Neutrosophic Fuzzy Interval Sets and its Extension through MCDM and Applications in E-Management

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Abstract
we are introducing the model-type operators over Interval-Valued Fuzzy Neutrosophic Sets with time moments [IVFNS] and learn a few of their properties with numerical examples to demonstrate the defined operations and operators. Also introduce various distance measures over the extension of interval neutrosophic sets as well as apply the introduced measures in ecological management in this direct to decide the type of corrosion disturbing some towns for valuable management to be taken, using this normalized distance measures. The extensions of neutrosophic connection values and non-connection values be not used for all time probable positive to our fulfillment, but this concept IVTNFS part has more significant responsibility at this point since the time progress with IVN-fuzzy sets provide the accurate solution in factual situations similarly, conclusion making, career deciding and so on. This is the main reason for taking in the extensions of neutrosophic sets.

Keywords: Intuitionistic Fuzzy Sets (IFS); Temporal Intuitionistic Fuzzy Sets (TIFS); Neutrosophic Set; Interval Valued Neutrosophic Set; Interval Valued Intuitionistic Fuzzy Sets (IVIFS) and Interval Valued Temporal Neutrosophic Fuzzy Sets (IVTNFS).

Subject Classification: 03E72, 03D45, 03B20, 03F55.

1. Introduction

In 1965, Lotfi A. Zadeh innovated the notion of fuzzy sets [26] whose elementary part is only a degree of membership. Further in 1986 [2], Krassimir T. Atanassov comprehensive this notion to IFS expending the degree of membership value with the degree of non-membership value, below the limit for the totality of two non-membership value and membership value is fewer than or else equal to 1. An unclear set be able to be well-thought-out as IFS, meanwhile, the totality of these evaluations is one. However, there exists a multiplicity of situations while the summation is lesser than 1, which way that here is a definite uncertainty in options of the non-membership function or the membership function. Since the origination of IFS, plentiful scholars have exposed their attention to the concept and also exploited it in numerous territories, including model finding, apparatus culture, similarity processing choice building, and so on. Lots of writers demonstrate several consequences make use of this concept of IFS. Further, in 1994, Atanassov K.T. introduced the latest operations that were defined on IFS[3]. Later in 2000, Supriya Kumar [8], A. R. Roy, and Ranjit Biswas proposed certain operations over IFS and also applied this concept to health diagnostics in 2001. Interval fuzzy Sets were introduced through Zadeh as an enlargement lead on unclear sets in the principles of the value of membership grade are time interval of quantities as a substitute for the numbers. Thus, the IVFS provides an extra satisfactory explanation of cognitive state than the traditionalistic FS. Therefore the IVFS is more essential in

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

real-life applications. In 1989, Gargov and K.T. Atanassov [5] presented the concept of IVIFS which is a generality of together IFS and IVFS with the succeeding development of IVIFS, numerous examiners stated their kindness to the conception and as well used it in a variety of areas. Afterward in 1994 [2], Krassimir T. Atanassov innovated the functions of IVIFS. Zeshui Xu [24] introduced methods planned for get-together intuitionistic fuzzy data with intervals and decisions made in 2007. In 2015, K. Rajesh and R. Srinivasan [10, 11, 12] introduced the idea of IVIFST namely the Second Type of IVIFS, and also studied their basic operations and operators. Further 2017 and 2019 the same authors introduced a variety of distance measures of IVIFSST and TIFS of the second type also applied the distance procedures in career determination and choosing a pattern & medical diagnosis, later in 2022, K. Rajesh [13] introduced Certain Level Operators of TIFS. Smarandache [19] has introduced the term Neutrosophic Set [NS] and its wide like Neutrosophic measure, Neutrosophic fit, Neutrosophic philosophy, the method of Multi-Moora, and so on. The Neutrosophic sets usage uncertainty as a liberated measure of non-membership and membership information.

In 2005, Florentin Smarandache [22] invited the notion of IVNS namely Neutrosophic Set with Interval, wherever the membership status of every part is related with neutrosophic mechanisms, i.e., reality, uncertainness & falseness of relationship positions. In 2024, K. Rajesh [14] introduced the conception of IV - interval valued temporal Neutrosophic Fuzzed Sets [IVTNFS] with numerical examples and also establish few operations of IVTNFS. IVFNS were applied in various fields like relational stores, inclination structures, neutrosophic expert systems reliability theory, and so on. The IVNFS of non-membership values and membership values be not all the time probable up in the direction of our satisfaction. Still, the IVTNFS component has an additional significant role here, for the reason that the period of IVNFS provides the greatest solvent to determination the fleeting distance in decision-making, career deciding, and so on. The extensions of Neutrosophic Sets is most significant in real-life conditions and its greatest in choice building similar to profession determination, image treatment, health opinion, etc. This is the main reason for taking in the widen of neutrosophic Set.

