Pentapartitioned Neutrosophic Binary Set And Its Properties

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Abstract

This study aims to propose a new kind of set, which we refer to as Pentapartitioned neutrosophic binary set. Additionally, we prove some of its basic properties.

Keywords: Neutrosophic set; Single valued neutrosophic set; Neutrosophic binary set; Pentapartitioned neutrosophic set.

1. Introduction

Zadeh [6] invented the idea of a fuzzy set (FS) to deal with various kinds of uncertainties that arise in numerous practical applications as well as complex systems including biological, behavioural and chemical, etc. In 1999, Smarandache [2] developed the field of neutrosophic set which is the extension of a fuzzy set that involves the degrees of truth, indeterminant and falsity to address the inconsistencies and ambiguities in knowledge that frequently arise in practical situations. Later, Rama Mallick and Pramanick defined the concept of pentapartitioned neutrosophic set (PNS) which divides the characteristics features into five components. The indeterminacy is split into three parts signifying contradiction, ignorance and unknown respectively. S. S. Surekha, J. Elekiah and G. Sindhu [4] introduced the notion of a neutrosophic binary set (NBS) in topological space and discussed some of its properties in 2022. In this article, we define a new set named as Pentapartitioned neutrosophic binary set (PNBS) by extending NBS and and procure some of its basic properties.

2. Preliminaries

Here we procure some basic definitions which is needed for our work.

Definition 2.1. A fuzzy set (FS) $\tilde{A}$ on the universe of discourse X is defined as:

$\tilde{A} = \{< u, \mu_{\tilde{A}}(u) >: u \in X\}$ where $\mu_{\tilde{A}}: X \to [0,1]$ and $0 \leq \mu_{\tilde{A}}(x) \leq 1$.

Definition 2.2. A *neutrosophic set* (NS) $\tilde{A}$ over X is defined as follows:

$\tilde{A} = \{< u, \mu_{\tilde{A}}(u), \sigma_{\tilde{A}}(u), \gamma_{\tilde{A}}(u) >: u \in X\}$,Where $\mu_{\tilde{A}}(u), \sigma_{\tilde{A}}(u), \gamma_{\tilde{A}}(u)$ are the truth, indeterminant, and falsity membership values of each $u \in X$. So,

$0 \leq \mu_{\tilde{A}}(u) + \sigma_{\tilde{A}}(u) + \gamma_{\tilde{A}} \leq 3$.

DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024

Definition 2.3. Assume that X be a fixed set. A *Pentapartitioned neutrosophic set* (PNS) $\tilde{A}$ over X is defined as follows:

$\tilde{A} = \{< u, \mu_{\tilde{A}}(u), \sigma_{\tilde{A}}(u), \vartheta_{\tilde{A}}(u), \phi_{\tilde{A}}(u), \gamma_{\tilde{A}}(u) >: u \in X\}$,Where $\mu_{\tilde{A}}(u), \sigma_{\tilde{A}}(u), \vartheta_{\tilde{A}}(u), \phi_{\tilde{A}}(u), \gamma_{\tilde{A}}(u)$ are the truth, contradiction, ignorance, unknown, and falsity membership values of each $u \in X$. So,

$0 \leq \mu_{\tilde{A}}(u) + \sigma_{\tilde{A}}(u) + \gamma_{\tilde{A}}(u) + \phi_{\tilde{A}}(u) + \gamma_{\tilde{A}}(u) \leq 5.$

3. Pentapartitioned neutrosophic binary sets

Definition 3.1. Let U and V be two universes of discourse. The *Pentapartitioned neutrosophic binary set* $(\tilde{A}_1, \tilde{A}_2) \subseteq (U, V)$ is given by

$$(\tilde{A}_1, \tilde{A}_2) = \begin{Bmatrix} < u, \mu_{\tilde{A}_1}(u), \sigma_{\tilde{A}_1}(u), \vartheta_{\tilde{A}_1}(u), \phi_{\tilde{A}_1}(u), \gamma_{\tilde{A}_1}(u) >: u \in U, \\ < v, \mu_{\tilde{A}_2}(v), \sigma_{\tilde{A}_2}(v), \vartheta_{\tilde{A}_2}(v), \phi_{\tilde{A}_2}(v), \gamma_{\tilde{A}_2}(v) >: v \in V \end{Bmatrix}$$

Where $\mu_{\tilde{A}_1}(u), \sigma_{\tilde{A}_1}(u), \vartheta_{\tilde{A}_1}(u), \phi_{\tilde{A}_1}(u), \gamma_{\tilde{A}_1}(u): U \to [0,1]$ are the degrees of the membership of truth, contradiction, ignorance, unknown, and falsity membership values of $u \in U$ and $\mu_{\tilde{A}_2}(u), \sigma_{\tilde{A}_2}(u), \vartheta_{\tilde{A}_2}(u), \phi_{\tilde{A}_2}(u), \gamma_{\tilde{A}_2}(u): V \to [0,1]$ are the degrees of the membership of truth, contradiction, ignorance, unknown, and falsity membership values of $v \in V$ such that $0 \leq \mu_{\tilde{A}_1}(u) + \sigma_{\tilde{A}_1}(u) + \vartheta_{\tilde{A}_1}(u) + \phi_{\tilde{A}_1}(u) + \gamma_{\tilde{A}_1}(u) \leq 5$ and $0 \leq \mu_{\tilde{A}_2}(v) + \sigma_{\tilde{A}_2}(v) + \vartheta_{\tilde{A}_2}(v) + \phi_{\tilde{A}_2}(v) + \gamma_{\tilde{A}_2}(v) \leq 5$.

Definition 3.2. A PNBS is said to be absolute PNBS iff its truth membership, contradiction membership, ignorance membership, unknown membership and falsity membership values are defined as follows: $\mu_{\tilde{A}_1}(u) = 1, \sigma_{\tilde{A}_1}(u) = 1, \vartheta_{\tilde{A}_1}(u) = 0, \phi_{\tilde{A}_1}(u) = 0, \gamma_{\tilde{A}_1}(u) = 0$ and $\mu_{\tilde{A}_2}(v) = 1, \sigma_{\tilde{A}_2}(v) = 1, \vartheta_{\tilde{A}_2}(v) = 0, \phi_{\tilde{A}_2}(v) = 0, \gamma_{\tilde{A}_2}(v) = 0$.

