Rare and Dense Sets in Fuzzy Neutrosophic Topological Spaces

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Abstract

The purpose of the current paper is study some new concept of sets and called fuzzy neutrosophic *rar* and fuzzy neutrosophic dense sets in fuzzy neutrosophic $t$opology and investigate some properties. In fact, the subject of fuzzy neutrosophic sets is already conducted by F. M. Mohammed et.al. [1-9]. However, the current study illustrates number of notable examples to shed the light on some novel attributes of recently established terms, as well as showing related interactions among these researches.

Keywords: fuzzy eutrosophic rare sets; fuzzy neutrosophic dense sets; fuzzy neutrosophic topology.

1.Introduction

Since the definition of the fuzzy set concept by Zadeh [10] has invaded almost all branches of mathematics. Then, the applications of fuzzy sets has been presents in many fields such as the theory of fuzzy topological spaces as soon as studied and developed by Chang [11]. Since then various concepts in general topology have been generalized by Chang's fuzzy topological spaces. Aftar that, as a generalization of fuzzy topological spaces developed in many directions by the concept of neutrosophic sets where the term of neutrosophic set was defined with membership, non-membership and indeterminacy degrees and founded by Smarandche [12] as a branch of scince and as extend the idea of fuzzy sets and intutionistic fuzzy sets, Then, the topological spaces of neutrosophic sets was study by A. Salama et.al.[13]. Then, a survey article of the developed the term $f$uzzy $n$eutrtrosophic $t$opological $s$paces has been published by Arockiarani [14,15].

Additionally, we may point out that the idea of "fuzzy neutrosophic sets" (that is now studied extensively. And our paper areas (The reader can check the papers [16-15]) when they studied the notions in $f$uzzy $n$eutrosophic $t$opology.

Hence, the aim of this paper is to introduce and study the concept of rere sets and dense sets in the case of $f$uzzy $n$eutrosophic. Several new characteristics and instances based on this specified idea have been addressed.

2. Basic definitions and terminologies

Definition 2.1 [15]: Let $X^*{}_N$ be a non-empty fixed set. The fuzzy neutrosophic set (F*NS), A_N^* is an object having the form $A_N^* = \{< x^*, \mu_{a^*N}(x^*), \sigma_{a^*N}(x^*), \nu_{a^*N}(x^*) >: x^* \in X^*{}_N\}$ where the function $\mu_{a^*}, \sigma_{a^*}, \nu_{a^*}: X^*{}_N \to [0,1]$ denote the degree of membership function (namely $\mu_{a^*N}(x^*)$), the degree of indeterminacy function (namely $\sigma_{a^*N}(x^*)$) and

the degree of non-membership function (namely $\nu_{a^*N}(x^*)$) respectively,

of each element $x^* \in X^*{}_N$ to the set A_N^* and $0 \le \mu_{a^*N}(x^*), \sigma_{a^*N}(x^*), \nu_{a^*N}(x^*) \ge 3$, for each $x^* \in X^*{}_N$.

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Remark 2.2 [3]: F*NS $A_N^* = \{< x^*, \mu_{a^*N}(x^*), \sigma_{a^*N}(x^*), \nu_{a^*N}(x^*) >: x^* \in X^*_N\}$ can be identified to an ordered triple < x, μ_{A_N}, σ_{A_N}, ν_{A_N} > in [0, 1] on X_N .

Lemma 2.3 [15]: Let X^*_N be a non-empty set the F*NS A_N^* and B_N^* be in

the form: $A_N^* = \{< x^*, \mu_{a^*N}(x^*), \sigma_{a^*N}(x^*), \nu_{a^*N}(x^*) >: x^* \in X^*_N\}$, and

$B_N^* = \{< x^*, \mu_{b^*N}(x^*), \sigma_{b^*N}(x^*), \nu_{b^*N}(x^*) >: x^* \in X^*_N\}$ on X^*_N. Then ,

i. $A_N^* \subseteq B_N^*$ $iff \mu_{a^*N}(x^*) \le \mu_{b^*N}(x^*), \sigma_{a^*N}(x^*) \le \sigma_{a^*N}(x^*)$ and $\nu_{a^*N}(x^*) \ge \nu_{b^*N}(x^*)$ for all $x^* \in X^*_N$,
ii. A_N^* =B_N^* iff $A_N^* \subseteq B_N^*$ and $B_N^* \subseteq A_N^*$,
iii. 1_N- A_N^* = $\{< x^*, \nu_{a^*N}(x^*), 1 - \sigma_{a^*N}(x^*), \mu_{a^*N}(x^*) >: x^* \in X^*_N\}$,
iv. A*∪B*={< Max($\mu_{a^*N}(x^*), \mu_{b^*N}(x^*)$), Max($\sigma_{a^*N}(x^*), \sigma_{b^*N}(x^*)$), Min($\nu_{a^*N}(x^*), \nu_{b^*N}(x^*)$) > : $x^* \in X^*_N$},
v. $A_N^* \cap B_N^*$ = {< Min($\mu_{a^*N}(x^*), \mu_{b^*N}(x^*)$, Min($\sigma_{a^*N}(x^*), \sigma_{b^*N}(x^*)$), Max($\nu_{a^*N}(x^*), \nu_{b^*N}(x^*)$) >: $x^* \in X^*_N$>},
vi. 0_N = < x, 0, 0, 1> and 1_N = < x, 1, 1, 0 > .

Definition 2.4 [15]: _Fuzzy neutrosophic topology (F*NT) on a non-empty set X^*_N is a family τ_N^* of fuzzy neutrosophic subsets in X^*_N satisfying the following axioms.

$i. 0_N, 1_N \in, \tau_N^*$.

$ii. A_N^* \cap B_N^* \in \tau_N^*$ for any $A_N^*, B_N^* \in \tau_N^*$.

$iii. \cup A_{N_j}^* \in \tau_N^*, \forall \left\{A_{N_j}^* : j \in J\right\} \subseteq \tau_N^*$.

the pair (X^*_N,τ_N^*) is called fuzzy neutrosophic topological space (F*NTS). The element of τ_N^* are called fuzzy neutrosophic-open sets (F*NOS). The complement of F*NOS in the F*NTS (X^*_N,τ_N^*) is

called fuzzy neutrosophic –closed set (F*NCLS).