The association of this manuscript is as follows: We present several essential definitions which will be presented in the paper in the 2nd section. In the 3rd section, we define some model-type operators over IVTNFS and also inspect a few of its properties. In the 4th section, we introduced various distance measures with examples and applied the measure in ecological management in section 5th and lastly, we concluded in the last segment.

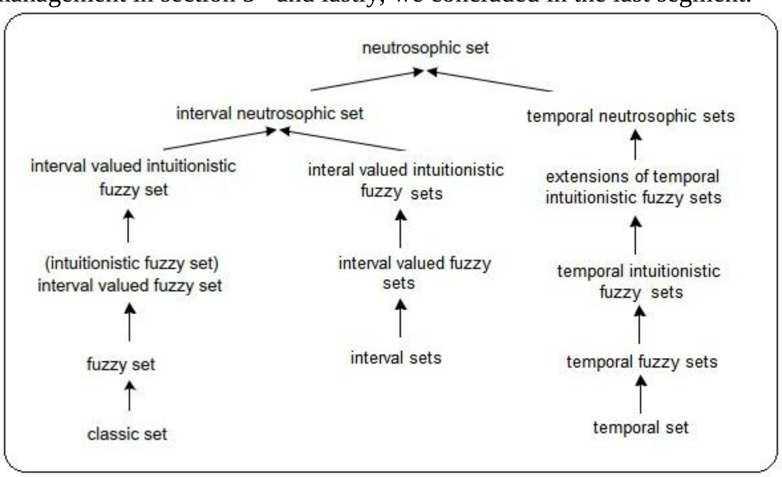

Figure 1: Branch of neutrosophic set

2. Preliminaries

We gave some fundamental definitions related to this research

Definition 2.1 [2] Let G be not an blank set. An IFS A in G is define the same as seeing that an item of the subsequent outline.

$$A = \{< x, \alpha_A(r), \beta_A(r) > | \, r \in G\}$$

anywhere the function $\square_A : G \to [0,1]$ and $\square_A : G \to [0,1]$ indicate the membership quantity and the non-membership quantity of the factor $r \in G$, in that order along with for each $r \in G$.

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

Definition 2.2 [6] Let G be not an blank set. An TIFS A in G is define the same as seeing that an item of the subsequent outline.

$$A = \{< x, \alpha_A(r,t), \beta_A(r,t) > | < r,t > \in G \times T\}$$

anywhere the function $\square_A : G \to [0,1]$ and $\square_A : G \to [0,1]$ indicate the membership quantity and the non-membership quantity of the factor $r \in G$, in that order along with for each $r \in G$ with and the time – moment $t \in T$.

$$0 \le \alpha_A(r,t) + \beta_A(r,t) \le 1.$$

Definition 2.3 [5]Let G bea not an unfilled set. An IVIFS-Interval Valued IFS A in G is define seeing that an item of subsequent outline.

$$A = \{< r, M_A(r,t), N_A(r,t) > | r \in G\}$$

where M_A & N_A: G → [0,1]. The interval $M_A(r,t)$ and $N_A(r,t)$indicate membership grade and non-membership grade of the component G to a deposit A, where $M_A(r,t) = [inf\ inf\ M_A\ (r,t), sup\ sup\ M_A\ (r,t)]$ and $N_A(r,t) = [inf\ inf\ N_A\ (r,t), sup\ sup\ N\ (r,t)]$ in the company of the circumstance to facilitate $M_{AU}(r,t) + N_{AU}(r,t) \le 1$ for the entire $r \in G$.

Definition 2.4 [18] Let G exist a entire set plus $r \in G$. A NS-set A inside G be characterize by the truth association value, indeterminacy association value and falseness association value functions in that order and its denote as $T_A, I_A$and F_A as well defined the NS in the following form

$$A = \{< r, T_A(r), I_A(r), F_A(r) > | r \in G\}$$

The function $T_A(r), I_A(r)$ and $F_A(r)$ be existentregular or else non-regular subsets of $]0^-, 1^+[$ .i.e., $T_A(r), I_A(r)$ & $F_A(r) : G \to]0^-, 1^+[$ refusal limitation is apply on the addition of $T_A(r)$, $I_A(r)$ and$F_A(r)$,

$$0^- \le (r) + I_{AU} + F_{AU} \le 3^+$$

intended for a fixed $r \in G$. $T_A(r), I_A(r)$ and $F_A(r)$is call the neutrosophic numeral.

Definition 2.5 [23] Let G be not an blank set. plus $r \in G$. A NS-set A inside G be characterize by the truth association value, indeterminacy association value and falseness association value functions in that order and its denote as $T_A, I_A$and F_A as well defined the NS in the following form

$$A = \{< r, \mu_A(r), T_A(r,\mu), I_A(r,\mu), F_A(r) > | r \in G\}$$

The function $T_A(r), I_A(r)$ and $F_A(r)$ be existentregular or else non-regular subsets of $]0^-, 1^+[$ .i.e., $T_A(r), I_A(r)$ & $F_A(r) : G \to]0^-, 1^+[$ refusal limitation is apply on the addition of $T_A(r)$, $I_A(r)$ and$F_A(r)$,

$$0^- \le (r) + I_{AU} + F_{AU} \le 3^+$$

intended for a fixed $r \in G$. $T_A(r), I_A(r)$ and $F_A(r)$is call the Neutrosophic FS.

