A New Neutrosophic Extended Rayliegh Distribution for Enhanced Productivity and Efficiency Across Industrial Sectors: A case study of Al-Kharj region

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Abstract

This paper introduces a new statistical distribution called the Neutrosophic Extended Rayleigh Distribution (NERD), which is specifically developed to handle uncertainty commonly found in industrial applications. We conduct a comprehensive examination of the statistical characteristics of NERD, including important measures such as the quantile function, moments, moment generating function, mean deviation, skewness, kurtosis, reliability measures, uncertainty measures, distributions of order statistics, and L-moments. Parameter estimation is conducted by maximum-likelihood estimation within a neutrosophic framework, guaranteeing resilient inference in practical situations. Through the application of NERD to actual industrial datasets, we evaluate its adaptability and efficiency in simulating industrial processes. A real case study of Al-Kharj region demonstrates the higher performance of NERD. This research highlights the capacity of NERD to greatly improve productivity and efficiency in several industrial sectors.

Keywords: Rayleigh distribution; neutrosophic probability; neutrosophic distribution; solar industry; renewable energy; Al-Kharj

1. Introduction

Statistics is essential for constructing effective statistical models that can accurately reflect different natural occurrences using established probability distributions. These distributions are crucial for simulating occurrences that are characterized by uncertainty and risk [1]. Nevertheless, the intricate nature of numerous natural occurrences might lead to classical distributions being inadequate in adequately representing the data. In response to this, a multitude of probability distributions have been created with the goal of offering more precise representations. However, even with these attempts, it is possible that existing probability distributions may not fully capture certain natural occurrences with accuracy [2]. The insufficiency has resulted in the development and adjustment of universal probability distributions. By including supplementary characteristics into established distributions, these generalized models provide better applicability for data related to natural occurrences and boost the precision in describing the extreme ends of the distribution. Rayleigh distribution (RD) was originally obtained by British physicist Lord Rayleigh [3]. It is a continuous probability distribution for positive-valued random variables. It is one of the most used distributions. The applications of this method are extensive and cover several fields such as project effort load modeling, life testing experiments, reliability analysis, communication theory, physical sciences, engineering, medical imaging science, applied statistics, and clinical studies [4]. The Rayleigh distribution is commonly used in project management and engineering to represent the distribution of effort loadings or the time intervals between consecutive failures in systems. dependability analysis facilitates the comprehension of the dependability and failure rates of components and systems as they evolve over time [5]. The Rayleigh distribution is employed in communication theory to represent the amplitude of messages in wireless communication channels. Furthermore, the Rayleigh distribution is used in medical imaging research and clinical

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studies to analyze data pertaining to the identification and description of malignancies, as well as in the examination of physiological processes [6]. The Rayleigh distribution is highly versatile and robust, making it essential for researchers and practitioners in several industries who want to analyze and comprehend different aspects of lifespan data and stochastic processes [7]. It is a special case of the Weibull distribution with a scale parameter of 2 The literature review in this research examines the development of statistical distributions and their use in modeling real-world data sets. The industrial sector has significant challenges in terms of analysis, modeling, and forecasting [8]. The Rayleigh distribution, Generalized Rayleigh distribution, and Weibull distribution are well-researched and widely studied due to their capacity to accurately describe various data patterns [9].

Researchers have also investigated the expansion and alterations of traditional distributions to improve their adaptability and usefulness [10]. The Generalized Rayleigh distribution includes extra shape parameters to suit a wider range of data distributions, resulting in enhanced fitting capabilities [11]. The Exponentiated Rayleigh distribution has an additional parameter to modify the tail behavior, allowing it to effectively represent data with either heavy or light tails [12]. Uncertainties and dangers are inherent aspects of real-life occurrences. Probability theory is the preferred method for dealing with uncertainties and accurately representing real-world events [13]. Nevertheless, the many intricacies, fluctuations, and diversities encountered in real-world situations frequently need the development of multiple statistical distributions. However, despite these efforts, there are still many important issues where real-life data do not follow standard probability distributions [14]. This has led to the need for the expansion and creation of extended statistical distributions [15]. In the literature, numerous generalized distributions have been identified, often distinguished by a higher number of parameters [16], [17], [18], [19], [20]. Adding more parameters to existing distributions improves the accuracy of the distribution being studied and strengthens the characteristics of the distribution's tail. The applications of this technology are wide-ranging and cover several domains such as modeling project effort loadings, conducting life testing trials, performing reliability analysis, studying communication theory, exploring the physical sciences, engineering, and medical imaging.

Furthermore, advancements in statistical methodology, such as maximum-likelihood estimation, have facilitated parameter estimation for complex distributions, enabling researchers to better infer model parameters from uncertain observed data [8]. This approach has become standard practice in distributional analysis, ensuring robust and reliable parameter estimates.

