



On Some Novel Generalizations of Weak Fuzzy Complex Numbers

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Abstract

The ring of weak fuzzy complex numbers is an extension of real numbers ring by using an algebraic element with fuzzy property. In this paper, we present two novel generalizations of weak fuzzy complex numbers, where the concepts of strong fuzzy complex numbers and split-complex weak fuzzy complex numbers will be defined for the first time with a general study of their elementary properties and special elements. On the other hand, we provide an algorithm to compute the demipotent elements in the ring of split-complex weak fuzzy complex numbers with many related examples that clarify the validity of our work.

Keywords: weak fuzzy complex number; strong fuzzy complex number; split-complex weak fuzzy complex number; idempotent element

1. Introduction

The generalizations of real numbers are considered interesting research areas, where It attracted the attention of many researchers worldwide due to its great importance in the study of the theory of both spaces and matrices [2-3,9].

One of the most recent generalizations of real numbers was the concept of weak fuzzy complex numbers presented in [1] as follows:

The set of *Weak Fuzzy Complex numbers* was defined as follows, where 'J' is the Weak Fuzzy Complex operator ($J \notin \mathbb{R}$):

$$F_J = \{x_0 + x_1 J ; x_0, x_1 \in \mathbb{R}, J^2 = t \in]0, 1[\}.$$

This novel generalization played a central role in the theory of Diophantine equations [6-8], and the classification of A-curves generated from the solutions of some vectorial equations [4-5].

In this work, we build some novel generalizations of weak fuzzy complex numbers, and these new algebraic classes of numbers will be very similar to the original structure, with many possible future applications.

Main Discussion

Definition:

The set of strong fuzzy complex numbers is defined as follows:

$$S_f = \{a + bJ + ci + diJ ; a, b, c, d \in \mathbb{R}, i^2 = -1, J^2 = t \in]0, 1[\},$$

Addition on S_f is defined as follows:

$$(+): S_f \times S_f \rightarrow S_f : (a_0 + b_0J + c_0i + d_0ij) + (a_1 + b_1J + c_1i + d_1ij) = (a_0 + a_1) + (b_0 + b_1)J + (c_0 + c_1)i + (d_0 + d_1)ij.$$

Multiplication on S_f is defined as follows:

$$\begin{aligned} (\times): S_f \times S_f \rightarrow S_f : (a_0 + b_0J + c_0i + d_0ij) \times (a_1 + b_1J + c_1i + d_1ij) &= a_0a_1 + a_0b_1J + a_0c_1i + a_0d_1ij \\ &+ b_0a_1J + b_0b_1J^2 + b_0c_1ij + b_0d_1ij^2 + c_0a_1i + c_0b_1ij + c_0c_1i^2 + c_0d_1i^2J + d_0a_1ij + d_0b_1ij^2 \\ &+ d_0c_1i^2J + d_0d_1i^2J^2 \\ &= (a_0a_1 + b_0b_1t - c_0c_1 - td_0d_1) + J(a_0b_1 + b_0a_1 - c_0d_1 - d_0c_1) + i(a_0c_1 + b_0d_1t + c_0a_1 - d_0b_1t) + \\ &ij(a_0d_1 + b_0c_1 + c_0b_1 + d_0a_1). \end{aligned}$$

Remark:

If $X = a + bJ + ci + dij \in S_f$, then $X = (a + ci) + J(b + di)$.

Definition:

Consider the mapping: $\varphi: S_f \rightarrow \mathbb{C} \times \mathbb{C}$ such that:

$$\varphi[(a + ci) + J(b + di)] = (a + ci - \sqrt{+}(b + di) . a + ci + \sqrt{+}(b + di))$$

Theorem:

The mapping (φ) is a ring isomorphism.

Proof:

First, we prove that $(S_f . + . \times)$ is a commutative ring.

It is clear that $(S_f . +)$ is a belian group.

Also, the multiplication ($\times$) is commutative, associative, and distributive over $(+)$, thus $(S_f . + . \times)$ is a commutative ring.