Definition 2.6 [23] Let G exist a universal set followed by an IVNFS A be define as the subsequent form

$$A = \{< r, [T_{AL}(r), T_{AU}(r)], [I_{AL}(r), I_{AU}(r)]\ [F_{AL}(r), F_{AU}(r)] > | r \in G\}$$

where the functions $T_A(r), I_A(r)$ & $F_A(r) : G \to]0^-, 1^+[$ and $0 \le T_{AU}(r) + F_{AU}(r) + I_{AU}(r) \le 3$.

Definition 2.7 [14] Let G be not an blank set, subsequently the extension of NS namely interval valued TNFS A in G is defined in the subsequent appearance

$$A = \{< r, [\mu_{AL}(r,t), \mu_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)], [I_{AL}(r,t), I_{AU}(r,t)]\ [F_{AL}(r,t), F_{AU}(r,t)] > | (r,t) \in G \times T\}$$

wherever the function $T_A(r,t), \mu_A(r,t), F_A(r,t)$& $I_A(r,t) : G \to]0^-, 1^+[$ in addition to

$$0 \le T_{AU}(r,t) + F_{AU}(r,t) + I_{AU}(r,t) \le 3$$

Definition 2.8 [14] Let A along with B exist any non-empty sets and its lies in the IVTNFS on G. Then A is call a subset of B, and its denote by $A \subseteq B$ if

$$T_{AL}(r,t) \le T_{BL}(r,t), T_{AU}(r,t) \le T_{BU}(r,t),$$
$$I_{AL}(r,t) \le I_{BL}(r,t), I_{AU}(r,t) \le I_{BU}(r,t),$$
$$F_{AL}(r,t) \le F_{BL}(r,t), F_{AU}(r,t) \le F_{BU}(r,t), \forall (r,t) \in G \times T$$

Definition 2.9 [14] A along with B be any not an blank set and its lies in the IVTNFS on G. Then, the junction of A and B is denote by $A \cap B$ and the combination of A and B is denote by $A \cup B$ in that order and it is defined have the subsequent structure

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

$$A \cap B = \{(r,t), < [\min[\mu_{AL}(r,t), \mu_{BL}(r,t)], \min[\mu_{AU}(r,t), \mu_{BU}(r,t)]], [\min[T_{AL}(r,t), T_{BL}(r,t)], \min[T_{AU}(r,t), T_{BU}(r,t)]], [\max[I_{AL}(r,t), I_{BL}(r,t)], \max[I_{AU}(r,t), I_{BU}(r,t)]] [\max[F_{AL}(r,t), F_{BL}(r,t)], \max[F_{AU}(r,t), F_{BU}(r,t)]] > |(r,t) \in G \times T\}$$

$$A \cup B = \{(r,t), < [\max[\mu_{AL}(r,t), \mu_{BL}(r,t)], \max[\mu_{AU}(r,t), \mu_{BU}(r,t)]], [\max[T_{AL}(r,t), T_{BL}(r,t)], \max[T_{AU}(r,t), T_{BU}(r,t)]], [\min[I_{AL}(r,t), I_{BL}(r,t)], \min[I_{AU}(r,t), I_{BU}(r,t)]] [\min[F_{AL}(r,t), F_{BL}(r,t)], \min[F_{AU}(r,t), F_{BU}(r,t)]] > |(r,t) \in G \times T\}$$

Definition 2.10 [14] Let A exist a any non - unfilled sets and its lies in the IVTNFS on G. Then, the accompaniment of A is denote by A^c and it is define in the subsequent structure

$$A^c = \{< r, [\mu_{AL}(r,t), \mu_{AU}(r,t)], [F_{AL}(r,t), F_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)], [1 - I_{AL}(r,t), 1 - I_{AU}(r,t)] > |(r,t) \in G \times T\}$$

3. New Operators over IVTNFS

We initiate the possibility and necessity operators over the extensions of Neutrosophic Sets with numerical examples and also established a few of its properties.