Recent research has also examined novel distributional forms that are impacted by neutrosophic statistics [21]. Neutrosophic statistics is a statistical discipline that specifically examines data that contains uncertainty, imprecision, and indeterminacy. It achieves this by employing the neutrosophic set theory. Neutrosophy, a word introduced by Florentin Smarandache in the later years of the 20th century, enhances the concepts of classical set theory by include the notion of indeterminacy [22], [23], [24], [25], [26]. This idea allows elements to have truth-membership values not just within the range of [0,1], but also within the range of [-1,1]. Neutrosophic statistics incorporates the notion of indeterminacy into statistical analysis, enabling the handling of incomplete, imprecise, and uncertain data [27]. The field provides strategies and approaches to manage data that demonstrates uncertainty or imprecision, which can arise in several practical situations such as decision-making, pattern recognition, image processing, and artificial intelligence [28]. Neutrosophic statistics aims to develop statistical models, inference methods, and estimate procedures that effectively handle the complexities associated with uncertain and ambiguous data. Various neutrosophic distributions have been developed specifically for the purpose of representing uncertain or ambiguous data.

This work has constructed a neutrosophic variant of the extended Rayleigh distribution, which is capable of accurately representing a wide range of real-world datasets. The introduction of the neutrosophic form of the extended Rayleigh distribution provides numerous benefits for accurately representing real-world data. Firstly, it utilizes the flexibility and adaptability of neutrosophic set theory to accurately express the uncertainty, imprecision, and indeterminacy that are inherent in several datasets. The neutrosophic form of the extended Rayleigh distribution offers a thorough framework for dealing with uncertainties and ambiguities in data. This makes it especially well-suited for applications in decision-making and issues where exact modeling of uncertainty is essential. In summary, the suggested model provides a robust and adaptable tool for statistical analysis, which has the capacity to enhance comprehension and elucidation of real-world datasets in various fields.

The subsequent sections of this paper are organized in the following manner: The definition of NERD is provided in Section 1. Section 2 explores statistical features, specifically focusing on a linear representation of its probability density function. The investigation of estimating methods is discussed in Section 3. In Section 4, a simulation study is done with the objective of defining the parameters of the suggested distribution estimate methods. Moreover, a genuine dataset from an industrial network is used to evaluate its practical application. The section titled 'Conclusion' provides a summary of the study's conclusions and highlights the main findings.

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2. Proposed Model

In this section, notions of the proposed model with some important statistical functions have been introduced. A random x is said to follow neutrosophic extended Rayleigh distribution if it has the following density function:

$$h(x) = (x^3)/(2\beta_n{}^4\sigma_n{}^2) * e^{\left(-x^4/(8\beta_n{}^4\sigma_n{}^2)\right)}, x > 0, \beta_n > 0, \sigma_n > 0 \quad (1)$$

where β_n and σ_n are two parameters of the proposed distribution Neutrosophic probability density functions are an extension of classical probability theory that incorporate three parameters: truth, indeterminacy, and falsity. These densities are used to quantify the degrees of membership, non-membership, and indeterminacy within a set, making them useful for modeling uncertain and imprecise information in fields such as decision-making, artificial intelligence, and risk analysis. The graphical representation of this model can be seen in Figure 1.