Definition 3.3. . A PNBS is said to be null PNBS iff its truth membership, contradiction membership, ignorance membership, unknown membership and falsity membership values are defined as follows:

$\mu_{\tilde{A}_1}(u) = 0, \sigma_{\tilde{A}_1}(u) = 0, \vartheta_{\tilde{A}_1}(u) = 1, \phi_{\tilde{A}_1}(u) = 1, \gamma_{\tilde{A}_1}(u) = 1$ and $\mu_{\tilde{A}_2}(v) = 0, \sigma_{\tilde{A}_2}(v) = 0, \vartheta_{\tilde{A}_2}(v) = 1, \phi_{\tilde{A}_2}(v) = 1, \gamma_{\tilde{A}_2}(v) = 1$.

Definition 3.4. Let $(\tilde{A}_1, \tilde{A}_2) = \begin{Bmatrix} < u, \mu_{\tilde{A}_1}(u), \sigma_{\tilde{A}_1}(u), \vartheta_{\tilde{A}_1}(u), \phi_{\tilde{A}_1}(u), \gamma_{\tilde{A}_1}(u) >: u \in U, \\ < v, \mu_{\tilde{A}_2}(v), \sigma_{\tilde{A}_2}(v), \vartheta_{\tilde{A}_2}(v), \phi_{\tilde{A}_2}(v), \gamma_{\tilde{A}_2}(v) >: v \in V \end{Bmatrix}$ be a PNBS on (U, V), the complement of the set $(\tilde{A}_1, \tilde{A}_2)^C$ is defined as:

$$(\tilde{A}_1, \tilde{A}_2)^C = \begin{Bmatrix} < u, \gamma_{\tilde{A}_1}(u), \phi_{\tilde{A}_1}(u), 1 - \vartheta_{\tilde{A}_1}(u), \sigma_{\tilde{A}_1}(u), \mu_{\tilde{A}_1}(u) >: u \in U, \\ < v, \gamma_{\tilde{A}_2}(v), \phi_{\tilde{A}_2}(v), 1 - \vartheta_{\tilde{A}_2}(v), \sigma_{\tilde{A}_2}(v), \mu_{\tilde{A}_2}(v) >: v \in V \end{Bmatrix}$$

Definition 3.5. Let $(\tilde{A}_1, \tilde{A}_2) = \{< \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} >, < \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} >\}$ and

$(\tilde{B}_1, \tilde{B}_2) = \{< \mu_{\tilde{B}_1}, \sigma_{\tilde{B}_1}, \vartheta_{\tilde{B}_1}, \phi_{\tilde{B}_1}, \gamma_{\tilde{B}_1} >, < \mu_{\tilde{B}_2}, \sigma_{\tilde{B}_2}, \vartheta_{\tilde{B}_2}, \phi_{\tilde{B}_2}, \gamma_{\tilde{B}_2} >\}$ be two PNBS on (U, V). Then A is contained in B, denoted by $(\tilde{A}_1, \tilde{A}_2) \subseteq (\tilde{B}_1, \tilde{B}_2)$ if and only if

$\mu_{\tilde{A}_1}(u) \leq \mu_{\tilde{B}_1}(u), \sigma_{\tilde{A}_1}(u) \leq \sigma_{\tilde{B}_1}(u), \vartheta_{\tilde{A}_1}(u) \geq \vartheta_{\tilde{B}_1}(u), \phi_{\tilde{A}_1}(u) \geq \phi_{\tilde{B}_1}(u), \gamma_{\tilde{A}_1}(u) \geq \gamma_{\tilde{B}_1}(u)$ for every $u \in U$ and $\mu_{\tilde{A}_2}(v) \leq \mu_{\tilde{B}_2}(v), \sigma_{\tilde{A}_2}(v) \leq \sigma_{\tilde{B}_2}(v), \vartheta_{\tilde{A}_2}(v) \geq \vartheta_{\tilde{B}_2}(v)$,

$\phi_{\tilde{A}_2}(v) \geq \phi_{\tilde{B}_2}(v), \gamma_{\tilde{A}_2}(v) \geq \gamma_{\tilde{B}_2}(v)$ for every $v \in V$.

Definition 3.6. Let $(\tilde{A}_1, \tilde{A}_2) = \begin{Bmatrix} < \mu_{\tilde{A}_1}(\mathrm{u}), \sigma_{\tilde{A}_1}(\mathrm{u}), \vartheta_{\tilde{A}_1}(\mathrm{u}), \phi_{\tilde{A}_1}(\mathrm{u}), \gamma_{\tilde{A}_1}(\mathrm{u}) >, \\ < \mu_{\tilde{A}_2}(\mathrm{v}), \sigma_{\tilde{A}_2}(\mathrm{v}), \vartheta_{\tilde{A}_2}(\mathrm{v}), \phi_{\tilde{A}_2}(\mathrm{v}), \gamma_{\tilde{A}_2}(\mathrm{v}) > \end{Bmatrix}$ and

$(\tilde{B}_1, \tilde{B}_2) = \begin{Bmatrix} < \mu_{\tilde{B}_1}(\mathrm{u}), \sigma_{\tilde{B}_1}(\mathrm{u}), \vartheta_{\tilde{B}_1}(\mathrm{u}), \phi_{\tilde{B}_1}(\mathrm{u}), \gamma_{\tilde{B}_1}(\mathrm{u}) >, \\ < \mu_{\tilde{B}_2}(\mathrm{v}), \sigma_{\tilde{B}_2}(\mathrm{v}), \vartheta_{\tilde{B}_2}(\mathrm{v}), \phi_{\tilde{B}_2}(\mathrm{v}), \gamma_{\tilde{B}_2}(\mathrm{v}) > \end{Bmatrix}$ be two PNBS on (U, V).Then their union $(\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2)$ and intersection $(\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2)$ are defined as:

$(\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) =$

DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024

$$\begin{Bmatrix} < \mu_{\tilde{A}_1}(u) \vee \mu_{\tilde{B}_1}(u), \sigma_{\tilde{A}_1}(u) \vee \sigma_{\tilde{B}_1}(u), \vartheta_{\tilde{A}_1}(u) \wedge \vartheta_{\tilde{B}_1}(u), \phi_{\tilde{A}_1}(u) \wedge \phi_{\tilde{B}_1}(u), \gamma_{\tilde{A}_1}(u) \wedge \gamma_{\tilde{B}_1}(u) >: u \in U, \\ < \mu_{\tilde{A}_2}(v) \vee \mu_{\tilde{B}_2}(v), \sigma_{\tilde{A}_2}(v) \vee \sigma_{\tilde{B}_2}(v), \vartheta_{\tilde{A}_2}(v) \wedge \vartheta_{\tilde{B}_2}(v), \phi_{\tilde{A}_2}(v) \wedge \phi_{\tilde{B}_2}(v), \gamma_{\tilde{A}_2}(v) \wedge \gamma_{\tilde{B}_2}(v) >: v \in V \end{Bmatrix}$$

$$(\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2) =$$
$$\begin{Bmatrix} < \mu_{\tilde{A}_1}(u) \wedge \mu_{\tilde{B}_1}(u), \sigma_{\tilde{A}_1}(u) \wedge \sigma_{\tilde{B}_1}(u), \vartheta_{\tilde{A}_1}(u) \vee \vartheta_{\tilde{B}_1}(u), \phi_{\tilde{A}_1}(u) \vee \phi_{\tilde{B}_1}(u), \gamma_{\tilde{A}_1}(u) \vee \gamma_{\tilde{B}_1}(u) >: u \in U, \\ < \mu_{\tilde{A}_2}(v) \wedge \mu_{\tilde{B}_2}(v), \sigma_{\tilde{A}_2}(v) \wedge \sigma_{\tilde{B}_2}(v), \vartheta_{\tilde{A}_2}(v) \vee \vartheta_{\tilde{B}_2}(v), \phi_{\tilde{A}_2}(v) \vee \phi_{\tilde{B}_2}(v), \gamma_{\tilde{A}_2}(v) \vee \gamma_{\tilde{B}_2}(v) >: v \in V \end{Bmatrix}$$

Proposition: PNBS satisfy the following properties over the universe (U, V) under the aforementioned set theoretic operations:

(1) Commutative law:

(a) $(\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) = (\tilde{B}_1, \tilde{B}_2) \cup (\tilde{A}_1, \tilde{A}_2)$

(b) $(\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2) = (\tilde{B}_1, \tilde{B}_2) \cap (\tilde{A}_1, \tilde{A}_2)$

Proof:

(a) $(\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) = (\tilde{B}_1, \tilde{B}_2) \cup (\tilde{A}_1, \tilde{A}_2)$, we know that,

$$(\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2)$$
$$= \begin{Bmatrix} < \mu_{\tilde{A}_1}(u) \vee \mu_{\tilde{B}_1}(u), \sigma_{\tilde{A}_1}(u) \vee \sigma_{\tilde{B}_1}(u), \vartheta_{\tilde{A}_1}(u) \wedge \vartheta_{\tilde{B}_1}(u), \phi_{\tilde{A}_1}(u) \wedge \phi_{\tilde{B}_1}(u), \gamma_{\tilde{A}_1}(u) \wedge \gamma_{\tilde{B}_1}(u) >: u \in U, \\ < \mu_{\tilde{A}_2}(v) \vee \mu_{\tilde{B}_2}(v), \sigma_{\tilde{A}_2}(v) \vee \sigma_{\tilde{B}_2}(v), \vartheta_{\tilde{A}_2}(v) \wedge \vartheta_{\tilde{B}_2}(v), \phi_{\tilde{A}_1}(v) \wedge \phi_{\tilde{B}_2}(v), \gamma_{\tilde{A}_2}(v) \wedge \gamma_{\tilde{B}_2}(v) >: v \in V \end{Bmatrix}$$

$$= \begin{Bmatrix} < \mu_{\tilde{A}_1}(u), \sigma_{\tilde{A}_1}(u), \vartheta_{\tilde{A}_1}(u), \phi_{\tilde{A}_1}(u), \gamma_{\tilde{A}_1}(u) > \vee < \mu_{\tilde{B}_1}(u), \sigma_{\tilde{B}_1}(u), \vartheta_{\tilde{B}_1}(u), \phi_{\tilde{B}_1}(u), \gamma_{\tilde{B}_1}(u) >: u \in U, \\ < \mu_{\tilde{A}_2}(v), \sigma_{\tilde{A}_2}(v), \vartheta_{\tilde{A}_2}(v), \phi_{\tilde{A}_2}(v), \gamma_{\tilde{A}_2}(v) > \vee < \mu_{\tilde{B}_2}(v), \sigma_{\tilde{B}_2}(v), \vartheta_{\tilde{B}_2}(v), \phi_{\tilde{B}_2}(v), \gamma_{\tilde{B}_2}(v) >: v \in V \end{Bmatrix}$$

Let $(p, q) \in (U, V)$ be arbitrary such that $(p, q) \in (\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2)$

$$\Rightarrow (p, q) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2} > \end{Bmatrix}$$

$$\Rightarrow (p, q) \in \begin{Bmatrix} < \mu_{\tilde{B}_1} \vee \mu_{\tilde{A}_1}, \sigma_{\tilde{B}_1} \vee \sigma_{\tilde{A}_1}, \vartheta_{\tilde{B}_1} \wedge \vartheta_{\tilde{A}_1}, \phi_{\tilde{B}_1} \wedge \phi_{\tilde{A}_1}, \gamma_{\tilde{B}_1} \wedge \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{B}_2} \vee \mu_{\tilde{A}_2}, \sigma_{\tilde{B}_2} \vee \sigma_{\tilde{A}_2}, \vartheta_{\tilde{B}_2} \wedge \vartheta_{\tilde{A}_2}, \phi_{\tilde{B}_2} \wedge \phi_{\tilde{A}_2}, \gamma_{\tilde{B}_2} \wedge \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$$\Rightarrow (p, q) \in (\tilde{B}_1, \tilde{B}_2) \cup (\tilde{A}_1, \tilde{A}_2)$$

$$\Rightarrow (\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) \subseteq (\tilde{B}_1, \tilde{B}_2) \cup (\tilde{A}_1, \tilde{A}_2) \ \ldots\ldots\ldots (1)$$