The mapping (φ) is well defined.

$$\text{If } X = (x_0 + x_1i) + J(x_2 + x_3i) = Y = (y_0 + y_1i) + J(y_2 + y_3i),$$

Then $x_i = y_i$ for all $0 \leq i \leq 3$, thus:

$$\begin{cases} (x_0 + x_1i) - \sqrt{+}(x_2 + x_3i) = (y_0 + y_1i) - \sqrt{+}(y_2 + y_3i) \\ (x_0 + x_1i) + \sqrt{+}(x_2 + x_3i) = (y_0 + y_1i) + \sqrt{+}(y_2 + y_3i) \end{cases}$$

Hence $\varphi(X) = \varphi(Y)$.

The mapping (φ) preserves addition and multiplication:

$$X + Y = [(x_0 + y_0) + (x_1 + y_1)i] + J[(x_2 + y_2) + (x_3 + y_3)i],$$

$\varphi(X + Y) = (A, B)$, where:

$$\begin{aligned} A &= (x_0 + y_0) + (x_1 + y_1)i - \sqrt{+}[(x_2 + y_2) + (x_3 + y_3)i] \\ &= [(x_0 + x_1i) - \sqrt{+}(x_2 + x_3i)] + [(y_0 + y_1i) - \sqrt{+}(y_2 + y_3i)] \end{aligned}$$

$$\begin{aligned} B &= (x_0 + y_0) + (x_1 + y_1)i + \sqrt{+}[(x_2 + y_2) + (x_3 + y_3)i] \\ &= [(x_0 + x_1i) + \sqrt{+}(x_2 + x_3i)] + [(y_0 + y_1i) + \sqrt{+}(y_2 + y_3i)] \end{aligned}$$

Thus $\varphi(X + Y) = \varphi(X) + \varphi(Y)$.

$$\text{Also, } X \cdot Y = (x_0 + x_1i)(y_0 + y_1i) + t(x_2 + x_3i)(y_2 + y_3i) + J[(x_0 + x_1i)(y_2 + y_3i) + (y_0 + y_1i)(x_2 + x_3i)] = A + B J.$$

$\varphi(X \cdot Y) = (C, D)$, where:

$$\begin{aligned} C &= A - \sqrt{+}B = (x_0 + x_1i)(y_0 + y_1i) + t(x_2 + x_3i)(y_2 + y_3i) - \sqrt{+}(x_0 + x_1i)(y_2 + y_3i) \\ &\quad - \sqrt{+}(y_0 + y_1i)(x_2 + x_3i) \\ &= [(x_0 + x_1i) - \sqrt{+}(x_2 + x_3i)] \times [(y_0 + y_1i) - \sqrt{+}(y_2 + y_3i)]. \end{aligned}$$

$$D = A + \sqrt{+}B = [(x_0 + x_1i) + \sqrt{+}(x_2 + x_3i)] \times [(y_0 + y_1i) + \sqrt{+}(y_2 + y_3i)].$$

Thus $\varphi(X \cdot Y) = \varphi(X) \cdot \varphi(Y)$.

$$\text{If } \varphi(X) = (0, 0), \text{ then: } \begin{cases} (x_0 + x_1i) - \sqrt{+}(x_2 + x_3i) = 0 \\ (x_0 + x_1i) + \sqrt{+}(x_2 + x_3i) = 0 \end{cases}$$

$$\text{Thus: } \begin{cases} x_0 + x_1i = 0 \\ x_2 + x_3i = 0 \end{cases} \Rightarrow x_0 = x_1 = x_2 = x_3 = 0 . \text{ and } X = 0$$

So that $k_{er}(\varphi) = \{0\}$.

On the other hand, for every $(a + bi . c + di) \in \mathbb{C} \times \mathbb{C}$, we have $X = \frac{1}{2}[a + c + (b + d)i] + \frac{1}{2\sqrt{+}}J[c - a + (d - b)i] \in S_f$ such that $\varphi(X) = (a + bi . c + di)$. Thus (φ) is a ring isomorphism.

Remark:

The inverse isomorphism is: $\varphi^{-1}: \mathbb{C} \times \mathbb{C} \rightarrow S_f$ such that: $\varphi^{-1}(a + bi \cdot c + di) = \frac{1}{2}[a + c + (b + d)i] + \frac{1}{2\sqrt{+}} J[c - a + (d - b)i]$.