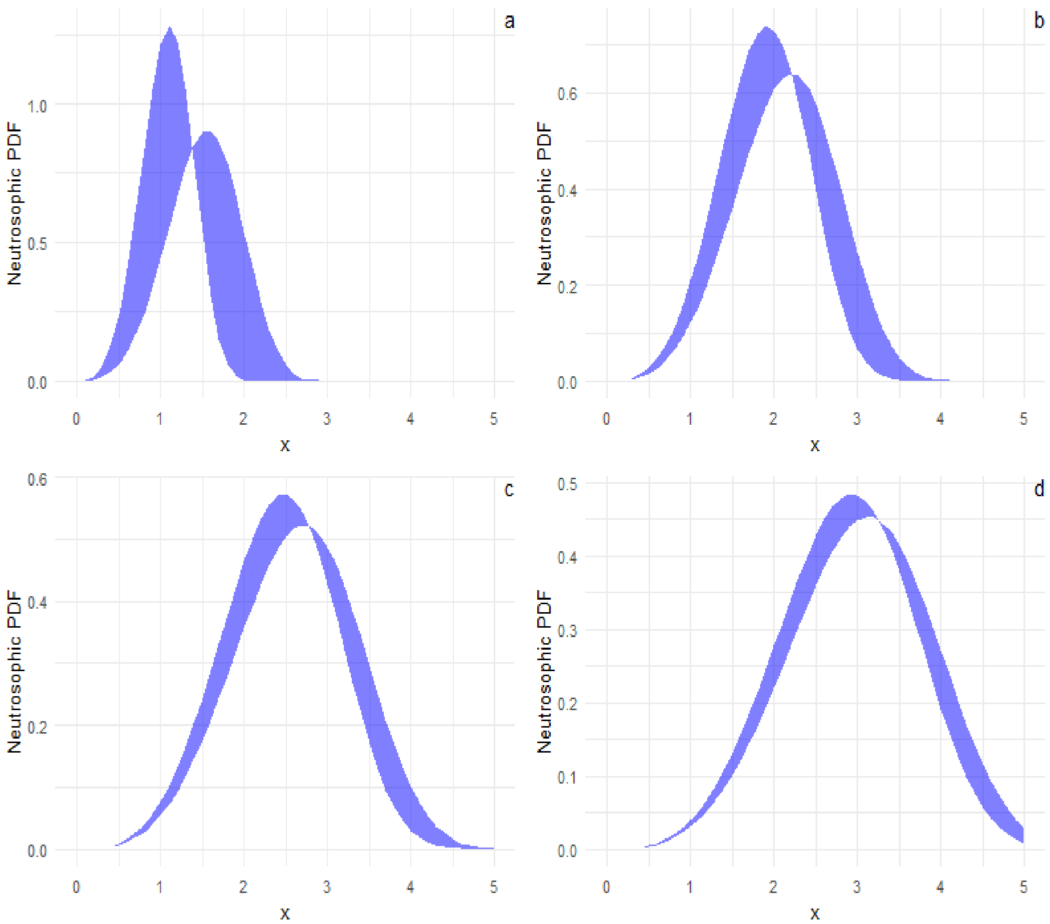

Figure 1: Density curves of the proposed model with (a) $\beta_n = 1$ and $\sigma_n = [0.5,1]$ (b) $\beta_n = 1$ and $\sigma_n = [1,1.5]$ (c) $\beta_n = 1$ and $\sigma_n = [2,2.5]$ (d) $\beta_n = 1$ and $\sigma_n = [3,3.5]$

Figure 1 provides plots that illustrate the neutrosophic probability density curves for varying values of the parameter $\sigma_n$where σ_n represents the range of uncertainty. Each plot corresponds to a different interval of σ_n, ranging while keeping the parameter β_n fixed at 1.0. The shaded blue regions represent the uncertainty in the PDF, with darker shades indicating higher uncertainty. These plots offer insights into how changes in σ_n affect the spread and uncertainty of the Neutrosophic PDF. With respect to a random variable X, the chance that its value will be less than or equal to x is given by the Cumulative Distribution Function (CDF). It is a vital tool in probability theory and statistical analysis, providing insightful information about data distribution. The suggested distribution's CDF function is described as follows:

$$H(x) = 1 - e^{\left(-x^4/(8*\beta_n{}^4*\sigma_n{}^2)\right)}; x > 0, \beta_n > 0, \sigma_n > 0 \quad (2)$$

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The CDF plots of the proposed distribution is depicted in Figure 2.