Let $(r, s) \in (U, V)$ be arbitrary such that $(r, s) \in (\tilde{B}_1, \tilde{B}_2) \cup (\tilde{A}_1, \tilde{A}_2)$

$$\Rightarrow (r, s) \in \begin{Bmatrix} < \mu_{\tilde{B}_1} \vee \mu_{\tilde{A}_1}, \sigma_{\tilde{B}_1} \vee \sigma_{\tilde{A}_1}, \vartheta_{\tilde{B}_1} \wedge \vartheta_{\tilde{A}_1}, \phi_{\tilde{B}_1} \wedge \phi_{\tilde{A}_1}, \gamma_{\tilde{B}_1} \wedge \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{B}_2} \vee \mu_{\tilde{A}_2}, \sigma_{\tilde{B}_2} \vee \sigma_{\tilde{A}_2}, \vartheta_{\tilde{B}_2} \wedge \vartheta_{\tilde{A}_2}, \phi_{\tilde{B}_2} \wedge \phi_{\tilde{A}_2}, \gamma_{\tilde{B}_2} \wedge \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$$\Rightarrow (r, s) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2} > \end{Bmatrix}$$

$$\Rightarrow (r, s) \in (\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) \ldots\ldots (2)$$

Therefore, from (1) and (2) we get,

$$(\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) = (\tilde{B}_1, \tilde{B}_2) \cup (\tilde{A}_1, \tilde{A}_2)$$

(b) Again by similar discussion, we can show that

$$(\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2) = (\tilde{B}_1, \tilde{B}_2) \cap (\tilde{A}_1, \tilde{A}_2)$$

DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024

(2) Associative law:

(c) $(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))=((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cup(\tilde{C}_1,\tilde{C}_2)$
(d) $(\tilde{A}_1,\tilde{A}_2)\cap((\tilde{B}_1,\tilde{B}_2)\cap(\tilde{C}_1,\tilde{C}_2))=((\tilde{A}_1,\tilde{A}_2)\cap(\tilde{B}_1,\tilde{B}_2))\cap(\tilde{C}_1,\tilde{C}_2)$

Proof:
(c) Now, to prove: $(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))=((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cup(\tilde{C}_1,\tilde{C}_2)$

Let $(p,q)\in(U,V)$ be arbitrary such that $(p,q)\in(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))$

$$\Rightarrow (p,q)\in(\tilde{A}_1,\tilde{A}_2)\cup\begin{Bmatrix}<\mu_{\tilde{B}_1}\vee\mu_{\tilde{C}_1},\sigma_{\tilde{B}_1}\vee\sigma_{\tilde{C}_1},\vartheta_{\tilde{B}_1}\wedge\vartheta_{\tilde{C}_1},\phi_{\tilde{B}_1}\wedge\phi_{\tilde{C}_1},\gamma_{\tilde{B}_1}\wedge\gamma_{\tilde{C}_1}>\\<\mu_{\tilde{B}_2}\vee\mu_{\tilde{C}_2},\sigma_{\tilde{B}_2}\vee\sigma_{\tilde{C}_2},\vartheta_{\tilde{B}_2}\wedge\vartheta_{\tilde{C}_2},\phi_{\tilde{B}_2}\wedge\phi_{\tilde{C}_2},\gamma_{\tilde{B}_2}\wedge\gamma_{\tilde{C}_2}>\end{Bmatrix}$$

$$\Rightarrow (p,q)\in\begin{Bmatrix}<\vee(\mu_{\tilde{A}_1},\mu_{\tilde{B}_1},\mu_{\tilde{C}_1}),\vee(\sigma_{\tilde{A}_1},\sigma_{\tilde{B}_1},\sigma_{\tilde{C}_1}),\wedge(\vartheta_{(\tilde{A}_1},\vartheta_{\tilde{B}_1},\vartheta_{\tilde{C}_1}),\wedge(\phi_{\tilde{A}_1},\phi_{\tilde{B}_1},\phi_{\tilde{C}_1}),\wedge(\gamma_{\tilde{A}_1},\gamma_{\tilde{B}_1},\gamma_{\tilde{C}_1})>,\\<\vee(\mu_{\tilde{A}_2},\mu_{\tilde{B}_2},\mu_{\tilde{C}_2}),\vee(\sigma_{\tilde{A}_2,}\sigma_{\tilde{B}_2},\sigma_{\tilde{C}_2}),\wedge(\vartheta_{\tilde{A}_2},\vartheta_{\tilde{B}_2},\vartheta_{\tilde{C}_2}),\wedge(\phi_{\tilde{A}_2},\phi_{\tilde{B}_2},\phi_{\tilde{C}_2}),\wedge(\gamma_{\tilde{A}_2},\gamma_{\tilde{B}_2},\gamma_{\tilde{C}_2})>\end{Bmatrix}\Rightarrow (p,q)\in$$

$$\begin{Bmatrix}<\mu_{\tilde{A}_1}\vee\mu_{\tilde{B}_1},\sigma_{\tilde{A}_1}\vee\sigma_{\tilde{B}_1},\vartheta_{\tilde{A}_1}\wedge\vartheta_{\tilde{B}_1},\phi_{\tilde{A}_1}\wedge\phi_{\tilde{B}_1},\gamma_{\tilde{A}_1}\wedge\gamma_{\tilde{B}_1}>,\\<\mu_{\tilde{A}_2}\vee\mu_{\tilde{B}_2},\sigma_{\tilde{A}_2}\vee\sigma_{\tilde{B}_2},\vartheta_{\tilde{A}_2}\wedge\vartheta_{\tilde{B}_2},\phi_{\tilde{A}_2}\wedge\phi_{\tilde{B}_2},\gamma_{\tilde{A}_2}\wedge\gamma_{\tilde{B}_2}>\end{Bmatrix}\cup(\tilde{C}_1,\tilde{C}_2)$$

$$\Rightarrow (p,q)\in((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cup(\tilde{C}_1,\tilde{C}_2)$$

$(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))\subseteq((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cup(\tilde{C}_1,\tilde{C}_2)$…..(3)

