Development of a novel uncertainty model for interval-valued Q-fuzzy soft sets: Application in design-making

Mohanad H. Jameel[1,*], Sinan O. Al-Salihi[1], Faisal Al-Sharqi[2,3]

[1]Department of Mathematics, College of Education for Pure Science, University of Tikrit, Tikrit, Iraq

[2]Department of Mathematics, Faculty of Education for Pure Sciences, University Of Anbar, Ramadi, Anbar, Iraq
[3]College of Engineering, National University of Science and Technology, Dhi Qar, Iraq

Emails: muhnd.h.jamil@st.tu.edu.iq; somar@tu.edu.iq; faisal.ghazi@uoanbar.edu.iq

Abstract

In actual life, dealing with uncertain information has become a challenge for researchers who strive day after day to develop more accurate mathematical tools for better dealing with this information. The Q-Fuzzy soft model can process uncertain information in two dimensions by dealing with the subjective judgments of users effectively. Therefore, this article aims to increase the effectiveness of the Q-fuzzy soft model and address the challenges of design-making under uncertain information by proposing a new model called the interval-valued Q-fuzzy soft (IV-Q-FSS) model. Under the IV-Q-FSSs, we discuss strongly set-theory operations such as subset, union of two IV-Q-FSSs, intersection of two IV-Q-FSSs, complement of IV-Q-FSS, AND operation, and OR operation for IV-Q-FSSs, and some operations like the possibility and necessity operations of an IV-Q-FSS. In addition, we hand over numerous properties held up by numerical examples that describe how they toil. Finally, this recently developed model has been successfully trying out in dealing with one of the design-making problems based on hypothetical data for a respiratory disease. This algorithm is built based on the aggregation operator for IV-Q-FSS data to break this issue (i.e., selecting the optimal alternative).

Keywords: fuzzy set; soft set; Q- fuzzy set; Q- fuzzy soft set; interval Q- fuzzy soft set; design-making

1. Introduction

Design-making innovation among multi-attributes requires a comprehensive understanding to address effectively. To meet that Zadeh [1] (1965) came up with the idea of a fuzzy set (FS) as an extension of a crisp set to describe how things belong to certain feelings even when they are unsure. Zadeh's FS is characterized by one part called single truth membership, such that its value belongs to a closed interval [0, 1]. In 1986, Atanassov [2] presented the intuitionistic fuzzy set (IFS) after pointing out that many types of uncertainty cannot be dealt with it via single-truth membership. Later, Smarandache pointed out in 1999 that FS and IFS are incapable of dealing with issues involving indeterminate and imprecise data. As a result, he built up the idea of the neutrosophic set (NS) [3] as a generalization of the crisp set, FS, and IFS. A neutrosophic structure consists of three functions: truthness, indeterminacy, and falsity, such that every element in the universal set has three membership functions, all of which lie in the closed interval [0, 1]. However, FS, IFS, and NS have inherent problems because they cannot give universal set elements (alternative set) more detail (attributes). To eliminate this weakness, Molodtsov [4] proposed a new mathematical tool called soft set (SS), which works to give the elements of the alternative set more detail. Later, an interval fuzzy soft set (IVFSSs) was explored by Yang et al.[5] as a hybrid notion that combines the advantages of both FS and SS both under interval. Researchers are busily engaged in developing the NSS environment [6-10].

For example, Liu et al. [11] defined similarity methods on FSS. Saleh [12] constructed the IVFSS's topology structure and discussed some of its properties. Al-sharqi et al.[13,14] established a link between the possibility degree and the FSS, IVFSS. Romdhini et al. [15] illustrated the sequence of IVFSS as a new form to address

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

uncertainty issues. Khan et al.[16] explored the idea of an m-polar interval-FSS and studied its different algebraic structures. Al-Sharqi et al.[17,18] extended the FSS from real space to complex space and applied these extensions to solve MCDM. Recently, Abu Qamar and Hassan [19] established a novel idea called Q-neutosophic soft set (Q-NSS) as an extension of Q-fuzzy soft set (Q-FSS) [20] and Q-intuitionistic fuzzy soft set (Q-IFSS)[21] by adding a second dimension of a universal set to FSS. In fact, this concept is more efficient in dealing with two-dimensional uncertainty issues that contain indeterminate data and are difficult to handle using Q-FSS and Q-IFSS. As a result, this idea inspired researchers around the world to contribute, such as Dalklç and Demirtaş [22] who demonstrated how to build a relationship between the two Q-FSS. Abuqamar and Hassan [23] described the algebraic structure of Q-NSS[24-30].

This approach has good capabilities compared to the works mentioned in this literature, but the outputs of this model are single values. As we mentioned previously, these values constitute an obstacle for the decision-maker and do not give him sufficient freedom to build numerical data that describes the data of the problem to be solved. This manuscript aimed to suggest techniques a new idea called IV-Q-FSSs, which stands for IV-Q-FSSs. These are a more developed form of Q-FSSs, and each membership function is unique to Q-NSSs given in interval form. This format gives the user more freedom and efficiency when dealing with everyday scenarios, especially those saturated with neutral, two-dimensional uncertainty information.

The main contributions shown in this work that were made to achieve these objectives are:

i. A new technique (IV-Q-FSSs) is proposed to contain the effects of uncertainty information in two-dimensional.

ii. To demonstrate the theoretical side of this model, we presented the basic operations, supported by an illustrative numerical example. In addition to presenting the basic properties and theories of IV-Q-FSSs.

iii. On the applied side, these techniques have been added to solve one of the decision-making problems in the medical field by proposing a multi-step algorithm that works on IV-Q-FSS data

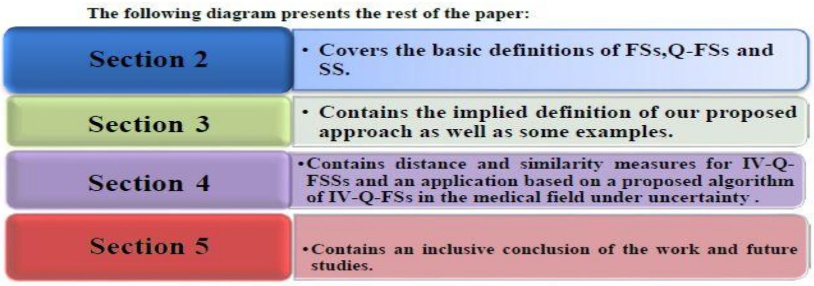

Fig 1: a representation of results.

2. Preliminaries

In this part, we recollect some critical notions related to our proposed approach like FS, Q-FS, IVFS and SS.

Definition 2.1. [1] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space (non-empty universal set). Then an FS $\mathcal{F}$ on $\mathfrak{U}$ is defined by following form:

$$\mathcal{F} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where $\mathcal{F}$ is a mapping defined as $\mathcal{F}: \mathfrak{U} \to [0,1]$ such that $\hat{P}^t \in [0,1]$ and called truth membership function (TMF).