Definition: 3.1 Let A be a any non - unfilled sets and its lies in the IVTNF on G. Then, the Necessity operator is define in the subsequent structure

$$\square(A) = \{< (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)], [I_{AL}(r,t), I_{AU}(r,t)], [F_{AL}(r,t), 1 - T_{AU}(r,t)] > |(r,t) \in G \times T\}$$

Example: 3.2 Let us choose a IVTN fuzzy set A on G given by

$$A = \{< [a, 0.2, 0.4], [0.6, 0.3], [0.3, 0.5], [0.2.0.3] >, < [b, 0.6, 0.8], [0.5, 0.6], [0.1, 0.4], [0.4, 0.7] >\}$$

Then,

$$\square(A) = \{< [a, 0.2, 0.4], [0.6, 0.3], [0.3, 0.5], [0.2.0.7] >, < [b, 0.6, 0.8], [0.5, 0.6], [0.1, 0.4], [0.4, 0.3] >\}$$

Definition: 3.3 Let A be any non - unfilled sets and it lies in the IVTNF on G. Then, the Possibility operator is defined in the subsequent structure

$$\lozenge(A) = \{< (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)], [T_{AL}(r,t), 1 - F_{AU}(r,t)], [I_{AL}(r,t), I_{AU}(r,t)], [F_{AL}(r,t), F_{AU}(r,t)] > |(r,t) \in G \times T\}$$

Example: 3.4 Let us contemplate a IVTNFS A on G given by

$$A = \{< [a, 0.3, 0.5], [0.4, 0.2], [0.4, 0.5], [0.2.0.6] >, < [b, 0.3, 0.4], [0.3, 0.6], [0.1, 0.8], [0.5, 0.4] >\}$$

Then,

$$\lozenge(A) = \{< [a, 0.3, 0.5], [0.4, 0.8], [0.4, 0.5], [0.2.0.6] >, < [b, 0.3, 0.4], [0.3, 0.4], [0.1, 0.8], [0.5, 0.4] >\}$$

Proposition 3.5 Let A is an IVTN fuzzy sets on G. After that, we can examine the subsequent

i. $\square(\square(A)) = \square(A)$

ii. $(\lozenge(A^c))^c = \square(A)$

iii. $(\square(A^c))^c = \lozenge(A)$

iv. $\lozenge(\lozenge(A)) = \lozenge(A)$

Proof: Let

$$A = \{< (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)], [I_{AL}(r,t), I_{AU}(r,t)], [F_{AL}(r,t), F_{AU}(r,t)] > |(r,t) \in G \times T\}$$

By the definition of the complement of a set, necessity operator and possibility operator,

$$A^c = \{< r, [\mu_{AL}(r,t), \mu_{AU}(r,t)], [F_{AL}(r,t), F_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)], [1 - I_{AL}(r,t), 1 - I_{AU}(r,t)] > |(r,t) \in G \times T\}$$

$$\square(A) = \{< (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)],$$

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

$$[\, I_{AL}(r,t)\ , I_{AU}(r,t)]\ ,[\, F_{AL}(r,t)\ ,1-\ T_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$

$$\Diamond\,(A) = \{ < (r,t), [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)]\ ,[\, T_{AL}(r,t)\ ,1-\ F_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t)\ , I_{AU}(r,t)]\ ,[\, F_{AL}(r,t)\ , F_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$

Apply the necessity operator to the necessity operator, we have

$$\Box(\Box(A))\ = \Box\{< (r,t), [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, T_{AL}(r,t)\ , T_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t), I_{AU}(r,t)]\ ,[\, F_{AL}(r,t)\ ,1-\ T_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$= \Box\{< (r,t), [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, T_{AL}(r,t), T_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t), I_{AU}(r,t)]\ ,[\, F_{AL}(r,t), F_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$= \{ < (r,t), [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, T_{AL}(r,t), T_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t), I_{AU}(r,t)], [\, F_{AL}(r,t)\ ,1-\ T_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$= \Box A$$

This is proved the first part of the proposition.

(ii) We know that the definition of complement A^c,

$$A^c = \{< r, [\mu_{AL}(r,t), \mu_{AU}(r,t)], [F_{AL}(r,t), F_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)],$$
$$[1 - I_{AL}(r,t), 1-\ I_{AU}(r,t)]\ > \ |(r,t) \in G\times T\}$$

Apply the possibility operator to the complement of a set A

$$\Diamond\,(A^c) = \Diamond\,\{(r,t), < [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, F_{AL}(r,t), F_{AU}(r,t)], [1-\ I_{AU}(r,t)\ , 1-\ I_{AL}(r,t)\ ], [(r,t) \in G\times T\}$$
$$= \{(r,t), < [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, F_{AL}(r,t)\ ,1-\ T_{AU}(r,t)]\ ,$$
$$[1-\ I_{AU}(r,t)\ , 1-\ I_{AL}(r,t)\ ], [(r,t)\ \in G\times T\}$$
$$= \{< (r,t), [\mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, T_{AL}(r,t), T_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t), I_{AU}(r,t)]\ ,[\, F_{AL}(r,t)\ ,1-\ T_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$= \Box A$$

This implies the second part of the proposition and the third part of the proof is similar.