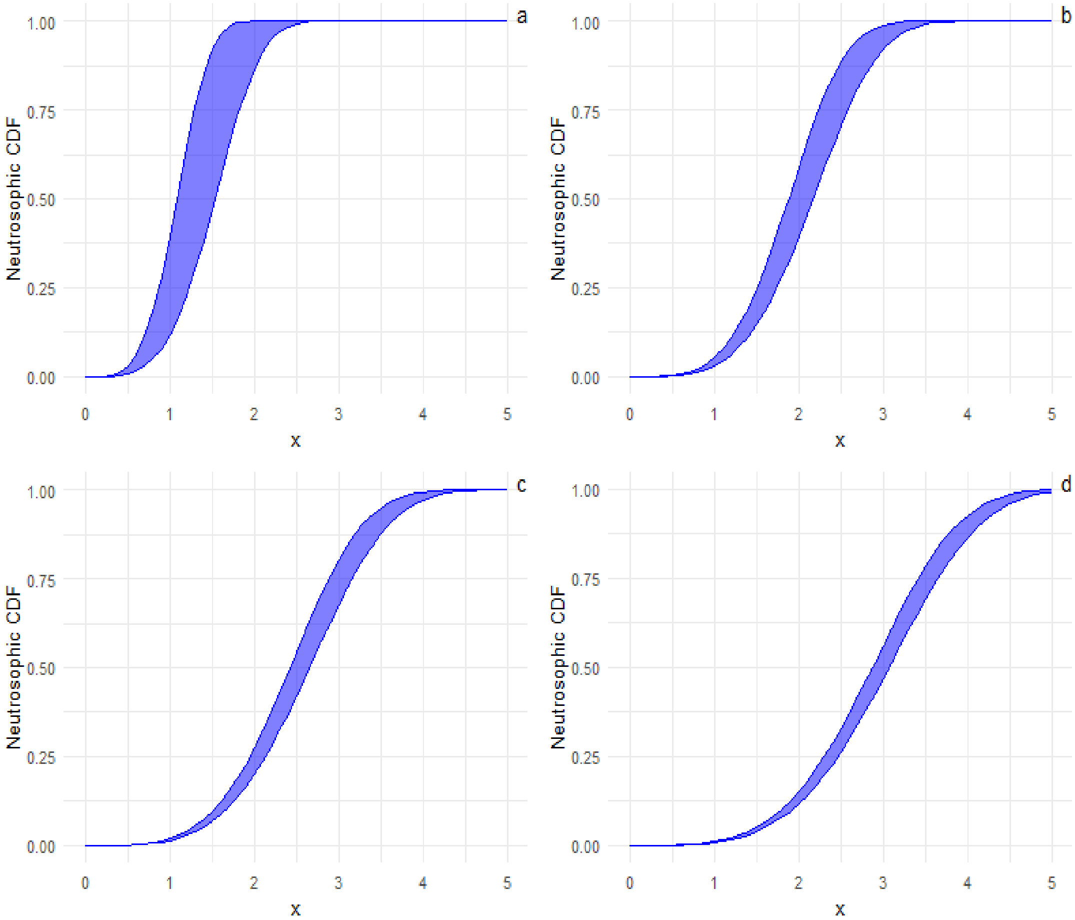

Figure 2: CDF curves of the proposed model with (a) $\beta_n = 1$ and $\sigma_n = [0.5,1]$ (b) $\beta_n = 1$ and $\sigma_n = [1,1.5]$ (c) $\beta_n = 1$ and $\sigma_n = [2,2.5]$ (d) $\beta_n = 1$ and $\sigma_n = [3,3.5]$

Figure2 illustrates the neutrosophic cumulative distribution function for varying intervals of σ_n, while keeping β_n constant. Each curve represents a different interval of uncertainty, with the shaded region between them indicating the level of uncertainty. This visualization provides insights into how changes in σ_n affect the cumulative distribution of the neutrosophic random variable x.

Survival function of the proposed model is defined as:

$$S(x) = e^{\left(-x^4/(8*\beta_n{}^4*\sigma_n{}^2)\right)} \quad (3)$$

The proposed model plays a crucial role in the analysis of reliability. The definition of the survival function is crucial due to its extensive use in survival research. The survival function, commonly represented as S(x), is a key concept in the fields of survival analysis and reliability engineering. It denotes the likelihood that a subject or system will endure beyond a specific time or occurrence. Essentially, it offers vital knowledge on the strength, dependability, and lifespan of many entities, including living creatures and mechanical parts. Analyzing the survival function allows analysts to make well-informed judgments regarding risk management, product development, and resource allocation in various domains like healthcare, engineering, and finance. The survival function of the suggested model is depicted in Figure 3.

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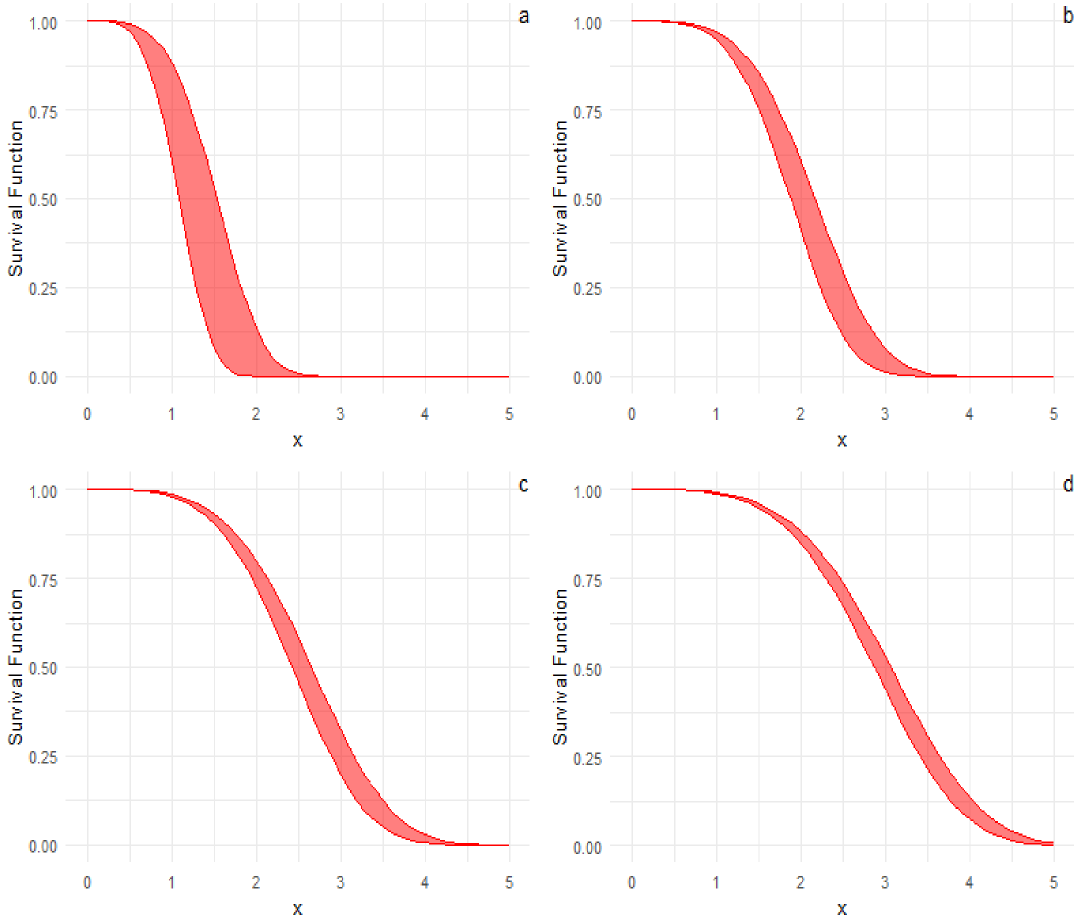