Special properties of S_f :

1] for $X = (x_0 + x_1i) + J(x_2 + x_3i) \in S_f, n \in \mathbb{N}$, we have:

$$X^n = \varphi^{-1}(\varphi(X))^n = \varphi^{-1}(x_0 + x_1i - \sqrt{+}(x_2 + x_3i).x_0 + x_1i + \sqrt{+}(x_2 + x_3i))^n = \varphi^{-1}(T^n.S^n) = \frac{1}{2}(T^n + S^n) + \frac{1}{2\sqrt{+}} J(S^n - T^n), \text{ where}$$

$$\begin{cases} S = x_0 + x_1i + \sqrt{+}(x_2 + x_3i) \\ T = x_0 + x_1i - \sqrt{+}(x_2 + x_3i) \end{cases}$$

2] X is invertible in S_f if and only if $S \neq 0, T \neq 0$ and $X^{-1} = \frac{1}{X} = \frac{1}{2}\left(\frac{1}{S} + \frac{1}{T}\right) + \frac{1}{2\sqrt{+}} J\left(\frac{1}{S} - \frac{1}{T}\right)$, where

$$\begin{cases} S = x_0 + x_1i + \sqrt{+}(x_2 + x_3i) \\ T = x_0 + x_1i - \sqrt{+}(x_2 + x_3i) \end{cases}$$

3] The n -th root:

The n -th root of X is $Y = (y_0 + y_1i) + J(y_2 + y_3i)$ such that $Y^n = X$.

$$\text{Hence } \varphi(Y^n) = \varphi(X) \Rightarrow \varphi(Y) = [\varphi(X)]^{\frac{1}{n}}$$

$$\Rightarrow \begin{cases} (y_0 + y_1i) - \sqrt{+}(y_2 + y_3i) = [x_0 + x_1i - \sqrt{+}(x_2 + x_3i)]^{\frac{1}{n}} = T^{\frac{1}{n}} \\ (y_0 + y_1i) + \sqrt{+}(y_2 + y_3i) = [x_0 + x_1i + \sqrt{+}(x_2 + x_3i)]^{\frac{1}{n}} = S^{\frac{1}{n}} \end{cases}$$

$$\text{So that: } \begin{cases} y_0 + y_1i = \frac{1}{2}(T^{\frac{1}{n}} + S^{\frac{1}{n}}) \\ (y_2 + y_3i) = \frac{1}{2\sqrt{+}}(S^{\frac{1}{n}} - T^{\frac{1}{n}}) \end{cases}$$

Example:

Let's find the square roots of $X = -1 + 0 \cdot J = -1$ for $J^2 = t = \frac{1}{2}$.

$\varphi(X) = (-1, -1)$, the square roots of -1 are $\{i, -i\}$.

Thus, the square roots of $\varphi(X)$ are: $\{(i, i), (i, -i), (-i, i), (-i, -i)\}$

Hence, the square roots of X are:

$$Y_1 = \varphi^{-1}(i, i) = i, Y_2 = \varphi^{-1}(-i, -i) = -i,$$

$$Y_3 = \varphi^{-1}(i, -i) = \frac{1}{2}(i - i) + \frac{1}{2\sqrt{+}} J[-i - i] = -\sqrt{2}iJ$$

$$Y_4 = \varphi^{-1}(-i, i) = \sqrt{2}iJ.$$

4] X is idempotent if and only if $X^2 = X$, thus:

$$[\varphi(X)]^2 = \varphi(X) \text{ . hence: } \begin{cases} T^2 = T \\ S^2 = S \end{cases} \Rightarrow T, S \in \{0, 1\}$$

For $T = S = 0$. then $X = 0$. for $T = S = 1$. then $X = 1$.

