2} \aleph_i)^{-1} - (\sum_{i=0}^{1} \aleph_i)^{-1}]P_2 + [(\sum_{i=1}^{3} \aleph_i)^{-1} - (\sum_{i=0}^{2} \aleph_i)^{-1}]P_3 + [(\sum_{i=1}^{4} \aleph_i)^{-1} - (\sum_{i=0}^{3} \aleph_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5} \aleph_i\right)^{-1} - (\sum_{i=0}^{4} \aleph_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6} \aleph_i)^{-1} - \left(\sum_{i=0}^{5} \aleph_i\right)^{-1}\right]P_6 + [(\sum_{i=1}^{7} \aleph_i)^{-1} - (\sum_{i=0}^{6} \aleph_i)^{-1}]P_7 + [(\sum_{i=1}^{8} \aleph_i)^{-1} - (\sum_{i=0}^{7} \aleph_i)^{-1}]P_8 + [(\sum_{i=1}^{9} \aleph_i)^{-1} - (\sum_{i=0}^{8} \aleph_i)^{-1}]P_9 + [(\sum_{i=1}^{10} \aleph_i)^{-1} - (\sum_{i=0}^{9} \aleph_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} \aleph_i)^{-1} - (\sum_{i=0}^{10} \aleph_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} \aleph_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} \aleph_i)^{-1} - (\sum_{i=0}^{12} \aleph_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} \aleph_i)^{-1} - (\sum_{i=0}^{13} \aleph_i)^{-1}]P_{14} + \left[\left(\sum_{i=1}^{15} \aleph_i\right)^{-1} - (\sum_{i=0}^{14} \aleph_i)^{-1}\right]P_{15} + \left[(\sum_{i=1}^{16} \aleph_i)^{-1} - \left(\sum_{i=0}^{15} \aleph_i\right)^{-1}\right]P_{16} + [(\sum_{i=1}^{17} \aleph_i)^{-1} - (\sum_{i=0}^{16} \aleph_i)^{-1}]P_{17} + [(\sum_{i=1}^{18} \aleph_i)^{-1} - (\sum_{i=0}^{17} \aleph_i)^{-1}]P_{18} + [(\sum_{i=1}^{19} \aleph_i)^{-1} - (\sum_{i=0}^{18} \aleph_i)^{-1}]P_{19} + [(\sum_{i=1}^{20} \aleph_i)^{-1} - (\sum_{i=0}^{19} \aleph_i)^{-1}]P_{20} + [(\sum_{i=1}^{21} \aleph_i)^{-1} - (\sum_{i=0}^{20} \aleph_i)^{-1}]P_{21} + [(\sum_{i=1}^{22} \aleph_i)^{-1} - (\sum_{i=0}^{21} \aleph_i)^{-1}]P_{22}$.

Proof

1). Let $\aleph = \aleph_0 + \sum_{i=1}^{22} \aleph_i P_i$, then $\aleph$ is invertible if and only if there exists $G = G_0 + \sum_{i=1}^{22} G_i P_i$ such that: $\aleph \times G = U_{n\times n}$, hence:

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$$\begin{cases} \aleph_0 G_0 = U_{n\times n} \\ \sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} I_i - \aleph_0 G_0 = O_{n\times n} \\ \sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i - \sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} G_i = O_{n\times n} \\ \sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} G_i - \sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i = O_{n\times n} \\ \sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} G_i - \sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} I_i = O_{n\times n} \\ \sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i - \sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} I_i = O_{n\times n} \\ \sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i - \sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i = O_{n\times n} \\ \sum_{i=0}^{7} \tau_i \sum_{i=0}^{7} G_i - \sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i = O_{n\times n} \\ \sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i - \sum_{i=0}^{7} \aleph_i \sum_{i=0}^{7} G_i = O_{n\times n} \\ \sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i - \sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i = O_{n\times n} \\ \sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i - \sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i = O_{n\times n} \\ \sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i - \sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i = O_{n\times n} \\ \sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i - \sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i = O_{n\times n} \\ \sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i - \sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i = O_{n\times n} \\ \sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i - \sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i = O_{n\times n} \\ \sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i - \sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i = O_{n\times n} \\ \sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i - \sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i = O_{n\times n} \\ \sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i - \sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i = O_{n\times n} \\ \sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i - \sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i = O_{n\times n} \\ \sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i - \sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i = O_{n\times n} \\ \sum_{i=0}^{20} \aleph_i \sum_{i=0}^{20} G_i - \sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i = O_{n\times n} \end{cases}$$