Take $(r,s)\in(U,V)$ be arbitrary such that $(r,s)\in((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cup(\tilde{C}_1,\tilde{C}_2)$

$$\Rightarrow (r,s)\in\begin{Bmatrix}<\mu_{\tilde{A}_1}\vee\mu_{\tilde{B}_1},\sigma_{\tilde{A}_1}\vee\sigma_{\tilde{B}_1},\vartheta_{\tilde{A}_1}\wedge\vartheta_{\tilde{B}_1},\phi_{\tilde{A}_1}\wedge\phi_{\tilde{B}_1},\gamma_{\tilde{A}_1}\wedge\gamma_{\tilde{B}_1}>,\\<\mu_{\tilde{A}_2}\vee\mu_{\tilde{B}_2},\sigma_{\tilde{A}_2}\vee\sigma_{\tilde{B}_2},\vartheta_{\tilde{A}_2}\wedge\vartheta_{\tilde{B}_2},\phi_{\tilde{A}_2}\wedge\phi_{\tilde{B}_2},\gamma_{\tilde{A}_2}\wedge\gamma_{\tilde{B}_2}>\end{Bmatrix}\cup(\tilde{C}_1,\tilde{C}_2)$$

$$\Rightarrow (r,s)\in\begin{Bmatrix}<\vee(\mu_{\tilde{A}_1},\mu_{\tilde{B}_1},\mu_{\tilde{C}_1}),\vee(\sigma_{\tilde{A}_1},\sigma_{\tilde{B}_1},\sigma_{\tilde{C}_1}),\wedge(\vartheta_{\tilde{A}_1},\vartheta_{\tilde{B}_1},\vartheta_{\tilde{C}_1}),\wedge(\phi_{\tilde{A}_1},\phi_{\tilde{B}_1},\phi_{\tilde{C}_1}),\wedge(\gamma_{\tilde{A}_1},\gamma_{\tilde{B}_1},\gamma_{\tilde{C}_1})>,\\<\vee(\mu_{\tilde{A}_2},\mu_{\tilde{B}_2},\mu_{\tilde{C}_2}),\vee(\sigma_{\tilde{A}_2,}\sigma_{\tilde{B}_2},\sigma_{\tilde{C}_2}),\wedge(\vartheta_{\tilde{A}_2},\vartheta_{\tilde{B}_2},\vartheta_{\tilde{C}_2}),\wedge(\phi_{\tilde{A}_2},\phi_{\tilde{B}_2},\phi_{\tilde{C}_2}),\wedge(\gamma_{\tilde{A}_2},\gamma_{\tilde{B}_2},\gamma_{\tilde{C}_2})>\end{Bmatrix}$$

$$\Rightarrow (r,s)\in(\tilde{A}_1,\tilde{A}_2)\cup\begin{Bmatrix}<\mu_{\tilde{B}_1}\vee\mu_{\tilde{C}_1},\sigma_{\tilde{B}_1}\vee\sigma_{\tilde{C}_1},\vartheta_{\tilde{B}_1}\wedge\vartheta_{\tilde{C}_1},\phi_{\tilde{B}_1}\wedge\phi_{\tilde{C}_1},\gamma_{\tilde{B}_1}\wedge\gamma_{\tilde{C}_1}>,\\<\mu_{\tilde{B}_2}\vee\mu_{\tilde{C}_2},\sigma_{\tilde{B}_2}\vee\sigma_{\tilde{C}_2},\vartheta_{\tilde{B}_2}\wedge\vartheta_{\tilde{C}_2},\phi_{\tilde{B}_2}\wedge\phi_{\tilde{C}_2},\gamma_{\tilde{B}_2}\wedge\gamma_{\tilde{C}_2}>\end{Bmatrix}$$

$$\Rightarrow (r,s)\in(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))$$

$((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cup(\tilde{C}_1,\tilde{C}_2)\subseteq(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))$…..(4)

From (3) and (4) we obtain that,

$$(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))=((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cup(\tilde{C}_1,\tilde{C}_2)$$

(d)Again By similar discussion, we can easily show that

$$(\tilde{A}_1,\tilde{A}_2)\cap((\tilde{B}_1,\tilde{B}_2)\cap(\tilde{C}_1,\tilde{C}_2))=((\tilde{A}_1,\tilde{A}_2)\cap(\tilde{B}_1,\tilde{B}_2))\cap(C_1,\tilde{C}_2)$$

(3) Distributive law:

(e) $(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cap(\tilde{C}_1,\tilde{C}_2))=((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{B}_1,\tilde{B}_2))\cap((\tilde{A}_1,\tilde{A}_2)\cup(\tilde{C}_1,\tilde{C}_2))$

(f) $(\tilde{A}_1,\tilde{A}_2)\cap((\tilde{B}_1,\tilde{B}_2)\cup(\tilde{C}_1,\tilde{C}_2))=((\tilde{A}_1,\tilde{A}_2)\cap(\tilde{B}_1,\tilde{B}_2))\cup((\tilde{A}_1,\tilde{A}_2)\cap(\tilde{C}_1,\tilde{C}_2))$

Proof:

(e) Let $(p,q)\in(U,V)$ be arbitrary such that $(p,q)\in(\tilde{A}_1,\tilde{A}_2)\cup((\tilde{B}_1,\tilde{B}_2)\cap(\tilde{C}_1,\tilde{C}_2))$

DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024

$$\Rightarrow (p,q) \in (\tilde{A}_1,\tilde{A}_2) \cup \begin{Bmatrix} < \mu_{\tilde{B}_1} \wedge \mu_{\tilde{C}_1}, \sigma_{\tilde{B}_1} \wedge \sigma_{\tilde{C}_1}, \vartheta_{\tilde{B}_1} \vee \vartheta_{\tilde{C}_1}, \phi_{\tilde{B}_1} \vee \phi_{\tilde{C}_1}, \gamma_{\tilde{B}_1} \vee \gamma_{\tilde{C}_1} >, \\ < \mu_{\tilde{B}_2} \wedge \mu_{\tilde{C}_2}, \sigma_{\tilde{B}_2} \wedge \sigma_{\tilde{C}_2}, \vartheta_{\tilde{B}_2} \vee \vartheta_{\tilde{C}_2}, \phi_{\tilde{B}_2} \vee \phi_{\tilde{C}_2}, \gamma_{\tilde{B}_2} \vee \gamma_{\tilde{C}_2} > \end{Bmatrix}$$

$$\Rightarrow (p,q) \in$$
$$\begin{Bmatrix} < \mu_{\tilde{A}_1} \vee (\mu_{\tilde{B}_1} \wedge \mu_{\tilde{C}_1}), \sigma_{\tilde{A}_1} \vee (\sigma_{\tilde{B}_1} \wedge \sigma_{\tilde{C}_1}), \vartheta_{\tilde{A}_1} \wedge (\vartheta_{\tilde{B}_1} \vee \vartheta_{\tilde{C}_1}), \phi_{\tilde{A}_1} \wedge (\phi_{\tilde{B}_1} \vee \phi_{\tilde{C}_1}), \gamma_{\tilde{A}_1} \wedge (\gamma_{\tilde{B}_1} \vee \gamma_{\tilde{C}_1}) >, \\ < \mu_{\tilde{A}_2} \vee (\mu_{\tilde{B}_2} \wedge \mu_{\tilde{C}_2}), \sigma_{\tilde{A}_2} \vee (\sigma_{\tilde{B}_2} \wedge \sigma_{\tilde{C}_2}), \vartheta_{\tilde{A}_2} \wedge (\vartheta_{\tilde{B}_2} \vee \vartheta_{\tilde{C}_2}), \phi_{\tilde{A}_2} \wedge (\phi_{\tilde{B}_2} \vee \phi_{\tilde{C}_2}), \gamma_{\tilde{A}_2} \wedge (\gamma_{\tilde{B}_2} \vee \gamma_{\tilde{C}_2}) > \end{Bmatrix}$$