Definition 2.2. [20] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space (non-empty universal set) and $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \dots, \mathfrak{q}_n\}$ be nonempty set. Then a Q-FS $\mathcal{F}_{\mathfrak{Q}}$ on the order pair $(\mathfrak{U}, \mathfrak{Q})$is defined by following form:

$$\mathcal{F}_{\mathfrak{Q}} = \{(u, \mathfrak{q}), \hat{P}^t(u, \mathfrak{q}) | (u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where $\mathcal{F}$ is a mapping defined as $\mathcal{F}_{\mathfrak{Q}}: \mathfrak{U} \times \mathfrak{Q} \to [0,1]$ such that $\hat{P}^t_{\mathfrak{Q}} \in [0,1]$ and called Q-truth membership function (TMF).

Definition 2.5. [5] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space (non-empty universal set). Then an IVFS N on $\mathfrak{U}$ is defined by following form:

$$\mathrm{N} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where $\hat{P}^t(\mathfrak{u}_j) = [\hat{P}^{t,l}(\mathfrak{u}_j), \hat{P}^{t,u}(\mathfrak{u}_j)]$, such that the domen of these terms is $\mathfrak{U}$ and the co-domen is $[0,1]$ and $\hat{P}^{t,l}(\mathfrak{u}_j), \hat{P}^{t,u}(\mathfrak{u}_j)$ are lower and upper of TMF with two stander conditions $0 \le \hat{P}^{t,l}(\mathfrak{u}_j) \le 1$ and $0 \le \hat{P}^{t.u}(\mathfrak{u}_j) \le 1$.

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

Definition 2.6. [5] Assume that
$N_1 = \{\mathfrak{u}_j, \hat{P}_1^t(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$, $N_2 = \{\mathfrak{u}_j, \hat{P}_2^t(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$ be two IVFS on initial points space (non-empty universal set) $\mathfrak{U}$
where $\hat{P}_1^t(\mathfrak{u}_j) = [\hat{P}_1^{t,l}(\mathfrak{u}_j), \hat{P}_1^{t,u}(\mathfrak{u}_j)]$ and $\hat{P}_2^t(\mathfrak{u}_j) = [\hat{P}_2^{t,l}(\mathfrak{u}_j), \hat{P}_2^{t,u}(\mathfrak{u}_j)]$ Then,

i. **Complement:** $N_1^C = \{\mathfrak{u}_j, [1 - \hat{P}_1^{t,u}(\mathfrak{u}_j), 1 - \hat{P}_1^{t,l}(\mathfrak{u}_j)], | \mathfrak{u}_j \in \mathfrak{U}\}$

ii. **Union:** $N_1 \cup N_2 = \{\mathfrak{u}_j, max[\hat{P}_1^{t,l}(\mathfrak{u}_j), \hat{P}_2^{t,l}(\mathfrak{u}_j)], \max[\hat{P}_1^{t,u}(\mathfrak{u}_j), \hat{P}_2^{t,u}(\mathfrak{u}_j)] | \mathfrak{u}_j \in \mathfrak{U}\}$.

iii. **Intersection:** $N_1 \cap N_2 = \{\mathfrak{u}_j, min[\hat{P}_1^{t,l}(\mathfrak{u}_j), \hat{P}_2^{t,l}(\mathfrak{u}_j)], \min[\hat{P}_1^{t,u}(\mathfrak{u}_j), \hat{P}_2^{t,u}(\mathfrak{u}_j)] | \mathfrak{u}_j \in \mathfrak{U}\}$.

iv. **Subset:** $N_1 \subseteq N_2$ if $\hat{P}_1^t(\mathfrak{u}_j) \leq \hat{P}_2^t(\mathfrak{u}_j)$ then $\hat{P}_1^{t,l}(\mathfrak{u}_j) \leq \hat{P}_2^{t,l}(\mathfrak{u}_j)$ and $\hat{P}_1^{t,u}(\mathfrak{u}_j) \leq \hat{P}_2^{t,u}(\mathfrak{u}_j)$.

Definition 2.7. [4] A pair $(\mathcal{F}, \bar{\mathrm{A}} \subseteq \mathcal{E})$ is named SSs over a non-empty universe of discourse $\mathfrak{U}$ if $\mathcal{F}: \bar{\mathrm{A}} \subseteq \mathcal{E} \longrightarrow P(\mathfrak{U})$, such that the term $P(\mathfrak{U})$ indicate the power set of $\mathfrak{U}$.

3. Interval Valued-Q-fuzzy Soft Sets (IV-Q-FSSs)

This section proposes the general framework definition of our concept IV-Q-FSS with fundamental operations like empty IV-Q-FSS, absolute IV-Q-FSS, subset IV-Q-FSS, and equality between two IV-Q-FSS. Also, to clarify our model more, we will give some numerical examples.

Definition 3.1. Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set), $\mathfrak{Q} \neq \emptyset, ie\ \ \mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_1, \mathfrak{q}_1, \dots, \mathfrak{q}_n\}$ and $\mathcal{E} = \{e_1, e_2, e_3, \dots, e_n\}$ be a set of attribute (parameters set). Let $\bar{\mathrm{A}} \subseteq \mathcal{E}$ be subset of attribute set, then a duet $(\hat{P}_{\mathfrak{Q}}, \bar{\mathrm{A}})$ is called a interval-valued $\mathfrak{Q}$-fuzzy soft set over the initial points space (non-empty universal set) $\mathfrak{U}$, where $\hat{P}_{\mathfrak{Q}}$ given as following mapping

$$\hat{P}_{\mathfrak{Q}}: \bar{\mathrm{A}} \to \mathfrak{Q} - IVNS(\mathfrak{U})$$

Then, the $IV - \mathfrak{Q} - FSS(\mathfrak{U})$ can be characterized by the following get form

$$(\hat{P}_{\mathfrak{Q}}, \bar{\mathrm{A}}) = \hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}}} = \{e \in \bar{\mathrm{A}}, < \hat{P}_{\mathfrak{Q}}^t(u, \mathfrak{q})(e) > | (u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where

$$\hat{P}_{\mathfrak{Q}}^t(u, \mathfrak{q})(\mathrm{e}) = [\hat{P}_{\mathfrak{Q}}^{t,l}(u, \mathfrak{q})(e), \hat{P}_{\mathfrak{Q}}^{t,u}(u, \mathfrak{q})(\mathrm{e})]$$

Such that , the terms here $\hat{P}_{\mathfrak{Q}}^{t,l}(u, \mathfrak{q})(\mathrm{e}), \hat{P}_{\mathfrak{Q}}^{t,u}(u, \mathfrak{q})(\mathrm{e})$ refer to true interval-valued membership function of objects $(u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}$, with two stander conditions $0 \leq \hat{P}_{\mathfrak{Q}}^{t,l}(u, \mathfrak{q})(e) \leq 1$ and $0 \leq \hat{P}_{\mathfrak{Q}}^{t,u}(u, \mathfrak{q})(\mathrm{e}) \leq 1$.