$$\text{For } T = 0, S = 1, \text{ we have: } \begin{cases} (x_0 + x_1i) - \sqrt{+}(x_2 + x_3i) = 0 \\ (x_0 + x_1i) + \sqrt{+}(x_2 + x_3i) = 1 \end{cases}$$

$$\Rightarrow x_0 = \frac{1}{2}, x_1 = 0, x_2 = \frac{1}{2\sqrt{+}}, x_3 = 0, \text{ thus } X = \frac{1}{2} + \frac{1}{2\sqrt{+}} J.$$

$$\text{For } T = 1, S = 0, \text{ we have: } \begin{cases} (x_0 + x_1i) - \sqrt{+}(x_2 + x_3i) = 1 \\ (x_0 + x_1i) + \sqrt{+}(x_2 + x_3i) = 0 \end{cases}$$

$$\Rightarrow x_0 = \frac{1}{2}, x_1 = 0, x_2 = \frac{-1}{2\sqrt{+}}, x_3 = 0, \text{ hence } X = \frac{1}{2} - \frac{1}{2\sqrt{+}} J.$$

Definition:

We define the set of weak fuzzy split-complex numbers as follows: $W_s = \{a + bJ + cK + dJK; a, b, c, d \in \mathbb{R}, K^2 = 1, J^2 = t \in]0, 1[\}$

Addition on W_s is defined as follows:

$$(a_0 + b_0J + c_0K + d_0JK) + (a_1 + b_1J + c_1K + d_1JK) \\ = (a_0 + a_1) + J(b_0 + b_1) + K(c_0 + c_1) + JK(d_0 + d_1)$$

Multiplication on W_s is defined as follows:

$$(a_0 + b_0J + c_0K + d_0JK) \times (a_1 + b_1J + c_1K + d_1JK) \\ = a_0a_1 + a_0b_1J + a_0c_1K + a_0d_1JK + b_0a_1J + b_0b_1J^2 + b_0c_1JK + b_0d_1J^2K$$

$$\begin{aligned}
& +c_0a_1K + c_0b_1JK + c_0c_1K^2 + c_0d_1JK^2 + d_0a_1JK + d_0b_1J^2K + d_0c_1JK^2 + d_0d_1J^2K^2 = \\
& (a_0a_1 + b_0b_1t + c_0c_1 + d_0d_1t) \\
& +J(a_0b_1 + b_0a_1 + c_0d_1 + d_0c_1) + K(a_0c_1 + b_0d_1t + c_0a_1 + d_0b_1t) + JK(a_0d_1 + b_0c_1 + c_0b_1 + d_0a_1).
\end{aligned}$$

Remark:

1] $(W_s, +, \times)$ is a commutative ring.

2] For every $X = (x_0 + x_1J + x_2K + x_3JK) \in W_s$, we can write X as follows:

$$X = (x_0 + x_2K) + J(x_1 + x_3K); x_i \in \mathbb{R} \quad 0 \leq i \leq 3.$$

$$\text{Or } X = (x_0 + x_1J) + K(x_2 + x_3J).$$

Definition:

We define the mapping $\varphi: W_s \rightarrow F_J \times F_J$ such that:

$$\varphi[(x_0 + x_1J) + K(x_2 + x_3J)] = (x_0 - x_2 + J(x_1 - x_3), x_0 + x_2 + J(x_1 + x_3))$$

Theorem:

$$(W_s, +, \times) \cong (F_J \times F_J, +, \times)$$

Proof:

The mapping is (φ) a ring isomorphism, that is because:

If $X = (x_0 + x_1J) + K(x_2 + x_3J) = Y = (y_0 + y_1J) + K(y_2 + y_3J)$, then $x_i = y_i$ for all $0 \leq i \leq 3$.

3. and $\varphi(X) = \varphi(Y)$.

It is clear that $\varphi(X + Y) = \varphi(X) + \varphi(Y)$.

$$X \times Y = (x_0 + x_1J)(y_0 + y_1J) + (x_2 + x_3J)(y_2 + y_3J) + K[(x_0 + x_1J)(y_2 + y_3J) + (x_2 + x_3J)(y_0 + y_1J)].$$

$$\varphi(X \times Y) = (A - B, A + B); \begin{cases} A = (x_0 + x_1J)(y_0 + y_1J) + (x_2 + x_3J)(y_2 + y_3J) \\ B = (x_0 + x_1J)(y_2 + y_3J) + (x_2 + x_3J)(y_0 + y_1J) \end{cases}$$

$$A - B = (x_0 + x_1J)(y_0 - y_2 + J(y_1 - y_3)) + (x_2 + x_3J)(y_2 - y_0 + J(y_3 - y_1))$$

$$= (x_0 - x_2 + (x_1 - x_3)J) \cdot (y_0 - y_2 + J(y_1 - y_3))$$

$$A + B = (x_0 + x_2 + J(x_1 + x_3)) \cdot (y_0 + y_2 + J(y_1 + y_3)), \text{ thus } \varphi(XY) = \varphi(X)\varphi(Y).$$

$$\text{If } \varphi(X) = (0, 0), \text{ then } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases}$$

So that $x_i = 0$ and $X = 0$.