Definition 2.5 [4]: Let (X_N^*, τ_N^*) be F*NTS and $A_N^* = x^*, \mu_{a^*N}, \sigma_{a^*N}, \nu_{a^*N>}$ be F*NS in X^*_N. Then the fuzzy neutrosophic-closure (F*NC) and the fuzzy neutrosophic-interior (F*NI) of A_N^* are defined by:

F*NC $(A_N^*) = \cap \{K: K\ is\ an\ F^*NCLS\ in\ X^*_N\ and\ A_N^* \subseteq K\}$

F*NI $(A_N^*) = \cup \{G: G\ is\ an\ F^*NOS\ in\ X^*_N\ and\ G \subseteq A_N^*\}$

Now that F*NC (A_N^*) is FN-closed set and F*NI (A_N^*) is F*N-open set in X^*_N. Further,

$i. A_N^*$ is F*NCLS in X^*_N iff F*NC(A_N^*) = A^*_N ,

$ii. A_N^*$ is F*NOS in X^*_N iff F*NI (A_N^*) = A_N^*.

Proposition 2.6 [6]: Let (X^*_N, τ_N^*) be F*NTS and A_N^*, B_N^* are F*NS in X^*_N. Then , the following properties hold:

i. $F^*NI(A_N^*) \subseteq A_N^*$ and $A_N^* \subseteq F^*NC(A^*)$,
ii. $A_N^* \subseteq B_N^* \Longrightarrow F^*NI(A_N^*) \subseteq F^*NI(B_N^*)\ and A_N^* \subseteq B_N^* \Longrightarrow F^*NC(A_N^*) \subseteq F^*NC(B_N^*)$,
iii. $F^*NI(F^*NI(A_N^*))$= $F^*NI(A_N^*)$ and $F^*NC(F^*NC(A_N^*) = F^*NC(A_N^*)$,
iv. $F^*NI(A_N^{*\,*} \cap B_N^*) = F^*NI(A_N^*) \cap F^*NI(B_N^*)$
$and\ F^*NC(A_N^* \cup B_N^*) = F^*NC(A_N^*) \cup F^*NC(B_N^*)$.

v. $F^*NI(1_N) = 1_N\ and F^*NC(0_N) = 0_N$.

Definition 2.7 [7]: F*NS S_N^* in F*NTS (X^*_N, τ_N^*) is called:

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i. Fuzzy neutrosophic semi-closed set (F^*NS-closed set) if $F^*NI(F^*NC(A^*)) \subseteq A^*$

ii. Fuzzy neutrosophic pre-closed set (F^*NP-closed set) if $F^*NC(F^*NI(A^*) \subseteq A^*$

iii. Fuzzy neutrosophic α-closed set (F^*Nα-closed set) if $F^*NC(F^*NI(F^*NC(A^*)))\subseteq A^*$

v. Fuzzy neutrosophic β-closed set ($F^*N\beta$ -closed set) if $F^*NI(F^*NC(F^*NI(A^*)))\subseteq A^*$

The complement of Fuzzy neutrosophic semi-closed set is Fuzzy neutrosophic semi-open set, Fuzzy neutrosophic pre-closed set is Fuzzy neutrosophic pre-open set , Fuzzy neutrosophic α-closed set is , Fuzzy neutrosophic α-open set and Fuzzy neutrosophic β-closed set is Fuzzy neutrosophic β-open set respectively.

3. Rare Sets and Dense Sets in Fuzzy Neutrosophic Topology.

Definition 3.1: A F^*NS A^* in F^*NTS (X^*_N,τ^*_N) is called:

i. Fuzzy neutrosophic rare set and denoted by (F^*NRA-S) in X^*_N if $F^*NI(A^*) = 0_N$ where $0_N =< x, 0,0,1 >$,

ii. Fuzzy neutrosophic dense set and denoted by (F^*ND-S) in X^*_N if $F^*NC(A^*) = 1_N$ where $1_N =< x, 1,1,0 >$.

Definition 3.2: A NS A^* is called neutrosophic nowhere dence set if $F^*NI(F^*NC(A^*)) = 0_N$.

Theorem 3.3: A F^*NS A^* of F^*NTS (X^*_N,τ^*_N) is F^*NRA-S iff if 1_N-A^* is F^*ND-S.
Proof: Suppose A^* is F^*NRA-S in X^*_N where $A^*=< A_1^*, A_2^*, A_3^* >$.
Then, $F^*NI(A^*)= 0_N$ or 1_N-$F^*NI(A^*) = 1_N$. Thus $F^*NC(A^*) = 1_N$.

Conversely, suppose 1_N-A^* is F^*ND-S is X^*_N. Then $F^*NC(A^*) = 1_N$, that is, 1_N-$F^*NI(A^*) = 1_N$. Thus $F^*NI(A^*) = 0_N$.

Theorem 3.4: A F^*NS A^* of F^*NTS (X^*_N,τ^*_N) is both an F^*NOS and a F^*NRA-S iff it is 0_N where $0_N =< x, 0,0,1 >$.
Proof: F^*NS is A^* is F^*NOS and F^*NRA-S so, $F^*NI(A^*) =A^*= 0_N$

Corollary 3.5: A F^*NS A^* of F^*NTS (X^*_N,τ^*_N) is both F^*NCL and F^*ND-S iff it is 1_N.

Remark 3.6: i. A F^*NS A^* of F^*NTS (X^*_N,τ^*_N) can be both F^*NRA-S and F^*ND-S .
ii. If F^*NS A^* of F^*NTS (X^*_N,τ^*_N) is both F^*NRA-S and F^*ND-S, then A^* is neither F^*NOS nor F^*NCLS, but the converse dose not hold from the following example.

