On Some Different Types of Algebraic Linear Equations In Several Different Symbolic 2-Plithogenic Rings

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Abstract

Plithogenic algebraic rings are considered novel generalizations of neutrosophic rings. This paper is dedicated to studying and solving many different types of symbolic 2-plithogenic linear algebraic equations where dual, split-complex, and weak fuzzy linear equations of symbolic 2-plithogenic type will be solved and discussed. Also, we illustrate many numerical examples to clarify the validity of our work.

Keywords: symbolic 2-plithogenic ring; weak fuzzy complex number; dual number; split-complex number

1. Introduction

The theory of algebraic equations is rich in a lot of different types of equations and algorithms that study the solutions of these equations. With the continuation of generalizations in mathematics, many numerical systems have appeared that expand the set of real or integer numbers such as neutrosophic numbers [12], split-complex numbers [1,5], dual numbers, and weak fuzzy complex numbers [2-3].

In [8], the concept of symbolic 2-plithogenic numbers was defined for the first time, and it was used with split-complex and weak fuzzy complex numbers to generate some new extended sets called symbolic 2-plithogenic dual, weak fuzzy, and split-complex numbers. [9-11]

Many researchers around the world have studied algebraic equations in these generalized numerical systems, studying Diophantine equations, algebraic equations, and even matrices [4,7,13].

These previous studies have prompted us to study three different kinds of algebraic linear equations in one variable, where we present many algorithms to solve symbolic 2-plithogenic dual, split-complex, and weak fuzzy complex linear algebraic equations in one variable. We will also support the theoretical results of this study with numerous numerical examples showing how the algorithms and proofs we have proved work.

2. Main discussion

Definition:

The symbolic 2-plithogenic dual linear equation with one Variable X is defined as follows:

$AX = B;\ A = (a_0 + a_1p_1 + a_2p_2) + t(a_0{}' + a_1{}'p_1 + a_2{}'p_2)$,
$B = (b_0 + b_1p_1 + b_2p_2) + t(b_0{}' + b_1{}'p_1 + b_2{}'p_2), X$
$= (x_0 + x_1p_1 + x_2p_2) + t(x_0{}' + x_1{}'p_1 + x_2{}'p_2), t^2 = 0.$

Remark:

The previous equation is solvable uniquely if and only if A is invertible.
According to [8], A is invertible if and only if:
$a_0 \neq 0,\ a_0 + a_1 \neq 0,\ a_0 + a_1 + a_2 \neq 0$

Algorithm for solution:

Let us assume that A is invertible, then:
$AX = (a_0 + a_1p_1 + a_2p_2)(x_0 + x_1p_1 + x_2p_2) + t[(a_0 + a_1p_1 + a_2p_2)(x_0{}' + x_1{}'p_1 + x_2{}'p_2) + (a_0{}' + a_1{}'p_1 + a_2{}'p_2)(x_0 + x_1p_1 + x_2p_2)]$,

Doi: https://doi.org/10.54216/JNFS.080105
Received: October 09, 2023 Revised: December 27, 2023 Accepted: January 25, 2024

$AX = B$ is equivalent to:
$(a_0 + a_1p_1 + a_2p_2)(x_0 + x_1p_1 + x_2p_2) = b_0 + b_1p_1 + b_2p_2$: (1)
$(a_0 + a_1p_1 + a_2p_2)\left(x_0' + x_1'^{p_1} + x_2'^{p_2}\right) + \left(a_0' + a_1'^{p_1} + a_2'^{p_2}\right)(x_0 + x_1p_1 + x_2p_2) = b_0' + b_1'p_1 + b_2'p_2$. (2)
By solving (1), we get:

$x_0 = \frac{b_0}{a_0}$, $x_0 + x_1 = \frac{b_0+b_1}{a_0+a_1} \Rightarrow$
$x_1 = \frac{b_0+b_1}{a_0+a_1} - \frac{b_0}{a_0}$,
$x_0 + x_1 + x_2 = \frac{b_0 + b_1 + b_2}{a_0 + a_1 + a_2} \Rightarrow$
$x_2 = \frac{b_0+b_1+b_2}{a_0+a_1+a_2} - \frac{b_0+b_1}{a_0+a_1}$,
To solve equation(2), we put:
$(a_0' + a_1'p_1 + a_2'p_2)(x_0 + x_1p_1 + x_2p_2) + b_0' + b_1'p_1 + b_2'p_2 = \ell_0 + \ell_1p_1 + \ell_2p_2$,
Then
$(a_0 + a_1p_1 + a_2p_2)(x_0' + x_1'p_1 + x_2'p_2) = \ell_0 + \ell_1p_1 + \ell_2p_2$ (3).
The solution of equation(3) is:
$x_0' = \frac{\ell_0}{a_0}$,
$X_0' + x_1' = \frac{\ell_0+\ell_1}{a_0+a_1} \Rightarrow$
$x_1' = \frac{\ell_0+\ell_1}{a_0+a_1} - \frac{\ell_0}{a_0}$,
$x_0' + x_1' + x_2' = \frac{\ell_0 + \ell_1 + \ell_2}{a_0 + a_1 + a_2} \Rightarrow$
$x_2' = \frac{\ell_0+\ell_1+\ell_2}{a_0+a_1+a_2} - \frac{\ell_0+\ell_1}{a_0+a_1}$.

Example:

Consider the dual symbolic 2-plithogenic linear equation:
$[(1 + 2p_1 + p_2) + t(3 - p_1 + 2p_2)]X = 1 + 8p_1 + 3p_2 + t(4 + 5p_1 + 3p_2)$.
We have:
$a_0 = 1, a_1 = 2, a_2 = 1, a_0' = 3, a_1' = -1, a_2' = 2$,
$b_0 = 1, b_1 = 8, b_2 = 3, b_0' = 4, b_1' = 5, b_2' = 3$,
$x_0 = \frac{b_0}{a_0} = 1$,
$x_1 = \frac{b_0+b_1}{a_0+a_1} - \frac{b_0}{a_0} = \frac{9}{3} - 1 = 2$,
$x_2 = \frac{b_0+b_1+b_2}{a_0+a_1+a_2} - \frac{b_0+b_1}{a_0+a_1} = \frac{12}{4} - \frac{9}{3} = 0$.
Now, we complete:
$\ell_0 + \ell_1p_1 + \ell_2p_2 = -(3 - p_1 + 2p_2)(1 + 2p_1) + 4 + 5p_1 + 3p_2 = -(3 + 6p_1 - p_1 - 2p_1 + 2p_2 + 4p_2) + 4 + 5p_1 + 3p_3 = 1 + 2p_1 - 3p_2$, thus
$\ell_0 = 1, \ell_1 = 2, \ell_2 = -3$.
$x_0' = \frac{\ell_0}{a_0} = \frac{1}{1} = 1$,
$x_1' = \frac{\ell_0+\ell_1}{a_0+a_1} - \frac{\ell_0}{a_0} = \frac{3}{3} - 1 = 0$,
$x_2' = \frac{\ell_0+\ell_1+\ell_2}{a_0+a_1+a_2} - \frac{\ell_0+\ell_1}{a_0+a_1} = \frac{0}{4} - \frac{3}{3} = -1$.
Thus: $X = (1 + 2p_1) + T(1 - p_2)$ is the solution.

Definition:

The split-complex symbolic 2-plithogenic linear equation with one variable is defined as follows:
AX=B, with
$A = (a_0 + a_1p_1 + a_2p_2) + J(a_0' + a_1'p_1 + a_2'p_2)$,
$B = (b_0 + b_1p_1 + b_2p_2) + J(b_0' + b_1'p_1 + b_2'p_2)$,
$X = (x_0 + x_1p_1 + x_2p_2) + J(x_0' + x_1'p_1 + x_2'p_2), J^2 = 1$.

Remark:

This previous equation is solvable if and only if A is invertible.
This is equivalent to:
$a_0 + a_0' \neq 0$,
$a_0 - a_0' \neq 0$,

Doi: https://doi.org/10.54216/JNFS.080105
Received: October 09, 2023 Revised: December 27, 2023 Accepted: January 25, 2024

$a_0 + a_1 + a_0' + a_1' \neq 0,$
$(a_0 + a_1) - (a_0' + a_1') \neq 0,$
$a_0 + a_1 + a_2 + a_0' + a_1' + a_2' \neq 0,$
$(a_0 + a_1 + a_2) - (a_0' + a_1' + a_2') \neq 0$

Algorithm for the solution:

Let's assume that A is invertible, then:

$AX = (a_0 + a_1p_1 + a_2p_2)(x_0 + x_1p_1 + x_2p_2) + (a_0' + a_1'p_1 + a_2'p_2)(x_0' + x_1'p_1 + x_2'p_2) + J[(a_0 + a_1p_1 + a_2p_2)(x_0' + x_1'p_1 + x_2'p_2) + (a_0' + a_1'p_1 + a_2'p_2)(x_0 + x_1p_1 + x_2p_2)],$

AX=B is equivalent to:

$(a_0 + a_1p_1 + a_2p_2)(x_0 + x_1p_1 + x_2p_2) + (a_0' + a_1'p_1 + a_2'p_2)(x_0' + x_1'p_1 + x_2'p_2) = b_0 + b_1p_1 + b_2p_2$ (1).

$(a_0 + a_1p_1 + a_2p_2)(x_0' + x_1'p_1 + x_2'p_2) + (a_0' + a_1'p_1 + a_2'p_2)(x_0 + x_1p_1 + x_2p_2) = b_0' + b_1'p_1 + b_2'p_2$ (2).

Equation (1) is equivalent to:

$a_0x_0 + a_0'x_0' = b_0$ (I).

$(a_0 + a_1)(x_0 + x_1) + (a_0' + a_1')(x_0' + x_1') = b_0 + b_1$ (II).

$(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + (a_0' + a_1' + a_2')(x_0' + x_1' + x_2') = b_0 + b_1 + b_2$ (III).

Equation (2) is equivalent to:

$a_0x_0' + a_0'x_0 = b_0'$ (IV),

$(a_0 + a_1)(x_0' + x_1') + (a_0' + a_1')(x_0 + x_1) = b_0' + b_1'$ (IIV),

$(a_0 + a_1 + a_2)(x_0' + x_1' + x_2') + (a_0' + a_1' + a_2')(x_0 + x_1 + x_2) = b_0' + b_1' + b_2'$ (IIIV),

By adding (I) to (IV), and subtracting (IV) from (I), we get:

$$\begin{cases}(a_0 + a_0')(x_0 + x_0') = b_0 + b_0' \\ (a_0 - a_0')(x_0 - x_0') = b_0 - b_0'\end{cases}$$

This implies that:

$$\begin{cases}x_0 + x_0' = \dfrac{b_0 + b_0'}{a_0 + a_0'} \\ x_0 - x_0' = \dfrac{b_0 - b_0'}{a_0 - a_0'}\end{cases}$$

So that,

$$\begin{cases}x_0 = \dfrac{1}{2}\left(\dfrac{b_0 + b_0'}{a_0 + a_0'} + \dfrac{b_0 - b_0'}{a_0 - a_0'}\right) \\ x_0' = \dfrac{1}{2}\left(\dfrac{b_0 + b_0'}{a_0 + a_0'} - \dfrac{b_0 - b_0'}{a_0 - a_0'}\right)\end{cases}$$

By adding (II) to (IIV), and subtracting (IIV) from (II), we get:

$$\begin{cases}(a_0 + a_1 + a_0' + a_1')(x_0 + x_1 + x_0' + x_1') = b_0 + b_1 + b_0' + b_1' \\ (a_0 + a_1 - a_0' - a_1')(x_0 + x_1 - x_0' - x_1') = b_0 + b_1 - b_0' - b_1'\end{cases}$$

So that:

$$\begin{cases}x_0 + x_1 + x_0' + x_1' = \dfrac{b_0 + b_1 + b_0' + b_1'}{a_0 + a_1 + a_0' + a_1'} \\ x_0 + x_1 - x_0' - x_1' = \dfrac{b_0 + b_1 - b_0' - b_1'}{a_0 + a_1 - a_0' - a_1'}\end{cases}$$

Hence,

Doi: https://doi.org/10.54216/JNFS.080105
Received: October 09, 2023 Revised: December 27, 2023 Accepted: January 25, 2024

$$\begin{cases} x_0 + x_1 = \frac{1}{2}\left(\frac{b_0 + b_1 + b_0' + b_1'}{a_0 + a_1 + a_0' + a_1'} + \frac{b_0 + b_1 - b_0' - b_1'}{a_0 + a_1 - a_0' - a_1'}\right) \\ x_0' + x_1' = \frac{1}{2}\left(\frac{b_0 + b_1 + b_0' + b_1'}{a_0 + a_1 + a_0' + a_1'} - \frac{b_0 + b_1 - b_0' - b_1'}{a_0 + a_1 - a_0' - a_1'}\right) \end{cases}$$