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$And\ \sum_{i=0}^{21}\aleph_i\sum_{i=0}^{21}G_i-\sum_{i=0}^{20}\aleph_i\sum_{i=0}^{20}G_i=O_{n\times n},\sum_{i=0}^{22}\aleph_i\sum_{i=0}^{22}G_i-\sum_{i=0}^{21}\aleph_i\sum_{i=0}^{21}G_i=O_{n\times n}$

$$\begin{cases}\aleph_0G_0=U_{n\times n}\\ \sum_{i=0}^{q}\aleph_i\sum_{i=0}^{q}G_i=U_{n\times n}\end{cases};\ 1\le q\le 22$$

Hence $\det\left(\sum_{i=0}^{q}\aleph_i\right)\neq 0$ for all $1\le q\le 22$, so that $\det(\aleph)$ is invertible in $22-SP_R$.

2). $\sum_{i=0}^{q}G_i=\left(\sum_{i=0}^{q}\aleph_i\right)^{-1}$for $1\le q\le 22$, therefor

$\aleph^{-1}={\aleph_0}^{-1}+\left[(\sum_{i=0}^{1}\aleph_i)^{-1}-{\aleph_0}^{-1}\right]P_1+[(\sum_{i=0}^{2}\aleph_i)^{-1}-(\sum_{i=0}^{1}\aleph_i)^{-1}]P_2+[(\sum_{i=0}^{3}\aleph_i)^{-1}-(\sum_{i=0}^{2}\aleph_i)^{-1}]P_3+[(\sum_{i=0}^{4}\aleph_i)^{-1}-(\sum_{i=0}^{3}\aleph_i)^{-1}]P_4+\left[\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}-(\sum_{i=0}^{4}\aleph_i)^{-1}\right]P_5+\left[(\sum_{i=0}^{6}\aleph_i)^{-1}-\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}\right]P_6+[(\sum_{i=0}^{7}\aleph_i)^{-1}-(\sum_{i=0}^{6}\aleph_i)^{-1}]P_7+[(\sum_{i=0}^{8}\aleph_i)^{-1}-(\sum_{i=0}^{7}\aleph_i)^{-1}]P_8+[(\sum_{i=0}^{9}\aleph_i)^{-1}-(\sum_{i=0}^{8}\aleph_i)^{-1}]P_9+[(\sum_{i=0}^{10}\aleph_i)^{-1}-(\sum_{i=0}^{9}\aleph_i)^{-1}]P_{10}+[(\sum_{i=0}^{11}\aleph_i)^{-1}-(\sum_{i=0}^{10}\aleph_i)^{-1}]P_{11}+[(\sum_{i=0}^{12}\aleph_i)^{-1}-(\sum_{i=0}^{11}\aleph_i)^{-1}]P_{12}+[(\sum_{i=0}^{13}\aleph_i)^{-1}-(\sum_{i=0}^{12}\aleph_i)^{-1}]P_{13}+[(\sum_{i=0}^{14}\aleph_i)^{-1}-(\sum_{i=0}^{13}\aleph_i)^{-1}]P_{14}+\left[\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}-(\sum_{i=0}^{14}\aleph_i)^{-1}\right]P_{15}+\left[(\sum_{i=0}^{16}\aleph_i)^{-1}-\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}\right]P_{16}+[(\sum_{i=0}^{17}\aleph_i)^{-1}-(\sum_{i=0}^{16}\aleph_i)^{-1}]P_{17}+[(\sum_{i=0}^{18}\aleph_i)^{-1}-(\sum_{i=0}^{17}\aleph_i)^{-1}]P_{18}+[(\sum_{i=0}^{19}\aleph_i)^{-1}-(\sum_{i=0}^{18}\aleph_i)^{-1}]P_{19}+[(\sum_{i=0}^{20}\aleph_i)^{-1}-(\sum_{i=0}^{19}\aleph_i)^{-1}]P_{20}+[(\sum_{i=0}^{21}\aleph_i)^{-1}-(\sum_{i=0}^{20}\aleph_i)^{-1}]P_{21}+[(\sum_{i=0}^{22}\aleph_i)^{-1}-(\sum_{i=0}^{21}\aleph_i)^{-1}]P_{22}.$