Example 3.7: Let X ={a ,b ,c} define F^*NS. S^*_N in X^*_N and $\tau^*_N = \{0_N, 1_N, S^*_N\}$ where,
$S^*_N =< x^*, a(0.3,0.5,0.3), b(0.4,0.2,0.2), c(0.5,0.3,0.2) >$, so
$M^*_N =< x^*, a(0.5,0.4,0.2), b(0.5,0.5,0.4), c(0.4,0.2,0.4) >$ is neither F^*NOS nor F^*NCLS, so

i. $F^*NI(M^*_N) =< x^*, a(0,0,1), b(0,0,1), c(0,0,0) >$
$\leq < x^*, a(0.5,0.4,0.2), b(0.5,0.5,0.4), c(0.4,0.2,0.4) >$
that is M^*_N is F^*NRA-S

ii. $F^*NC(M^*_N) =< x^*, a(0.5,0.4,02), b(0.5,0.5,0.4), c(0.4,0.2,0.4)>$
$\leq < x^*, a(1,1,0), b(1,1,0), c(1,1,0) >$ which means M^*_N is F^*ND-S.

Theorem 3.8: A F^*NS $A^*$of F^*NTS (X^*_N,τ^*_N) is both a F^*NOS contained in A^* nor F^*NCLS containing

A^*, except for 0_N and 1_N respectively.

Proof: Let A^* be both F^*NRA-S and F^*ND-S. Then, by definition, $F^*NC(A^*) = 1_N$ and $F^*NI(A^*) = 0_N \Longrightarrow A^*$ contains neither F^*NOS except 0_N nor contained in F^*NCLS except 1_N .

Theorem 3.9: In F^*NTS (X^*_N,τ^*_N), we have:

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i. 1_N is F^*ND-S, but it is not F^*NRA-S.
ii. 0_N is F^*NRA-S, but it is not F^*ND-S.
iii. arbitrary intersection (resp . union) of F^*NRA-S is F^*NRA-S.

Proof: (i) and (ii) are obvious.

iii) Let $A^* = \wedge_{\alpha\in\Delta} \mu_\alpha$ be an arbitrary intersection of F^*NRA-S s , that is $F^*NI(\mu_\alpha) = 0_N$ for each $\alpha \in \Delta$. Then, $\wedge_{\alpha\in\Delta} \mu_\alpha = 0_N$.

we have $0_N = \wedge_{\alpha\in\Delta} \mu_\alpha \geq F^*NI(\wedge_{\alpha\in\Delta} \mu_\alpha) \Longrightarrow 0_N \geq F^*I(\mu)$ or $F^*NI(\mu) = 0_N$

Similarly, Let $A^* = \wedge_{\alpha\in\Delta} \mu_\alpha$ be an arbitrary union of F^*ND-S, that is $F^*NC(\mu_\alpha) = 1_N$ for each $\alpha \in \Delta$.Then, $\wedge_{\alpha\in\Delta} \mu_\alpha = 1_N$.we have $1_N = \wedge_{\alpha\in\Delta} \mu_\alpha \leq F^*NC(\wedge_{\alpha\in\Delta} \mu_\alpha) \Longrightarrow F^*NC(\mu) = 1_N$.

Theorem 3.10: A F^*NS A^* of F^*NTS (X^*_N, τ^*_N) is a F^*ND-S iff for every F^*NOS (resp. F^*NCLS) μ satisfying $A^* \leq \mu$ (resp. $\mu \leq A^*$), we have $F^*NC(A^*) \geq A^*$ (resp. $F^*NI(A^*) \leq \mu$).

Proof: First assume that A^* is F^*ND-S and take F^*NOS μ with $A^* \leq \mu$.

Then, $F^*NC(A^*) = 1_N \geq \mu$

Conversely, suppose the necessary conditions hold and take $\mu = 1_N$ then, μ is F^*NOS and $A^* \leq \mu$. Thus $F^*NC(A^*) \geq \mu = 1_N$; that is $F^*NC(A^*) = 1_N$. So A^* F^*ND-S.

Suppose $A^*$is F^*NRA-S. Thus $F^*NI(A^*) \leq \mu = 0_N$; that is $F^*NI(A^*) = 0_N$.

Then, μ is F^*NOS and $A^* \geq \mu$ and take $\mu = 0_N$.

Remark 3.11: i. A F^*NS A^* of F^*NTS (X^*_N, τ^*) is F^*NRA-S, if there exists no F^*NOS other than 0_N contained in A^*.

ii. The union (resp. intersection) of F^*ND-S (resp. F^*NRAS) and F^*NCLS (resp. F^*NOS) is F^*ND-S (resp. F^*NRA-S).

Proof : i. Let A^* be F^*ND-S and μ a F^*NCLS. If v is F^*NOS with $A^* \vee \mu \leq v$, then, $A^* \leq v$. Thus $F^*NC(A^*) \geq v$. Now $F^*NC(A^* \vee \mu) \geq F^*NC(A^*) \vee F^*NC(\mu) \geq \mu \vee v \geq v$.

So, the union of F^*ND-S and F^*NCLS is F^*ND-S.

ii) Let A^* be F^*NRA-S and μ a F^*NOS. If v is F^*NCLS with $v \leq A^* \vee \mu$, then $v \leq A^*$.Thus $F^*NI(A^*) \leq v$

Now, $F^*NI(A^* \wedge \mu) = F^*NI(A^*) \wedge F^*NI(\mu) \leq \mu \wedge v \leq v$.

So the intersection of F^*NRA-S and F^*NOS is F^*NRA-S.

Theorem 3.12: $F^*NC(A^*)$ (resp. $F^*NI(A^*)$) is F^*ND-S (resp.F^*NRA-S) whenever A^* is F^*ND-S (resp. F^*NRA-S).

Proof:

i. Let A^* be F^*ND-S. Then, $F^*NC(A^*) = 1_N \Longrightarrow F^*NC(A^*) = F^*NC(1_N) = 1_N$. Thus $F^*NC(A^*)$ is F^*ND-S.
ii. Let A^* be F^*NRA-S. Then, $F^*NI(A^*) = 0_N \Longrightarrow F^*NI(A^*) = F^*NI(0_N) = 0_N$. Thus $F^*NC(A^*)$ is F^*NRA-S.

Definition 3.13: A F^*NS A^* of F^*NTS (X^*_N, τ^*_N) is said to be F^*N closed RA-S (resp. F^*N open D-S) in X^*_N, if the F^*NS A^* is both F^*NCLS and F^*NRA-S (resp. F^*NOS and F^*ND-S) in X^*_N.