Figure 3: Survival functions of the proposed model with (a) $\beta_n = 1$ and $\sigma_n = [0.5,1]$ (b) $\beta_n = 1$ and $\sigma_n = [1,1.5]$ (c) $\beta_n = 1$ and $\sigma_n = [2,2.5]$ (d) $\beta_n = 1$ and $\sigma_n = [3,3.5]$

Figure 3 shows the survival function $S(x)$, which represents the probability that a subject or system will survive beyond a certain time or event, for varying intervals of σ_n, while keeping) β_n constant. Each curve represents a different level of uncertainty, with the shaded region between them indicating the range of survival probabilities. This visualization provides insights into how changes in σ_n affect the durability and reliability of the system under consideration.

3. Some Key Statistical Properties

In this section, we have derived some important statistical properties that describe the analytical structure and neutrosophic framework.

Theorem 1 If X follows the NERD then $E(\mathrm{X}) = 1.524\sigma_n\sqrt{\beta_n}$

$$E(X) = E[X] = \int_0^\infty \mathrm{x} * (x^3)/(2\beta_n{}^4\sigma_n{}^2) * e^{\left(-x^4/(8\beta_n{}^4\sigma_n{}^2)\right)}dx \quad (4)$$

$\mathrm{E(X)} = 1.524\sigma_n\sqrt{\beta_n}$=$[\mu_l, \mathrm{u_u}]$

where μ_l and $\mathrm{u_u}$ are upper and lower values of indeterminate mean.

Theorem 2 Derive the variance of the proposed model NERD

$$Var(X) = E(X^2) - [E(X)]^2$$

Now $E(X^2) = \int_0^\infty \mathrm{x}^2 * (x^3)/(2\beta_n{}^4\sigma_n{}^2) * e^{\left(-x^4/(8\beta_n{}^4\sigma_n{}^2)\right)}dx = (2.5066282 * \beta_n{}^2 * \sigma_n)/2$

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Thus $Var(X) = (2.5066282 * \beta_n{}^2 * \sigma_n) - [1.524\beta_n\sqrt{\sigma_n}]^2 = 0.986 * \beta_n{}^2\sigma_n$

Theorem 3 Derive the rth moment expression for the NERD.

$$E(X^r) = \int_0^\infty x^r h(x)dx; r = 1,2,3,.. \quad (5)$$

$$E(X^r) = \int_0^\infty x^r * \left(x^3/(2\beta^4\sigma^2)\right) * e^{\left(-x^4/(8\beta^4\sigma^2)\right)}dx \quad (6)$$

Eq (6) further can be simplified as:

$$E(X^r) = 8^{r/4}\beta_n^r\sigma^{r/2}\left[\left(\frac{r}{4}+1\right)=[E(X^r)_l,\ E(X^r)_u]\right. \quad (7)$$

Eq (7) can be used to derive expressions for higher moments of the proposed model.

Theorem 4 Derive the expression for coefficients of skewness and kurtosis.

The coefficient of skewness is defined as:

$$\beta_1 = \frac{\mu_3^{2/3}}{\mu_2} \quad (8)$$

Moments about origin μ_2 and μ_2 can be found from Eq (7). Thus the coefficient of sknewness can be written as:

$$\beta_1 = \frac{\left[-0.0068\sigma^{3/2}\beta^3\right]^{2/3}}{0.183\beta^2\sigma}$$

Similarly the kurtosis coefficient is given by:

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \quad (9)$$

Putting the values of μ_2 and μ_4 from (7) provide, kurtosis coefficient:

$$\beta_2 = \frac{[0.0916\beta^4\sigma^2]}{[0.182\beta^2\sigma]^2} = 2.745 \quad (10)$$

Thus, kurtosis coefficient is a crisp value.

Other properties of the proposed such as Shannon entropy, Renyi entropy and random number generator can be expressed as:

$$E\left[-log\left(h(x)\right)\right] = \int_0^\infty log\left(h(x)\right)h(x)dx = -\log\left[\frac{\left(8\beta_n{}^4\sigma_n{}^2\right)^{3/4}}{2\beta_n{}^4\sigma_n{}^2}\right] + \frac{3}{4}\gamma + 1 \quad (11)$$

Eq (10) measures the Shannon entroy for the proposed model.

$$R(x) = \frac{1}{1-\mu}log\left[1 - \frac{\left(2\beta_n{}^4\sigma_n{}^2\right)^{(1-\mu)}\left(8\beta_n{}^4\sigma_n{}^2\right)^{3/4(1-\mu)}}{\mu^{\left(\frac{3}{4}\mu+\frac{1}{4}\right)}}\right] \quad (12)$$

Eq (11) indicates the Renyi entropy of the proposed model and usually used to dynamical uncertainty of the probability distribution.

