New approach towards $(\eth_1, \eth_2, \eth_3)$ neutrosophic normal interval valued set applied to sin trigonometric aggregating operator and its generalization

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Abstract

We introduce the concept of sine trigonometric $(\eth_1, \eth_2, \eth_3)$ neutrosophic normal interval valued set. An identifying sine trigonometric $(\eth_1, \eth_2, \eth_3)$ neutrosophic normal interval valued set is a combination of $(\eth_1, \eth_2, \eth_3)$ neutrosophic interval valued set and neutrosophic interval valued set. We communicate the new aggregating operator such as sine trigonometric $(\eth_1, \eth_2, \eth_3)$ neutrosophic normal interval valued weighted averaging, sine trigonometric $(\eth_1, \eth_2, \eth_3)$ neutrosophic normal interval valued weighted geometric, sine trigonometric generalized $(\eth_1, \eth_2, \eth_3)$ neutrosophic normal interval valued weighted averaging and sine trigonometric generalized $(\eth_1, \eth_2, \eth_3)$ neutrosophic normal interval valued weighted geometric.

Keywords: Aggregating operator; weighted averaging; weighted geometric; generalized weighted averaging and generalized weighted geometric.

1 Introduction

There are many uncertainties namely fuzzy set (FS),[1] intuitionistic FS (IFS),[8] interval-valued FS (IVFS),[2] Pythagorean FS (PyFS),[3] Pythagorean IVFS (PyIVFS),[4] neutrosophic set (NS)[5] and interval-valued NS (IVNS)[6] are put forward. There is only one grade of membership for an element in FS, such as 0 or 1. FS applications include fuzzy time series using clustering algorithms and fuzzy c-numbers.[7] Atanassov developed IFS logic, and the sum of membership grade (MG) and non-membership grade (NMG) does not exceed 1.[8] Occasionally, we use the decision-making (DM) method to solve a single problem: the sum of MG and NMG exceeds 1. Therefore, Yager[3] explained PyFS as an extension of IFS whose sum of squares of MG and NMG do not exceed 1. According to Akram et al.,[9–11] PyFS can be used for a wide range of applications. AOs in PyIVFS have been studied by Rahman et al.,[12] which we extend to group DMs. Khan[13] introduced a PyFS for the Einstein choquet integral operator application under the PyIVFS-Einstein AO. Rahman et al.[14] introduced the MAGDM approach under the PyIVFS-Einstein AO. Recently, Palanikumar et al. discussed

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the new aggregating operators[15]-.[19] The PyNIVF technique is based on the AOs developed by Yang et al.[20] Smarandache[5] introduced the NS several years ago. Neutrosophy refers to the knowledge of neutrality, and neutrality is described as the difference between FS and IFS. It is a truth grade (TG), an indeterminacy grade (IG), and a falsehood grade (FG). It ranges from 0 to 1. The NS is a generalization of the classical set, which includes FS, IVFS, and so on. Smarandache et al.[21] developed the PyNIVS with several applications. Medical diagnostics[22] and context analysis[23] were both the subjects of a single-valued NS application. By enhancing distances for IFSs, such as HD and ED and applying them to MADM situations, Ejegwa[24] produced many similar features to PyFSs. Shami[25] discussed the new notion of $(2,1)$-FS properties, weighted AOs and their applications to MCDM methods. Recently, Shami et al.[26] introduced the concept of generalized frame for orthopair FSs and (m,n)-FSs and their applications to MCDM. Yang[27] et al. discussed the concept of fuzzy c-numbers clustering procedures for fuzzy data. In many applications and AOs and its algebraic structures are explained[28]-.[30]

The paper is divided into five sections. Section 1 contains the introduction. In section 2 describes the preliminaries. In Section 3 deals fundamental operations of ST $(\eth_1,\eth_2,\eth_3)$ NNIVN. The section 4 describes the new aggregating operators such as ST $(\eth_1,\eth_2,\eth_3)$ NNIVWA, ST $(\eth_1,\eth_2,\eth_3)$ NNIVWG, STG $(\eth_1,\eth_2,\eth_3)$ NNIVWA and STG $(\eth_1,\eth_2,\eth_3)$ NNIVWG.

2 Preliminaries

The fundamental definitions for PyIVFS and PyNIVS are presented below in this section.

Definition 2.1. [4] The PyIVFS $\Re = \left\{\varsigma, \left\langle \Xi^{\mathcal{T}}_{\Re}(\varsigma), \Xi^{\mathcal{F}}_{\Re}(\varsigma)\right\rangle \middle| \varsigma \in \mathscr{U}\right\}$, where $\Xi^{\mathcal{T}}_{\Re}, \Xi^{\mathcal{F}}_{\Re} : \mathscr{U} \to Int([0,1])$ denotes MG and NMG of $\varsigma \in \mathscr{U}$ to $\Re$, respectively with $0 \le (\Xi^{\mathcal{T}+}_{\Re}(\varsigma))^2 + (\Xi^{\mathcal{F}+}_{\Re}(\varsigma))^2 \le 1$. For $\Re = \left\langle \left[\Xi^{\mathcal{T}-}_{\Re}, \Xi^{\mathcal{T}+}_{\Re}\right], \left[\Xi^{\mathcal{F}-}_{\Re}, \Xi^{\mathcal{F}+}_{\Re}\right]\right\rangle$ is called a Pythagorean interval-valued FN (PyIVFN).

Definition 2.2. [5] The NS $\Re$ in $\mathscr{U}$ is $\Re = \left\{\varsigma, \langle \Xi^{\mathcal{T}}_{\Re}(\varsigma), \Xi^{\mathcal{I}}_{\Re}(\varsigma), \Xi^{\mathcal{F}}_{\Re}(\varsigma)\rangle \middle| \varsigma \in \mathscr{U}\right\}$, where $\Xi^{\mathcal{T}}_{\Re}, \Xi^{\mathcal{I}}_{\Re}, \Xi^{\mathcal{F}}_{\Re} : \mathscr{U} \to [0,1]$ represents TG, IG and FG of $\varsigma \in \mathscr{U}$ to $\Re$, respectively with $0 \le \Xi^{\mathcal{T}}_{\Re}(\varsigma) + \Xi^{\mathcal{I}}_{\Re}(\varsigma) + \Xi^{\mathcal{F}}_{\Re}(\varsigma) \le 3$. For $\Re = \langle \Xi^{\mathcal{T}}_{\Re}, \Xi^{\mathcal{I}}_{\Re}, \Xi^{\mathcal{F}}_{\Re}\rangle$ is called a neutrosophic number (NN).

Definition 2.3. [4] Let $\Re = \left\langle [\Xi^{\mathcal{T}-}, \Xi^{\mathcal{T}+}], [\Xi^{\mathcal{F}-}, \Xi^{\mathcal{F}+}]\right\rangle$,$\Re_1 = \left\langle [\Xi^{\mathcal{T}-}_1, \Xi^{\mathcal{T}+}_1], [\Xi^{\mathcal{F}-}_1, \Xi^{\mathcal{F}+}_1]\right\rangle$ and $\Re_2 = \left\langle [\Xi^{\mathcal{T}-}_2, \Xi^{\mathcal{T}+}_2], [\Xi^{\mathcal{F}-}_2, \Xi^{\mathcal{F}+}_2]\right\rangle$ be any three PyIVFNs and $\Lambda > 0$. Then

1. $\Re_1 \sqcup \Re_2 = \left[\begin{array}{c}\left[\sqrt{(\Xi^{\mathcal{T}-}_1)^2 + (\Xi^{\mathcal{T}-}_2)^2 - (\Xi^{\mathcal{T}-}_1)^2 \cdot (\Xi^{\mathcal{T}-}_2)^2}, \sqrt{(\Xi^{\mathcal{T}+}_1)^2 + (\Xi^{\mathcal{T}+}_2)^2 - (\Xi^{\mathcal{T}+}_1)^2 \cdot (\Xi^{\mathcal{T}+}_2)^2}\right], \\ \left[\Xi^{\mathcal{F}-}_1 \cdot \Xi^{\mathcal{F}-}_2,\ \Xi^{\mathcal{F}+}_1 \cdot \Xi^{\mathcal{F}+}_2\right]\end{array}\right]$,

2. $\Re_1 \sqcap \Re_2 = \left[\begin{array}{c}\left[\Xi^{\mathcal{T}-}_1 \cdot \Xi^{\mathcal{T}-}_2,\ \Xi^{\mathcal{T}+}_1 \cdot \Xi^{\mathcal{T}+}_2\right], \\ \left[\sqrt{(\Xi^{\mathcal{F}-}_1)^2 + (\Xi^{\mathcal{F}-}_2)^2 - (\Xi^{\mathcal{F}-}_1)^2 \cdot (\Xi^{\mathcal{F}-}_2)^2}, \sqrt{(\Xi^{\mathcal{F}+}_1)^2 + (\Xi^{\mathcal{F}+}_2)^2 - (\Xi^{\mathcal{F}+}_1)^2 \cdot (\Xi^{\mathcal{F}+}_2)^2}\right]\end{array}\right]$,

3. $\Lambda \cdot \Re = \left[\left[\sqrt{1 - \left(1 - (\Xi^{\mathcal{T}-})^2\right)^{\Lambda}}, \sqrt{1 - \left(1 - (\Xi^{\mathcal{T}+})^2\right)^{\Lambda}}\ \right], \left[(\Xi^{\mathcal{F}-})^{\Lambda}, (\Xi^{\mathcal{F}+})^{\Lambda}\right]\right]$,

4. $\Re^{\Lambda} = \left[\left[(\Xi^{\mathcal{T}-})^{\Lambda}, (\Xi^{\mathcal{T}+})^{\Lambda}\right], \left[\sqrt{1 - \left(1 - (\Xi^{\mathcal{F}-})^2\right)^{\Lambda}}, \sqrt{1 - \left(1 - (\Xi^{\mathcal{F}+})^2\right)^{\Lambda}}\ \right]\right]$.