Theorem 3.14: A F^*NS A^* of F^*NTS (X^*_N, τ^*_N) is F^*N closed RA-S, iff A^* is F^*NCLS which does not contain any F^*NOS other than 0_N.

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Proof: Let A^* be F^*N closed RA-S in X^*_N. Then we have $F^*NI(A^*) = 0_N$ and $F^*NC(A^*) = A^*$

This shows that A^* is F^*NCLS which does not contain any F^*NOS other than 0_N.

Theorem 3.15: A F^*NS A^* of F^*NTS (X^*_N, τ_N^*) is F^*N open D-S, iff A^* is F^*NCLS which does not contain any F^*N open D-S other than 1_N.

Proof: Let A^* be F^*N open D-S in X^*_N .then, we get $F^*NI(A^*) = A^*$ and $F^*NC(A^*) = 1_N$.

This shows that A^* is F^*NOS, which does not contain any F^*NCLS other than 1_N.

Definition 3.16: Let (X^*_N, τ_N^*) be F^*NTS. Then we define $Fr(A^*) = F^*NC(A^*) \wedge F^*NC(A^{*^c})$

Example 3.17: Let X ={a ,b ,c} define $\tau_N^* = \{0_N, 1_N, S^*_N\}$ and the F^*NS. S^*_N in X^*_N by,

$S^*_N = < x^*, a(0.3,0.5,0.3), b(0.4,0.2,0.2), c(0.5,0.3,0.2) >$

Now, if $\lambda^*_N = < x^*, a(0.5,0.4,0.2), b(0.5,0.5,0.4), c(0.4,0.2,0.4) >$ so,

$F^*NC(\lambda^*_N) = < x^*, a(0.5,0.4,0.2), b(0.5,0.5,0.4), c(0.4,0.2,0.4) >$

$\leq < x^*, a(1,1,0), b(1,1,0), c(1,1,0) >$ so,

$Fr(\lambda^*_N) = F^*NC(\lambda^*_N) \wedge F^*NC(\lambda^*_N)^c$

$= 1_N \wedge 1_N = 1_N$

Theorem 3.18: If a F^*NS A^* of F^*NTS (X^*_N, τ_N^*) is F^*ND-S. Then, $Fr(A^*) = 1_N - F^*NI(A^*)$.

Proof: Let A^* be F^*ND-S in X^*_N, i.e, $F^*NC(A^*) = 1_N$.

Then, Fr. $(A^*) = F^*NC(A^*) \wedge F^*NC(1_N - A^*)$

$= 1_N \wedge F^*NC(1_N - A^*)$

$= F^*NC(1_N - A^*)$

$= 1_N - F^*NI(A^*)$.

Theorem 3.19: If F^*NS A^* of F^*NTS (X^*_N, τ_N^*) is F^*NRA-S then, $Fr(A^*) = F^*NC(A^*)$.

Proof: Let A^* is F^*NRA-S in X^*_N. Then, $F^*NI(A^*) = 0_N$

Thus we have, $Fr(A^*) = F^*NC(A^*) \wedge F^*NC(1_N - A^*)$

$= F^*NC(A^*) \wedge (1_N - F^*NI(A^*))$

$= F^*NC(A^*) \wedge (1_N - 0_N)$

$= F^*NC(A^*)$.

Theorem 3.20: A F^*NS A^* of F^*NTS (X^*_N, τ_N^*) is F^*ND-S and F^*NRA-S iff $Fr(A^*) = 1_N$.

Proof: Let $Fr(A^*) = 1_N$ .Then $F^*NC(A^*) \wedge F^*NC(1_N - A^*) = 1_N$,

i.e, $F^*NC(A^*) = 1_N$ (*)

$F^*NC(1_N - A^*) = 1_N$ (**)

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By (*), A^* is F^*ND-S in X^*_N, from (**) we have F^*NC ($1_N - A^*$) = 1_N. F^*NI (A^*) = 1_N, i.e, F^*NI (A^*) = 0_N.

Thus A^* is F^*NRA-S. The converse follows from Theorem 3.16.

Theorem 3.21: In F^*NTS (X^*_N, τ_N^*) we have the following statement:

i. 0_N is F^*N closed RA-S in X^*_N.
ii. arbitrary intersections of F^*N closed RA-S in X^*_N are F^*N closed RA-S in X^*_N.

Proof: (i) by definition

(ii) Let $(A_i^*)_{i\in\Gamma}$ be a collection of F^*N closed RA-Ss in X^*_N, i.e,

F^*NC (A_i^*) = A_i^* and F^*NI (A_i^*) = 0_N for each $\in \Gamma$

Then, F^*NC ($\wedge_{i\in\Gamma} A_i^*$) = $\wedge_{i\in\Gamma}$ F^*NC (A_i^*) = $\wedge_{i\in\Gamma} A_i^*$. This proves that arbitrary inter- section of F^*NCLS is F^*NCLS.

Since each A_i^* is F^*NRA-S in X^*_N, F^*NI (A_i^*) = 0_N.

Thus, F^*NI ($\wedge_{i\in\Gamma} A_i^*$) = $\wedge_{i\in\Gamma}$ F^*NI (A_i^*) = 0_N.

4. Fuzzy Neutrosophic Generalized Semi –Closur and Fuzzy Neutrosophic Generalized Semi –Interior Operations

Definition 4.1: Let (X^*_N, τ_N^*) be F^*NTS, $A^* \subseteq X^*_N$ then, A^* is called:

i. fuzzy neutrosophic semi-interior and denoted by F^*NSI (A^*) if F^*NSI (A^*) = $A^* \cap F^*NC$ (F^*NI (A^*)).
ii. fuzzy neutrosophic semi-closure and denoted by F^*NSI (A^*) if F^*NSC (A^*) = $A^* \cup F^*NI$ (F^*NC (A^*)).