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2} > \end{Bmatrix}$$
$$\cap \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{C}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{C}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{C}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{C}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{C}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{C}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{C}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{C}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{C}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{C}_2} > \end{Bmatrix}$$

$$\Rightarrow (p,q) \in ((\tilde{A}_1,\tilde{A}_2) \cup (\tilde{B}_1,\tilde{B}_2)) \cap ((\tilde{A}_1,\tilde{A}_2) \cup (C_1,\tilde{C}_2))$$

$(\tilde{A}_1,\tilde{A}_2) \cup ((\tilde{B}_1,\tilde{B}_2) \cap (\tilde{C}_1,\tilde{C}_2)) \subseteq ((\tilde{A}_1,\tilde{A}_2) \cup (\tilde{B}_1,\tilde{B}_2)) \cap ((\tilde{A}_1,\tilde{A}_2) \cup (C_1,\tilde{C}_2))$……(5)

Let $(r,s) \in (U,V)$ be arbitrary such that $(r,s) \in ((\tilde{A}_1,\tilde{A}_2) \cup (\tilde{B}_1,\tilde{B}_2)) \cap ((\tilde{A}_1,\tilde{A}_2) \cup (C_1,\tilde{C}_2))$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2} > \end{Bmatrix}$$
$$\cap \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{C}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{C}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{C}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{C}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{C}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{C}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{C}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{C}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{C}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{C}_2} > \end{Bmatrix}$$

$$\Longrightarrow (p,q) \in$$

$$\begin{Bmatrix} < \mu_{\tilde{A}_1} \vee (\mu_{\tilde{B}_1} \wedge \mu_{\tilde{C}_1}), \sigma_{\tilde{A}_1} \vee (\sigma_{\tilde{B}_1} \wedge \sigma_{\tilde{C}_1}), \vartheta_{\tilde{A}_1} \wedge (\vartheta_{\tilde{B}_1} \vee \vartheta_{\tilde{C}_1}), \phi_{\tilde{A}_1} \wedge (\phi_{\tilde{B}_1} \vee \phi_{\tilde{C}_1}), \gamma_{\tilde{A}_1} \wedge (\gamma_{\tilde{B}_1} \vee \gamma_{\tilde{C}_1}) >, \\ < \mu_{\tilde{A}_2} \vee (\mu_{\tilde{B}_2} \wedge \mu_{\tilde{C}_2}), \sigma_{\tilde{A}_2} \vee (\sigma_{\tilde{B}_2} \wedge \sigma_{\tilde{C}_2}), \vartheta_{\tilde{A}_2} \wedge (\vartheta_{\tilde{B}_2} \vee \vartheta_{\tilde{C}_2}), \phi_{\tilde{A}_2} \wedge (\phi_{\tilde{B}_2} \vee \phi_{\tilde{C}_2}), \gamma_{\tilde{A}_2} \wedge (\gamma_{\tilde{B}_2} \vee \gamma_{\tilde{C}_2}) > \end{Bmatrix}$$

$$\Rightarrow (r,s) \in (\tilde{A}_1,\tilde{A}_2) \cup \begin{Bmatrix} < \mu_{\tilde{B}_1} \wedge \mu_{\tilde{C}_1}, \sigma_{\tilde{B}_1} \wedge \sigma_{\tilde{C}_1}, \vartheta_{\tilde{B}_1} \vee \vartheta_{\tilde{C}_1}, \phi_{\tilde{B}_1} \vee \phi_{\tilde{C}_1}, \gamma_{\tilde{B}_1} \vee \gamma_{\tilde{C}_1} >, \\ < \mu_{\tilde{B}_2} \wedge \mu_{\tilde{C}_2}, \sigma_{\tilde{B}_2} \wedge \sigma_{\tilde{C}_2}, \vartheta_{\tilde{B}_2} \vee \vartheta_{\tilde{C}_2}, \phi_{\tilde{B}_2} \vee \phi_{\tilde{C}_2}, \gamma_{\tilde{B}_2} \vee \gamma_{\tilde{C}_2} > \end{Bmatrix}$$

$$\Rightarrow (r,s) \in (\tilde{A}_1,\tilde{A}_2) \cup ((\tilde{B}_1,\tilde{B}_2) \cap (\tilde{C}_1,\tilde{C}_2))$$

$((\tilde{A}_1,\tilde{A}_2) \cup (\tilde{B}_1,\tilde{B}_2)) \cap ((\tilde{A}_1,\tilde{A}_2) \cup (\tilde{C}_1,\tilde{C}_2)) \subseteq (\tilde{A}_1,\tilde{A}_2) \cup ((\tilde{B}_1,\tilde{B}_2) \cap (\tilde{C}_1,\tilde{C}_2))$……..(6)

From (5) and (6), we get

$$(\tilde{A}_1,\tilde{A}_2) \cup ((\tilde{B}_1,\tilde{B}_2) \cap (\tilde{C}_1,\tilde{C}_2)) = ((\tilde{A}_1,\tilde{A}_2) \cup (\tilde{B}_1,\tilde{B}_2)) \cap ((\tilde{A}_1,\tilde{A}_2) \cup (\tilde{C}_1,\tilde{C}_2))$$

(f) Similarly, it is easy to show that

$$(\tilde{A}_1,\tilde{A}_2) \cap ((\tilde{B}_1,\tilde{B}_2) \cup (\tilde{C}_1,\tilde{C}_2)) = ((\tilde{A}_1,\tilde{A}_2) \cap (\tilde{B}_1,\tilde{B}_2)) \cup ((\tilde{A}_1,\tilde{A}_2) \cap (\tilde{C}_1,\tilde{C}_2))$$

(4) Idempotent law:

(g) $(\tilde{A}_1,\tilde{A}_2) \cap (\tilde{A}_1,\tilde{A}_2) = (\tilde{A}_1,\tilde{A}_2)$