Definition 2.4. [30] The PyNIVS $\Re$ in $\mathscr{U}$ is $\Re = \left\{\varsigma, \left\langle \Xi^{\mathcal{T}}_{\Re}(\varsigma), \Xi^{\mathcal{I}}_{\Re}(\varsigma), \Xi^{\mathcal{F}}_{\Re}(\varsigma)\right\rangle \middle| \varsigma \in \mathscr{U}\right\}$, where $\Xi^{\mathcal{T}}_{\Re}, \Xi^{\mathcal{I}}_{\Re}, \Xi^{\mathcal{F}}_{\Re} : \mathscr{U} \to Int([0,1])$ represent the TG, IG and FG of $\varsigma \in \mathscr{U}$ to $\Re$, respectively with $0 \le (\Xi^{\mathcal{T}}_{\Re}(\varsigma))^2 + (\Xi^{\mathcal{I}}_{\Re}(\varsigma))^2 + (\Xi^{\mathcal{F}}_{\Re}(\varsigma))^2 \le 2$, it is observe that $0 \le (\Xi^{\mathcal{T}+}_{\Re}(\varsigma))^2 + (\Xi^{\mathcal{I}+}_{\Re}(\varsigma))^2 + (\Xi^{\mathcal{F}+}_{\Re}(\varsigma))^2 \le 2$.
For $\Re = \left\langle \left[\Xi^{\mathcal{T}-}_{\Re}, \Xi^{\mathcal{T}+}_{\Re}\right], \left[\Xi^{\mathcal{I}-}_{\Re}, \Xi^{\mathcal{I}+}_{\Re}\right], \left[\Xi^{\mathcal{F}-}_{\Re}, \Xi^{\mathcal{F}+}_{\Re}\right]\right\rangle$ is called a Pythagorean neutrosophic interval-valued number (PyNIVN).

Definition 2.5. [30] Let $\Re = \left\langle \left[\Xi^{\mathcal{T}-}_{\Re}, \Xi^{\mathcal{T}+}_{\Re}\right], \left[\Xi^{\mathcal{I}-}_{\Re}, \Xi^{\mathcal{I}+}_{\Re}\right], \left[\Xi^{\mathcal{F}-}_{\Re}, \Xi^{\mathcal{F}+}_{\Re}\right]\right\rangle$ be the PyNIVN. Then the score function of $\Re$ is $S(\Re) = \left(\frac{(\Xi^{\mathcal{T}-})^2 + (\Xi^{\mathcal{T}+})^2}{2} - \frac{(\Xi^{\mathcal{I}-})^2 + (\Xi^{\mathcal{I}+})^2}{2} + 2 - \frac{(\Xi^{\mathcal{F}-})^2 + (\Xi^{\mathcal{F}+})^2}{2}\right)$, where $S(\Re) \in [-1,1]$.

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Definition 2.6. [30] Let $(\ell,\hbar)\in N,\Re=\left\langle(\ell,\hbar);[\Xi^{\mathcal{T}-},\Xi^{\mathcal{T}+}],[\Xi^{\mathcal{I}-},\Xi^{\mathcal{I}+}],[\Xi^{\mathcal{F}-},\Xi^{\mathcal{F}+}]\right\rangle$ is a Pythagorean neutrosophic normal interval-valued number (PyNNIVN), TG, IG and FG are defined as $\left[\Xi^{\mathcal{T}-},\Xi^{\mathcal{T}+}\right]=\left[\Xi^{\mathcal{T}-}e^{-\left(\frac{y-\ell}{\hbar}\right)^2},\Xi^{\mathcal{T}+}e^{-\left(\frac{y-\ell}{\hbar}\right)^2}\right]$, $\left[\Xi^{\mathcal{I}-},\Xi^{\mathcal{I}+}\right]=\left[\Xi^{\mathcal{I}-}e^{-\left(\frac{y-\ell}{\hbar}\right)^2},\Xi^{\mathcal{I}+}e^{-\left(\frac{y-\ell}{\hbar}\right)^2}\right]$ and $\left[\Xi^{\mathcal{F}-},\Xi^{\mathcal{F}+}\right]=\left[1-(1-\Xi^{\mathcal{F}-})e^{-\left(\frac{y-\ell}{\hbar}\right)^2},\ 1-(1-\Xi^{\mathcal{F}+})e^{-\left(\frac{y-\ell}{\hbar}\right)^2}\right]$,$y\in Y$ respectively, Here Y is a non-empty set with $\left[\Xi^{\mathcal{T}-},\Xi^{\mathcal{T}+}\right],\left[\Xi^{\mathcal{I}-},\Xi^{\mathcal{I}+}\right],\left[\Xi^{\mathcal{F}-},\Xi^{\mathcal{F}+}\right]\in Int([0,1])$ and $0\le\left(\Xi^{\mathcal{T}+}(\varsigma)\right)^2+\left(\Xi^{\mathcal{I}+}(\varsigma)\right)^2+\left(\Xi^{\mathcal{F}+}(\varsigma)\right)^2\le 2$.

Definition 2.7. [27] The set of real number R and $Z(\varsigma)=e^{-\left(\frac{\varsigma-\ell}{\hbar}\right)^2},(\hbar>0)$ be the fuzzy numbers. Hence, $Z=(\ell,\hbar)$ is called the normal fuzzy number (NFN) and N denote normal FS.

Definition 2.8. [7] Let $Z_1=(\ell_1,\hbar_1)\in N$and $Z_2=(\ell_2,\hbar_2)\in N$,$(\hbar_1,\hbar_2>0)$, then Z_1 and Z_2 is $\mathbb{D}(Z_1,Z_2)=\sqrt{(\ell_1-\ell_2)^2+\frac{1}{2}(\hbar_1-\hbar_2)^2}$

3 Basic operations

There are connections between ST- $(\eth_1,\eth_2,\eth_3)$ NIVN and NFN. Thereafter, ST- $(\eth_1,\eth_2,\eth_3)$ NNIVN was defined along with its basic operations, here our convenience $\aleph=\pi/2$.

Definition 3.1. Let $(\ell,\hbar)\in N$, $\Re=\left\langle(\ell,\hbar);[\Xi^{\mathcal{T}-},\Xi^{\mathcal{T}+}],[\Xi^{\mathcal{I}-},\Xi^{\mathcal{I}+}],[\Xi^{\mathcal{F}-},\Xi^{\mathcal{F}+}]\right\rangle$ be the $(\eth_1,\eth_2,\eth_3)$ NNIVN. We define a ST- $(\eth_1,\eth_2,\eth_3)$ NNIVN set is $\sin\Re=\Big\{\Big[\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{T}-}(\varsigma))^{\eth_1}\right),\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{T}+}(\varsigma))^{\eth_1}\right)\Big],$ $\Big[\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{I}-}(\varsigma))^{\eth_2}\right),\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{I}+}(\varsigma))^{\eth_2}\right)\Big],\Big[1-\sin\left(\aleph\cdot(1-\Xi_{\Re}^{\mathcal{F}-}(\varsigma))^{\eth_3}\right),1-\sin\left(\aleph\cdot(1-\Xi_{\Re}^{\mathcal{F}+}(\varsigma))^{\eth_3}\right)\Big]\Big\}$. Clearly, $\sin\Re$ is a $(\eth_1,\eth_2,\eth_3)$ NNIVN, where TG, IG and FG of $(\eth_1,\eth_2,\eth_3)$ NNIVN are defined correspondingly, $\sin\left(\aleph\cdot\Xi_{\Re}^{\mathcal{T}+}(\varsigma)\right):\mathscr{U}\to[0,1]$ such that $0\le\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{T}+}(\varsigma))^{\eth_1}\right)\le 1$ and $\sin\left(\aleph\cdot\Xi_{\Re}^{\mathcal{I}+}(\varsigma)\right):\mathscr{U}\to[0,1]$ such that $0\le\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{I}+}(\varsigma))^{\eth_2}\right)\le 1$ and $1-\sin\left(\aleph\cdot\left(1-\Xi_{\Re}^{\mathcal{F}+}(\varsigma)\right)\right):\mathscr{U}\to[0,1]$ such that $0\le 1-\sin\left(\aleph\cdot\left((1-\Xi_{\Re}^{\mathcal{F}+}(\varsigma))^{\eth_3}\right)\right)\le 1$. Therefore, $\sin\Re=\Big\{\Big[\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{T}-}(\varsigma))^{\eth_1}\right),\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{T}+}(\varsigma))^{\eth_1}\right)\Big],$ $\Big[\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{I}-}(\varsigma))^{\eth_2}\right),\sin\left(\aleph\cdot(\Xi_{\Re}^{\mathcal{I}+}(\varsigma))^{\eth_2}\right)\Big],\Big[1-\sin\left(\aleph\cdot(1-\Xi_{\Re}^{\mathcal{F}-}(\varsigma))^{\eth_3}\right),1-\sin\left(\aleph\cdot(1-\Xi_{\Re}^{\mathcal{F}+}(\varsigma))^{\eth_3}\right)\Big]\Big\}$ is a $(\eth_1,\eth_2,\eth_3)$ NNIVN, where $\left[\Xi_{\Re}^{\mathcal{T}-},\Xi_{\Re}^{\mathcal{T}+}\right]=\left[\Xi_{\Re}^{\mathcal{T}-}e^{-\left(\frac{y-\ell}{\hbar}\right)^2},\Xi_{\Re}^{\mathcal{T}+}e^{-\left(\frac{y-\ell}{\hbar}\right)^2}\right]$,
$\left[\Xi_{\Re}^{\mathcal{I}-},\Xi_{\Re}^{\mathcal{I}+}\right]=\left[\Xi_{\Re}^{\mathcal{I}-}e^{-\left(\frac{y-\ell}{\hbar}\right)^2},\Xi_{\Re}^{\mathcal{I}+}e^{-\left(\frac{y-\ell}{\hbar}\right)^2}\right]$ and $\left[\Xi_{\Re}^{\mathcal{F}-},\Xi_{\Re}^{\mathcal{F}+}\right]=\left[\Xi_{\Re}^{\mathcal{F}-}e^{-\left(\frac{y-\ell}{\hbar}\right)^2},\Xi_{\Re}^{\mathcal{F}+}e^{-\left(\frac{y-\ell}{\hbar}\right)^2}\right]$,
$y\in Y$, where Y is a non-empty set.