For every $Y = (a + bJ, c + dJ) \in F_J \times F_J$, there exists

$$X = \frac{1}{2}(a + c + (b + d)J) + \frac{1}{2}(c - a + (d - b)J) \in W_s \text{ such that } \varphi(X) = Y, \text{ thus } \varphi \text{ is a ring isomorphism.}$$

Special properties of W_s :

1] $X = (x_0 + x_1J) + K(x_2 + x_3J)$ is invertible in W_s if and only if:

$$\begin{cases} x_0 - x_2 + J(x_1 - x_3) \\ x_0 + x_2 + J(x_1 + x_3) \end{cases} \text{ are invertible in } F_J$$

$$\text{Also, } X^{-1} = \frac{1}{X} = \varphi^{-1}(\varphi(X))^{-1} = \frac{1}{2} \left[((x_0 - x_2) + J(x_1 - x_3))^{-1} + (x_0 + x_2 + J(x_1 + x_3))^{-1} \right] +$$

$$\frac{1}{2} K \left[(x_0 + x_2 + J(x_1 + x_3))^{-1} - (x_0 - x_2 + J(x_1 - x_3))^{-1} \right]$$

2] X is idempotent if and only if: $\varphi(X^2) = \varphi(X)$.

$$\text{Let } a + bJ \in F_J \text{ with } (a + bJ)^2 = a + bJ, \text{ then } \begin{cases} (a - \sqrt{+}b)^2 = a - \sqrt{+}b \\ (a + \sqrt{+}b)^2 = a + \sqrt{+}b \end{cases} \Rightarrow \begin{cases} a - \sqrt{+}b \in \{0, 1\} \\ a + \sqrt{+}b \in \{0, 1\} \end{cases}$$

$$\Rightarrow a + bJ \in \left\{ 0, 1, \frac{1}{2} + \frac{1}{2\sqrt{+}}J, \frac{1}{2} - \frac{1}{2\sqrt{+}}J \right\}.$$

$$\text{Thus } \begin{cases} x_0 - x_2 + J(x_1 - x_3) \\ x_0 + x_2 + J(x_1 + x_3) \end{cases} \in \left\{ 0, 1, \frac{1}{2} + \frac{1}{2\sqrt{+}}J, \frac{1}{2} - \frac{1}{2\sqrt{+}}J \right\}$$

The possible cases are:

Case (1):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases} \text{ then } x_0 = x_2 = x_1 = x_3 = 0 \text{ and } X = 0$$

Case (2):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases} \text{ then } x_0 = 1, x_2 = x_3 = x_1 = 0 \text{ and } X = 1$$

Case (3):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases} \text{ . then } \begin{cases} x_0 = \frac{1}{2} \cdot x_2 = 0 \\ x_1 = \frac{1}{2\sqrt{+}} \cdot x_3 = 0 \end{cases} \text{ . so that } X = \frac{1}{2} + \frac{1}{2\sqrt{+}}J.$$

Case (4):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases} \text{ . then } X = \frac{1}{2} - \frac{1}{2\sqrt{+}}J.$$

Case (5):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases} \text{ . then } \begin{cases} x_0 = \frac{1}{2} \cdot x_2 = \frac{1}{2} \\ x_1 = x_3 = 0 \end{cases} \text{ . and } X = \frac{1}{2} + \frac{1}{2}K.$$

Case (6):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases} \text{ . then } X = \frac{1}{2} - \frac{1}{2}K.$$