Definition 4.2: A F^*NS A^* in F^*NTS (X^*_N, τ_N^*) is called:

i. fuzzy neutrosophic semi-rare set and denoted by, F^*NS-rs in X^*_N if F^*NSI (A^*) = 0_N where, $0_N = < x, 0,0,1 >$.
ii. fuzzy neutrosophic semi- dense set and denoted by, F^*NS-ds in X^*_N if F^*NSC (A^*) = 1_N where, $1_N = < x, 1,1,0 >$

Example 4.3: Let X^*_N = {a, b, c} define F^*NS. A^*_N in X^*_N by:

$A^*_N = < x^*$, a (0.3, 0.5, 0.3), b (0.4, 0.6, 0.2), c (0.5, 0.3, 0.2) $>$ and let, τ_N^* = {0_N, 1_N, A^*_N} if

$K^*_N = < x^*$, a (0.5, 0.5,0.2), b (0.5, 0.5, 0.3), c (0.5, 0.3, 0.4) $>$ so,

i-F^*NSI (K^*_N) = $K^*_N \cap F^*NC$ (F^*NI (K^*_N))

= $K^*_N \cap F^*NC$ (0_N)

= $K^*_N \cap 0_N = 0_N$

That is K^*_N is F^*NS-rs.

ii-F^*NSC (K^*_N) = $K^*_N \cup F^*NI$ (F^*NC (K^*_N))

= $K^*_N \cup F^*NI$ (1_N)

= $K^*_N \cup 1_N = 1_N$

That is, K^*_N is F^*NS-ds.

Definition 4.4: Let (X^*_N, τ_N^*) be F^*NTS. A NS A^* in (X^*_N, τ_N^*) is said to be:

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i. generalized fuzzy neutrosophic semi-closed set if NC (A^*) ⊆ G^* where $A^* \subseteq G^*$ and G^* is a F^*NOS. The complement of a generalized fuzzy neutrosophic closed set is called a generalized fuzzy neutrosophic open set.

ii. fuzzy neutrosophic generalized semi –closure and fuzzy neutrosophic generalized semi- interior of A^* are defined by:

$F^*NG^*SC(A^*) = \cap \{G^*:G^*$ is a generalized fuzzy neutrosophic semi- closed set in X^*_N and $A^* \subseteq G^*\}$.

$F^*NG^*SI(A^*) = \cup \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-open set in X^*_N and $A^* \supseteq G^*\}$.

Proposition 4.5: Let (X^*_N,τ_N^*) be any NTS and let A^* and B^* be NSs in (X^*_N,τ_N^*). Then the fuzzy neutrosophic generalized semi-closure operator satisfy the following properties:

i. $A^* \subseteq F^*NG^*SC(A^*)$.
ii. $F^*NG^*SI(A^*) \subseteq A^*$.
iii. $A^* \subseteq B^* \Rightarrow F^*NG^*SI(A^*) \subseteq F^*NG^*SI(B^*)$.
iv. $A^* \subseteq B^* \Rightarrow F^*NG^*SI(A^*) \subseteq F^*NG^*SI(B^*)$.
v. $F^*NG^*SC(A^* \cup B^*) = F^*NG^*SC(A^*) \cup F^*NG^*SC(B^*)$.
vi. $F^*NG^*SI(A^* \cap B^*) = F^*NG^*SI(A^*) \cap F^*NG^*SC(B^*)$.
vii. $(F^*NG^*SC(A^*))^c = F^*NG^*SI(A^*)^c$.
viii. $(F^*NG^*SI(A^*))^c = F^*NG^*SI(A^*)^c$.

Proof: i. $F^*NG^*SC(A^*) = \cap \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-closed set in X^*_N and $A^* \subseteq G_*\}$. Thus, $A^* \subseteq F^*NG^*SC(A^*)$.

ii- $F^*NG^*SI(A^*) = \cup \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-open set in X^*_N and $A^* \supseteq G^*\}$.

Thus $F^*NG^*SI(A^*) \subseteq A^*$.

iii- $F^*NG^*SC(B^*) = \cap \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-closed set in X^*_N and $B^* \subseteq G^*\}$,

$\supseteq \{G^*:G^*$is a generalized fuzzy neutrosophic semi-closed set in X^*_N and $A^* \subseteq G^*\}$, $\supseteq NG^*SC(A^*)$. Thus, $F^*NG^*SC(A^*) \subseteq F^*NG^*SC(B^*)$.

iv- $F^*NG^*SI(B^*) = \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-open set in X^*_N and $B^* \supseteq G^*\}$,

$\supseteq \cup \{G^*:G^*$ is generalized fuzzy neutrosophic semi-open set in X^*_N and A^* $G^*\}$,

$\supseteq F^*NG^*SI(A^*)$. Thus, $F^*NG^*SI(A^*) \subseteq F^*NG^*SI(B^*)$.

v- $F^*NG^*SC(A^* \cup B^*) = \cap \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-closed set in X^*_N and $A^* \cup B^* \subseteq G^*\} \subseteq (\cap \{G^*:G^*$ is generalized fuzzy neutrosophic semi-closed set in X^*_N and ,

$A^* \subseteq G^*\}) \cup (\cap \{G^*:G^*$ is generalized fuzzy neutrosophic semi-closed set in X^*_N and $B^* \subseteq G^*\}) = F^*NG^*SC(A^*) \cup F^*NG^*SC(B^*)$. Thus, $F^*NG^*SC(A^* \cup B^*) = F^*NG^*SC(A^*) \cup F^*NG^*SC(B^*)$.

vi- $F^*NG^*SI(A^* \cap B^*) = \cup \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-open set in X^*_N and $A^* \cap B^* \supseteq G^*\} \subseteq (\cup \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-closed set in X^*_N and,

$A^* \supseteq G^*\}) \cap (\cup \{G^*:G^*$ is a generalized fuzzy neutrosophic semi-open set in X^*_N and $B^* \supseteq G^*\}) = F^*NG^*SI(A^*) \cap F^*NG^*SI(B^*)$. Thus, $F^*NG^*SI(A^* \cap B^*) = F^*NG^*SI(A^*) \cap F^*NG^*SI(B^*)$.

Vii- $F^*NG^*SC(A^*) = \cap \{G^*:G^*$ is generalized fuzzy neutrosophic semi-closed set in X^*_N and $A^* \subseteq G^*\}$and,

$(F^*NG^*SC(A^*))^c = \cup \{G^{*^c}: G^{*^c}$is a generalized fuzzy neutrosophic semi-open set in X^*_N and $A^{*^c} \supseteq G^{*^c}\}$

$= F^*NG^*SI(A^c)$.