Now, to shed more light on the above definition, we present below the following numerical example, which describes the mechanism of action of our approach presented in this work.

Example 3.2. Assume that we are interested in analyzing the attractiveness of three houses that one person is thinking of buying one of them. Now, let us analyze this attractiveness according to our model (IV-Q-FSS), therefore we assume that the three houses present as following universal set $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3\}$ and $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$ be a set constituting two cities under consideration and $\mathcal{E} = \{e_1, e_2, e_3\}$ be a collection of

$\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}}} =$

$$\left\{\left(\mathrm{e}_1, \frac{\langle[0.2,0.8]\rangle}{(\mathfrak{u}_1, \mathfrak{q}_1)}, \frac{\langle[0.1,0.4]\rangle}{(\mathfrak{u}_1, \mathfrak{q}_2)}\right.\right.$$

$$\left.\frac{\langle[0.1,0.5]\rangle}{(\mathfrak{u}_3, \mathfrak{q}_1)}, \frac{\langle[0.4,0.8]\rangle}{(\mathfrak{u}_3, \mathfrak{q}_2)}\right)$$

$$\left(\mathrm{e}_2, \frac{\langle[0.1,0.8]\rangle}{(\mathfrak{u}_1, \mathfrak{q}_1)}, \frac{\langle[0.1,0.8]\rangle}{(\mathfrak{u}_1, \mathfrak{q}_2)}\right.$$

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

$\frac{\langle[0.5,0.8]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.1,0.2]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}$

$\frac{\langle,[0.2,0.5]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.4,0.5]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)}\Big)$

$\Big(\mathfrak{e}_3, \frac{\langle,[0.3,0.6]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.4,0.7]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}$

$\frac{\langle[0.3,0.6]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.5,0.6]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}$

$\frac{\langle[0.3,0.6]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)}\Big)\Big\}$

Definition 3.3 Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < \hat{P}^t_{\mathfrak{Q}}(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$ be a $IV-Q-FSS$ on initial point space (universal set). Then $\hat{P}_{Q_{\bar{A}}}$knowing as $IV-Q-FS-nullset\ and\ refer\ as \hat{P}_{\emptyset_{(0)}} if\ \ \hat{P}_{\emptyset_{(0)}}(u,\mathfrak{q}) = \{([0,0])\}$.

Example 2.4. The term $\hat{P}_{\emptyset_{(0)}(\mathfrak{u}_3,\mathfrak{q}_2)}(\mathfrak{e}_1) = \left(\mathfrak{e}_1, \frac{([0,0])}{(\mathfrak{u}_3,\mathfrak{q}_2)}\right)$ is consider $IV-Q-FS-nullset$ on $\mathfrak{U}$.

Definition 3.5 Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < \hat{P}^t_{\mathfrak{Q}}(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$ be a $IV-Q-FSS$ on initial point space (universal set). Then $\hat{P}_{Q_{\bar{A}}}$knowing as $IV-Q-FS-$ absolute $set\ and\ refer\ as\ \hat{P}_{\emptyset_{(1)}} if\ \ \hat{P}_{\emptyset_{(1)}}(u,\mathfrak{q}) = \{([1,1])\}$.

Example 3.6. The term $\hat{P}_{\emptyset_{(1)}(\mathfrak{u}_3,\mathfrak{q}_2)(\mathfrak{e}_1)} = \left(\mathfrak{e}_1, \frac{([1,1])}{(\mathfrak{u}_3,\mathfrak{q}_2)}\right)$ is consider $IV-Q-FS-$ absolute set on $\mathfrak{U}$.

Definition 3.7 Let $\hat{P}_{Q_{\bar{A}}} and \hat{P}_{Q_{\bar{B}}}$be two $IV-Q-FSSs$ on non empty universal set (initial points space) $\mathfrak{U}$ with $\mathfrak{Q}$ .Then we say that $\hat{P}_{Q_{\bar{A}}}$is $IV-Q-FSS\ subset\ of\ \hat{P}_{Q_{\bar{B}}}$ and refer to this relation as $\hat{P}_{Q_{\bar{A}}} \subseteq \hat{P}_{Q_{\bar{B}}}$if fulifed the following conditions

For $A \subseteq B$ and $\hat{P}_{Q_{\bar{A}}} \subseteq \hat{P}_{Q_{\bar{B}}}$for all $e \in A\ and\ B$, $(u,q) \in \mathcal{U} \times Q$

Then

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q}) \le \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q}), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q}) \le \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q}).$$

Example 3.8. Consider the two terms in example 3.2, where $B = \{\mathfrak{e}_1\}$,such that

$\hat{P}_{Q_{\bar{A}}(\mathfrak{u}_1,\mathfrak{q}_2)}(\mathfrak{e}_1) = \left(\mathfrak{e}_1, \frac{\langle[0.1,0.4]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}\right)$

$\hat{P}_{Q_{\bar{B}}(\mathfrak{u}_1,\mathfrak{q}_2)}(\mathfrak{e}_1) = \left(\mathfrak{e}_1, \frac{\langle[0.2,0.5]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}\right)$

Then, it's clear $\hat{P}_{Q_{\bar{A}}} \subseteq \hat{P}_{Q_{\bar{B}}}$.

Definition 3.9. Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < \hat{P}^t_{\mathfrak{Q}}(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

$\in IV-Q-FSS(\mathfrak{U})$. Then, its complement given as $\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} or\ c\hat{P}_{\mathfrak{Q}_{\bar{A}}}$ and its defined as following:.

$\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < {P^t_{\mathfrak{Q}}}^c(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

Or $c\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < c\hat{P}^t_{\mathfrak{Q}}(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

Where

$${P^t_{\mathfrak{Q}}}^c(u,\mathfrak{q})(e) = \left[{P^{t,l}_{\mathfrak{Q}_{\bar{A}}}}^c(u,\mathfrak{q})(e), {P^{t,u}_{\mathfrak{Q}_{\bar{A}}}}^c(u,\mathfrak{q})(e)\right] = \left[1 - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), 1 - \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right],$$

Or

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

$c\hat{P}^t_{\mathfrak{Q}}(u,\mathfrak{q})(e) = \left[c\hat{P}^{t,l}_{\mathfrak{Q}}(u,\mathfrak{q})(e), c\hat{P}^{t,u}_{\mathfrak{Q}}(u,\mathfrak{q})(e)\right] = \left[1 - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), 1 - \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right],$

Based on a above definition we sat that this $\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} or\ c\hat{P}_{\mathfrak{Q}_{\bar{A}}}$ is the complement of $\mathrm{IV-Q-FSS}(\mathfrak{U})$.