(h) $(\tilde{A}_1,\tilde{A}_2) \cup (\tilde{A}_1,\tilde{A}_2) = (\tilde{A}_1,\tilde{A}_2)$

Proof:

(g) Take $(p,q) \in (U,V)$ be arbitrary such that $(p,q) \in (\tilde{A}_1,\tilde{A}_2) \cap (\tilde{A}_1,\tilde{A}_2)$

DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \wedge \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1} \wedge \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1} \vee \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1} \vee \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} \vee \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2} \wedge \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2} \wedge \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2} \vee \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2} \vee \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} \vee \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$\Rightarrow (p,q) \in (\tilde{A}_1, \tilde{A}_2)$…..(7)

Take $(r,s) \in (U,V)$ be arbitrary such that $(r,s) \in (\tilde{A}_1, \tilde{A}_2)$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \wedge \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1} \wedge \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1} \vee \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1} \vee \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} \vee \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2} \wedge \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2} \wedge \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2} \vee \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2} \vee \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} \vee \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$\Rightarrow (r,s) \in (\tilde{A}_1, \tilde{A}_2) \cap (\tilde{A}_1, \tilde{A}_2)$….(8)

From (7) and (8), we get

$$(\tilde{A}_1, \tilde{A}_2) \cap (\tilde{A}_1, \tilde{A}_2) = (\tilde{A}_1, \tilde{A}_2)$$

(h) Similarly, we can show that,

$$(\tilde{A}_1, \tilde{A}_2) \cup (\tilde{A}_1, \tilde{A}_2) = (\tilde{A}_1, \tilde{A}_2)$$

(5) Absorption law:

(i) $(\tilde{A}_1, \tilde{A}_2) \cup ((\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2)) = (\tilde{A}_1, \tilde{A}_2)$

(j) $(\tilde{A}_1, \tilde{A}_2) \cap ((\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2)) = (\tilde{A}_1, \tilde{A}_2)$

Proof:

(i) Let $(p,q) \in (U,V)$ be arbitrary such that $(p,q) \in (\tilde{A}_1, \tilde{A}_2) \cup ((\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2))$

$$\Rightarrow (p,q) \in (\tilde{A}_1, \tilde{A}_2) \cup \begin{Bmatrix} < \mu_{\tilde{A}_1} \wedge \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \wedge \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \vee \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \vee \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \vee \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \wedge \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \wedge \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \vee \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \vee \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \vee \gamma_{\tilde{B}_2} > \end{Bmatrix}$$

$\Rightarrow (p,q) \in$

$$\begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \left(\mu_{\tilde{A}_1} \wedge \mu_{\tilde{B}_1}\right), \sigma_{\tilde{A}_1} \vee \left(\sigma_{\tilde{A}_1} \wedge \sigma_{\tilde{B}_1}\right), \vartheta_{\tilde{A}_1} \wedge \left(\vartheta_{\tilde{A}_1} \vee \vartheta_{\tilde{B}_1}\right), \phi_{\tilde{A}_1} \wedge \left(\phi_{\tilde{A}_1} \vee \phi_{\tilde{B}_1}\right), \gamma_{\tilde{A}_1} \wedge \left(\gamma_{\tilde{A}_1} \vee \gamma_{\tilde{B}_1}\right) >, \\ < \mu_{\tilde{A}_2} \vee \left(\mu_{\tilde{A}_2} \wedge \mu_{\tilde{B}_2}\right), \sigma_{\tilde{A}_2} \vee \left(\sigma_{\tilde{A}_2} \wedge \sigma_{\tilde{B}_2}\right), \vartheta_{\tilde{A}_2} \wedge \left(\vartheta_{\tilde{A}_2} \vee \vartheta_{\tilde{B}_2}\right), \phi_{\tilde{A}_2} \wedge \left(\phi_{\tilde{A}_2} \vee \phi_{\tilde{B}_2}\right), \gamma_{\tilde{A}_2} \wedge \left(\gamma_{\tilde{A}_2} \vee \gamma_{\tilde{B}_2}\right) > \end{Bmatrix}$$

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$\Rightarrow (p,q) \in \left(\tilde{A}_1, \tilde{A}_2\right)$……(9)

Let $(r,s) \in (U,V)$ be arbitrary such that $(r,s) \in (\tilde{A}_1, \tilde{A}_2)$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$\Rightarrow (r,s) \in$

DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024

$$\begin{Bmatrix} < \mu_{\tilde{A}_1} \vee (\mu_{\tilde{A}_1} \wedge \mu_{\tilde{B}_1}), \sigma_{\tilde{A}_1} \vee (\sigma_{\tilde{A}_1} \wedge \sigma_{\tilde{B}_1}), \vartheta_{\tilde{A}_1} \wedge (\vartheta_{\tilde{A}_1} \vee \vartheta_{\tilde{B}_1}), \phi_{\tilde{A}_1} \wedge (\phi_{\tilde{A}_1} \vee \phi_{\tilde{B}_1}), \gamma_{\tilde{A}_1} \wedge (\gamma_{\tilde{A}_1} \vee \gamma_{\tilde{B}_1}) >, \\ < \mu_{\tilde{A}_2} \vee (\mu_{\tilde{A}_2} \wedge \mu_{\tilde{B}_2}), \sigma_{\tilde{A}_2} \vee (\sigma_{\tilde{A}_2} \wedge \sigma_{\tilde{B}_2}), \vartheta_{\tilde{A}_2} \wedge (\vartheta_{\tilde{A}_2} \vee \vartheta_{\tilde{B}_2}), \phi_{\tilde{A}_2} \wedge (\phi_{\tilde{A}_2} \vee \phi_{\tilde{B}_2}), \gamma_{\tilde{A}_2} \wedge (\gamma_{\tilde{A}_2} \vee \gamma_{\tilde{B}_2}) > \end{Bmatrix}$$

$$\Rightarrow (r,s) \in (\tilde{A}_1, \tilde{A}_2) \cup \begin{Bmatrix} < \mu_{\tilde{A}_1} \wedge \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \wedge \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \vee \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \vee \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \vee \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \wedge \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \wedge \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \vee \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \vee \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \vee \gamma_{\tilde{B}_2} > \end{Bmatrix}$$

$\Rightarrow (r,s) \in (\tilde{A}_1, \tilde{A}_2) \cup ((\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2))$…..(10)

From (9) and (10), we conclude that

$$(\tilde{A}_1, \tilde{A}_2) \cup ((\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2)) = (\tilde{A}_1, \tilde{A}_2)$$