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Thus, $(F^*NG^*SC(A^*))^c = F^*NG^*SI(A^c)$.

viii- $F^*NG^*SI(A^*) = \cup \{G^*: G^*$ is generalized fuzzy neutrosophic semi-open set in X^*_N and $A^* \supseteq G^*\}$ and,

$(F^*NG^*SI(A^*))^c = \cap \{G^{*c}: G^{*c}$ is generalized fuzzy neutrosophic semi-closed set in X^*_N and $A^{*c} \subseteq G^{*c}\}$

$= F^*NG^*SC(A^{*c})$. Thus, $(F^*NG^*SI(A^*))^c = F^*NG^*SC(A^{*c})$.

Proposition 4.6: Let (X^*_N, τ^*_N) be a NTS. If B^* is generalized fuzzy neutrosophis semi-closed set and $B^* \subseteq A^* \subseteq F^*NSC(B^*)$ then A^*, is generalized fuzzy neutrosophic semi-closed set.

Proof: Let G^* be a F^*NOS in (X^*_N, τ^*_N) such that, $A^* \subseteq G^*$. Since $B^* \subseteq A^*$ so, $B^* \subseteq G^*$.

But, B^* is generalized fuzzy neutrosophic semi-closed set and $NSC(B^*) \subseteq G^*$.

So, $F^*NSC(A^*) \subseteq F^*NSC(B^*)$.

Since, $F^*NSC(A^*) \subseteq F^*NSC(B^*) \subseteq G^*$, $F^*NSC(A^*) \subseteq G^*$.Hence, A^* is generalized fuzzy neutrosophic semi-closed set.

Proposition 4.7: Let (X^*_N, τ^*_N) be a NTS. A F^*NS A^* is generalized fuzzy neutrosophic semi-open set iff $B^* \subseteq F^*NSI(A^*)$, whenever B^* is an fuzzy neutrosophic semi-closed set and $B^* \subseteq A^*$.

Proof: Let A^* be a generalized fuzzy neutrosophic semi-open set and B^* be generalized fuzzy neutrosophic semi-closed set such that $B^* \subseteq A^*$.

Now, $B^* \subseteq A^* \Rightarrow A^{*c} \subseteq B^{*c}$ and since A^{*c} is generalized fuzzy neutrosophic semi-closed set then,

$F^*NSC(A^{*c}) \subseteq B^{*c}$.This means that $B^* = \left(B^{*c}\right)^c \subseteq \left(F^*NSC(A^{*c})\right)^c$.

But, $\left(F^*NSC(A^{*c})\right)^c = F^*NSI(A^*)$. Hence, $B^* \subseteq F^*NSI(A^*)$.

Conversely, let A^* be a NS such that $B^* \subseteq F^*NSI(A^*)$ whenever, B^* is F^*NCS and $B^* \subseteq A^*$.

Let $A^{*c} \subseteq B^*$ whenever, B^* is a NOS. Now, $A^{*c} \subseteq B^* \Longrightarrow B^{*c} \subseteq A^*$.

Hence by assumption, $B^{*c} \subseteq F^*NSI(A^*)$. That is, $\left(F^*NSI(A^{*c})\right)^c \subseteq B^*$.

But, $\left(F^*NSI(A^{*c})\right)^c = F^*NSC(A^{*c})$.

Hence, $F^*NSC(A^{*c}) \subseteq B^*$. This means that A^{*c} is a generalized fuzzy neutrosophic semi-closed set.

Therefore, A^* is a generalized fuzzy neutrosophic semi-open set.

Proposition 4.8: If $NSI(A^*) \subseteq B^* \subseteq A^*$ and if A^* is generalized fuzzy neutrosophic semi-open set then, B^* is also a generalized fuzzy neutrosophic semi-open set.

Proof: $A^{*c} \subseteq B^{*c} \subseteq (F^*NSI(A^*))^c = F^*NSC(A^{*c})$.

Since A^* is a generalized fuzzy neutrosophic semi-open set then, A^{*c} is generalized fuzzy neutrosophic semi-closed set.

By proposition 2.2.8, B^{*c} is generalized fuzzy neutrosophic semi-closed set. That is, B^* is a generalized fuzzy neutrosophic semi-open set.

Definition 4.9: Let X^*_N be an NTS, an NS A^* in X^*_N is called:

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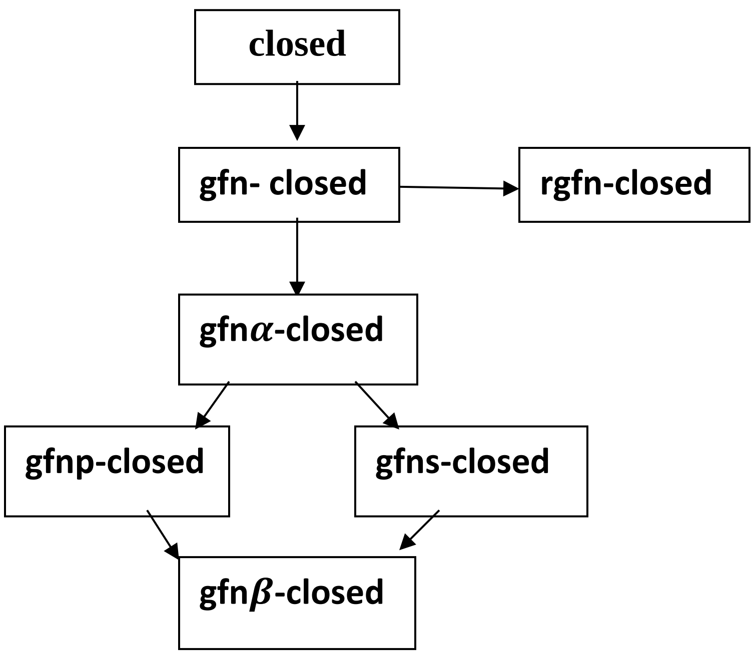
i. generalized fuzzy neutrosophic closed and denoted by, gfn-closed, if FNC (A^*) ⊆ U^*, whenever A^* ⊆ U^* and U^* is F^*NOS.
ii. regular generalized fuzzy neutrosophic closed and denoted by, rgfn-closed, if C (A^*) ⊆ U^* whenever, A^* ⊆ U^* and U^* is F^*NROS.
iii. semi- generalized fuzzy neutrosophic closed and denoted by, sgfn- closed, if SNC(A^*) ⊆ U^*, whenever A^* ⊆ U^* and U^* is F^*NSOS.
iv. generalized fuzzy neutrosophic semi-closed (and denoted by, gfns-closed), if NSC (A^*) ⊆ U^*, whenever A^* ⊆ U^* and U^* is F^*NOS,
v. generalized fuzzy neutrosophic α-closed and denoted by, gfnα-closed if, $N\alpha$C (A^*) ⊆ U^* whenever, A^* ⊆ U^* and U^* is F^*NαOS.
vi. α- generalized fuzzy neutrosophic closed and denoted by, αgfn-closed if αNC (A^*) ⊆ U^*, whenever A^* ⊆ U^* and U^* is F^*NOS.
vii. Generalized fuzzy neutrosophic β-closed and denoted by, gfnβ-closed if NβC (A^*) ⊆ U^*, whenever A^* ⊆ U^* and U^* is F^*NOS.
viii. generalized fuzzy neutrosophic pre-closed and denoted by, gfnp-closed if NPC (A^*) ⊆ U^*, whenever A^* ⊆ U^* and U^* is F^*NOS.