(iv) Apply the possibility operator to the possibility operator, we have

$$\Diamond(\Diamond\,(A)) = \Diamond\,\{< (r,t), [\mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [T_{AL}(r,t)\ ,1-\ \ F_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t)\ , I_{AU}(r,t)]\ ,[\, F_{AL}(r,t)\ , F_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$= \Diamond\,\{ < (r,t), [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, T_{AL}(r,t)\ , T_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t)\ , I_{AU}(r,t)]\ ,[\, F_{AL}(r,t), F_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$= \{ < (r,t), [\, \mu_{AL}(r,t)\ , \mu_{AU}(r,t)], [\, T_{AL}(r,t)\ ,1-\ F_{AU}(r,t)]\ ,$$
$$[\, I_{AL}(r,t)\ , I_{AU}(r,t)]\ ,[\, F_{AL}(r,t)\ , F_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$= \Diamond\,(A).$$

This is proved the fourth part of this proposition.

Proposition 3.6 Let A along with B be any two IVTN fuzzy sets on G. Then, we have the following

i. $\Box(A \cup B) = \Box A \cup \Box B$

ii. $\Box(A \cap B) = \Box A \cap \Box B$

iii. $\Diamond\,(A \cup B) = \Diamond\,A \cup \Diamond\,B$

iv. $\Diamond\,(A \cap B) = \Diamond\,A\ \cap \Diamond\,B$

Proof: Let

$$A\ = \{ < (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)]\ ,[\, T_{AL}(r,t), T_{AU}(r,t)]\ ,$$
$$[I_{AL}(r,t)\ , I_{AU}(r,t)]\ ,[\, F_{AL}(r,t), F_{AU}(r,t)]\ \ > |(r,t) \in G\times T\}$$
$$B\ = \{ < (r,t), [\mu_{BL}(r,t), \mu_{BU}(r,t)]\ ,[\, T_{BL}(r,t), T_{BU}(r,t)]\ ,$$
$$[I_{BL}(r,t), I_{BU}(r,t)], [\, F_{BL}(r,t), F_{BU}(r,t)]\ \ > |(r,t) \in G\times T\}$$

Union between two sets is defined by

$$A \cup B = \{(r,t), < [\max[\mu_{AL}(r,t), \mu_{BL}(r,t)]\ , \max[\mu_{AU}(r,t), \mu_{BU}(r,t)]\ ],$$
$$[\max[T_{AL}(r,t), T_{BL}(r,t)]\ , \max[T_{AU}(r,t), T_{BU}(r,t)]\ ],$$
$$[\min[I_{AL}(r,t), I_{BL}(r,t)], \min[I_{AU}(r,t), I_{BU}(r,t)]\ ]$$
$$[\min[F_{AL}(r,t), F_{BL}(r,t)]\ , \min[F_{AU}(r,t), F_{BU}(r,t)]\ ] > |(r,t) \in G\times T\}$$

Apply the necessity operator we've

$$\Box(A \cup B) = \Box\{(r,t), < [\max[\mu_{AL}(r,t), \mu_{BL}(r,t)]\ , \max[\mu_{AU}(r,t), \mu_{BU}(r,t)]\ ],$$
$$[\max[T_{AL}(r,t), T_{BL}(r,t)]\ , \max[T_{AU}(r,t), T_{BU}(r,t)]\ ],$$

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

$$[\min[I_{AL}(r,t), I_{BL}(r,t)], \min[I_{AU}(r,t), I_{BU}(r,t)]]$$
$$[\min[F_{AL}(r,t), F_{BL}(r,t)], \min[F_{AU}(r,t), F_{BU}(r,t)]] > |(r,t) \in G \times T\}$$
$$= \{(r,t), < [\max[\mu_{AL}(r,t), \mu_{BL}(r,t)], \max[\mu_{AU}(r,t), \mu_{BU}(r,t)]],$$
$$[\max[T_{AL}(r,t), T_{BL}(r,t)], \max[T_{AU}(r,t), T_{BU}(r,t)]],$$
$$[\min[I_{AL}(r,t), I_{BL}(r,t)], \min[I_{AU}(r,t), I_{BU}(r,t)]]$$
$$[\min[F_{AL}(r,t), F_{BL}(r,t)], 1 - \max[T_{AU}(r,t), T_{BU}(r,t)]] > |(r,t) \in G \times T\}$$
$$= \{ < (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)], [T_{AL}(r,t), T_{AU}(r,t)], [I_{AL}(r,t), I_{AU}(r,t)],$$
$$[F_{AL}(r,t), 1 - T_{AU}(r,t)] > |(r,t) \in G \times T\} \cup \{ < (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)],$$
$$[T_{AL}(r,t), T_{AU}(r,t)], [I_{AL}(r,t), I_{AU}(r,t)], [F_{AL}(r,t), 1 - T_{AU}(r,t)] > |(r,t) \in G \times T\}$$
$$= \Box A \cup \Box B.$$

This implies the first part of the proposition, the second part of the verification is similar.