(j) Again by similar discussion,we can show that

$$(\tilde{A}_1, \tilde{A}_2) \cap ((\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2)) = (\tilde{A}_1, \tilde{A}_2)$$

(6) Involution law:

(k) $\left((\tilde{A}_1, \tilde{A}_2)^C \right)^C = (\tilde{A}_1, \tilde{A}_2)$

Proof:

(k) Take $(p,q) \in (U,V)$ be arbitrary such that $(p,q) \in \left((\tilde{A}_1, \tilde{A}_2)^C \right)^C$

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \gamma_{\tilde{A}_1}, \phi_{\tilde{A}_1}, 1 - \vartheta_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \mu_{\tilde{A}_1} >, \\ < \gamma_{\tilde{A}_2}, \phi_{\tilde{A}_2}, 1 - \vartheta_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \mu_{\tilde{A}_2} > \end{Bmatrix}^C$$

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$\Rightarrow (p,q) \in (\tilde{A}_1, \tilde{A}_2)$…….(11)

Take $(r,s) \in (U,V)$ be arbitrary such that $(r,s) \in (\tilde{A}_1, \tilde{A}_2)$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \mu_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \vartheta_{\tilde{A}_1}, \phi_{\tilde{A}_1}, \gamma_{\tilde{A}_1} >, \\ < \mu_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \vartheta_{\tilde{A}_2}, \phi_{\tilde{A}_2}, \gamma_{\tilde{A}_2} > \end{Bmatrix}$$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \gamma_{\tilde{A}_1}, \phi_{\tilde{A}_1}, 1 - \vartheta_{\tilde{A}_1}, \sigma_{\tilde{A}_1}, \mu_{\tilde{A}_1} >, \\ < \gamma_{\tilde{A}_2}, \phi_{\tilde{A}_2}, 1 - \vartheta_{\tilde{A}_2}, \sigma_{\tilde{A}_2}, \mu_{\tilde{A}_2} > \end{Bmatrix}^C$$

$\Rightarrow (r,s) \in \left((\tilde{A}_1, \tilde{A}_2)^C \right)^C$…..(12)

From (11) and (12), we get

$$\left((\tilde{A}_1, \tilde{A}_2)^C \right)^C = (\tilde{A}_1, \tilde{A}_2)$$

(7) De Morgan's law:

(l) $\left((\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) \right)^C = (\tilde{A}_1, \tilde{A}_2)^C \cap (\tilde{B}_1, \tilde{B}_2)^C$

(m) $((\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2))^C = (\tilde{A}_1, \tilde{A}_2)^C \cup (\tilde{B}_1, \tilde{B}_2)^C$

Proof:

(l) Take $(p,q) \in (U,V)$ be arbitrary such that $(p,q) \in \left((\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2) \right)^C$

DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2} > \end{Bmatrix}^C$$

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, 1-(\vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}), \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1} >, \\ < \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, 1-(\vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}), \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2} > \end{Bmatrix}$$

$$\Rightarrow (p,q) \in \begin{Bmatrix} < \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, (1-\vartheta_{\tilde{A}_1}) \vee (1-\vartheta_{\tilde{B}_1}), \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1} >, \\ < \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, (1-\vartheta_{\tilde{A}_2}) \vee (1-\vartheta_{\tilde{B}_2}), \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2} > \end{Bmatrix}$$

$\Rightarrow (p,q) \in (\tilde{A}_1, \tilde{A}_2)^C \cap (\tilde{B}_1, \tilde{B}_2)^C$……(13)

Take $(r,s) \in (U,V)$ be arbitrary such that $(r,s) \in (\tilde{A}_1, \tilde{A}_2)^C \cap (\tilde{B}_1, \tilde{B}_2)^C$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, (1-\vartheta_{\tilde{A}_1}) \vee (1-\vartheta_{\tilde{B}_1}), \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1} >, \\ < \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, (1-\vartheta_{\tilde{A}_2}) \vee (1-\vartheta_{\tilde{B}_2}), \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2} > \end{Bmatrix}$$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, 1-(\vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}), \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1} >, \\ < \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, 1-(\vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}), \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2} > \end{Bmatrix}$$

$$\Rightarrow (r,s) \in \begin{Bmatrix} < \mu_{\tilde{A}_1} \vee \mu_{\tilde{B}_1}, \sigma_{\tilde{A}_1} \vee \sigma_{\tilde{B}_1}, \vartheta_{\tilde{A}_1} \wedge \vartheta_{\tilde{B}_1}, \phi_{\tilde{A}_1} \wedge \phi_{\tilde{B}_1}, \gamma_{\tilde{A}_1} \wedge \gamma_{\tilde{B}_1} >, \\ < \mu_{\tilde{A}_2} \vee \mu_{\tilde{B}_2}, \sigma_{\tilde{A}_2} \vee \sigma_{\tilde{B}_2}, \vartheta_{\tilde{A}_2} \wedge \vartheta_{\tilde{B}_2}, \phi_{\tilde{A}_2} \wedge \phi_{\tilde{B}_2}, \gamma_{\tilde{A}_2} \wedge \gamma_{\tilde{B}_2} > \end{Bmatrix}^C$$

$\Rightarrow (r,s) \in ((\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2))^C$…..(14)

From (13) and (14),we get

$$((\tilde{A}_1, \tilde{A}_2) \cup (\tilde{B}_1, \tilde{B}_2))^C = (\tilde{A}_1, \tilde{A}_2)^C \cap (\tilde{B}_1, \tilde{B}_2)^C$$

(m) Similarly, by the above discussion, we can prove that

$$((\tilde{A}_1, \tilde{A}_2) \cap (\tilde{B}_1, \tilde{B}_2))^C = (\tilde{A}_1, \tilde{A}_2)^C \cup (\tilde{B}_1, \tilde{B}_2)^C$$

4. Conclusion

In this paper, we have developed pentapartitioned neutrosophic binary set and it is an extension of NBS. The concept of commutative law, associative law, absorption law, idempotent law, etc. have been defined on pentapartitioned neutrosophic binary sets. In the future, the proposed set can be applied in different algebraic structures.

Funding: “This research received no external funding”

Conflicts of Interest: “The authors declare no conflict of interest.”

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DOI: https://doi.org/10.54216/IJNS.240221
Received: October 21, 2023 Revised: February 15, 2024 Accepted: April 28, 2024