A F^*NS A^* in X^*_N is gfn-open (resp. sgfn-open, rgfn-open, gfns-open, gfnα-open, αgfn-open, gfnβ-open and gfnp-open), if A^{*^c} is gfn-closed (resp. sgfn-closed, rgfn-closed, gfns-closed, gn$f\alpha$-closed, αgfn-closed, gfnβ-closed and gfnp-closed) but, not conversely.



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Example 4.10: Let X^*_N = {a, b, c} define F^*NS. η^*_N in X^*_N

η^*_N= < x^*, a (0.3, 0.5, 0.3), b (0.4, 0.2, 0.2), c (0.5, 0.3, 0.2) >

U^* = < x^*, a (0.4, 0.4, 0.2), b (0.5, 0.3, 0.3), c (0.4, 0.2, 0.3) >

τ^*_N = {0_N, 1_N, η^*_N} if

A^* = < x^*, a (0.3, 0.4, 0.3), b (0.4, 0.2, 0.4), c (0.4, 0.2, 0.2) >

i- C (A^*) ⊆ U^* = 1_N ⊊ U^*. So, A^* is not gfn-closed and not rgfn- closed

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ii- $F^*SNC(A^*) = (A^* \cup F^*NI(F^*NC(A^*))$

$= (A^* \cup F^*NI(1_N))$

$= (A^* \cup 1_N) = 1_N \subsetneq U^*$.

Then, A^* is not sgfn-closed and not gfns-closed .

iii- $F^*N\alpha C(A^*) = (A^* \cup (F^*NI(F^*NC(A^*)))$

$= (A^* \cup (I(1_N))$

$= (A^* \cup 1_N) = 1_N \subsetneq U^*$

So, A^* is not gfnα-closed and not αgfn-closed

iv- $F^*N\beta C(A^*) = (A^* \cup F^*NI(F^*NC(F^*NI(A^*)))$

$= (A^* \cup F^*NI(F^*NC(0_N)))$

$= (A^* \cup F^*NI(0_N)) = (A^* \cup 0_N) = A^* \subseteq U^*$

So, A^* is gfnβ-closed

v- $F^*NPC(A^*) = (A^* \cup F^*NC(F^*NI(A^*)))$

$= (A^* \cup F^*NC(0_N))$

$= (A^* \cup 0_N) = A^* \subseteq U^*$

Therefore, A^* is gfnp-closed.

Theorem 4.11: Let X^*_N be any F^*NTS. Afuzzy neutrosophic subset A^* of X^*_N is gfn-open iff, for each F^*NCLS F^* in X^*_N such that $F^* \subseteq F^*NI(A^*)$ whenever, $F^* \subseteq A^*$.

Proof: ⇒ Suppose that A^* is gfn-open set in X^*_N, and let F^* be any F^*NCLS such that $F^* \subseteq A^*$, so by definition, ${A^*}^c$ is gfn-closed set in X^*_N. Therefore, for each F^*NOS U^* say $U^* = {F^*}^c$ in X^*_N, ${A^*}^c \subseteq {F^*}^c$ then,

$F^*NC({A^*}^c) \subseteq {F^*}^c$, so $\left({F^*}^c\right)^c = F^* \subseteq \left(F^*NC({A^*}^c)\right)^c = F^*NI(A^*)$.

⇐ Suppose that for each F^*NCLS $F^* \subseteq$ then $F^* \subseteq F^*NI(A^*)$, we need to demonstrate that A^* is gfn-open, i.e. we need to demonstrate that ${A^*}^c$ is gfn- closed, let U^* be any F^*NOS in X^*_N such that ${A^*}^c \subseteq U^*$, we need to demonstrate that $F^*NC({A^*}^c) \subseteq U^*$.

For if, since U^* is F^*NOS, then ${U^*}^c$is F^*NCLS and ${U^*}^c \subseteq A^*$, so by hypothesis ${U^*}^c \subseteq F^*NI(A^*)$. Therefore, $\left(F^*NI({A^*}^c)\right)^c = F^*NC({A^*}^c) \subseteq \left({U^*}^c\right)^c = U^*$. So by definition we get that ${A^*}^c$ is gfn-closed.