Case (7):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases} \text{ . then } \begin{cases} x_0 = \frac{1}{4} \cdot x_2 = \frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \cdot x_3 = \frac{1}{4\sqrt{+}} \end{cases} \text{ . so that } X = \left(\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right) + K\left(\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right).$$

Case (8):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases} \text{ . then } \begin{cases} x_0 = \frac{1}{4} \cdot x_2 = -\frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \cdot x_3 = \frac{-1}{4\sqrt{+}} \end{cases} \text{ . so that } X = \left(\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right) + K\left(-\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right).$$

Case (9):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 0 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases} \text{ . then } \begin{cases} x_0 = \frac{1}{4} \cdot x_2 = \frac{1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} \cdot x_3 = \frac{-1}{4\sqrt{+}} \end{cases} \text{ . so that } X = \left(\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right) + K\left(\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right).$$

Case (10):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 0 \end{cases} \text{ . then } \begin{cases} x_0 = \frac{1}{4} \cdot x_2 = -\frac{1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} \cdot x_3 = \frac{1}{4\sqrt{+}} \end{cases} \text{ . so that } X = \left(\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right) + K\left(-\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right).$$

Case (11):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases} \text{ . then } \begin{cases} x_0 = \frac{3}{4} \cdot x_2 = -\frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \cdot x_3 = \frac{1}{4\sqrt{+}} \end{cases} \text{ . and } X = \left(\frac{3}{4} + \frac{1}{4\sqrt{+}}J\right) + K\left(-\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right).$$

Case (12):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases} \text{ . then } \begin{cases} x_0 = \frac{3}{4} \cdot x_2 = \frac{1}{4} \\ x_1 = \frac{1}{4\sqrt{+}} \cdot x_3 = \frac{-1}{4\sqrt{+}} \end{cases} \text{ . and } X = \left(\frac{3}{4} + \frac{1}{4\sqrt{+}}J\right) + K\left(\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right).$$

Case (13):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = 1 \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases} \text{ then } \begin{cases} x_0 = \frac{3}{4} \cdot x_2 = -\frac{1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} \cdot x_3 = \frac{-1}{4\sqrt{+}} \end{cases} \text{ and } X = \left(\frac{3}{4} - \frac{1}{4\sqrt{+}}J\right) + K\left(-\frac{1}{4} - \frac{1}{4\sqrt{+}}J\right).$$

Case (14):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = 1 \end{cases} \text{ then } \begin{cases} x_0 = \frac{3}{4} \cdot x_2 = \frac{1}{4} \\ x_1 = \frac{-1}{4\sqrt{+}} \cdot x_3 = \frac{1}{4\sqrt{+}} \end{cases} \text{ and } X = \left(\frac{3}{4} - \frac{1}{4\sqrt{+}}J\right) + K\left(\frac{1}{4} + \frac{1}{4\sqrt{+}}J\right).$$

Case (15):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \end{cases} \text{ then } \begin{cases} x_0 = \frac{1}{2} \cdot x_2 = 0 \\ x_1 = 0 \cdot x_3 = \frac{-1}{2\sqrt{+}} \end{cases} \text{ and } X = \frac{1}{2} + KJ\left(\frac{-1}{2\sqrt{+}}\right).$$

Case (16):

$$\text{If } \begin{cases} x_0 - x_2 + J(x_1 - x_3) = \frac{1}{2} - \frac{1}{2\sqrt{+}}J \\ x_0 + x_2 + J(x_1 + x_3) = \frac{1}{2} + \frac{1}{2\sqrt{+}}J \end{cases} \text{ then } \begin{cases} x_0 = \frac{1}{2} \cdot x_2 = 0 \\ x_1 = 0 \cdot x_3 = \frac{1}{2\sqrt{+}} \end{cases} \text{ and } X = \frac{1}{2} + \frac{1}{2\sqrt{+}}KJ.$$

2. Conclusion

In this paper, we presented two novel generalizations of weak fuzzy complex numbers, where the concepts of strong fuzzy complex numbers and split-complex weak fuzzy complex numbers are defined for the first time with a general study of their elementary properties and special elements. On the other hand, we provided an algorithm to compute the dempotent elements in the ring of split-complex weak fuzzy complex numbers with many related examples.

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