Example3.10. Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2\}$ be initial point (universal set) , $\mathfrak{Q} = \{q_1, q_2\}$ and $\bar{A} = \{e_1, e_2\}$. Then

$\hat{P}_{\mathfrak{Q}_{\bar{A}}} =$

$\left\{\left(\mathrm{e}_1, \frac{\langle[0.2,0.8],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.4],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}\right.\right.$

$\left.\frac{\langle[0.1,0.5],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}\right)$

$\left(\mathrm{e}_2, \frac{\langle[0.1,0.8],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.8],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}\right.$

$\left.\left.\frac{\langle[0.1,0.4],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.1,0.6],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}\right)\right\}$

Then the complement operation defining $\hat{P}_{\mathfrak{Q}_{\bar{A}}}\ as\ P^c_{\mathfrak{Q}_{\bar{A}}}\ or\ cP_{\mathfrak{Q}_{\bar{A}}}$ basedon definition as following

$\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} = c\hat{P}_{\mathfrak{Q}_{\bar{A}}} =$

$\left\{\left(\mathrm{e}_1, \frac{\langle[0.2,0.8],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.6,0.9]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}\right.\right.$

$\left.\frac{\langle[0.5,0.9],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.2,0.6]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}\right)$

$\left(\mathrm{e}_2, \frac{\langle[0.2,0.9],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.2,0.9],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}\right.$

$\left.\left.\frac{\langle[0.6,0.9],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.9],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}\right)\right\}$

Proposition 3.11 If $\hat{P}_{\mathfrak{Q}_{\bar{A}}} \in \mathrm{IV-Q-FSS}(\mathcal{X})$. Then $c\left(c\hat{P}_{\mathfrak{Q}_{\bar{A}}}\right) = \left(\hat{P}^c_{\mathfrak{Q}_{\bar{A}}}\right) or \left(\hat{P}^c_{\mathfrak{Q}_{\bar{A}}}\right)^c = \hat{P}_{\mathfrak{Q}_{\bar{A}}}$

Proof: From above definition, we have

$\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < \hat{P}^t_{\mathfrak{Q}}(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

Then,

$\hat{P}^c_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < {P^t_{\mathfrak{Q}}}^c(u,\mathfrak{q})(e), > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

$= \left\{e \in \bar{A}, \langle\left[{P^{t,l}_{\mathfrak{Q}_{\bar{A}}}}^c(u,\mathfrak{q})(e), {P^{t,u}_{\mathfrak{Q}_{\bar{A}}}}^c(u,\mathfrak{q})(e)\right]\rangle\, |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\right\}$

$\left(\hat{P}^c_{\mathfrak{Q}_{\bar{A}}}\right)^c = \left\{e \in \bar{A}, \langle\left[\left(1 - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right)^c, \left(1 - \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right)^c\right]\rangle : |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\right\}$

$= \left\{e \in \bar{A}, \langle\left[\left(1 - \left(1 - \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right)\right), \left(1 - \left(1 - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right)\right)\right]\rangle : |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\right\}$

$= \{e \in \bar{A}, < \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right] > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

$= \hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < \hat{P}^t_{\mathfrak{Q}}(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$

Hence, we get $c\left(c\hat{P}_{\mathfrak{Q}_{\bar{A}}}\right) = \hat{P}_{\mathfrak{Q}_{\bar{A}}}$.

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

Definition 3.12 The union of two $IV-QFSS$ $\hat{P}_{\mathfrak{Q}_{\bar{C}}}$ and written as $\hat{P}_{\mathfrak{Q}_{\bar{A}}} \cup \hat{P}_{\mathfrak{Q}_{\bar{B}}} = \hat{P}_{\mathfrak{Q}_{\bar{C}}}$, where $\bar{C} = \bar{A} \cup \bar{B}$ and for all $c \in \bar{C}$, $(\mathfrak{u}, q) \in \mathfrak{U} \times \mathfrak{Q}$,the three $IV-Q-FSS$ member ships function given as follows :.

$$\hat{P}^{t}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q}) \begin{cases} P^{t}_{\mathfrak{Q}_{\bar{A}}}(\mathfrak{u},q) = \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right] & if\ c \in \bar{A}-\bar{B} \\ P^{t}_{\mathfrak{Q}_{\bar{B}}}(\mathfrak{u},q) = \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right] & if\ c \in \bar{B}-\bar{A} \\ P^{t}_{\mathfrak{Q}_{\bar{\bar{C}}}}(\mathfrak{u},q) = \max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e)\right] & if\ c \in \bar{A}\cap\bar{B} \end{cases}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right], \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \max\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

Here. The max represents the largest value of $IV-QFSS$ and min represents the smallest value of $IV-QFSS$.

Definition 3.13 The intersection of two $IV-QFSS$ $\hat{P}_{\mathfrak{Q}_{\bar{C}}}$ and written as $\hat{P}_{\mathfrak{Q}_{\bar{A}}} \cup \hat{P}_{\mathfrak{Q}_{\bar{B}}} = \hat{P}_{\mathfrak{Q}_{\bar{C}}}$, where $\bar{C} = \bar{A} \cap \bar{B}$ and for all $c \in \bar{C}$, $(\mathfrak{u}, q) \in \mathfrak{U} \times \mathfrak{Q}$,the three $IV-Q-FSS$ member ships function given as follows :.

$$\hat{P}^{t}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q}) \begin{cases} P^{t}_{\mathfrak{Q}_{\bar{A}}}(\mathfrak{u},q) = \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e)\right] & if\ c \in \bar{A}-\bar{B} \\ P^{t}_{\mathfrak{Q}_{\bar{B}}}(\mathfrak{u},q) = \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right] & if\ c \in \bar{B}-\bar{A} \\ P^{t}_{\mathfrak{Q}_{\bar{\bar{C}}}}(\mathfrak{u},q) = \min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e)\right] & if\ c \in \bar{A}\cap\bar{B} \end{cases}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right], \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}}}(u,\mathfrak{q})(e) = \min\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(u,\mathfrak{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}}}(u,\mathfrak{q})(e)\right],$$

Here. The min represents the largest value of $IV-QFSS$ and min represents the smallest value of $IV-QFSS$.

Example 3.14. Let $\mathcal{X} = \{u_1, u_2\}$ be non-empty initial universal set, $\mathfrak{L} = \{e_1, e_2\} and\ Q = \{\mathfrak{q}_1\}. then\ , if\ \bar{A} = \{e_1\} \subseteq \mathfrak{L}$,

$\bar{B} = \{e_1, e_2\} \subseteq \mathfrak{L}\ , then\ the\ two\ IV-Q-FSSs\ (\hat{P}_Q, \bar{A}), (\hat{P}_Q, \bar{B})$ will be analyze as following

$(\hat{P}_Q, \bar{A}) =$

$$\left\{\left(\mathfrak{e}_1, \frac{\langle[0.4,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.5,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}\right)\right\}$$

$(\hat{P}_Q, \bar{B}) =$

$$\left\{\left(\mathfrak{e}_1, \frac{\langle[0.6,0.7]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.4,0.6]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}\right)\right.$$

$$\left.\left(\mathfrak{e}_2 \frac{\langle[0.6,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.6,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}\right)\right\}$$