Definition 3.2. Let $\Re=\left\langle(\ell,\hbar);[\Xi^{\mathcal{T}-},\Xi^{\mathcal{T}+}],[\Xi^{\mathcal{I}-},\Xi^{\mathcal{I}+}],[\Xi^{\mathcal{F}-},\Xi^{\mathcal{F}+}]\right\rangle$,$\Re_1=\left\langle(\ell_1,\hbar_1);[\Xi_1^{\mathcal{T}-},\Xi_1^{\mathcal{T}+}],[\Xi_1^{\mathcal{I}-},\Xi_1^{\mathcal{I}+}],[\Xi_1^{\mathcal{F}-},\Xi_1^{\mathcal{F}+}]\right\rangle$ and $\Re_2=\left\langle(\ell_2,\hbar_2);[\Xi_2^{\mathcal{T}-},\Xi_2^{\mathcal{T}+}],[\Xi_2^{\mathcal{I}-},\Xi_2^{\mathcal{I}+}],[\Xi_2^{\mathcal{F}-},\Xi_2^{\mathcal{F}+}]\right\rangle$ be any three ST- $(\eth_1,\eth_2,\eth_3)$ NNIVNs and $\Lambda>0$. Then

$$1.\ \sin\Re_1\sqcup\sin\Re_2=\left[\begin{array}{c}(\ell_1+\ell_2,\hbar_1+\hbar_2);\\ \left[\begin{array}{c}\sqrt[\eth_1]{\begin{array}{c}\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}+\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\\-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\cdot\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\end{array}},\\ \sqrt[\eth_1]{\begin{array}{c}\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}+\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\\-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\cdot\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\end{array}}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[\eth_2]{\begin{array}{c}\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}+\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\\-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\cdot\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\end{array}},\\ \sqrt[\eth_2]{\begin{array}{c}\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}+\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\\-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\cdot\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\end{array}}\end{array}\right],\\ \left[\begin{array}{c}\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}-})^{\eth_3}\right)\cdot\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}-})^{\eth_3}\right),\\ \sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}+})^{\eth_3}\right)\cdot\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}+})^{\eth_3}\right)\end{array}\right]\end{array}\right],$$

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2. $$\sin \Re_1 \sqcap \sin \Re_2 = \left[\begin{array}{c} (\ell_1 \cdot \ell_2, \hbar_1 \cdot \hbar_2); \\ \left[\begin{array}{c} \sin^2\left((\aleph)\cdot\left(\Xi_1^{\mathcal{T}-}\right)^{\eth_1}\right)\cdot\sin^2\left((\aleph)\cdot\left(\Xi_2^{\mathcal{T}-}\right)^{\eth_1}\right), \\ \sin^2\left((\aleph)\cdot\left(\Xi_1^{\mathcal{T}+}\right)^{\eth_1}\right)\cdot\sin^2\left((\aleph)\cdot\left(\Xi_2^{\mathcal{T}+}\right)^{\eth_1}\right) \end{array} \right], \\ \left[\begin{array}{c} \sqrt[\eth_2]{\begin{array}{c} \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2} + \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2} \\ - \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2} \cdot \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2} \end{array}}, \\ \sqrt[\eth_2]{\begin{array}{c} \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2} + \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2} \\ - \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2} \cdot \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2} \end{array}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[\eth_3]{\begin{array}{c} \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}-})^{\eth_3}\right)\right)^{\eth_3} + \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}-})^{\eth_3}\right)\right)^{\eth_3} \\ - \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}-})^{\eth_3}\right)\right)^{\eth_3} \cdot \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}-})^{\eth_3}\right)\right)^{\eth_3} \end{array}}, \\ \sqrt[\eth_3]{\begin{array}{c} \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}+})^{\eth_3}\right)\right)^{\eth_3} + \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}+})^{\eth_3}\right)\right)^{\eth_3} \\ - \left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}+})^{\eth_3}\right)\right)^{\eth_3} \cdot \left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}+})^{\eth_3}\right)\right)^{\eth_3} \end{array}} \end{array} \right] \end{array} \right],$$

3. $$\Lambda \cdot \sin \Re = \left[\begin{array}{c} (\Lambda \cdot \ell, \Lambda \cdot \hbar); \\ \left[\sqrt[\eth_1]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}}, \sqrt[\eth_1]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}} \right], \\ \left[\sqrt[\eth_2]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}}, \sqrt[\eth_2]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}} \right], \\ \left[\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\right)\right)^{\eth_3}, \left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\right)\right)^{\eth_3} \right] \end{array} \right],$$

4. $$(\sin \Re)^{\Lambda} = \left[\begin{array}{c} (\ell^{\Lambda}, \hbar^{\Lambda}); \\ \left[\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}, \left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1} \right], \\ \left[\sqrt[\eth_2]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}}, \sqrt[\eth_2]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}} \right], \\ \left[\sqrt[\eth_3]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\right)\right)^{\eth_3}}, \sqrt[\eth_3]{1-\left(1-\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\right)\right)^{\eth_3}} \right] \end{array} \right].$$

4 New aggregation operators

ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA, ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWG, ST-G $(\eth_1, \eth_2, \eth_3)$ NNIVWA, and ST-G $(\eth_1, \eth_2, \eth_3)$ NNIVWG are introduced in this section.

4.1 ST- $(\eth_1, \eth_2, \eth_3)$ NNIV weighted averaging(ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA) operator

Definition 4.1. Let $\Re_i = \left\langle (\ell_i, \hbar_i); [\Xi_i^{\mathcal{T}-}, \Xi_i^{\mathcal{T}+}], [\Xi_i^{\mathcal{I}-}, \Xi_i^{\mathcal{I}+}], [\Xi_i^{\mathcal{F}-}, \Xi_i^{\mathcal{F}+}] \right\rangle$ be the set of ST- $(\eth_1, \eth_2, \eth_3)$ NNIVNs, $\mathscr{W} = (\kappa_1, \kappa_2, ..., \kappa_n)$, and $\sqcup_{i=1}^{n} \kappa_{i=1}$. Prove that ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA $(\Re 1, \Re 2, ..., \Re n) = \sqcup_{i=1}^{n} \kappa_i \sin \Re_i$ $(i = 1, 2, ..., n)$.

Theorem 4.2. *Let $\Re_i = \left\langle (\ell_i, \hbar_i); [\Xi_i^{\mathcal{T}-}, \Xi_i^{\mathcal{T}+}], [\Xi_i^{\mathcal{I}-}, \Xi_i^{\mathcal{I}+}], [\Xi_i^{\mathcal{F}-}, \Xi_i^{\mathcal{F}+}] \right\rangle$ be a finite collection of ST- $(\eth_1, \eth_2, \eth_3)$ NNIVNs. Prove that ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA $(\Re_1, \Re_2, ..., \Re_n)$=*

$$\left[\begin{array}{c} \left(\sqcup_{i=1}^{n} \kappa_i \ell_i, \sqcup_{i=1}^{n} \kappa_i \hbar_i \right); \\ \left[\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)^{\kappa_i}}, \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\kappa_i}} \right], \\ \left[\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}}, \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}} \right], \\ \left[\sqcap_{i=1}^{n} \left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})^{\eth_3}\right)\right)^{\kappa_i}, \sqcap_{i=1}^{n} \left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})^{\eth_3}\right)\right)^{\kappa_i} \right] \end{array} \right].$$

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Proof. The proof follows from mathematical induction. Put $n = 2$, then ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA$(\Re_1, \Re_2) = \kappa_1 \cdot \sin \Re_1 \sqcup \kappa_2 \cdot \sin \Re_2$, since

$$\kappa_1 \cdot \sin \Re_1 = \begin{bmatrix} \Big(\kappa_1 \ell_1, \kappa_1 \hbar_1\Big); \\ \left[\sqrt[\eth_1]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\big)\Big)^{\kappa_1}}, \sqrt[\eth_1]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\big)\Big)^{\kappa_1}} \right], \\ \left[\sqrt[\eth_2]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\big)\Big)^{\kappa_1}}, \sqrt[\eth_2]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\big)\Big)^{\kappa_1}} \right], \\ \left[\big(\sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_1}, \big(\sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_1} \right] \end{bmatrix}$$

$$\kappa_2 \cdot \sin \Re_2 = \begin{bmatrix} \Big(\kappa_2 \ell_2, \kappa_2 \hbar_2\Big); \\ \left[\sqrt[\eth_1]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\big)\Big)^{\kappa_2}}, \sqrt[\eth_1]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\big)\Big)^{\kappa_2}} \right], \\ \left[\sqrt[\eth_2]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\big)\Big)^{\kappa_2}}, \sqrt[\eth_2]{1 - \Big(1 - \sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\big)\Big)^{\kappa_2}} \right], \\ \left[\big(\sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_2}, \big(\sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_2} \right] \end{bmatrix}.$$

Now, $\kappa_1 \cdot \sin \Re_1 \sqcup \kappa_2 \cdot \sin \Re_2 =$

$$\begin{bmatrix} \Big(\kappa_1 \ell_1 + \kappa_2 \ell_2, \kappa_1 \hbar_1 + \kappa_2 \hbar_2\Big); \\ \left[\begin{array}{c} \sqrt[\eth_1]{\begin{array}{c} \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\Big)\Big)^{\kappa_1}\Big) + \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\Big)\Big)^{\kappa_2}\Big) \\ -\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\Big)\Big)^{\kappa_1}\Big)\cdot\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\Big)\Big)^{\kappa_2}\Big) \end{array}}, \\ \sqrt[\eth_1]{\begin{array}{c} \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\Big)\Big)^{\kappa_1}\Big) + \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\Big)\Big)^{\kappa_2}\Big) \\ -\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\Big)\Big)^{\kappa_1}\Big)\cdot\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\Big)\Big)^{\kappa_2}\Big) \end{array}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[\eth_2]{\begin{array}{c} \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\Big)\Big)^{\kappa_1}\Big) + \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\Big)\Big)^{\kappa_2}\Big) \\ -\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\Big)\Big)^{\kappa_1}\Big)\cdot\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\Big)\Big)^{\kappa_2}\Big) \end{array}}, \\ \sqrt[\eth_2]{\begin{array}{c} \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\Big)\Big)^{\kappa_1}\Big) + \Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\Big)\Big)^{\kappa_2}\Big) \\ -\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\Big)\Big)^{\kappa_1}\Big)\cdot\Big(1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\Big)\Big)^{\kappa_2}\Big) \end{array}} \end{array} \right], \\ \left[\begin{array}{c} \big(\sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_1}\cdot\big(\sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_2}, \\ \big(\sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_1}\cdot\big(\sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_2} \end{array} \right] \end{bmatrix}$$

$$= \begin{bmatrix} \Big(\kappa_1 \ell_1 + \kappa_2 \ell_2, \kappa_1 \hbar_1 + \kappa_2 \hbar_2\Big); \\ \left[\begin{array}{c} \sqrt[\eth_1]{1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\Big)\Big)^{\kappa_1}\Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\Big)\Big)^{\kappa_2}}, \\ \sqrt[\eth_1]{1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\Big)\Big)^{\kappa_1}\Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\Big)\Big)^{\kappa_2}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[\eth_2]{1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\Big)\Big)^{\kappa_1}\Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\Big)\Big)^{\kappa_2}}, \\ \sqrt[\eth_2]{1 - \Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\Big)\Big)^{\kappa_1}\Big(1 - \sin^2\Big((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\Big)\Big)^{\kappa_2}} \end{array} \right], \\ \left[\begin{array}{c} \big(\sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_1}\cdot\big(\sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_2}, \\ \big(\sin^2\big((\aleph)\cdot(\Xi_1^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_1}\cdot\big(\sin^2\big((\aleph)\cdot(\Xi_2^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_2} \end{array} \right] \end{bmatrix}$$

ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA $(\Re_1, \Re_2) =$

$$\begin{bmatrix} \Big(\sqcup_{i=1}^{2} \kappa_i \ell_i, \sqcup_{i=1}^{2} \kappa_i \hbar_i\Big); \\ \left[\sqrt[\eth_1]{1 - \sqcap_{i=1}^{2}\Big(1 - \sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\big)\Big)^{\kappa_i}}, \sqrt[\eth_1]{1 - \sqcap_{i=1}^{2}\Big(1 - \sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\big)\Big)^{\kappa_i}} \right], \\ \left[\sqrt[\eth_2]{1 - \sqcap_{i=1}^{2}\Big(1 - \sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\big)\Big)^{\kappa_i}}, \sqrt[\eth_2]{1 - \sqcap_{i=1}^{2}\Big(1 - \sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\big)\Big)^{\kappa_i}} \right], \\ \left[\sqcap_{i=1}^{2}\big(\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_i}, \sqcap_{i=1}^{2}\big(\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_i} \right] \end{bmatrix}.$$

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It is valid for $n \geq 3$ and hence ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA $(\Re_1, \Re_2, ..., \Re_l) =$

$$\left[\begin{array}{c} \left(\sqcup_{i=1}^{l}\kappa_i\ell_i, \sqcup_{i=1}^{l}\kappa_i\hbar_i\right); \\ \left[\sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)^{\kappa_i}}, \sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\kappa_i}}\right], \\ \left[\sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}}, \sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}}\right], \\ \left[\sqcap_{i=1}^{l}\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})^{\eth_3}\right)\right)^{\kappa_i}, \sqcap_{i=1}^{l}\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})^{\eth_3}\right)\right)^{\kappa_i}\right] \end{array}\right].$$

Put $n = l+1$, then ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA $(\Re_1, \Re_2, ..., \Re_l, \Re_{l+1}) =$

$$\left[\begin{array}{c} \left(\sqcup_{i=1}^{l}\kappa_i\ell_i + \kappa_{l+1}\ell_{l+1}, \sqcup_{i=1}^{l}\kappa_i\hbar_i + \kappa_{l+1}\hbar_{l+1}\right); \\ \left[\begin{array}{c} \sqrt[\eth_1]{\begin{array}{c}\sqcup_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}-})^{\eth_1}\right)^{\kappa_{l+1}}\right)\right) \\ -\sqcap_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}-})^{\eth_1}\right)^{\kappa_{l+1}}\right)\right)\end{array}}, \\ \sqrt[\eth_1]{\begin{array}{c}\sqcup_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}+})^{\eth_1}\right)^{\kappa_{l+1}}\right)\right) \\ -\sqcap_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}+})^{\eth_1}\right)^{\kappa_{l+1}}\right)\right)\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\eth_2]{\begin{array}{c}\sqcup_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}-})^{\eth_2}\right)^{\kappa_{l+1}}\right)\right) \\ -\sqcap_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}-})^{\eth_2}\right)^{\kappa_{l+1}}\right)\right)\end{array}}, \\ \sqrt[\eth_2]{\begin{array}{c}\sqcup_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}\right)+\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}+})^{\eth_2}\right)^{\kappa_{l+1}}\right)\right) \\ -\sqcap_{i=1}^{l}\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}\right)\cdot\left(1-\left(1-\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}+})^{\eth_2}\right)^{\kappa_{l+1}}\right)\right)\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqcap_{i=1}^{l}\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})^{\eth_3}\right)\right)^{\kappa_i}\cdot\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{F}-})^{\eth_3}\right)\right)^{\kappa_{l+1}}, \\ \sqcap_{i=1}^{l}\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})^{\eth_3}\right)\right)^{\kappa_i}\cdot\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{F}+})^{\eth_3}\right)\right)^{\kappa_{l+1}} \end{array}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\sqcup_{i=1}^{l+1}\kappa_i\ell_i, \sqcup_{i=1}^{l+1}\kappa_i\hbar_i\right); \\ \left[\sqrt[\eth_1]{1-\sqcap_{i=1}^{l+1}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)^{\kappa_i}}, \sqrt[\eth_1]{1-\sqcap_{i=1}^{l+1}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\kappa_i}}\right], \\ \left[\sqrt[\eth_2]{1-\sqcap_{i=1}^{l+1}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}}, \sqrt[\eth_2]{1-\sqcap_{i=1}^{l+1}\left(1-\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}}\right], \\ \left[\sqcap_{i=1}^{l+1}\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})^{\eth_3}\right)\right)^{\kappa_i}, \sqcap_{i=1}^{l+1}\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})^{\eth_3}\right)\right)^{\kappa_i}\right] \end{array}\right].$$

Theorem 4.3. *If all* $\Re_i = \left\langle(\ell_i, \hbar_i); [\sin(\aleph\cdot\Xi_i^{\mathcal{T}-}), \sin(\aleph\cdot\Xi_i^{\mathcal{T}+})], [\sin(\aleph\cdot\Xi_i^{\mathcal{I}-}), \sin(\aleph\cdot\Xi_i^{\mathcal{I}+})], [\sin(\aleph\cdot\Xi_i^{\mathcal{F}-}), \sin(\aleph\cdot\Xi_i^{\mathcal{F}+})]\right\rangle (i = 1, 2, ..., n)$ *are equal and* $[\sin^2(\aleph\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}), \sin^2(\aleph\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1})] = [(\sin(\aleph\cdot(\Xi_i^{\mathcal{T}-})))^{\eth_1}, (\sin(\aleph\cdot(\Xi_i^{\mathcal{T}+})))^{\eth_1}]$ *and* $[\sin^2(\aleph\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}), \sin^2(\aleph\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2})] = [(\sin(\aleph\cdot(\Xi_i^{\mathcal{I}-})))^{\eth_2}, (\sin(\aleph\cdot(\Xi_i^{\mathcal{I}+})))^{\eth_2}]$*, then ST-* $(\eth_1, \eth_2, \eth_3)$ *NNIVWA* $(\Re_1, \Re_2, ..., \Re_n) = \Re$ *(idempotency property).*

Proof. Since $(\ell_i, \hbar_i) = (\ell, \hbar)$, $[\sin(\aleph\cdot\Xi_i^{\mathcal{T}-}), \sin(\aleph\cdot\Xi_i^{\mathcal{T}+})] = [\sin(\aleph\cdot\Xi^{\mathcal{T}-}), \sin(\aleph\cdot\Xi^{\mathcal{T}+})]$, $[\sin(\aleph\cdot\Xi_i^{\mathcal{I}-}), \sin(\aleph\cdot\Xi_i^{\mathcal{I}+})] = [\sin(\aleph\cdot\Xi^{\mathcal{I}-}), \sin(\aleph\cdot\Xi^{\mathcal{I}+})]$ and $[\sin(\aleph\cdot\Xi_i^{\mathcal{F}-}), \sin(\aleph\cdot\Xi_i^{\mathcal{F}+})] = [\sin(\aleph\cdot\Xi^{\mathcal{F}-}), \sin(\aleph\cdot\Xi^{\mathcal{F}+})]$, for

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$i = 1, 2, ..., n$ and $\sqcup_{i=1}^{n}\kappa_i = 1$. Now, ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA $(\Re_1, \Re_2, ..., \Re_n)$

$$= \left[\begin{array}{c} \Big(\sqcup_{i=1}^{n}\kappa_i\ell_i, \sqcup_{i=1}^{n}\kappa_i\hbar_i\Big); \\ \left[\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\big)\Big)^{\kappa_i}}, \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\big)\Big)^{\kappa_i}}\right], \\ \left[\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\Big(1-\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\big)\Big)^{\kappa_i}}, \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\Big(1-\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\big)\Big)^{\kappa_i}}\right], \\ \left[\sqcap_{i=1}^{n}\big(\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{F}-})^{\eth_3}\big)\big)^{\kappa_i}, \sqcap_{i=1}^{n}\big(\sin^2\big((\aleph)\cdot(\Xi_i^{\mathcal{F}+})^{\eth_3}\big)\big)^{\kappa_i}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \Big(\ell\sqcup_{i=1}^{n}\kappa_i, \hbar\sqcup_{i=1}^{n}\kappa_i\Big); \\ \left[\sqrt[\eth_1]{1-\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\big)\Big)^{\sqcup_{i=1}^{n}\kappa_i}}, \sqrt[\eth_1]{1\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\big)\Big)^{\sqcup_{i=1}^{n}\kappa_i}}\right], \\ \left[\sqrt[\eth_2]{1-\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\big)\Big)^{\sqcup_{i=1}^{n}\kappa_i}}, \sqrt[\eth_2]{1\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\big)\Big)^{\sqcup_{i=1}^{n}\kappa_i}}\right], \\ \left[\big(\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\big)\big)^{\sqcup_{i=1}^{n}\kappa_i}, \big(\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\big)\big)^{\sqcup_{i=1}^{n}\kappa_i}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} (\ell, \hbar); \\ \left[\sqrt[\eth_1]{1-\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\big)\Big)}, \sqrt[\eth_1]{1-\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\big)\Big)}\right], \\ \left[\sqrt[\eth_2]{1-\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\big)\Big)}, \sqrt[\eth_2]{1-\Big(1-\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\big)\Big)}\right], \\ \left[\big(\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\big)\big), \big(\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\big)\big)\right] \end{array}\right]$$