Theorem

Let $\aleph=\aleph_0+\sum_{i=1}^{23}\aleph_iP_i$ be a 23-plithogenic matrix of size $n\times n$, hence:

1. $\aleph$ is invertible if and only if det $\aleph$ is an invertible 23-plithogenic real number.
2. $\aleph^{-1}={\aleph_0}^{-1}+\left[(\sum_{i=0}^{1}\aleph_i)^{-1}-{\aleph_0}^{-1}\right]P_1+[(\sum_{i=0}^{2}\aleph_i)^{-1}-(\sum_{i=0}^{1}\aleph_i)^{-1}]P_2+[(\sum_{i=1}^{3}\aleph_i)^{-1}-(\sum_{i=0}^{2}\aleph_i)^{-1}]P_3+[(\sum_{i=1}^{4}\aleph_i)^{-1}-(\sum_{i=0}^{3}\aleph_i)^{-1}]P_4+\left[\left(\sum_{i=1}^{5}\aleph_i\right)^{-1}-(\sum_{i=0}^{4}\aleph_i)^{-1}\right]P_5+\left[(\sum_{i=1}^{6}\aleph_i)^{-1}-\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}\right]P_6+[(\sum_{i=1}^{7}\aleph_i)^{-1}-(\sum_{i=0}^{6}\aleph_i)^{-1}]P_7+[(\sum_{i=1}^{8}\aleph_i)^{-1}-(\sum_{i=0}^{7}\aleph_i)^{-1}]P_8+[(\sum_{i=1}^{9}\aleph_i)^{-1}-(\sum_{i=0}^{8}\aleph_i)^{-1}]P_9+[(\sum_{i=1}^{10}\aleph_i)^{-1}-(\sum_{i=0}^{9}\aleph_i)^{-1}]P_{10}+[(\sum_{i=1}^{11}\aleph_i)^{-1}-(\sum_{i=0}^{10}\aleph_i)^{-1}]P_{11}+[(\sum_{i=1}^{12}\aleph_i)^{-1}-(\sum_{i=0}^{11}\tau_i)^{-1}]P_{12}+[(\sum_{i=1}^{13}\aleph_i)^{-1}-(\sum_{i=0}^{12}\aleph_i)^{-1}]P_{13}+[(\sum_{i=1}^{14}\aleph_i)^{-1}-(\sum_{i=0}^{13}\aleph_i)^{-1}]P_{14}+\left[\left(\sum_{i=1}^{15}\aleph_i\right)^{-1}-(\sum_{i=0}^{14}\aleph_i)^{-1}\right]P_{15}+\left[(\sum_{i=1}^{16}\aleph_i)^{-1}-\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}\right]P_{16}+[(\sum_{i=1}^{17}\aleph_i)^{-1}-(\sum_{i=0}^{16}\aleph_i)^{-1}]P_{17}+[(\sum_{i=1}^{18}\aleph_i)^{-1}-(\sum_{i=0}^{17}\aleph_i)^{-1}]P_{18}+[(\sum_{i=1}^{19}\aleph_i)^{-1}-(\sum_{i=0}^{18}\aleph_i)^{-1}]P_{19}+[(\sum_{i=1}^{20}\aleph_i)^{-1}-(\sum_{i=0}^{19}\aleph_i)^{-1}]P_{20}+[(\sum_{i=1}^{21}\aleph_i)^{-1}-(\sum_{i=0}^{20}\aleph_i)^{-1}]P_{21}+[(\sum_{i=1}^{22}\aleph_i)^{-1}-(\sum_{i=0}^{21}\aleph_i)^{-1}]P_{22}+[(\sum_{i=1}^{23}\aleph_i)^{-1}-(\sum_{i=0}^{22}\aleph_i)^{-1}]P_{23}.$

Proof

1). Let $\aleph=\aleph_0+\sum_{i=1}^{23}\aleph_iP_i$, then $\aleph$ is invertible if and only if there exists $G=G_0+\sum_{i=1}^{23}G_iP_i$ such that: $\aleph\times G=U_{n\times n}$, hence:

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$$\begin{cases} \aleph_0 G_0 = U_{n\times n} \\ \sum_{i=0}^{1}\aleph_i \sum_{i=0}^{1} I_i - \aleph_0 G_0 = O_{n\times n} \\ \sum_{i=0}^{2}\aleph_i \sum_{i=0}^{2} G_i - \sum_{i=0}^{1}\aleph_i \sum_{i=0}^{1} G_i = O_{n\times n} \\ \sum_{i=0}^{3}\aleph_i \sum_{i=0}^{3} G_i - \sum_{i=0}^{2}\aleph_i \sum_{i=0}^{2} G_i = O_{n\times n} \\ \sum_{i=0}^{4}\aleph_i \sum_{i=0}^{4} G_i - \sum_{i=0}^{3}\aleph_i \sum_{i=0}^{3} I_i = O_{n\times n} \\ \sum_{i=0}^{5}\aleph_i \sum_{i=0}^{5} G_i - \sum_{i=0}^{4}\aleph_i \sum_{i=0}^{4} I_i = O_{n\times n} \\ \sum_{i=0}^{6}\aleph_i \sum_{i=0}^{6} G_i - \sum_{i=0}^{5}\aleph_i \sum_{i=0}^{5} G_i = O_{n\times n} \\ \sum_{i=0}^{7}\tau_i \sum_{i=0}^{7} G_i - \sum_{i=0}^{6}\aleph_i \sum_{i=0}^{6} G_i = O_{n\times n} \\ \sum_{i=0}^{8}\aleph_i \sum_{i=0}^{8} G_i - \sum_{i=0}^{7}\aleph_i \sum_{i=0}^{7} G_i = O_{n\times n} \\ \sum_{i=0}^{9}\aleph_i \sum_{i=0}^{9} G_i - \sum_{i=0}^{8}\aleph_i \sum_{i=0}^{8} G_i = O_{n\times n} \\ \sum_{i=0}^{10}\aleph_i \sum_{i=0}^{10} G_i - \sum_{i=0}^{9}\aleph_i \sum_{i=0}^{9} G_i = O_{n\times n} \\ \sum_{i=0}^{11}\aleph_i \sum_{i=0}^{11} G_i - \sum_{i=0}^{10}\aleph_i \sum_{i=0}^{10} G_i = O_{n\times n} \\ \sum_{i=0}^{12}\aleph_i \sum_{i=0}^{12} G_i - \sum_{i=0}^{11}\aleph_i \sum_{i=0}^{11} G_i = O_{n\times n} \\ \sum_{i=0}^{13}\aleph_i \sum_{i=0}^{13} G_i - \sum_{i=0}^{12}\aleph_i \sum_{i=0}^{12} G_i = O_{n\times n} \\ \sum_{i=0}^{14}\aleph_i \sum_{i=0}^{14} G_i - \sum_{i=0}^{13}\aleph_i \sum_{i=0}^{13} G_i = O_{n\times n} \\ \sum_{i=0}^{15}\aleph_i \sum_{i=0}^{15} G_i - \sum_{i=0}^{14}\aleph_i \sum_{i=0}^{14} G_i = O_{n\times n} \\ \sum_{i=0}^{16}\aleph_i \sum_{i=0}^{16} G_i - \sum_{i=0}^{15}\aleph_i \sum_{i=0}^{15} G_i = O_{n\times n} \\ \sum_{i=0}^{17}\aleph_i \sum_{i=0}^{17} G_i - \sum_{i=0}^{16}\aleph_i \sum_{i=0}^{16} G_i = O_{n\times n} \\ \sum_{i=0}^{18}\aleph_i \sum_{i=0}^{18} G_i - \sum_{i=0}^{17}\aleph_i \sum_{i=0}^{17} G_i = O_{n\times n} \\ \sum_{i=0}^{19}\aleph_i \sum_{i=0}^{19} G_i - \sum_{i=0}^{18}\aleph_i \sum_{i=0}^{18} G_i = O_{n\times n} \\ \sum_{i=0}^{20}\aleph_i \sum_{i=0}^{20} G_i - \sum_{i=0}^{19}\aleph_i \sum_{i=0}^{19} G_i = O_{n\times n} \end{cases}$$