Then, $(\hat{P}_{\mathfrak{Q}}, \hat{A}) \cup (\hat{P}_{\mathfrak{Q}}, \hat{B}) =$

$$\left\{\left(\mathfrak{e}_1, \frac{\langle[0.4,0.7]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.6,0.9],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}\right)\right.$$

$$\left.\left(\mathfrak{e}_2, \frac{\langle[0.6,1]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.3,0.5]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}\right)\right\}$$

$(\hat{P}_{\mathfrak{Q}}, \hat{A}) \cap (\hat{P}_{\mathfrak{Q}}, \hat{B}) =$

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

$\left\{\left(e_1, \frac{\langle[0.2,0.5]\rangle}{(u_1,q_1)}, \frac{\langle[0.1,0.6]\rangle}{(u_1,q_2)}\right)\right.$

$\left.\left(e_2, \frac{\langle[0.6,1]\rangle}{(u_1,q_1)}, \frac{\langle[0.3,0.5]\rangle}{(u_2,q_1)}\right)\right\}$

Proposition 3. 15. Let $(\hat{P}_{\mathfrak{Q}},\hat{A}),(\hat{P}_{\mathfrak{Q}},\hat{B}) and (\hat{P}_{\mathfrak{Q}},C)$ be three $IV-Q-FSSs\ on\ non-universal\ set\ U$. The, the following points are satisfied:

1. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B})=(\hat{P}_{\mathfrak{Q}},\hat{B}) \cup (\hat{P}_{\mathfrak{Q}},\hat{A})$
2. $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cap (\hat{P}_{\mathfrak{Q}},\hat{B})=(\hat{P}_{\mathfrak{Q}},\hat{B}) \cap (\hat{P}_{\mathfrak{Q}},\hat{A})$

Proof (1). Now we will show that $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B})=(\hat{P}_{\mathfrak{Q}},\hat{B}) \cup (\hat{P}_{\mathfrak{Q}},\hat{A})$based on Definition 3.12 Also, in this case we will consider the case $c \in \bar{A} \cap \bar{B}$ and other case are trivial .

Now, take the left side $(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B}), then$

$$(\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B}) = \{\max\{\hat{P}^t_{\mathfrak{Q}_{\bar{A}}}(u,q), \hat{P}^t_{\mathfrak{Q}_{\bar{B}}}(u,q): (u,q) \in U \times Q >$$

$$=\left\{< c(\max\left[\max P^{t,i}_{Q_{\bar{A}}}(u,q), P^{t,l}_{Q_{\bar{B}}}(u,q)\right], \left[\max\ [P^{t,u}_{Q_{\bar{A}}}(u,q), P^{t,u}_{Q_{\bar{B}}}(u,q)\right]\right\}$$

$$,\min\left\{\ \min\left[P^{i,l}_{Q_{\bar{A}}}(u,q), P^{i,l}_{Q_{\bar{B}}}(u,q)\right], (u,q) \in U \times Q\right\},$$

$$= \{< c\ (\max\{\hat{P}^t_{\mathfrak{Q}_{\bar{A}}}(u,t), \hat{P}^t_{\mathfrak{Q}_{\bar{B}}}(u,t), (u,q) \in U \times Q >\}$$

$$= (\hat{P}_{\mathfrak{Q}},\hat{A}) \cup (\hat{P}_{\mathfrak{Q}},\hat{B})$$

Definition 3.16. Assume that $(\hat{P}_{\mathfrak{Q}},\hat{A})\ and\ (\hat{P}_{\mathfrak{Q}},\hat{B})\ are\ two\ IV-Q-FSSs$ on initial point space (non-empty universal set) U,then $(\hat{P}_{\mathfrak{Q}},\hat{A})\ AND(\hat{P}_{\mathfrak{Q}},\hat{B})\ is\ an\ IV-Q-NSSs$ and denoted by $(\hat{P}_{\mathfrak{Q}},\hat{A})\wedge(\hat{P}_{\mathfrak{Q}},\hat{B})$ and it defined by the following formalh $(\hat{P}_{\mathfrak{Q}},\hat{A})\ AND(\hat{P}_{\mathfrak{Q}},\hat{B}) = (\hat{P}_{\mathfrak{Q}},\bar{A} \times \hat{B}), where$

$$\hat{P}_{\mathfrak{Q}}(\bar{a},\bar{b})^{(u,q)} = \hat{P}_{\mathfrak{Q}_{(\bar{a})}}{}^{(u,q)} \cap \hat{P}_{\mathfrak{Q}_{(\bar{b})}}{}^{(u,q)}$$

For all $(\bar{a},\bar{b}) \in \bar{A} \times \bar{B}$, where $\cap$ *is tha intersection* operation of two $IV-Q-FSSs$ on initial points pace (non-empty universal set)

Now, based on the intersection definition the three $IV-Q-FSSs$ membership function defined as following

$$P^t_{Q_{(\bar{a},\bar{b})}}(u,q) = \min\{P^t_{Q_{(\bar{a})}}(u,q), P^t_{Q_{(\bar{b})}}(u,q)\} =$$
$$\min\left\{\min\left[P^{t,l}_{Q_{(\bar{a})}}(u,q), P^{t,l}_{Q_{(\bar{b})}}(u,q)\right], \min\left[P^{t,u}_{Q_{(\bar{a})}}(u,q), P^{t,u}_{Q_{(\bar{b})}}(u,q)\right]\right\},$$

Definition 3.17. Assume that $(\hat{P}_{\mathfrak{Q}},\hat{A})\ and\ (\hat{P}_{\mathfrak{Q}},\hat{B})\ are\ two\ IV-Q-FSSs$ on initial point space (non-empty universal set) U,then $(\hat{P}_{\mathfrak{Q}},\hat{A})\ OR(\hat{P}_{\mathfrak{Q}},\hat{B})\ is\ an\ IV-Q-FSSs$ and denoted by $(\hat{P}_{\mathfrak{Q}},\hat{A})\vee(\hat{P}_{\mathfrak{Q}},\hat{B})$ and it defined by the following formalh $(\hat{P}_{\mathfrak{Q}},\hat{A})OR(\hat{P}_{\mathfrak{Q}},\hat{B}) = (\hat{P}_{\mathfrak{Q}},\bar{A} \times \hat{B}), where$

$$\hat{P}_{\mathfrak{Q}}(\bar{a},\bar{b})^{(u,q)} = \hat{P}_{\mathfrak{Q}_{(\bar{a})}}{}^{(u,q)} \cup \hat{P}_{\mathfrak{Q}_{(\bar{b})}}{}^{(u,q)}$$

For all $(\bar{a},\bar{b}) \in \bar{A} \times \bar{B}$, where $\cup$ *is tha union*operation of two $IV-Q-FSSs$ on initial points pace (non-empty universal set) U.