$$= \sin\Re.$$

Theorem 4.4. *Let* $\Re_i = \Big\langle(\ell_{ij}, \hbar_{ij}); [\Xi_{ij}^{\mathcal{T}-}, \Xi_{ij}^{\mathcal{T}+}], [\Xi_{ij}^{\mathcal{I}-}, \Xi_{ij}^{\mathcal{I}+}][\Xi_{ij}^{\mathcal{F}-}, \Xi_{ij}^{\mathcal{F}+}]\Big\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the ST-* $(\eth_1, \eth_2, \eth_3)$ *NNIVWAs. Since* $\overleftarrow{\ell} = \inf \ell_{ij}$, $\overrightarrow{\ell} = \sup \ell_{ij}$, $\overleftarrow{\hbar} = \sup \hbar_{ij}$, $\overrightarrow{\hbar} = \inf \hbar_{ij}$, $\overleftarrow{\Xi^{\mathcal{T}-}} = \inf \Xi_{ij}^{\mathcal{T}^-}$, $\overrightarrow{\Xi^{\mathcal{T}-}} = \sup \Xi_{ij}^{\mathcal{T}^-}$, $\overleftarrow{\Xi^{\mathcal{T}+}} = \inf \Xi_{ij}^{\mathcal{T}^+}$, $\overrightarrow{\Xi^{\mathcal{T}+}} = \sup \Xi_{ij}^{\mathcal{T}^+}$, $\overleftarrow{\Xi^{\mathcal{I}-}} = \inf \Xi_{ij}^{\mathcal{I}^-}$, $\overrightarrow{\Xi^{\mathcal{I}-}} = \sup \Xi_{ij}^{\mathcal{I}^-}$, $\overleftarrow{\Xi^{\mathcal{I}+}} = \inf \Xi_{ij}^{\mathcal{I}^+}$, $\overrightarrow{\Xi^{\mathcal{I}+}} = \sup \Xi_{ij}^{\mathcal{I}^+}$, $\overleftarrow{\Xi^{\mathcal{F}-}} = \inf \Xi_{ij}^{\mathcal{F}^-}$, $\overrightarrow{\Xi^{\mathcal{F}-}} = \sup \Xi_{ij}^{\mathcal{F}^-}$, $\overleftarrow{\Xi^{\mathcal{F}+}} = \inf \Xi_{ij}^{\mathcal{F}^+}$, $\overrightarrow{\Xi^{\mathcal{F}+}} = \sup \Xi_{ij}^{\mathcal{F}^+}$, $j = 1, 2, ..., i_j$ *and* $1 \leq i \leq n$. *Prove that,*

$$\begin{aligned} \Big\langle(\overleftarrow{\ell}, \overleftarrow{\hbar}); [\overleftarrow{\Xi^{\mathcal{T}-}}, \overleftarrow{\Xi^{\mathcal{T}+}}], [\overleftarrow{\Xi^{\mathcal{I}-}}, \overleftarrow{\Xi^{\mathcal{I}+}}], [\overrightarrow{\Xi^{\mathcal{F}-}}, \overrightarrow{\Xi^{\mathcal{F}+}}]\Big\rangle &\leq ST-(\eth_1, \eth_2, \eth_3)\ NNIVWA(\Re_1, \Re_2, ..., \Re_n) \\ &\leq \Big\langle(\overrightarrow{\ell}, \overrightarrow{\hbar}); [\overrightarrow{\Xi^{\mathcal{T}-}}, \overrightarrow{\Xi^{\mathcal{T}+}}], [\overrightarrow{\Xi^{\mathcal{I}-}}, \overrightarrow{\Xi^{\mathcal{I}+}}], [\overleftarrow{\Xi^{\mathcal{F}-}}, \overleftarrow{\Xi^{\mathcal{F}+}}]\Big\rangle. \end{aligned}$$

This is known as boundedness property.

Proof. Since, $\overleftarrow{\Xi^{\mathcal{T}-}} = \inf \Xi_{ij}^{\mathcal{T}^-}$, $\overrightarrow{\Xi^{\mathcal{T}-}} = \sup \Xi_{ij}^{\mathcal{T}^-}$, $\overleftarrow{\Xi^{\mathcal{T}+}} = \inf \Xi_{ij}^{\mathcal{T}^+}$, $\overrightarrow{\Xi^{\mathcal{T}+}} = \sup \Xi_{ij}^{\mathcal{T}^+}$ and $\overleftarrow{\Xi^{\mathcal{T}-}} \leq \Xi_{ij}^{\mathcal{T}^-} \leq \overrightarrow{\Xi^{\mathcal{T}-}}$ and $\overleftarrow{\Xi^{\mathcal{T}+}} \leq \Xi_{ij}^{\mathcal{T}^+} \leq \overrightarrow{\Xi^{\mathcal{T}+}}$. We have, $\overleftarrow{\Xi^{\mathcal{T}-}} + \overleftarrow{\Xi^{\mathcal{T}+}}$

$$\begin{aligned} &= \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\overleftarrow{\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\big)}\Big)^{\kappa_i}} + \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\overleftarrow{\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\big)}\Big)^{\kappa_i}} \\ &\leq \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\sin^2\Big((\aleph)\cdot(\Xi_{ij}^{\mathcal{T}^-})^{\eth_1}\Big)\Big)^{\kappa_i}} + \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\sin^2\Big((\aleph)\cdot(\Xi_{ij}^{\mathcal{T}^+})^{\eth_1}\Big)\Big)^{\kappa_i}} \\ &\leq \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\overrightarrow{\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\big)}\Big)^{\kappa_i}} + \sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\Big(1-\overrightarrow{\sin^2\big((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\big)}\Big)^{\kappa_i}} \\ &= \overrightarrow{\Xi^{\mathcal{T}-}} + \overrightarrow{\Xi^{\mathcal{T}+}}. \end{aligned}$$

Since, $\overleftarrow{\Xi^{\mathcal{I}-}} = \inf \Xi_{ij}^{\mathcal{I}^-}$, $\overrightarrow{\Xi^{\mathcal{I}-}} = \sup \Xi_{ij}^{\mathcal{I}^-}$, $\overleftarrow{\Xi^{\mathcal{I}+}} = \inf \Xi_{ij}^{\mathcal{I}^+}$, $\overrightarrow{\Xi^{\mathcal{I}+}} = \sup \Xi_{ij}^{\mathcal{I}^+}$ and $\overleftarrow{\Xi^{\mathcal{I}-}} \leq \Xi_{ij}^{\mathcal{I}^-} \leq \overrightarrow{\Xi^{\mathcal{I}-}}$ and

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$\overleftarrow{\Xi^{\mathcal{I}+}} \leq \Xi_{ij}^{\mathcal{I}^+} \leq \overrightarrow{\Xi^{\mathcal{I}+}}$. We have, $\overleftarrow{\Xi^{\mathcal{I}-}} + \overleftarrow{\Xi^{\mathcal{I}+}}$

$$\begin{aligned}
&= \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overleftarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\right)}\right)^{\kappa_i}} + \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overleftarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\right)}\right)^{\kappa_i}} \\
&\leq \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{I}^-})^{\eth_2}\right)\right)^{\kappa_i}} + \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{I}^+})^{\eth_2}\right)\right)^{\kappa_i}} \\
&\leq \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overrightarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\right)}\right)^{\kappa_i}} + \sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overrightarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\right)}\right)^{\kappa_i}} \\
&= \overrightarrow{\Xi^{\mathcal{I}-}} + \overrightarrow{\Xi^{\mathcal{I}+}}.
\end{aligned}$$

Since, $\overleftarrow{\Xi^{\mathcal{F}-}} = \inf \Xi_{ij}^{\mathcal{F}^-}$, $\overrightarrow{\Xi^{\mathcal{F}-}} = \sup \Xi_{ij}^{\mathcal{F}^-}$, $\overleftarrow{\Xi^{\mathcal{F}+}} = \inf \Xi_{ij}^{\mathcal{F}^+}$, $\overrightarrow{\Xi^{\mathcal{F}+}} = \sup \Xi_{ij}^{\mathcal{F}^+}$ and $\overleftarrow{\Xi^{\mathcal{F}-}} \leq \Xi_{ij}^{\mathcal{F}^-} \leq \overrightarrow{\Xi^{\mathcal{F}-}}$ and $\overleftarrow{\Xi^{\mathcal{F}+}} \leq \Xi_{ij}^{\mathcal{F}^+} \leq \overrightarrow{\Xi^{\mathcal{F}+}}$. We have,

$$\begin{aligned}
\overleftarrow{\Xi^{\mathcal{F}-}} + \overleftarrow{\Xi^{\mathcal{F}+}} &= \sqcap_{i=1}^{n}\overleftarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\right)\right)}^{\kappa_i} + \sqcap_{i=1}^{n}\overleftarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\right)\right)}^{\kappa_i} \\
&\leq \sqcap_{i=1}^{n}\left(\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{F}^-})^{\eth_3}\right)\right)^{\kappa_i} + \sqcap_{i=1}^{n}\left(\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{F}^+})^{\eth_3}\right)\right)^{\kappa_i} \\
&\leq \sqcap_{i=1}^{n}\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\right)\right)^{\kappa_i}} + \sqcap_{i=1}^{n}\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\right)\right)^{\kappa_i}} \\
&= \overrightarrow{\Xi^{\mathcal{F}-}} + \overrightarrow{\Xi^{\mathcal{F}+}}.
\end{aligned}$$