We know that

$$A \cap B = \{(r,t), < [\min[\mu_{AL}(r,t), \mu_{BL}(r,t)], \min[\mu_{AU}(r,t), \mu_{BU}(r,t)]],$$
$$[\min[T_{AL}(r,t), T_{BL}(r,t)], \min[T_{AU}(r,t), T_{BU}(r,t)]],$$
$$[\max[I_{AL}(r,t), I_{BL}(r,t)], \max[I_{AU}(r,t), I_{BU}(r,t)]]$$
$$[\max[F_{AL}(r,t), F_{BL}(r,t)], \max[F_{AU}(r,t), F_{BU}(r,t)]] > |(r,t) \in G \times T\}$$

Apply the possibility operator

$$\Diamond (A \cap B) = \Diamond \{(r,t), < [\min[\mu_{AL}(r,t), \mu_{BL}(r,t)], \min[\mu_{AU}(r,t), \mu_{BU}(r,t)]],$$
$$[\min[T_{AL}(r,t), T_{BL}(r,t)], \min[T_{AU}(r,t), T_{BU}(r,t)]],$$
$$[\max[I_{AL}(r,t), I_{BL}(r,t)], \max[I_{AU}(r,t), I_{BU}(r,t)]]$$
$$[\max[F_{AL}(r,t), F_{BL}(r,t)], \max[F_{AU}(r,t), F_{BU}(r,t)]] > |(r,t) \in G \times T\}$$
$$= \{(r,t), < [\min[\mu_{AL}(r,t), \mu_{BL}(r,t)], \min[\mu_{AU}(r,t), \mu_{BU}(r,t)]],$$
$$[\min[T_{AL}(r,t), T_{BL}(r,t)], 1 - \min[F_{AU}(r,t), F_{BU}(r,t)]],$$
$$[\max[I_{AL}(r,t), I_{BL}(r,t)], \max[I_{AU}(r,t), I_{BU}(r,t)]]$$
$$[\max[F_{AL}(r,t), F_{BL}(r,t)], \max[F_{AU}(r,t), F_{BU}(r,t)]] > |(r,t) \in G \times T\}$$
$$= \{< (r,t), [\mu_{AL}(r,t), \mu_{AU}(r,t)], [T_{AL}(r,t), 1 - F_{AU}(r,t)],$$
$$[I_{AL}(r,t), I_{AU}(r,t)], [F_{AL}(r,t), F_{AU}(r,t)] > |(r,t) \in G \times T\} \cap$$
$$\{< (r,t), [\mu_{BL}(r,t), \mu_{BU}(r,t)], [T_{BL}(r,t), 1 - F_{BU}(r,t)],$$
$$[I_{BL}(r,t), I_{BU}(r,t)], [F_{BL}(r,t), F_{BU}(r,t)] > |(r,t) \in G \times T\}$$
$$= \Diamond A \cap \Diamond B.$$

This implies that the fourth part is proved and the third part of proof is similar.

4. Various distance measure over IVTN Fuzzy Sets

Here, we define various distance measures over the extension of neutrosophic set with numerical example.

Definition 4.1. Let G exist non - unfilled set such that IVTNFS $A, B, C \in G$. The IVTNFS is the distance dealing among A & B is $d: G \times G \rightarrow [0,1]$; if $d(A,B)$ satisfy the subsequent conditions.

A_1: $0 \leq d(A,B) \leq 1$

A_2: $d(A,B) = 0$ iff $A = B$

A_3: $d(A,B) = d(B,A)$

A_4: $d(A,C) + d(B,C) \geq d(A,B)$

A_5: if $A \subseteq B \subseteq C$, then $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$.

Definition 4.2. Let A along with B exist any two IVTNFS on $G = \{x_1, x_2, \ldots, x_n\}$, then the variety of distance dealings are define as the subsequent form

4.2.1 Hamming Distance

$$d_{HIVTNFS}(A,B) = \frac{1}{8}\sum_{j=1}^{n}\left\{\begin{array}{c} |\mu_{AL}(x_j,t) - \mu_{BL}(x_j,t)| + |\mu_{AU}(x_j,t) - \mu_{BU}(x_j,t)| + \\ |T_{AL}(x_j,t) - T_{BL}(x_j,t)| + |T_{AU}(x_j,t) - T_{BU}(x_j,t)| + \\ |I_{AL}(x_j,t) - I_{BL}(x_j,t)| + |I_{AU}(x_j,t) - I_{BU}(x_j,t)| + \\ |F_{AL}(x_j,t) - F_{BL}(x_j,t)| + |F_{AU}(x_j,t) - F_{BU}(x_j,t)| \end{array}\right\}$$