Now, based on the *union* definition the three $IV-Q-FSSs$ membership function defined as following

$$P^t_{Q_{(\bar{a},\bar{b})}}(u,q) = \text{mix}\{P^t_{Q_{(\bar{a})}}(u,q), P^t_{Q_{(\bar{b})}}(u,q)\} = \text{mix}\left\{\text{mix}\left[P^{t,l}_{Q_{(\bar{a})}}(u,q), P^{t,l}_{Q_{(\bar{b})}}(u,q)\right], \text{mix}\left[P^{t,u}_{Q_{(\bar{a})}}(u,q), P^{t,u}_{Q_{(\bar{b})}}(u,q)\right]\right\},$$

Example 3.18. Let $\mathcal{X} = \{u_1,u_2\}$ be non-empty initial universal set, $\mathfrak{L} = \{e_1,e_2,e_3\} and\ Q = \{q_1\}. then\ , if\ \bar{A} = \{e_1\} \subseteq \mathfrak{L}$,

$$\bar{B} = \{e_2,e_3\} \subseteq \mathfrak{L}\ , then\ the\ two\ IV-Q-FSSs\ (\hat{P}_Q,\bar{A}),(\hat{P}_Q,\bar{B})$$

Will be analyzed as following

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

$(\hat{P}_Q, \bar{A}) =$

$\left\{\left(e_1, \frac{\langle[0.2,0.8]\rangle}{(u_1, q_1)}, \frac{\langle[0.1,0.6]\rangle}{(u_2, q_1)}\right)\right\}$

$(\hat{P}_Q, \bar{B}) =$

$\left\{\left(e_2, \frac{\langle[0.3,0.5]\rangle}{(u_1, q_1)}, \frac{\langle[0.6,0.9]\rangle}{(u_2, q_1)}\right)\right.$

$\left.\left(e_3 \frac{\langle[0.6,1]\rangle}{(u_1, q_1)}, \frac{\langle[0.3,0.5]\rangle}{(u_2, q_1)}\right)\right\}$

Then,

$(\hat{P}_{\mathfrak{Q}}, \hat{A})\ AND(\hat{P}_{\mathfrak{Q}}, \hat{B}) = (\hat{P}_{\mathfrak{Q}}, \bar{A} \times \hat{B}) =$

$\left\{\left((e_1, e_2), \frac{\langle[0.2,0.5]\rangle}{(u_1, q_1)}, \frac{\langle[0.1,0.3]\rangle}{(u_1, q_2)}\right)\right.$

$\left.\left((e_1, e_3), \frac{\langle[0.3,0.5]\rangle}{(u_2, q_1)}, \frac{\langle[0.3,0.5]\rangle}{(u_2, q_2)}\right)\right\}$

And $(\hat{P}_{\mathfrak{Q}}, \hat{A})\ OR(\hat{P}_{\mathfrak{Q}}, \hat{B}) = (\hat{P}_{\mathfrak{Q}}, \bar{A} \times \hat{B}) =$

$\left\{\left((e_1, e_2), \frac{\langle[0.3,0.8],\rangle}{(u_1, q_1)}, \frac{\langle[0.6,0.9]\rangle}{(u_1, q_2)}\right)\right.$

$\left.\left((e_1, e_3), \frac{\langle[0.6,1]\rangle}{(u_2, q_1)}, \frac{\langle[0.3,0.6]\rangle}{(u_2, q_2)}\right)\right\}$

Proposition 3.19 Assume that $(\hat{P}_Q, \bar{A}), (\hat{P}_Q, \bar{B}) and\ (\hat{P}_Q, \bar{C})$ be three $IV - Q - FSSs$ no non-empty initial universal set U. Then following point (properties) will be satisfied:

1. $(\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C}) = \left((\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B})\right) \wedge (\hat{P}_Q, \bar{C})$
2. $(\hat{P}_Q, \bar{A}) \vee (\hat{P}_Q, \bar{B}) \vee (\hat{P}_Q, \bar{C}) = \left((\hat{P}_Q, \bar{A}) \vee (\hat{P}_Q, \bar{B})\right) \vee (\hat{P}_Q, \bar{C})$

Proof.1. Assume that $\bar{a} \in \bar{A}, \bar{b} \in \bar{B}$ and the thired one $\bar{c} \in \bar{C}$ and $(\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C}) = (\hat{P}_Q, \bar{B} \times \bar{C}), sach\ that$ $\hat{P}_Q(\bar{b}, \bar{c}) = \hat{P}_Q(\bar{b}) \cap \hat{P}_Q(\bar{c})$

Now , we have $(\hat{P}_Q, \bar{A}) \wedge \left((\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C})\right) = (\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B} \times \bar{C}) = (\hat{P}_Q, \bar{A} \times \bar{B} \times \bar{C}),$

Such that

$(\hat{P}_Q, \bar{a} \times \bar{b} \times \bar{c}) = \hat{P}_Q(a) \cap \hat{P}_Q(b, c) = \hat{P}_Q(a) \cap \hat{P}_Q(b) \cap \hat{P}_Q(c)$

Also we have $(\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B}) = (\hat{P}_Q, \bar{A} \times \bar{B})$ such that

$\hat{P}_Q(\bar{a}, \bar{b}) = \hat{P}_Q(\bar{a}) \cap \hat{P}_Q(\bar{b})$

Therefor $\left((\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B})\right) \wedge (\hat{P}_Q, \bar{C}) = (\hat{P}_Q, \bar{A} \times \bar{B}) \wedge (\hat{P}_Q, \bar{C})$

$= (\hat{P}_Q, \bar{A} \times \bar{B} \times \bar{C})\ where\ (\hat{P}_Q, \bar{a} \times \bar{b} \times \bar{c}) = \hat{P}_Q(\bar{a}, \bar{b}) \cap \hat{P}_Q(\bar{c}) = \hat{P}_Q(\bar{a}) \cap \hat{P}_Q(\bar{b}) \cap \hat{P}_Q(\bar{c}).$

Hence $(\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B}) \wedge (\hat{P}_Q, \bar{C}) = \left((\hat{P}_Q, \bar{A}) \wedge (\hat{P}_Q, \bar{B})\right) \wedge (\hat{P}_Q, \bar{C})$

Proof 2. Same proof (1)

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

4. An Application of IV-Q-FSs

In this portion, our model is employed to handle design-making problems under uncertainty information. we will work on creating an algorithm consisting of several sequential steps that analyze the algebraic structure of our proposed model and the data it represents.

Now we will provide some definitions that will be useful to us in building the above algorithm.