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And $\sum_{i=0}^{21}\aleph_i\sum_{i=0}^{21}G_i-\sum_{i=0}^{20}\aleph_i\sum_{i=0}^{20}G_i=O_{n\times n},\sum_{i=0}^{22}\aleph_i\sum_{i=0}^{22}G_i-\sum_{i=0}^{21}\aleph_i\sum_{i=0}^{21}G_i=O_{n\times n},\sum_{i=0}^{23}\aleph_i\sum_{i=0}^{23}G_i-\sum_{i=0}^{22}\aleph_i\sum_{i=0}^{22}G_i=O_{n\times n}$

$$\begin{cases}\aleph_0G_0=U_{n\times n}\\ \sum_{i=0}^{q}\aleph_i\sum_{i=0}^{q}G_i=U_{n\times n}\end{cases};\ 1\le q\le 23$$

Hence $\det\left(\sum_{i=0}^{q}\aleph_i\right)\neq 0$ for all $1\le q\le 23$, so that $\det(\aleph)$ is invertible in $23-SP_R$.

2). $\sum_{i=0}^{q}G_i=\left(\sum_{i=0}^{q}\aleph_i\right)^{-1}$for $1\le q\le 23$, therefor

$\aleph^{-1}={\aleph_0}^{-1}+\left[(\sum_{i=0}^{1}\aleph_i)^{-1}-{\aleph_0}^{-1}\right]P_1+[(\sum_{i=0}^{2}\aleph_i)^{-1}-(\sum_{i=0}^{1}\aleph_i)^{-1}]P_2+[(\sum_{i=0}^{3}\aleph_i)^{-1}-(\sum_{i=0}^{2}\aleph_i)^{-1}]P_3+[(\sum_{i=0}^{4}\aleph_i)^{-1}-(\sum_{i=0}^{3}\aleph_i)^{-1}]P_4+\left[\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}-(\sum_{i=0}^{4}\aleph_i)^{-1}\right]P_5+\left[(\sum_{i=0}^{6}\aleph_i)^{-1}-\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}\right]P_6+[(\sum_{i=0}^{7}\aleph_i)^{-1}-(\sum_{i=0}^{6}\aleph_i)^{-1}]P_7+[(\sum_{i=0}^{8}\aleph_i)^{-1}-(\sum_{i=0}^{7}\aleph_i)^{-1}]P_8+[(\sum_{i=0}^{9}\aleph_i)^{-1}-(\sum_{i=0}^{8}\aleph_i)^{-1}]P_9+[(\sum_{i=0}^{10}\aleph_i)^{-1}-(\sum_{i=0}^{9}\aleph_i)^{-1}]P_{10}+[(\sum_{i=0}^{11}\aleph_i)^{-1}-(\sum_{i=0}^{10}\aleph_i)^{-1}]P_{11}+[(\sum_{i=0}^{12}\aleph_i)^{-1}-(\sum_{i=0}^{11}\aleph_i)^{-1}]P_{12}+[(\sum_{i=0}^{13}\aleph_i)^{-1}-(\sum_{i=0}^{12}\aleph_i)^{-1}]P_{13}+[(\sum_{i=0}^{14}\aleph_i)^{-1}-(\sum_{i=0}^{13}\aleph_i)^{-1}]P_{14}+\left[\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}-(\sum_{i=0}^{14}\aleph_i)^{-1}\right]P_{15}+\left[(\sum_{i=0}^{16}\aleph_i)^{-1}-\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}\right]P_{16}+[(\sum_{i=0}^{17}\aleph_i)^{-1}-(\sum_{i=0}^{16}\aleph_i)^{-1}]P_{17}+[(\sum_{i=0}^{18}\aleph_i)^{-1}-(\sum_{i=0}^{17}\aleph_i)^{-1}]P_{18}+[(\sum_{i=0}^{19}\aleph_i)^{-1}-(\sum_{i=0}^{18}\aleph_i)^{-1}]P_{19}+[(\sum_{i=0}^{20}\aleph_i)^{-1}-(\sum_{i=0}^{19}\aleph_i)^{-1}]P_{20}+[(\sum_{i=0}^{21}\aleph_i)^{-1}-(\sum_{i=0}^{20}\aleph_i)^{-1}]P_{21}+[(\sum_{i=0}^{22}\aleph_i)^{-1}-(\sum_{i=0}^{21}\aleph_i)^{-1}]P_{22}+[(\sum_{i=0}^{23}\aleph_i)^{-1}-(\sum_{i=0}^{22}\aleph_i)^{-1}]P_{23}.$