Since, $\overleftarrow{\ell} = \inf \ell_{ij}$, $\overrightarrow{\ell} = \sup \ell_{ij}$, $\overleftarrow{\hbar} = \sup \hbar_{ij}$, $\overrightarrow{\hbar} = \inf \hbar_{ij}$ and $\overleftarrow{\ell} \leq \ell_{ij} \leq \overrightarrow{\ell}$ and $\overrightarrow{\hbar} \leq \hbar_{ij} \leq \overleftarrow{\hbar}$.
Hence, $\sqcup_{i=1}^{n}\kappa_i\overleftarrow{\ell} \leq \sqcup_{i=1}^{n}\kappa_i\ell_{ij} \leq \sqcup_{i=1}^{n}\kappa_i\overrightarrow{\ell}$ and $\sqcup_{i=1}^{n}\kappa_i\overrightarrow{\hbar} \leq \sqcup_{i=1}^{n}\kappa_i\hbar_{ij} \leq \sqcup_{i=1}^{n}\kappa_i\overleftarrow{\hbar}$.
Therefore,

$$\begin{aligned}
&\frac{\sqcup_{i=1}^{n}\kappa_i\overleftarrow{\ell}}{2} \times \left[\frac{\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\overleftarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\right)}\right)^{\kappa_i}}\right)^2 + \left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\overleftarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\right)}\right)^{\kappa_i}}\right)^2}{-\frac{\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overleftarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\right)}\right)^{\kappa_i}}\right)^2 + \left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overleftarrow{\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\right)}\right)^{\kappa_i}}\right)^2}{2} + 2 - \frac{\left(\sqcap_{i=1}^{n}\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\right)\right)^{\kappa_i}}\right)^2 + \left(\sqcap_{i=1}^{n}\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\right)\right)^{\kappa_i}}\right)^2}{2}}\right] \\
\leq\ &\frac{\sqcup_{i=1}^{n}\kappa_i\ell_{ij}}{2} \times \left[\frac{\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{T}^-})^{\eth_1}\right)\right)^{\kappa_i}}\right)^2 + \left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{T}^+})^{\eth_1}\right)\right)^{\kappa_i}}\right)^2}{-\frac{\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{I}^-})^{\eth_2}\right)\right)^{\kappa_i}}\right)^2 + \left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{I}^+})^{\eth_2}\right)\right)^{\kappa_i}}\right)^2}{2} + 2 - \frac{\left(\sqcap_{i=1}^{n}\left(\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{F}^-})^{\eth_3}\right)\right)^{\kappa_i}\right)^2 + \left(\sqcap_{i=1}^{n}\left(\sin^2\left((\aleph)\cdot(\Xi_{ij}^{\mathcal{F}^+})^{\eth_3}\right)\right)^{\kappa_i}\right)^2}{2}}\right] \\
\leq\ &\frac{\sqcup_{i=1}^{n}\kappa_i\overrightarrow{\ell}}{2} \times \left[\frac{\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}-})^{\eth_1}\right)\right)}\right)^{\kappa_i}}\right)^2 + \left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{T}+})^{\eth_1}\right)\right)}\right)^{\kappa_i}}\right)^2}{-\frac{\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}-})^{\eth_2}\right)\right)}\right)^{\kappa_i}}\right)^2 + \left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\overrightarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{I}+})^{\eth_2}\right)\right)}\right)^{\kappa_i}}\right)^2}{2} + 2 - \frac{\left(\sqcap_{i=1}^{n}\overleftarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}-})^{\eth_3}\right)\right)}^{\kappa_i}\right)^2 + \left(\sqcap_{i=1}^{n}\overleftarrow{\left(\sin^2\left((\aleph)\cdot(\Xi^{\mathcal{F}+})^{\eth_3}\right)\right)}^{\kappa_i}\right)^2}{2}}\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
\left\langle(\overleftarrow{\ell}, \overleftarrow{\hbar}); [\overleftarrow{\Xi^{\mathcal{T}-}}, \overleftarrow{\Xi^{\mathcal{T}+}}], [\overleftarrow{\Xi^{\mathcal{I}-}}, \overleftarrow{\Xi^{\mathcal{I}+}}], [\overrightarrow{\Xi^{\mathcal{F}-}}, \overrightarrow{\Xi^{\mathcal{F}+}}]\right\rangle &\leq ST-(\eth_1, \eth_2, \eth_3)\ \text{NNIVWA}(\Re_1, \Re_2, ..., \Re_n) \\
&\leq \left\langle(\overrightarrow{\ell}, \overrightarrow{\hbar}); [\overrightarrow{\Xi^{\mathcal{T}-}}, \overrightarrow{\Xi^{\mathcal{T}+}}], [\overrightarrow{\Xi^{\mathcal{I}-}}, \overrightarrow{\Xi^{\mathcal{I}+}}], [\overleftarrow{\Xi^{\mathcal{F}-}}, \overleftarrow{\Xi^{\mathcal{F}+}}]\right\rangle.
\end{aligned}$$

Theorem 4.5. *Let* $\Re_i = \left\langle(\ell_{t_{ij}}, \hbar_{t_{ij}}); [\Xi_{t_{ij}}^{\mathcal{T}-}, \Xi_{t_{ij}}^{\mathcal{T}+}], [\Xi_{t_{ij}}^{\mathcal{I}-}, \Xi_{t_{ij}}^{\mathcal{I}+}], [\Xi_{t_{ij}}^{\mathcal{F}-}, \Xi_{t_{ij}}^{\mathcal{F}+}]\right\rangle$ *and* $\mathscr{W}_i = \left\langle(\ell_{h_{ij}}, \hbar_{h_{ij}});\ [\Xi_{h_{ij}}^{\mathcal{T}-}, \Xi_{h_{ij}}^{\mathcal{T}+}], [\Xi_{h_{ij}}^{\mathcal{I}-}, \Xi_{h_{ij}}^{\mathcal{I}+}], [\Xi_{h_{ij}}^{\mathcal{F}-}, \Xi_{h_{ij}}^{\mathcal{F}+}]\right\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the ST-*$(\eth_1, \eth_2, \eth_3)$ *NNIVWAs. For any i, if* $\ell_{t_{ij}} \leq \hbar_{h_{ij}}$,

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$\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{T}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{T}+}\right)^2\right)\le\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{T}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{T}+}\right)^2\right)$ *and* $\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{I}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{I}+}\right)^2\right)\le\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{I}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{I}+}\right)^2\right)\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{F}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{F}+}\right)^2\right)\ge\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{F}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{F}+}\right)^2\right)$ *or* $\Re_i\le\mathscr{W}_i$. *Prove that ST-* $(\eth_1,\eth_2,\eth_3)$ *NNIVWA* $(\Re_1,\Re_2,...,\Re_n)\le$ *ST-* $(\eth_1,\eth_2,\eth_3)$ *NNIVWA* $(\mathscr{W}_1,\mathscr{W}_2,...,\mathscr{W}_n)$ *(monotonicity property).*

Proof. For every i, $\ell_{t_{ij}}\le\hbar_{h_{ij}}$. Therefore, $\sqcup_{i=1}^n\ell_{t_{ij}}\le\sqcup_{i=1}^n\hbar_{h_{ij}}$.
For any i, $\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{T}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{T}+}\right)^2\right)\le\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{T}-}\right)^2\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{T}+}\right)^2\right)$.
Therefore, $1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{T}-})^2\right)+1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{T}+})^2\right)\ge1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{T}-})^2\right)+1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{T}+})^2\right)$.
Hence, $\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{T}-})^2\right)\right)^{\kappa_i}+\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{T}+})^2\right)\right)^{\kappa_i}\ge$
$\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{T}-})^2\right)\right)^{\kappa_i}+\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{T}+})^2\right)\right)^{\kappa_i}$
implies that $\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot\left(\Xi_{t_i}^{\mathcal{T}-}\right)^{\eth_1}\right)\right)^{\kappa_i}}+\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot\left(\Xi_{t_i}^{\mathcal{T}+}\right)^{\eth_1}\right)\right)^{\kappa_i}}$
$\le\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot\left(\Xi_{h_i}^{\mathcal{T}-}\right)^{\eth_1}\right)\right)^{\kappa_i}}+\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot\left(\Xi_{h_i}^{\mathcal{T}+}\right)^{\eth_1}\right)\right)^{\kappa_i}}$.
For any i, $\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{I}-}\right)^{\eth_2}\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{I}+}\right)^{\eth_2}\right)\le\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{I}-}\right)^{\eth_2}\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{I}+}\right)^{\eth_2}\right)$.
Therefore, $1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{I}-})^{\eth_2}\right)+1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{I}+})^{\eth_2}\right)\ge$
$1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{I}-})^{\eth_2}\right)+1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{I}+})^{\eth_2}\right)$.
Hence, $\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}+\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}\ge$
$\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}+\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}$.
Hence, $\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}}+\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{t_i}^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}}$
$\le\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{I}-})^{\eth_2}\right)\right)^{\kappa_i}}+\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{h_i}^{\mathcal{I}+})^{\eth_2}\right)\right)^{\kappa_i}}$.
For any i, $\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{F}-}\right)^{\eth_3}\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{t_{ij}}^{\mathcal{F}+}\right)^{\eth_3}\right)\ge\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{F}-}\right)^{\eth_3}\right)+\sin^2\left((\aleph)\cdot\left(\Xi_{h_{ij}}^{\mathcal{F}+}\right)^{\eth_3}\right)$.
Therefore, $2-\frac{\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{t_{ij}}^{\mathcal{F}-})^{\eth_3}\right)\right)^2+\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{t_{ij}}^{\mathcal{F}+})^{\eth_3}\right)\right)^2}{2}\le$
$2-\frac{\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{h_{ij}}^{\mathcal{F}-})^{\eth_3}\right)\right)^2+\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{h_{ij}}^{\mathcal{F}+})^{\eth_3}\right)\right)^2}{2}$.
Now,

$$\frac{\sqcup_{i=1}^n\ell_{tij}}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ti}^{\mathcal{T}^-})^{\eth_1}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ti}^{\mathcal{T}^+})^{\eth_1}\right)\right)^{\kappa_i}}\right)^2}{2}\\-\frac{\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ti}^{\mathcal{I}^-})^{\eth_2}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{ti}^{\mathcal{I}^+})^{\eth_2}\right)\right)^{\kappa_i}}\right)^2}{2}\\+2-\frac{\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{t_{ij}}^{\mathcal{F}-})^{\eth_3}\right)\right)^2+\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{t_{ij}}^{\mathcal{F}+})^{\eth_3}\right)\right)^2}{2}\end{array}\right]$$

$$\le\frac{\sqcup_{i=1}^n\ell_{hij}}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{hi}^{\mathcal{T}^-})^{\eth_1}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{hi}^{\mathcal{T}^+})^{\eth_1}\right)\right)^{\kappa_i}}\right)^2}{2}\\-\frac{\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{hi}^{\mathcal{I}^-})^{\eth_2}\right)\right)^{\kappa_i}}\right)^2+\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^n\left(1-\sin^2\left((\aleph)\cdot(\Xi_{hi}^{\mathcal{I}^+})^{\eth_2}\right)\right)^{\kappa_i}}\right)^2}{2}\\+2-\frac{\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{h_{ij}}^{\mathcal{F}-})^{\eth_3}\right)\right)^2+\left(\sqcap_{i=1}^n\sin^2\left((\aleph)\cdot(\Xi_{h_{ij}}^{\mathcal{F}+})^{\eth_3}\right)\right)^2}{2}\end{array}\right].$$

Hence, ST- $(\eth_1,\eth_2,\eth_3)$ NNIVWA $(\Re_1,\Re_2,...,\Re_n)\le$ ST- $(\eth_1,\eth_2,\eth_3)$ NNIVWA $(\mathscr{W}_1,\mathscr{W}_2,...,\mathscr{W}_n)$.