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

4.2.2 Normalized $d_{HIVTNFS}(A,B)$

$$d_{NHIVTNFS}(A,B) = \frac{1}{8n}\sum_{j=1}^{n}\left\{\begin{array}{c}|\mu_{AL}(x_j,t) - \mu_{BL}(x_j,t)| + |\mu_{AU}(x_j,t) - \mu_{BU}(x_j,t)| + \\ |T_{AL}(x_j,t) - T_{BL}(x_j,t)| + |T_{AU}(x_j,t) - T_{BU}(x_j,t)| + \\ |I_{AL}(x_j,t) - I_{BL}(x_j,t)| + |I_{AU}(x_j,t) - I_{BU}(x_j,t)| + \\ |F_{AL}(x_j,t) - F_{BL}(x_j,t)| + |F_{AU}(x_j,t) - F_{BU}(x_j,t)|\end{array}\right\}$$

4.2.3 Euclidean Distance

$$d_{EIVTNFS}(A,B) = \frac{1}{4}\sum_{j=1}^{n}\left\{\begin{array}{c}\left(\mu_{AL}(x_j,t) - \mu_{BL}(x_j,t)\right)^2 + \left(\mu_{AU}(x_j,t) - \mu_{BU}(x_j,t)\right)^2 + \\ \left(T_{AL}(x_j,t) - T_{BL}(x_j,t)\right)^2 + \left(T_{AU}(x_j,t) - T_{BU}(x_j,t)\right)^2 + \\ \left(I_{AL}(x_j,t) - I_{BL}(x_j,t)\right)^2 + \left(I_{AU}(x_j,t) - I_{BU}(x_j,t)\right)^2 + \\ \left(F_{AL}(x_j,t) - F_{BL}(x_j,t)\right)^2 + \left(F_{AU}(x_j,t) - F_{BU}(x_j,t)\right)^2\end{array}\right\}^{0.5}$$

4.2.4 Normalized $d_{EIVTNFS}(A,B)$

$$d_{EIVTNFS}(A,B) = \frac{1}{4\sqrt{n}}\sum_{j=1}^{n}\left\{\begin{array}{c}\left(\mu_{AL}(x_j,t) - \mu_{BL}(x_j,t)\right)^2 + \left(\mu_{AU}(x_j,t) - \mu_{BU}(x_j,t)\right)^2 + \\ \left(T_{AL}(x_j,t) - T_{BL}(x_j,t)\right)^2 + \left(T_{AU}(x_j,t) - T_{BU}(x_j,t)\right)^2 + \\ \left(I_{AL}(x_j,t) - I_{BL}(x_j,t)\right)^2 + \left(I_{AU}(x_j,t) - I_{BU}(x_j,t)\right)^2 + \\ \left(F_{AL}(x_j,t) - F_{BL}(x_j,t)\right)^2 + \left(F_{AU}(x_j,t) - F_{BU}(x_j,t)\right)^2\end{array}\right\}^{0.5}$$

Example 4.3. Let us choose two IVTNFS A & B in $G = \{a, b, c\}$ as follows:
$A = \{\langle[0.6, 0.3], [0.2, 0.2], [0.2, 0.5], [0.5, 0.3]\rangle, \langle[0.3, 0.5], [0.4, 0.3], [0.1, 0.7], [0.6, 0.2]\rangle, \langle[0.1, 0.5], [0.4, 0.3], [0.1, 0.3], [0.2, 0.6]\rangle\}$
$B = \{\langle[0.5, 0.2], [0.4, 0.3], [0.1, 0.5], [0.4, 0.3]\rangle, \langle[0.1, 0.3], [0.2, 0.2], [0.2, 0.5], [0.5, 0.4]\rangle \langle[0.4, 0.6], [0.5, 0.7], [0.1, 0.4], [0.7, 0.2]\rangle\}$
Then,
i) $d_{HIVTNFS}(A,B)$ is 0.49
ii) $d_{NHIVTNFS}(A,B)$ is 0.02
iii) $d_{EIVTNFS}(A,B)$ is 0.25
iv) $d_{NEIVTNFS}(A,B)$ is 0.15.

Since the above outcome, we assume that the $d_{NHIVTNFS}(A,B)$ give the most excellent result. Because it gives very short distance for that reason, we are used the $d_{NHIVTNFS}(A,B)$- distance in the application used for its soaring pace of guarantee in the conditions of precision.

5. Applications of IVTNFuzzy Sets in ecological management

The concept IVTNFS branch has more significant responsibility here since the instant progress through interval in the NFS provides the accurate clarification in existent - time situation like, choice creation, career deciding and many other fields. Let us think about a multi - characteristic cluster management difficulties in the IVTNFS nature, the common process for the multi - characteristic cluster management can be shortened below.

Assume that there bea four towns, K given as a set $K = \{K_1, K_2, K_3, K_4\}$ to be surveyed for the four set of corrosion types given as: Q = {water corrosion, gully corrosion, wind corrosion and tunnel corrosion}, which contain the set of causes: R = {cultivation, boorish design, grazing, overflow water, reduced vegetation}. Since the study understanding, Table 1 contains each corrosion and its causes in IVTNFS values.