Theorem

Let $\aleph=\aleph_0+\sum_{i=1}^{24}\aleph_iP_i$ be a 24-plithogenic matrix of size $n\times n$, hence:

1. $\aleph$ is invertible if and only if $\det\aleph$ is an invertible 24-plithogenic real number.
2. $\aleph^{-1}={\aleph_0}^{-1}+\left[(\sum_{i=0}^{1}\aleph_i)^{-1}-{\aleph_0}^{-1}\right]P_1+[(\sum_{i=0}^{2}\aleph_i)^{-1}-(\sum_{i=0}^{1}\aleph_i)^{-1}]P_2+[(\sum_{i=1}^{3}\aleph_i)^{-1}-(\sum_{i=0}^{2}\aleph_i)^{-1}]P_3+[(\sum_{i=1}^{4}\aleph_i)^{-1}-(\sum_{i=0}^{3}\aleph_i)^{-1}]P_4+\left[\left(\sum_{i=1}^{5}\aleph_i\right)^{-1}-(\sum_{i=0}^{4}\aleph_i)^{-1}\right]P_5+\left[(\sum_{i=1}^{6}\aleph_i)^{-1}-\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}\right]P_6+[(\sum_{i=1}^{7}\aleph_i)^{-1}-(\sum_{i=0}^{6}\aleph_i)^{-1}]P_7+[(\sum_{i=1}^{8}\aleph_i)^{-1}-(\sum_{i=0}^{7}\aleph_i)^{-1}]P_8+[(\sum_{i=1}^{9}\aleph_i)^{-1}-(\sum_{i=0}^{8}\aleph_i)^{-1}]P_9+[(\sum_{i=1}^{10}\aleph_i)^{-1}-(\sum_{i=0}^{9}\aleph_i)^{-1}]P_{10}+[(\sum_{i=1}^{11}\aleph_i)^{-1}-(\sum_{i=0}^{10}\aleph_i)^{-1}]P_{11}+[(\sum_{i=1}^{12}\aleph_i)^{-1}-(\sum_{i=0}^{11}\tau_i)^{-1}]P_{12}+[(\sum_{i=1}^{13}\aleph_i)^{-1}-(\sum_{i=0}^{12}\aleph_i)^{-1}]P_{13}+[(\sum_{i=1}^{14}\aleph_i)^{-1}-(\sum_{i=0}^{13}\aleph_i)^{-1}]P_{14}+\left[\left(\sum_{i=1}^{15}\aleph_i\right)^{-1}-(\sum_{i=0}^{14}\aleph_i)^{-1}\right]P_{15}+\left[(\sum_{i=1}^{16}\aleph_i)^{-1}-\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}\right]P_{16}+[(\sum_{i=1}^{17}\aleph_i)^{-1}-(\sum_{i=0}^{16}\aleph_i)^{-1}]P_{17}+[(\sum_{i=1}^{18}\aleph_i)^{-1}-(\sum_{i=0}^{17}\aleph_i)^{-1}]P_{18}+[(\sum_{i=1}^{19}\aleph_i)^{-1}-(\sum_{i=0}^{18}\aleph_i)^{-1}]P_{19}+[(\sum_{i=1}^{20}\aleph_i)^{-1}-(\sum_{i=0}^{19}\aleph_i)^{-1}]P_{20}+[(\sum_{i=1}^{21}\aleph_i)^{-1}-(\sum_{i=0}^{20}\aleph_i)^{-1}]P_{21}+[(\sum_{i=1}^{22}\aleph_i)^{-1}-(\sum_{i=0}^{21}\aleph_i)^{-1}]P_{22}+[(\sum_{i=1}^{23}\aleph_i)^{-1}-(\sum_{i=0}^{22}\aleph_i)^{-1}]P_{23}+[(\sum_{i=1}^{24}\aleph_i)^{-1}-(\sum_{i=0}^{23}\aleph_i)^{-1}]P_{24}.$