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4.2 ST- $(\eth_1, \eth_2, \eth_3)$ NNIV Weighted Geometric(ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWG) Operator

Definition 4.6. Let $\Re_i = \left\langle (\ell_i, \hbar_i); [\Xi_i^{\mathcal{T}-}, \Xi_i^{\mathcal{T}+}], [\Xi_i^{\mathcal{I}-}, \Xi_i^{\mathcal{I}+}], [\Xi_i^{\mathcal{F}-}, \Xi_i^{\mathcal{F}+}] \right\rangle$ be the collection of ST- $(\eth_1, \eth_2, \eth_3)$ NNIVNs. Then ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWG $(\Re_1, \Re_2, ..., \Re_n) = \sqcap_{i=1}^{n} (\sin \Re_i)^{\kappa_i}$ $(i = 1, 2, ..., n)$.

Theorem 4.7. *Let* $\Re_i = \left\langle (\ell_i, \hbar_i); [\Xi_i^{\mathcal{T}-}, \Xi_i^{\mathcal{T}+}], [\Xi_i^{\mathcal{F}-}, \Xi_i^{\mathcal{F}+}] \right\rangle$ *be the set of ST-* $(\eth_1, \eth_2, \eth_3)$ *NNIVNs. Prove that ST-* $(\eth_1, \eth_2, \eth_3)$ *NNIVWG* $(\Re_1, \Re_2, ..., \Re_n) =$

$$\left[\begin{array}{c} \left(\sqcap_{i=1}^{n} \ell_i^{\kappa_i}, \sqcap_{i=1}^{n} \hbar_i^{\kappa_i} \right); \\ \left[\sqcap_{i=1}^{n} \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{T}-})^{\eth_1} \right) \right)^{\kappa_i}, \sqcap_{i=1}^{n} \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{T}+})^{\eth_1} \right) \right)^{\kappa_i} \right], \\ \left[\sqrt[\eth_2]{1 - \sqcap_{i=1}^{n} \left(1 - \sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{I}-})^{\eth_2} \right) \right)^{\kappa_i}}, \sqrt[\eth_2]{1 - \sqcap_{i=1}^{n} \left(1 - \sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{I}+})^{\eth_2} \right) \right)^{\kappa_i}} \right], \\ \left[\sqrt[\eth_3]{1 - \sqcap_{i=1}^{n} \left(1 - \sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{F}-})^{\eth_3} \right) \right)^{\kappa_i}}, \sqrt[\eth_3]{1 - \sqcap_{i=1}^{n} \left(1 - \sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{F}+})^{\eth_3} \right) \right)^{\kappa_i}} \right] \end{array}\right].$$

Proof. The theorem 4.2 provides the proof.

Corollary 4.8. *Idempotency, boundedness and monotonicity are satisfied by the ST-* $(\eth_1, \eth_2, \eth_3)$ *NNIVWG operator.*

Proof. The proof is provided by the following Theorem 4.3, Theorem 4.4 and Theorem 4.5.

4.3 Generalized ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWA (ST-G $(\eth_1, \eth_2, \eth_3)$ NNIVWA) Operator

Definition 4.9. Let $\Re_i = \left\langle (\ell_i, \hbar_i); [\Xi_i^{\mathcal{T}-}, \Xi_i^{\mathcal{T}+}], [\Xi_i^{\mathcal{I}-}, \Xi_i^{\mathcal{I}+}], [\Xi_i^{\mathcal{F}-}, \Xi_i^{\mathcal{F}+}] \right\rangle$ be the collection of ST- $(\eth_1, \eth_2, \eth_3)$ NNIVN. The ST-G $(\eth_1, \eth_2, \eth_3)$ NNIVWA operator is ST-G $(\eth_1, \eth_2, \eth_3)$ NNIVWA $(\Re_1, \Re_2, ..., \Re_n) = \Big(\sqcup_{i=1}^{n} \kappa_i (\sin \Re_i)^{\Lambda} \Big)^{1/\Lambda}$.

Theorem 4.10. *Let* $\Re_i = \left\langle (\ell_i, \hbar_i); [\Xi_i^{\mathcal{T}-}, \Xi_i^{\mathcal{T}+}], [\Xi_i^{\mathcal{I}-}, \Xi_i^{\mathcal{I}+}], [\Xi_i^{\mathcal{F}-}, \Xi_i^{\mathcal{F}+}] \right\rangle$ *be the set of ST-* $(\eth_1, \eth_2, \eth_3)$ *NNIVNs. Prove that ST-G* $(\eth_1, \eth_2, \eth_3)$ *NNIVWA* $(\Re_1, \Re_2, ..., \Re_n) =$

$$\left[\begin{array}{c} \left(\left(\sqcup_{i=1}^{n} \kappa_i \ell_i^{\Lambda} \right)^{1/\Lambda}, \left(\sqcup_{i=1}^{n} \kappa_i \hbar_i^{\Lambda} \right)^{1/\Lambda} \right); \\ \left[\begin{array}{c} \left(\sqrt[\eth_1]{1 - \sqcap_{i=1}^{n} \left(1 - \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{T}-})^{\eth_1} \right) \right)^{\eth_1} \right)^{\kappa_i}} \right)^{1/\eth_1}, \\ \left(\sqrt[\eth_1]{1 - \sqcap_{i=1}^{n} \left(1 - \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{T}+})^{\eth_1} \right) \right)^{\eth_1} \right)^{\kappa_i}} \right)^{1/\eth_1} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[\eth_2]{1 - \sqcap_{i=1}^{n} \left(1 - \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{I}-})^{\eth_2} \right) \right)^{\eth_2} \right)^{\kappa_i}} \right)^{1/\eth_2}, \\ \left(\sqrt[\eth_2]{1 - \sqcap_{i=1}^{n} \left(1 - \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{I}+})^{\eth_2} \right) \right)^{\eth_2} \right)^{\kappa_i}} \right)^{1/\eth_2} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[\eth_3]{1 - \left(1 - \left(\sqcap_{i=1}^{n} \left(\sqrt[\eth_3]{1 - \left(1 - \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{F}-}) \right) \right) \right)^{\eth_3}} \right)^{\eth_3} \right)^{\kappa_i} \right)^{\eth_3}}^{1/\eth_3}, \\ \sqrt[\eth_3]{1 - \left(1 - \left(\sqcap_{i=1}^{n} \left(\sqrt[\eth_3]{1 - \left(1 - \left(\sin^2 \left((\aleph) \cdot (\Xi_i^{\mathcal{F}+}) \right) \right) \right)^{\eth_3}} \right)^{\eth_3} \right)^{\kappa_i} \right)^{\eth_3}}^{1/\eth_3} \end{array}\right] \end{array}\right].$$

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Proof. It is necessary to demonstrate that, $\sqcup_{i=1}^{n}\kappa_i(\sin\Re_i)^{\Lambda} =$

$$\left[\begin{array}{c} \left(\left(\sqcup_{i=1}^{n}\kappa_i\ell_i^{\Lambda}\right),\left(\sqcup_{i=1}^{n}\kappa_i\hbar_i^{\Lambda}\right)\right); \\ \left[\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}},\sqrt[\eth_1]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}}\right], \\ \left[\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}},\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right], \\ \left[\sqcap_{i=1}^{n}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i},\sqcap_{i=1}^{n}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\right] \end{array}\right].$$

Put $n=2$, $\kappa_1(\sin\Re_1)^{\Lambda}\sqcup\kappa_2(\sin\Re_2)^{\Lambda} =$

$$\left[\begin{array}{c} \left(\kappa_1\ell_1^{\Lambda}+\kappa_2\ell_2^{\Lambda},\kappa_1\hbar_1^{\Lambda}+\kappa_2\hbar_2^{\Lambda}\right); \\ \left[\begin{array}{c} \sqrt[\eth_1]{\begin{array}{c}\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}+\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}, \\ -\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}\cdot\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}\end{array}}, \\ \sqrt[\eth_1]{\begin{array}{c}\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}+\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1} \\ -\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}\cdot\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}\end{array}} \end{array}\right] \\ \left[\begin{array}{c} \sqrt[\eth_2]{\begin{array}{c}\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}+\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}, \\ -\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}\cdot\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}\end{array}}, \\ \sqrt[\eth_2]{\begin{array}{c}\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}+\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2} \\ -\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}\cdot\left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}\end{array}} \end{array}\right] \\ \left[\begin{array}{c}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_1}\cdot\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_1}, \\ \left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{F}+})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_1}\cdot\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_2^{\mathcal{F}+})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_1}\end{array}\right] \end{array}\right]$$

implies that,

$$\left[\begin{array}{c} \left(\sqcup_{i=1}^{2}\kappa_i\ell_i^{\Lambda},\sqcup_{i=1}^{2}\kappa_i\hbar_i^{\Lambda}\right); \\ \left[\sqrt[\eth_3]{1-\sqcap_{i=1}^{2}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_3}\right)\right)^{\eth_3}\right)^{\kappa_i}},\sqrt[\eth_3]{1-\sqcap_{i=1}^{2}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_3}\right)\right)^{\eth_3}\right)^{\kappa_i}}\right], \\ \left[\sqrt[\eth_3]{1-\sqcap_{i=1}^{2}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_3}\right)\right)^{\eth_3}\right)^{\kappa_i}},\sqrt[\eth_3]{1-\sqcap_{i=1}^{2}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_3}\right)\right)^{\eth_3}\right)^{\kappa_i}}\right], \\ \left[\sqcap_{i=1}^{2}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i},\sqcap_{i=1}^{2}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\right] \end{array}\right].$$