The function that maps probabilities to values in the context of the cumulative distribution function for any probability distribution can be referred to as the quantile function. Quantile function of the proposed distribution is given by:

$$Q = H^{-1}(x) = \left[8\beta_n{}^4\sigma_n{}^2 log\left(\frac{1}{1-u}\right)\right]^{\frac{1}{4}}; 0 < u < 1. \quad (13)$$

Random data can easily be generated from Eq (13) if we assume that u follows the uniform distribution.

Table 1: Characteristics of the proposed model for selected values of β_n and σ_n

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Table 1 demonstrates that when there is ambiguity in the parameters of the proposed model, it leads to interval values for some attributes, such as the average and variability. These properties are directly affected by the unspecified factors. Nevertheless, the values of skewness and kurtosis remain constant regardless of the alternative parameter sets utilized, as they are computed using precise formulas and are impervious to fluctuations in the parameters. It is crucial to consider the uncertainty of parameters while examining the features of a model, since it might have a substantial effect on specific statistical aspects while keeping others unaffected.

4. Estimation Procedure

In this section, we derive maximum-likelihood estimators for the parameters of proposed distribution. Let $x_1, x_2, \dots x_n$ represent a random sample from the NERD with parameters β_n and σ_n. The log likelihood function for the observed sample can be expressed as follows:

Parameter values	Characteristics of the proposed model	Computed values
$\beta_n = 1, \sigma_n = [1,3]$	Mean	[1.52, 4.57]
	Variance	[0.98, 2.95]
	Skewness	[-0.19, -0.19]
	Kurtosis	[2.74,2.74]
$\beta_n = 1, \sigma_n = [2,5]$	Mean	[3.05, 7.62]
	Variance	[1.97, 4.93]
	Skewness	[-0.19, -0.19]
	Kurtosis	[2.74, 2.74]
$\beta_n = 1, \sigma_n = [4,7]$	Mean	[6.09, 10.66]
	Variance	[3.94, 6.90]
	Skewness	[-0.19, -0.19]
	Kurtosis	[2.74, 2.74]

$$L = -\frac{\sum_{i=1}^{n} x_i^4}{8\beta_n{}^4\sigma_n{}^2} - nlog2 + \sum_{i=1}^{n} log x_i^3 - 4nlog\beta_n - 2nlog\sigma_n \tag{14}$$

Taking partial derivate of Eq (14) with respect to unknown parametric values:

$$\frac{\partial L}{\partial \beta} = \frac{-\sum_{i=1}^{n} x_i^4}{2\beta_n{}^5\sigma_n{}^2} - \frac{4n}{\beta_n} \tag{15}$$

$$\frac{\partial L}{\partial \sigma} = -\frac{2n}{\sigma_n} + \frac{\sum_{i=1}^{n} x_i^4}{4\beta_n{}^4\sigma_n{}^3} \tag{16}$$

The equations (15) and (16) defy analytical solution, thus necessitating the employment of the Nelder-Mead method via the R package for numerical resolution.

Second partial derivatives of Eq (14) provide:

$$\frac{\partial^2 L}{\partial \beta^2} = \frac{4}{\beta_n^2} - \frac{x^4}{2\sigma_n{}^2\beta_n{}^6} \tag{17}$$

$$\frac{\partial^2 L}{\partial \sigma^2} = \frac{2}{\sigma^2} - \frac{x^4}{4\beta_n^4\sigma_n^4} \tag{18}$$

Thus, Fisher Information matric can be constructed as:

$$I = \begin{bmatrix} \frac{4}{\sigma_n{}^2} & \frac{8}{\beta_n\sigma_n} \\ \frac{8}{\beta_n\sigma_n} & \frac{16}{\beta_n^2} \end{bmatrix} \tag{19}$$

The Fisher information matrix, as defined in Eq (19), is a mathematical tool utilized in statistics to measure the quantity of information that an observable random variable conveys about an unknown parameter in a statistical model. It offers valuable information about the accuracy of parameter estimates and is essential in statistical inference, especially in maximum likelihood estimation.

5. Simulation Analysis

In this section, a simulation study is conducted to evaluate the performance of the estimators. Using the R statistical program, 10,000 replications are generated from NERD with parameters $\beta_n = 1$ and $\sigma_n = [1,3]$ using the inverse

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transformation method. According to inverse transformation approach, random number can be generated according to the expression as given below:

$$x = \left(\left(-8 * \beta_n{}^4 * \sigma_n{}^2 * log(1-u)\right)^{(1/4)}\right) \quad (20)$$

The inverse CDF method is a highly efficient technique used to generate random numbers that follow a specific probability distribution. It takes use of the opposite relationship between cumulative probabilities and their corresponding quantiles. The inverse CDF method ensures that the generated random numbers conform to the PDF of the selected distribution. The use of random number generation becomes particularly beneficial in situations where there are difficulties or restrictions in getting analytical solutions. Furthermore, it is appropriate for a wide range of distributions, making it a valuable tool in statistical modeling and simulation. However, it is important to recognize that this method may not work well for distributions that have complex or computationally intensive inverse CDF functions. However, for most used distributions, it offers a simple and dependable method of generating random values. We have produced 10,000 random numbers using the distribution described in the suggested formula provided in Eq (19). In this simulation, we have assumed that random data is created from the suggested model using precise values of β_n and uncertain values of σ_n. The results of the first 8 randomly generated values are given in Table 2.