Proof

1). Let $\aleph=\aleph_0+\sum_{i=1}^{24}\aleph_iP_i$, then $\aleph$ is invertible if and only if there exists $G=G_0+\sum_{i=1}^{24}G_iP_i$ such that: $\aleph\times G=U_{n\times n}$, hence:

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$$
\begin{cases}
\aleph_0 G_0 = U_{n\times n} \\
\sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} I_i - \aleph_0 G_0 = O_{n\times n} \\
\sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i - \sum_{i=0}^{1} \aleph_i \sum_{i=0}^{1} G_i = O_{n\times n} \\
\sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} G_i - \sum_{i=0}^{2} \aleph_i \sum_{i=0}^{2} G_i = O_{n\times n} \\
\sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} G_i - \sum_{i=0}^{3} \aleph_i \sum_{i=0}^{3} I_i = O_{n\times n} \\
\sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i - \sum_{i=0}^{4} \aleph_i \sum_{i=0}^{4} I_i = O_{n\times n} \\
\sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i - \sum_{i=0}^{5} \aleph_i \sum_{i=0}^{5} G_i = O_{n\times n} \\
\sum_{i=0}^{7} \tau_i \sum_{i=0}^{7} G_i - \sum_{i=0}^{6} \aleph_i \sum_{i=0}^{6} G_i = O_{n\times n} \\
\sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i - \sum_{i=0}^{7} \aleph_i \sum_{i=0}^{7} G_i = O_{n\times n} \\
\sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i - \sum_{i=0}^{8} \aleph_i \sum_{i=0}^{8} G_i = O_{n\times n} \\
\sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i - \sum_{i=0}^{9} \aleph_i \sum_{i=0}^{9} G_i = O_{n\times n} \\
\sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i - \sum_{i=0}^{10} \aleph_i \sum_{i=0}^{10} G_i = O_{n\times n} \\
\sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i - \sum_{i=0}^{11} \aleph_i \sum_{i=0}^{11} G_i = O_{n\times n} \\
\sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i - \sum_{i=0}^{12} \aleph_i \sum_{i=0}^{12} G_i = O_{n\times n} \\
\sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i - \sum_{i=0}^{13} \aleph_i \sum_{i=0}^{13} G_i = O_{n\times n} \\
\sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i - \sum_{i=0}^{14} \aleph_i \sum_{i=0}^{14} G_i = O_{n\times n} \\
\sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i - \sum_{i=0}^{15} \aleph_i \sum_{i=0}^{15} G_i = O_{n\times n} \\
\sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i - \sum_{i=0}^{16} \aleph_i \sum_{i=0}^{16} G_i = O_{n\times n} \\
\sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i - \sum_{i=0}^{17} \aleph_i \sum_{i=0}^{17} G_i = O_{n\times n} \\
\sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i - \sum_{i=0}^{18} \aleph_i \sum_{i=0}^{18} G_i = O_{n\times n} \\
\sum_{i=0}^{20} \aleph_i \sum_{i=0}^{20} G_i - \sum_{i=0}^{19} \aleph_i \sum_{i=0}^{19} G_i = O_{n\times n}
\end{cases}
$$

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And $\sum_{i=0}^{21}\aleph_i\sum_{i=0}^{21}G_i-\sum_{i=0}^{20}\aleph_i\sum_{i=0}^{20}G_i=O_{n\times n},\sum_{i=0}^{22}\aleph_i\sum_{i=0}^{22}G_i-\sum_{i=0}^{21}\aleph_i\sum_{i=0}^{21}G_i=O_{n\times n},\sum_{i=0}^{23}\aleph_i\sum_{i=0}^{23}G_i-\sum_{i=0}^{22}\aleph_i\sum_{i=0}^{22}G_i=O_{n\times n},\sum_{i=0}^{24}\aleph_i\sum_{i=0}^{24}G_i-\sum_{i=0}^{23}\aleph_i\sum_{i=0}^{23}G_i=O_{n\times n}$

$$\begin{cases}\aleph_0G_0=U_{n\times n}\\ \sum_{i=0}^{q}\aleph_i\sum_{i=0}^{q}G_i=U_{n\times n}\end{cases};\ 1\le q\le 24$$

Hence $\det(\sum_{i=0}^{q}\aleph_i)\neq 0$ for all $1\le q\le 24$, so that $\det(\aleph)$ is invertible in $24-SP_R$.