In general, $\sqcup_{i=1}^{l}\kappa_i(\sin\Re_i)^{\Lambda} =$

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$$\left[\begin{array}{c}\left(\sqcup_{i=1}^{l}\kappa_i\ell_i^{\Lambda},\sqcup_{i=1}^{l}\kappa_i\hbar_i^{\Lambda}\right);\\ \left[\sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}},\ \sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}}\right],\\ \left[\sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}},\ \sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right],\\ \left[\sqcap_{i=1}^{l}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i},\sqcap_{i=1}^{l}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\right]\end{array}\right].$$

If $n=l+1$, then $\sqcup_{i=1}^{l}\kappa_i(\sin\Re_i)^{\Lambda}+\kappa_{l+1}(\sin\Re_{l+1})^{\Lambda}=\sqcup_{i=1}^{l+1}\kappa_i(\sin\Re_i)^{\Lambda}$.
Now, $\sqcup_{i=1}^{l}\kappa_i(\sin\Re_i)^{\Lambda}+\kappa_{l+1}(\sin\Re_{l+1})^{\Lambda}=\kappa_1(\sin\Re_1)^{\Lambda}\sqcup\kappa_2(\sin\Re_2)^{\Lambda}\sqcup...\sqcup\kappa_l(\sin\Re_l)^{\Lambda}\sqcup\kappa_{l+1}(\sin\Re_{l+1})^{\Lambda}$

$$=\left[\begin{array}{c}\left(\sqcup_{i=1}^{l}\kappa_i\ell_i^{\Lambda}+\kappa_{l+1}\ell_{l+1}^{\Lambda},\sqcup_{i=1}^{l}\kappa_i\hbar_i^{\Lambda}+\kappa_{l+1}\hbar_{l+1}^{\Lambda}\right);\\ \left[\begin{array}{c}\sqrt[\eth_1]{\begin{array}{r}\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)\right)^{\eth_1}\right)^{\kappa_i}}\right)^{\eth_1}+\\ \left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1},\\ -\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)\right)^{\eth_1}\right)^{\kappa_i}}\right)^{\eth_1}\cdot\\ \left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}\end{array}},\\ \sqrt[\eth_1]{\begin{array}{r}\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}}\right)^{\eth_1}+\\ \left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}\\ -\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}}\right)^{\eth_1}\cdot\\ \left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_1}}\right)^{\eth_1}\end{array}}\end{array}\right]\\ \left[\begin{array}{c}\sqrt[\eth_2]{\begin{array}{r}\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{\eth_2}+\\ \left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2},\\ -\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{\eth_2}\cdot\\ \left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}\end{array}},\\ \sqrt[\eth_2]{\begin{array}{r}\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{\eth_2}+\\ \left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}\\ -\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{l}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{\eth_2}\cdot\\ \left(\sqrt[\eth_2]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{I}+})^{\eth_2}\right)\right)^{2\eth_2}\right)^{\kappa_1}}\right)^{\eth_2}\end{array}}\end{array}\right]\\ \left[\begin{array}{c}\sqcap_{i=1}^{l}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\cdot\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_1},\\ \sqcap_{i=1}^{l}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\cdot\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_{l+1}^{\mathcal{F}+})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_1}\end{array}\right]\end{array}\right]$$

Thus, $\sqcup_{i=1}^{l+1}\kappa_i(\sin\Re_i)^{\Lambda}=$

$$\left[\begin{array}{c}\left(\sqcup_{i=1}^{l+1}\kappa_i\ell_i^{\Lambda},\sqcup_{i=1}^{l+1}\kappa_i\hbar_i^{\Lambda}\right);\\ \left[\sqrt[\eth_1]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}},\ \sqrt[\eth_1]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}}\right],\\ \left[\sqrt[\eth_2]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}},\ \sqrt[\eth_2]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_1^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right],\\ \left[\sqcap_{i=1}^{l+1}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i},\sqcap_{i=1}^{l+1}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\right]\end{array}\right].$$

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Hence, $\left(\sqcup_{i=1}^{l+1}\kappa_i(\sin\Re_i)^{\Lambda}\right)^{1/\Lambda}=$

$$\left[\begin{array}{c}\left(\left(\sqcup_{i=1}^{l+1}\kappa_i\ell_i^{\Lambda}\right)^{1/\Lambda},\left(\sqcup_{i=1}^{l+1}\kappa_i\hbar_i^{\Lambda}\right)^{1/\Lambda}\right);\\ \left[\begin{array}{c}\left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}}\right)^{1/\eth_1},\\ \left(\sqrt[\eth_1]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})^{\eth_1}\right)\right)^{\eth_1}\right)^{\kappa_i}}\right)^{1/\eth_1}\end{array}\right],\\ \left[\begin{array}{c}\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{1/\eth_2},\\ \left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{l+1}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{1/\eth_2}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[\eth_3]{1-\left(1-\left(\sqcap_{i=1}^{l+1}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\right)^2\right)^{1/\eth_3}},\\ \sqrt[\eth_3]{1-\left(1-\left(\sqcap_{i=1}^{l+1}\left(\sqrt[\eth_3]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})\right)\right)^{\eth_3}\right)^{\eth_3}}\right)^{\kappa_i}\right)^2\right)^{1/\eth_3}}\end{array}\right]\end{array}\right].$$

It is true for any l.

A ST- $(\eth_1,\eth_2,\eth_3)$ NNIVWG operator is substituted for ST-G $(\eth_1,\eth_2,\eth_3)$ NNIVWA when $\Lambda=1$.

Corollary 4.11. *Idempotent, bounded, and monotonic properties are fulfilled by the ST-G $(\eth_1,\eth_2,\eth_3)$ NNIVWA operator.*

Proof. Proof is provided by Theorem 4.3, Theorem 4.4 and Theorem 4.5.

4.4 Generalized ST- $(\eth_1,\eth_2,\eth_3)$ NNIVWG (ST-G $(\eth_1,\eth_2,\eth_3)$ NNIVWG) Operator

Definition 4.12. Let $\Re_i=\left\langle(\ell_i,\hbar_i);[\Xi_i^{\mathcal{T}-},\Xi_i^{\mathcal{T}+}],[\Xi_i^{\mathcal{I}-},\Xi_i^{\mathcal{I}+}],[\Xi_i^{\mathcal{F}-},\Xi_i^{\mathcal{F}+}]\right\rangle$ be a finite collection of ST- $(\eth_1,\eth_2,\eth_3)$ NNIVNs. Then ST-G $(\eth_1,\eth_2,\eth_3)$ NNIVWG $(\Re_1,\Re_2,...,\Re_n)=\frac{1}{\Lambda}\left(\sqcap_{i=1}^{n}(\Lambda\cdot\sin\Re_i)^{\kappa_i}\right)$ $(i=1,2,...,n)$.

Theorem 4.13. *Let $\Re_i=\left\langle(\ell_i,\hbar_i);[\Xi_i^{\mathcal{T}-},\Xi_i^{\mathcal{T}+}],[\Xi_i^{\mathcal{I}-},\Xi_i^{\mathcal{I}+}],[\Xi_i^{\mathcal{F}-},\Xi_i^{\mathcal{F}+}]\right\rangle$ be the set of ST- $(\eth_1,\eth_2,\eth_3)$ NNIVNs. Prove that ST-G $(\eth_1,\eth_2,\eth_3)$ NNIVWG $(\Re_1,\Re_2,...,\Re_n)$=*

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$$\left[\begin{array}{c}\left(\frac{1}{\Lambda}\sqcap_{i=1}^{n}(\Lambda\ell_i)^{\kappa_i},\frac{1}{\Lambda}\sqcap_{i=1}^{n}(\Lambda\hbar_i)^{\kappa_i}\right);\\ \left[\begin{array}{c}\sqrt[\eth_1]{1-\left(1-\left(\sqcap_{i=1}^{n}\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}-})\right)\right)^{\eth_1}\right)^{\eth_1}}\right)^{\kappa_i}\right)^{\eth_1}\right)^{1/\eth_1}},\\ \sqrt[\eth_1]{1-\left(1-\left(\sqcap_{i=1}^{n}\left(\sqrt[\eth_1]{1-\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{T}+})\right)\right)^{\eth_1}\right)^{\eth_1}}\right)^{\kappa_i}\right)^{\eth_1}\right)^{1/\eth_1}}\end{array}\right],\\ \left[\begin{array}{c}\left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}-})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{1/\eth_2},\\ \left(\sqrt[\eth_2]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{I}+})^{\eth_2}\right)\right)^{\eth_2}\right)^{\kappa_i}}\right)^{1/\eth_2}\end{array}\right],\\ \left[\begin{array}{c}\left(\sqrt[\eth_3]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}-})^{\eth_3}\right)\right)^{\eth_3}\right)^{\kappa_i}}\right)^{1/\eth_3},\\ \left(\sqrt[\eth_3]{1-\sqcap_{i=1}^{n}\left(1-\left(\sin^2\left((\aleph)\cdot(\Xi_i^{\mathcal{F}+})^{\eth_3}\right)\right)^{\eth_3}\right)^{\kappa_i}}\right)^{1/\eth_3}\end{array}\right]\end{array}\right].$$

Proof. Theorem 4.10 provides the proof.

The ST- $(\eth_1, \eth_2, \eth_3)$ NNIVWG operator is substituted for the ST-G $(\eth_1, \eth_2, \eth_3)$ NNIVWG operator when $\Lambda = 1$.

Corollary 4.14. *Idempotent, boundedness, and monotonicity properties must be fulfilled by the ST-G $(\eth_1, \eth_2, \eth_3)$ NNIVWG operator.*

Proof. The proof is provided by the following Theorem 4.3, Theorem 4.4 and Theorem 4.5.

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