Table 2 Random data generated from the NERD

Selected Parameters	Random Numbers
$\beta_n = 1$	[2.22, 2.72], [2.30, 2.82], [2.31, 2.83], [2.48, 3.03],
σ_n=[2,3]	[2.44, 2.98], [2.77, 3.39], [2.51, 3.08], [1.34, 1.64]
$\beta_n = 1$	[3.14, 3.51], [3.25, 3.64], [3.27, 3.65], [3.50, 3.91]
σ_n=[4,5]	[3.45, 3.85], [3.91, 4.37], [3.55, 3.97],[1.90, 2.12]
$\beta_n = 1$	[3.85, 4.45], [3.99, 4.60], [4.00, 4.62], [4.29, 4.95]
σ_n=[6,8]	[4.22, 4.87], [4.79, 5.53], [4.35, 5.02], [2.33, 2.68]
$\beta_n = 1$	[4.97, 5.45], [5.15, 5.64], [5.17, 5.66], [5.54, 6.06]
σ_n=[10,12]	[5.45, 5.97], [6.18, 6.77], [5.62, 6.15], [3.00, 3.29]

Table 2 displays the outcomes of a simulation research that assesses the generation of random data using the NERD model. The study finds different values of σ_n while keeping β_n constant. The random numbers are generated using the inverse transformation approach, which exploits the inverse correlation between cumulative probabilities and their corresponding quantiles.

This guarantees that the randomly produced integers adhere to the probability density function of the specified distribution. Each row in Table 2 represents a distinct combination of parameters (β_n, σ_n), where σ_n falls within the intervals [2,3], [4,5], [6,8], and [10,12]. 10,000 replications of random numbers are created for each combination. The lower and upper bounds of the intervals define the range of uncertainty that is considered for the values of σ_n in the simulation. The simulation intends to evaluate the impact of changes in the value of σ_n on the distribution of the randomly generated integers. By examining the randomly produced numbers inside various intervals of σ_n, one can obtain valuable information about the behavior and effectiveness of the NERD model under different parameter configurations.

6. Real Application

Industrial growth is crucial for strengthening a nation's economy and effectively utilizing its workforce to achieve national goals. Saudi Arabia experienced a rapid and continuous period of industrialization that defined the country's development over a period of forty years. To attain sustained growth and development in the manufacturing sector, several crucial variables are necessary. One component that plays a role is the energy source. Saudi Arabia has launched the ambitious Saudi Vision 2030, which seeks to decrease the country's dependence on oil by promoting the development of promising private energy companies and generating opportunities that contribute to the national economy. In the industrial sector, the government is aggressively encouraging the transfer of technology within the renewable energy industries. This strategic decision is expected to result in the localization of significant parts of the renewable energy value chain within Saudi Arabia.

Expanding power generation in Saudi Arabia is crucial to accommodate the anticipated growth in electricity demand. According to IRENA renewable energy statistics 2022, solar industry growth in Saudi Arabia is given in Figure 4.

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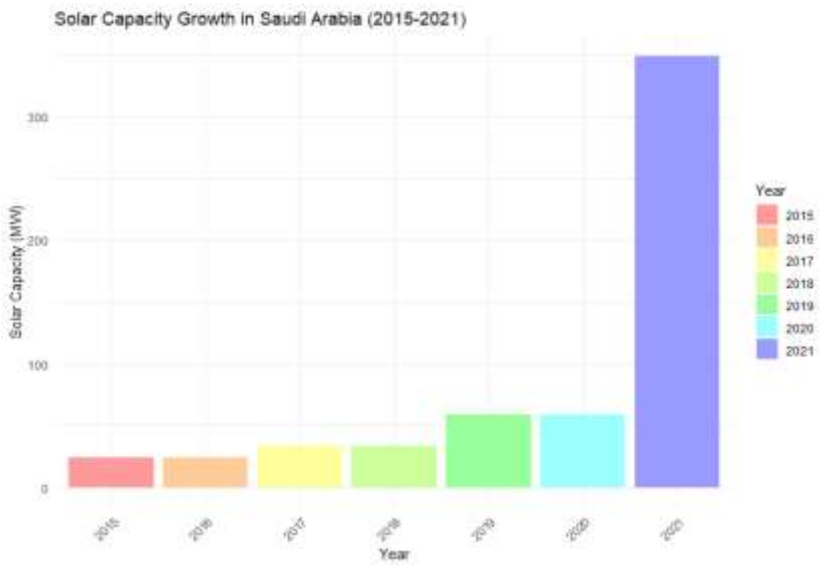