Definition 4.1 Let $(\hat{P}_Q, \bar{A})$ be IV-QFSS on non-empty initial universal set U. Then, an IV-QNS aggregation operator of $(\hat{P}_Q, \bar{A})$ and denoted by $\breve{\Pi}_Q^{agg}$ is defined by

$\breve{\Pi}_Q^{agg} = \left\{< \bar{a}\left[(u,q), \hat{P}_{Q(\bar{a})}^{t,agg}(u,q)\right] : (u,q) \in U \times Q >\right\}$

Where $\hat{P}_Q^{t,agg} : U \times Q \to [0,1]$, such that

$$\hat{P}_Q^{t,l,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q((u,q))}^{t,l}, \hat{P}_Q^{t,u,agg} = \frac{1}{|\bar{A}|} \cdot \sum_{(u,q)\in U\times Q} \hat{P}_{Q(u,q)}^{t,u},$$

Now, using the above definitions, we lever up the following algorithm for a decision medical field method:

Algorithm
Step 1. Put up an IV-Q-FSSs on U.
Step 2. Calculate IV-Q-FS aggregation operator
Step 3. Convert IV-QFS aggregation operator $(\hat{P}_Q^{t,l}, \hat{P}_Q^{t,u})$ to SV-QFS aggregation operator $(\hat{P}_Q^t)$,i.e. $\hat{P}_Q^t = \frac{\hat{P}_Q^{t,l} + {}_Q^{t,u}}{2}$
.
Step 4. The optimal decision is the element available in M, such that $M = max_{(u,q)\in U\times Q}\{\hat{P}_Q^t\}$.

Now, we provide a case study related to the medical field for IV-Q-FSS strategic decision-making method.

Assume that Mr. Ahmed wants to buy a modern house which commensurate with qualifications with the trait in $\bar{A}$ to the greatest extent from obtainable houses in $\mathfrak{U}$.
Now, Suppose that that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \mathfrak{u}_4\}$ includes four modern houses, $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$ where $\mathfrak{q}_1$ =suitable and $\mathfrak{q}_2$ =unsuitable,while $\bar{A} \subseteq \mathcal{E} = \{\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4\}$ be a set of traits contains four trait such that $\bar{a}_1$ = size, $\bar{a}_2$ = number of floors, $\bar{a}_3$ = location, $\bar{a}_4$ = Available services.

Now the decision maker will evaluate these modern homes according to the criteria mentioned above. So this evaluation grade is in the form of IV-Q-FSS.

$$\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \left\{\left(\bar{a}_1, \frac{\langle[0.2,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.4]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)} \frac{\langle[0.3,0.6],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.6],\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)} \frac{\langle[0.6,0.8],\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.3,0.7],\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)} \frac{\langle[0.1,0.5],\rangle}{(\mathfrak{u}_4,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],\rangle}{(\mathfrak{u}_4,\mathfrak{q}_2)}\right)\right.$$

$$\left(\mathfrak{e}_2, \frac{\langle[0.1,0.8],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.8],\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)} \frac{\langle,[0.2,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)} \frac{\langle[0.6,0.8]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.3,0.7],\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)} \frac{\langle[0.1,0.4],\rangle}{(\mathfrak{u}_4,\mathfrak{q}_1)}, \frac{\langle[0.1,0.6],\rangle}{(\mathfrak{u}_4,\mathfrak{q}_2)}\right)$$

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

$\left(e_3, \frac{\langle[0.1,0.8],\rangle}{(u_1,q_1)}, \frac{\langle[0.1,0.8],\rangle}{(u_1,q_2)}\right.$
$\frac{\langle[0.5,0.8],\rangle}{(u_2,q_1)}, \frac{\langle[0.1,0.2],\rangle}{(u_2,q_2)}$
$\frac{\langle[0.6,0.8],\rangle}{(u_3,q_1)}, \frac{\langle[0.3,0.7],\rangle}{(u_3,q_2)}$
$\left.\frac{\langle[0.1,0.4],\rangle}{(u_4,q_1)}, \frac{\langle[0.1,0.6],\rangle}{(u_4,q_2)}\right)$
$\left(e_4, \frac{\langle[0.7,0.9],\rangle}{(u_1,q_1)}, \frac{\langle[0.4,0.7],\rangle}{(u_1,q_2)}\right.$
$\frac{\langle[0.1,0.8],\rangle}{(u_2,q_1)}, \frac{\langle[0.5,0.6],\rangle}{(u_2,q_2)}$
$\frac{\langle[0.3,0.5]\rangle}{(u_3,q_1)}, \frac{\langle[0.3,0.7],\rangle}{(u_3,q_2)}$
$\left.\left.\frac{\langle[0.4,0.6],\rangle}{(u_4,q_1)}, \frac{\langle[0.4,0.8],\rangle}{(u_4,q_2)}\right)\right\}$

Step 2. The IV-Q-FS aggregation operator is given as

$\breve{\Pi}^{agg}_{ivQ-NS} =$
$\{((u_1,q_1), \langle[0.275,0.825],\rangle),$
$((u_1,q_2), \langle[0.124,0.342],\rangle),$
$((u_2,q_1), \langle\langle[0.335,0.673],\rangle\rangle),$
$((u_2,q_2), \langle\langle[0.453,0.765],\rangle\rangle),$
$((u_3,q_1), \langle\langle[0.237,0.763],\rangle\rangle),$
$((u_3,q_2), \langle\langle[0.287,0.325],\rangle\rangle),$
$((u_4,q_1), \langle\langle[0.128,0.342],\rangle\rangle),$
$((u_4,q_2), \langle\langle[0.234,0.432],\rangle\rangle)\}$

Step 3. Convert IV-Q-FS aggregation operator ($\hat{P}^{t,l}_Q, \hat{P}^{t,u}_Q$) to SV-QFS aggregation operator ($\hat{P}^t_Q$).

$\breve{\Pi}^{agg}_{ivQ-FS} =$
$\{((u_1,q_1), \langle 0.515\rangle), ((u_1,q_2), \langle 0.426\rangle),$
$((u_2,q_1), \langle 0.492\rangle), ((u_2,q_2), \langle 0.506\rangle),$
$((u_3,q_1), \langle 0.500\rangle), ((u_3,q_2), \langle 0.379\rangle),$
$((u_4,q_1), \langle 0.501\rangle), ((u_4,q_2), \langle 0.478\rangle)\}$

Step 5. The optimal decision is the element available inM_i, such that

$M_1 = max_{(u_1,q_{1,2})\in U\times Q}\{0.515, 0.426\} = 0.515.$
$M_2 = max_{(u_2,q_{1,2})\in U\times Q}\{0.492, 0.506\} = 0.506.$
$M_3 = max_{(u_3,q_{1,2})\in U\times Q}\{0.500, 0.379\} = 0.500.$
$M_4 = max_{(u_4,q_{1,2})\in U\times Q}\{0.501, 0..478\} = 0.501.$

it is clear that all patients $\mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}$ are infected except the patient $\mathbf{u_1}$.

5. Conclusion

IV-Q-FSS is a useful tool for dealing with Q-two-dimensional universal information in interval form. It is made up of three NS membership degrees in interval form. Also, this tool was created to deal with the relationship between parameters in the SS environment when these parameters play a key role in the deep description of two-dimensional universal information. So, in this paper, we suggested an interval-valued Q-fuzzy soft set (IV-Q-FSS) mean set theory. This theory includes special operations like the necessity and possibility operations of an IV-Q-FSS, as well as operations like the complement of an IV-Q-FSS, the union of two IV-Q-NSSs, the intersection of two IV-Q-FSSs, and the AND and OR operation of two IV-Q-FSSs. In addition, we presented many properties supported by numerical examples that explain how they work. Finally, this new model has been successfully tested in dealing with one of the design-making problems based on hypothetical data for a respiratory disease when a new algorithm based on the aggregation operator for IV-Q-FSS data was built to solve this issue.