Theorem 4.12: Let X^*_N be F^*NTS. An neutrosophic subset A^* in X^*_N is gfns-open (resp. αfgn-open, gfnp-open, gfnβ-open) iff for each F^*NCLS F^* in X^*_N such that $F^* \subseteq A^*$, then $F^* \subseteq$ gfns $I(A^*)$ (resp. $F^* \subseteq \alpha$gfn,

$F^* \subseteq$gfnp $I(A^*)$, $F^* \subseteq$ gfnβ $I(A^*)$)

Proof: ⟹ Suppose that A^* is gfns-open (resp. αgfn-open, gfnp-open, gfnβ-open) in X^*_N, and let F^* be any NCLS such that $F^* \subseteq A^*$, so by definition, ${A^*}^c$ is gfns-closed (resp. αgfn-closed, gfnp-closed, gfnβ-closed) set in X^*_N. Therefore, for each F^*NOS U^* say $U^* = {F^*}^c$ in X^*_N, ${A^*}^c \subseteq {F^*}^c$, then $F^*NSC({A^*}^c) \subseteq {F^*}^c$, (resp. $F^*N\alpha C({A^*}^c) \subseteq {F^*}^c$, $F^*NPC({A^*}^c) \subseteq {F^*}^c$,$F^*N\beta C({A^*}^c) \subseteq {F^*}^c$, so $\left({F^*}^c\right)^c = F^* \subseteq \left(F^*NSC({A^*}^c)\right)^c = F^*NSI(A^*)$ (resp. $\left({F^*}^c\right)^c = F^* \subseteq$

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$\left(FN\alpha C(A^{*^c})\right)^c = F^*N\alpha$I (A*), $\left(F^{*^c}\right)^c$ = F* ⊆ $\left(F^*NPC\ (A^{*^c})\right)^c$ = F*NPI (A*), $\left(F^{*^c}\right)^c$ = F* ⊆ $\left(F^*N\beta C\ (A^{*^c})\right)^c$ = $F^*N\beta$I (A*).

⟸ suppose that for each F*NCLS F* ⊆ A* then F* ⊆ F*NSI (A*), (resp. F* ⊆ $NF^*\alpha$I (A*), F* ⊆ F*NPI (A*), F* ⊆ $F^*N\beta$I (A*), (we need to demonstrate that A* is gfns-open,) (resp. αgfn-open, gfnp-open, gfnβ-open) i.e. we need to demonstrate that A^{*^c} is gfns-closed, (resp. αfgn-closed, gfnp-closed, gfnβ-closed),

Let U* be any F*NOS in X*N such that A^{*^c} ⊆ U*, we need to demonstrate that F*NSC (A^{*^c}) ⊆ U* (resp. $F^*N\alpha$C (A^{*^c}) ⊆ U*, F*NPC (A^{*^c}) ⊆U*, F*NβC (A^{*^c}) ⊆ U*. For if, since U* is F*NOS, then U^{*^c} is F*NCLS and U^{*^c} ⊆ A*, so by hypothesis U^{*^c} ⊆ F*NSI (A*) (resp. U^{*^c} ⊆ $F^*N\alpha$I (A*), U^{*^c} ⊆ F*NPI (A*), U^{*^c} ⊆ $F^*N\beta$I (A*)). Therefore, $\left(F^*NSI\ (A^*)\right)^c$ = F*NSC (A^{*^c}) ⊆ $\left(U^{*^c}\right)^c$ = U* (resp. $\left(F^*N\alpha I\ (A^*)\right)^c$ = $F^*N\alpha$C (A^{*^c}) ⊆ $\left(U^{*^c}\right)^c$ = U*.

So, by definition we get that, A^{*^c} is gfns-closed (resp. gfnα-closed, gfnp-closed, gfnβ-closed).

Theorem 4.13: Let X*N be NTS. A fuzzy neutrosophic subset A* in X*N is sgfn-open (resp. gfnα-open) iff for each F*NCLS F* in X*N such that F* ⊆ A*, then F* ⊆ F*NSI (A*) (resp. F* ⊆ F*NαI (A*)).

⟹ Suppose that A* is sgfn-open (resp. gfnα-open) set in X*N, and let F* be any F*NCLS such that F* ⊆ A*, so by definition, A^{*^c} is gfns-closed (resp. gnα-closed) set in X*N. Therefore, for each F*NS-open set U*(resp. F*Nα-open set) U* say U* = F^{*^c} in X*N, A^{*^c} ⊆ F^{*^c}, then F*NSC (A^{*^c}) ⊆ F^{*^c}, (resp. F*NαC (A^{*^c}) ⊆ F^{*^c} so $\left(F^{*^c}\right)^c$ = F* ⊆ $\left(F^*NSC\ (A^{*^c})\right)^c$ = F*NSI (A*) (resp. $\left(F^{*^c}\right)^c$ = F* ⊆ $\left(F^*N\alpha C\ (A^{*^c})\right)^c$ = $F^*N\alpha$I (A*).

⟸ Suppose that for each F*NS-closed set F* ⊆ A* (resp. F*Nα-closed set) then F* ⊆ F*NSI (A*), (resp. F* ⊆ $F^*N\alpha I\ (A^*)$ we need to demonstrate that A* is sgfn-open, (resp. gfnα-open) i.e. we need to demonstrate that A^{*^c} is sgfn-closed (resp. gfnα-closed), let U* be any F*NS-open set (resp. F*Nα-closed set). In X*N such that A^{*^c} ⊆ U*, we need to demonstrate that F*NC (A^{*^c}) ⊆ U*.For if, since U* is F*NS-open set, (resp. F*Nα-open set) then U^{*^c} is F*NS-closed set (resp. F*Nα-closed set) and U^{*^c} ⊆ A*, so by hypothesis U^{*^c} ⊆ F*NSI (A*), (resp. U^{*^c} ⊆ F*NαI (A*),

Therefore, $(F^*NSI\ (A^*))^c$ = F*NSC (A^{*^c}) ⊆ $\left(U^{*^c}\right)^c$ = U* (resp. $(F^*N\alpha I\ (A^*))^c$ = F*NαC (A^{*^c}) ⊆ $\left(U^{*^c}\right)^c$ = U*. So by definition we get that, A^{*^c} is sgfn-closed (resp. gfnα-closed).

5.Conclusion

In this study, we have analyzed some properties of rare and dense sets in the light of the theory of $f$uzzy $n$eutrosophic $s$ets which has been defined. Clearly, some of these results are more general than fuzzy set and intutionistic fuzzy set. The study has proposed some characteristics recently established and attributes of this concept. Some relations between the defined sets with others based of $f$uzzy $n$eutrosophic $t$opological $s$paces has been clarified.

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