Figure 4: Solar energy industry in Saudi Arabia

According to Global Data, in 2022, solar photovoltaic (PV) capacity accounted for 13.0% of all power plant installations worldwide, amounting to a total of 1,109 gigawatts (GW). By 2030, it is projected that the percentage will increase to 30%, and the number of installations will reach 4,002 GW. Saudi Arabia now possesses 0.08% of the worldwide solar photovoltaic (PV) capacity. These are the five largest operational solar PV power plants in Saudi Arabia, as per the data obtained from the Global Data power plants database. The Al-Kharj region is highly suitable for harnessing solar energy due to its ample solar irradiation, vast dry territory, and extended hours of sunlight. The Kingdom's objective is to substantially increase solar power output to meet a large amount of the country's future energy needs and promote economic growth.

Temperature and climate data are essential variables in evaluating the viability of solar power systems. The creation of solar energy is heavily reliant on the availability of sunshine, which is impacted by local meteorological conditions, seasonal variations, and geographical positioning. This study examines the climate and weather conditions in the Al-Kharj region to determine the suitability of implementing additional government solar projects in this area. Al-Kharj has a subtropical desert climate, characterized by an average annual temperature of 27.5ºC (81.5ºF), which is 0.49% higher than the average temperature in Saudi Arabia. The district normally experiences an average annual rainfall of 8.82 millimeters (0.35 inches) distributed among 20.4 wet days, which constitutes around 5.59% of the year. The temperature data for the latest year in Al-Kharj city has been documented and presented in Table 3.

Table 4: Low and high temperature of Al-Kharj region

Time	Temperature in C^0
Jan	[1.96, 29.4]
February	[0.98, 34.3]
March	[5.88, 36.26]
April	[12.74, 39.20]
May	[19.6, 42.14]
June	[22.54, 45.08]
July	[23.52, 46.06]
August	[24.5, 45.08]
September	[13.72, 43.12]
October	[3.92, 39.20]
Number	[0.98, 33.32]
December	[0.98, 29.40]

Table 4 illustrates that Al-Kharj's annual temperature fluctuations, which range from 1.96°C to 46.06°C, indicate ideal circumstances for solar energy projects in the area. The abundant sunshine and unobstructed sky provide constant solar irradiance levels, even in the face of seasonal variations, thereby optimizing the potential for energy

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generation. Solar panels function at maximum efficiency from May to August, when temperatures are at their highest; the cooler months provide reprieve without jeopardizing the profitability of solar energy. Al-Kharj is a suitable site for the installation of solar energy projects since, all things considered, its climate offers the perfect conditions for utilizing solar energy. Based on the uncertain information provided in Table 4, the proposed model provides summary statistics of the temperature data given in Table 5.

Table 5: Summary statistics of temperature data of Al-Kharj region

Summary statistics	Estimated values
β_n	[5.61, 123.79]
σ_n	[3.41, 3.45]
Neutrosophic Mean	[12.68, 18.92]
Neutrosophic Variance	[106.24, 556.22]

Table 5 shows that temperature condition in Al-Kharj region is ideal for solar system installation. the climate is characterized by a subtropical desert environment, solar energy holds significant potential for power generation. With abundant sunlight throughout the year due to minimal cloud cover and a high solar irradiance level, the region is conducive to the efficient operation of solar power systems.

Despite the high temperatures experienced, careful consideration of solar panel placement and technology can mitigate efficiency losses associated with heat. Additionally, the low precipitation levels minimize the risk of weather-related damage to solar infrastructure, further enhancing the feasibility of solar energy adoption in Al-Kharj.

7. Conclusions

This study has introduced the neutrosophic extended form of the Rayleigh distribution. This is a novel statistical distribution specifically tailored to address uncertainties prevalent in industrial contexts. We have shown the adaptability and efficiency of NERD in modeling solar industrial processes through a thorough examination of its statistical properties, including crucial metrics like the quantile function, moments, moment generating function, mean deviation, skewness, kurtosis, reliability measures, uncertainty measures, distributions of order statistics, and L-moments. For parameter estimation, using maximum-likelihood estimation in a neutrosophic environment guarantees robust inference in practical situations. We have demonstrated NERD's exceptional performance and its ability to greatly increase productivity and efficiency throughout an economy's sector by applying it to a real dataset on temperature. The progression of statistical distributions from their classical to their generalized forms highlights the ongoing endeavors to depict and comprehend intricate real-world occurrences more accurately. The integration of neutrosophic statistics into distributional modeling enhances its capacity to manage the uncertainty, imprecision, and indeterminacy present in several datasets, providing an effective framework for problem-solving and decision-making across diverse fields. With the help of this research, we have created a useful statistical analysis tool that will help practitioners and scholars better comprehend and evaluate real-world information. The NERD is especially well-suited for situations where accurate uncertainty modeling is essential since it provides a thorough framework for handling uncertainties and ambiguities.

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