2). $\sum_{i=0}^{q}G_i=\left(\sum_{i=0}^{q}\aleph_i\right)^{-1}$ for $1\le q\le 24$, therefor

$$\aleph^{-1}={\aleph_0}^{-1}+\left[(\textstyle\sum_{i=0}^{1}\aleph_i)^{-1}-{\aleph_0}^{-1}\right]P_1+[(\sum_{i=0}^{2}\aleph_i)^{-1}-(\sum_{i=0}^{1}\aleph_i)^{-1}]P_2+[(\sum_{i=0}^{3}\aleph_i)^{-1}-(\sum_{i=0}^{2}\aleph_i)^{-1}]P_3+[(\sum_{i=0}^{4}\aleph_i)^{-1}-(\sum_{i=0}^{3}\aleph_i)^{-1}]P_4+\left[\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}-(\sum_{i=0}^{4}\aleph_i)^{-1}\right]P_5+\left[(\sum_{i=0}^{6}\aleph_i)^{-1}-\left(\sum_{i=0}^{5}\aleph_i\right)^{-1}\right]P_6+[(\sum_{i=0}^{7}\aleph_i)^{-1}-(\sum_{i=0}^{6}\aleph_i)^{-1}]P_7+[(\sum_{i=0}^{8}\aleph_i)^{-1}-(\sum_{i=0}^{7}\aleph_i)^{-1}]P_8+[(\sum_{i=0}^{9}\aleph_i)^{-1}-(\sum_{i=0}^{8}\aleph_i)^{-1}]P_9+[(\sum_{i=0}^{10}\aleph_i)^{-1}-(\sum_{i=0}^{9}\aleph_i)^{-1}]P_{10}+[(\sum_{i=0}^{11}\aleph_i)^{-1}-(\sum_{i=0}^{10}\aleph_i)^{-1}]P_{11}+[(\sum_{i=0}^{12}\aleph_i)^{-1}-(\sum_{i=0}^{11}\aleph_i)^{-1}]P_{12}+[(\sum_{i=0}^{13}\aleph_i)^{-1}-(\sum_{i=0}^{12}\aleph_i)^{-1}]P_{13}+[(\sum_{i=0}^{14}\aleph_i)^{-1}-(\sum_{i=0}^{13}\aleph_i)^{-1}]P_{14}+\left[\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}-(\sum_{i=0}^{14}\aleph_i)^{-1}\right]P_{15}+\left[(\sum_{i=0}^{16}\aleph_i)^{-1}-\left(\sum_{i=0}^{15}\aleph_i\right)^{-1}\right]P_{16}+[(\sum_{i=0}^{17}\aleph_i)^{-1}-(\sum_{i=0}^{16}\aleph_i)^{-1}]P_{17}+[(\sum_{i=0}^{18}\aleph_i)^{-1}-(\sum_{i=0}^{17}\aleph_i)^{-1}]P_{18}+[(\sum_{i=0}^{19}\aleph_i)^{-1}-(\sum_{i=0}^{18}\aleph_i)^{-1}]P_{19}+[(\sum_{i=0}^{20}\aleph_i)^{-1}-(\sum_{i=0}^{19}\aleph_i)^{-1}]P_{20}+[(\sum_{i=0}^{21}\aleph_i)^{-1}-(\sum_{i=0}^{20}\aleph_i)^{-1}]P_{21}+[(\sum_{i=0}^{22}\aleph_i)^{-1}-(\sum_{i=0}^{21}\aleph_i)^{-1}]P_{22}+[(\sum_{i=0}^{23}\aleph_i)^{-1}-(\sum_{i=0}^{22}\aleph_i)^{-1}]P_{23}+[(\sum_{i=0}^{24}\aleph_i)^{-1}-(\sum_{i=0}^{23}\aleph_i)^{-1}]P_{24}.$$

3. Conclusion

In this scientific work, we studied the Invertibility properties of all plithogenic square matrices of high orders between 20 and 24, where we presented the mathematical conditions and formulas for computing the inverses of all plithogenic square matrices with real plithogenic entries. This goal will be completed by proving many theorems that describe the Invertibility of all classic matrix parts of the corresponding symbolic m-plithogenic matrix.'

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