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

References

[1] Zadeh, L. A. (1965). Fuzzy sets. Information Control, 8(3): 338-353.
[2] Atanassov, K. T. (1986). Instuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1): 87- 96.
[3] Smarandache, F. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Lgics, American Research Press, Reoboth, NM, USA, 1998.
[4] Molodtsov, D. (1999). Soft set theory—first results. Computers & mathematics with applications, 37(4-5), 19-31.
[5] Yang, X., Lin, T. Y., Yang, J., Li, Y., & Yu, D. (2009). Combination of interval-valued fuzzy set and soft set. Computers & Mathematics with Applications, 58(3), 521-527.
[6] Ashraf Al-Quran, Faisal Al-Sharqi, Zahari Md. Rodzi, Mona Aladil, Rawan A. shlaka, Mamika Ujianita Romdhini, Mohammad K. Tahat, Obadah Said Solaiman. (2023). The Algebraic Structures of Q-Complex Neutrosophic Soft Sets Associated with Groups and Subgroups. International Journal of Neutrosophic Science, 22 (1), 60-76.
[7] Z. bin M. Rodzi et al., "Integrated Single-Valued Neutrosophic Normalized Weighted Bonferroni Mean (SVNNWBM)-DEMATEL for Analyzing the Key Barriers to Halal Certification Adoption in Malaysia," Int. J. Neutrosophic Sci., vol. 21, no. 3, pp. 106–114, 2023.
[8] Zail, S. H., Abed, M. M., & Faisal, A. S. (2022). Neutrosophic BCK-algebra and Ω-BCK-algebra. International Journal of Neutrosophic Science, 19(3), 8-15.
[9] A. Al-Quran, F. Al-Sharqi, A. U. Rahman and Z. M. Rodzi, The q-rung orthopair fuzzy-valued neutrosophic sets: Axiomatic properties, aggregation operators and applications. AIMS Mathematics, 9(2), 5038-5070, 2024.
[10] F. Al-Sharqi, A. Al-Quran and Z. M. Rodzi, Multi-Attribute Group Decision-Making Based on Aggregation Operator and Score Function of Bipolar Neutrosophic Hypersoft Environment, Neutrosophic Sets and Systems, 61(1), 465-492, 2023.
[11] Liu, Z., Qin, K., & Pei, Z. (2014). Similarity measure and entropy of fuzzy soft sets. The Scientific World Journal, 2014.
[12] Saleh, S. (2012). On category of interval valued fuzzy topological spaces. Ann. Fuzzy Math. Inform, 4(2), 385-392.
[13] Al-Sharqi, F., Al-Quran, A., & Romdhini, M. U. (2023). Decision-making techniques based on similarity measures of possibility interval fuzzy soft environment. Iraqi Journal for Computer Science and Mathematics, 4(4), 18-29.
[14] F. Al-Sharqi, Y. Al-Qudah and N. Alotaibi, Decision-making techniques based on similarity measures of possibility neutrosophic soft expert sets, Neutrosophic Sets and Systems, vol. 55, pp. 358-382, 2023.
[15] M. U. Romdhini, F. Al-Sharqi, A. Nawawi, A. Al-Quran and H. Rashmanlou, Signless Laplacian Energyof Interval-Valued Fuzzy Graph and its Applications, Sains Malaysiana 52(7), 2127-2137, 2023
[16] Khan, A., Yang, M. S., Haq, M., Shah, A. A., & Arif, M. (2022). A new approach for normal parameter reduction using σ-algebraic soft sets and its application in multi-attribute decision making. Mathematics, 10(8), 1297.
[17] A. Al Quran, A. G. Ahmad, F. Al-Sharqi, A. Lutfi, Q-Complex Neutrosophic Set, International Journal of Neutrosophic Science, vol. 20(2), pp.08-19, 2023.
[18] F. Al-Sharqi, A.G. Ahmad, A. Al-Quran, Interval-valued neutrosophic soft expert set from real space to complex space, Computer Modeling in Engineering and Sciences, vol. 132(1), pp. 267–293, 2022.
[19] Abu Qamar, M., & Hassan, N. (2019). An approach toward a Q-neutrosophic soft set and its application in decision making. Symmetry, 11(2), 139.
[20] Adam, F., & Hassan, N. (2014). Q-fuzzy soft set. Appl. Math. Sci, 8(174), 8689-8695.
[21] Broumi, S. (2015). Q-intuitionistic fuzzy soft sets. Journal of New Theory, (5), 81-91.
[22] Dalkılıç, O., and Demirtaş, N. (2022). A novel perspective for Q-neutrosophic soft relations and their application in decision making. Artificial Intelligence Review, 1-21.
[23] Abuqamar, M., and Hassan, N. (2022). The Algebraic Structure of Normal Groups Associated with Q-Neutrosophic Soft Sets. Neutrosophic Sets and Systems, 48, 328-338.
[24] F. Al-Sharqi, M. U. Romdhini, A. Al-Quran, Group decision-making based on aggregation operator and score function of Q-neutrosophic soft matrix, Journal of Intelligent and Fuzzy Systems, vol. 45, pp.305–321, 2023.
[25] Jamiatun Nadwa Ismail et al. The Integrated Novel Framework: Linguistic Variables in Pythagorean Neutrosophic Set with DEMATEL for Enhanced Decision Support. Int. J. Neutrosophic Sci., vol. 21, no. 2, pp. 129-141, 2023.
[26] Abed, M. M., Al-Jumaili, A. F., Al-sharqi, F. G. Some mathematical structures in a topological group. Journal of Algebra and Applied Mathematics. 2018, 16(2), 99-117.

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024

[27] Al-Hijjawi, S., Ahmad, A. G., and Alkhazaleh, S. (2022). Time Q-Neutrosophic Soft Expert Set. International Journal of Neutrosophic Science (IJNS), 19(1).
[28] M. M. Abed, F. Al-Sharqi, and S. H. Zail, "A Certain Conditions on Some Rings Give PP Ring," in Journal of Physics: Conference Series, 2021, vol. 1818, no. 1, p. 12068.
[29] Al-Sharqi, F.; Al-Quran, A.; Ahmad, A. G. Mapping on Interval Complex Neutrosophic Soft Sets. International Journal of Neutrosophic Science, 2022, 19(4), 77-85.
[30] F. Al-Sharqi, A.G. Ahmad, A. Al-Quran, Interval complex neutrosophic soft relations and their application in decision-making, Journal of Intelligent and Fuzzy Systems, vol. 43 , pp. 745-771, 2022.

DOI: https://doi.org/10.54216/IJNS.240306
Received: November 24, 2023 Revised: February 22, 2024 Accepted: May 11, 2024