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

Table 1:Corrosion vs Causes

	Cultivation	reduced vegetation	grazing	overflow water
Water corrosion	[0.2,0.2], [0.4,0.3], [0.5,0.2], [0.4,0.6]	[0.3,0.4], [0.2,0.3], [0.6,0.2], [0.1,0.2]	[0.4,0.4], [0.2,0.4], [0.5,0.6], [0.3,0.2]	[0.3,0.5], [0.2,0.1], [0.4,0.3], [0.4,0.2]
Gully corrosion	[0.3,0.5], [0.4,0.3], [0.2,0.4], [0.2,0.6]	[0.2,0.3], [0.3,0.5], [0.4,0.4], [0.8,0.4]	[0.3,0.3], [0.3,0.4], [0.5,0.4], [0.4,0.5]	[0.2,0.8], [0,0.2], [0.4,0.2], [0.3,0.3]
Wind corrosion	[0.2,0.5], [0.1,0.3], [0.5,0.2], [0.4,0.7]	[0.4,0.3], [0.5,0.2], [0.3,0.2], [0.6,0.4]	[0.4,0.5], [0.3,0.4], [0.2,0.3], [0.2,0.3]	[0.3,0.5], [0.6,0.4], [0.3,0.4], [0.3,0.1]
Tunnel corrosion	[0.5,0.4], [0.5,0.2], [0.4,0.3], [0.6,0.8]	[0.5,0.2], [0.1,0.6], [0.4,0.2], [0.3,0.4]	[0.1,0.2], [0.4,0.5], [0.4,0.3], [0.3,0.6]	[0.2,0.4], [0.3,0.6], [0.5,0.5], [0.2,0.1]

In Table 1, is describedR_i is describe by four numbers in conditions of the interval, i.e., fuzz membership μ_i, Truth values T_i, Indeterminacy $I_i$and Falsity F_i. deliberate for the sake of control compute, we supposed to facilitate the survey grades were collect from the towns and analyzed, the obtained grades be obtainable in Table 1.

Table 2:Towns vs Causes

	Cultivation	reduced vegetation	Grazing	overflow water
K_1	[0.6,0.2], [0.4,0.3], [0.4,0.4], [0.2,0.4]	[0.5,0.6], [0.1,0.2], [0.5,0.4], [0.3,0.6]	[0.4,0.3], [0.5,0.2], [0.2,0.4], [0.3,0.7]	[0.4,0.3], [0.5,0.2], [0.6,0.2], [0.3,0.4]
K_2	[0.4,0.3], [0.2,0.5], [0.6,0.3], [0.3,0.5]	[0.2,0.3], [0.4,0.2], [0.4,0.2], [0.4,0.2]	[0.4,0.2], [0.2,0.2], [0.3,0.2], [0.4,0]	[0.5,0.2], [0.1,0.6], [0.2,0.2], [0.3,0.4]
K_3	[0,0.2], [0.5,0.2], [0.4,0.2], [0.3,0.2]	[0.4,0.3], [0.6,0.7], [0.6,0.1], [0.3,0.8]	[0.1,0.6], [0.7,0.5], [0.2,0.4], [0.6,0.2]	[0.4,0.5], [0.4,0.3], [0.5,0.2], [0.5,0]
K_4	[0.4,0.2], [0.3,0.5], [0.3,0.2], [0.4,0.5]	[0.6,0.3], [0.4,0.5], [0.4,0.3], [0.2,0.5]	[0.5,0.4], [0.5,0.2], [0.3,0.3], [0.5,0.5]	[0.2,0.3], [0.3,0.8], [0.2,0.4], [0.5,0.4]

by using this$d_{NHIVTNFS}(A,B)$– distance over IVTNFS to calculate the shortest distance among the towns and the corrosions with high opinion to every of the causes, we obtain the subsequent tables as exposed below.

Table 3:the shortest distance among towns vs corrosions

	Water corrosion	Gully corrosion	Wind corrosion	Tunnel corrosion
K_1	0.17	**0.16**	0.18	0.18

DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

K_2	**0.17**	0.18	0.23	0.18
K_3	0.19	**0.16**	0.19	0.17
K_4	0.18	0.20	0.20	**0.16**

From Table 3, town K_1 & K_3 is to be managed from Gully corrosion; town K_2 is to be managed by water corrosion and town K_4 is to be managed from Tunnel corrosion.

6. Conclusion

The extensions of IVNFS is a branch of neutrosophy that studied in the beginning, environment as well as their connections through in the various ideational spectra and the span of neutralities. In this study, we are introduced the set-theoretic model type operator over the extensions of (IVNFS) neutrosophic sets. Further studied a few of its properties with few numerical examples. Also, introduce different distance measures over the extension of IVTNFS and further apply these introduced measures in ecological management in this direction to decide the type of corrosion disturbing some towns for valuable management to be taken, using these normalized distance measures.

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Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024

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DOI: https://doi.org/10.54216/IJNS.240225
Received: December 29, 2023 Revised: February 08, 2024 Accepted: May 07, 2024