By a similar argument, we get:

$$\begin{cases} x_0 + x_1 + x_2 = \frac{1}{2}\left(\frac{b_0 + b_1 + b_2 + b_0' + b_1' + b_2'}{a_0 + a_1 + a_2 + a_0' + a_1' + a_2'} + \frac{b_0 + b_1 + b_2 - b_0' - b_1' - b_2'}{a_0 + a_1 + a_2 - a_0' - a_1' - a_2'}\right) \\ x_0' + x_1' + x_2' = \frac{1}{2}\left(\frac{b_0 + b_1 + b_2 + b_0' + b_1' + b_2'}{a_0 + a_1 + a_2 + a_0' + a_1' + a_2'} - \frac{b_0 + b_1 + b_2 - b_0' - b_1' - b_2'}{a_0 + a_1 + a_2 - a_0' - a_1' - a_2'}\right) \end{cases}$$

Example:

Consider the symbolic 2-plithogenic split-complex linear equation:

$[(3 + 2p_1 + p_2) + J(1 + p_1 + p_2)]X = (9 + 6p_1 + 6p_2) + J(3 + 3p_1 + 9p_2)$,

We have:

$a_0 = 3, a_1 = 2, a_2 = 1$,

$b_0 = 9, b_1 = 6, b_2 = 6$,

$a_0' = 1, a_1' = 1, a_2' = 1$,

$b_0' = 3, b_1' = 3, b_2' = 9$.

The solution is:

$$x_0 = \frac{1}{2}\left(\frac{b_0 + b_0'}{a_0 + a_0'} + \frac{b_0 - b_0'}{a_0 - a_0'}\right) = \frac{1}{2}\left(\frac{12}{4} + \frac{6}{2}\right) = 3$$

$$x_0' = \frac{1}{2}\left(\frac{b_0 + b_0'}{a_0 + a_0'} - \frac{b_0 - b_0'}{a_0 - a_0'}\right) = \frac{1}{2}(3 - 3) = 0$$

$$x_0 + x_1 = \frac{1}{2}\left(\frac{b_0 + b_1 + b_0' + b_1'}{a_0 + a_1 + a_0' + a_1'} + \frac{b_0 + b_1 - b_0' - b_1'}{a_0 + a_1 - a_0' - a_1'}\right) = \frac{1}{2}\left(\frac{21}{7} + \frac{9}{3}\right) = 3$$

Thus

$x_1 = 3 - x_0 = 3 - 3 = 0$

$$x_0' + x_1' = \frac{1}{2}\left(\frac{b_0 + b_1 + b_0' + b_1'}{a_0 + a_1 + a_0' + a_1'} - \frac{b_0 + b_1 - b_0' - b_1'}{a_0 + a_1 - a_0' - a_1'}\right) = \frac{1}{2}(3 - 3) = 0$$

Thus

$x_1' = 0 - x_0' = 0$

$x_0 + x_1 + x_2 = \frac{1}{2}\left(\frac{b_0+b_1+b_2+b_0'+b_1'+b_2'}{a_0+a_1+a_2+a_0'+a_1'+a_2'} + \frac{b_0+b_1+b_2-b_0'-b_1'-b_2'}{a_0+a_1+a_2-a_0'-a_1'-a_2'}\right) = \frac{1}{2}\left(\frac{36}{9} + \frac{3}{3}\right) = 3$,

thus $x_2 = 3 - x_0 - x_1 = 0$.

$x_0' + x_1' + x_2' = \frac{1}{2}\left(\frac{b_0+b_1+b_2+b_0'+b_1'+b_2'}{a_0+a_1+a_2+a_0'+a_1'+a_2'} - \frac{b_0+b_1+b_2-b_0'-b_1'-b_2'}{a_0+a_1+a_2-a_0'-a_1'-a_2'}\right) = \frac{1}{2}\left(\frac{36}{9} - \frac{6}{3}\right) = 1$,

thus $x_2' = 1 - x_0' - x_1' = 1$, hence $X = 3 + p_2J$.

Definition:

The symbolic 2-plithogenic weak fuzzy complex linear equation with one variable is defined as follows:

AX=B, where

$A = (a_0 + a_1p_1 + a_2p_2) + J(a_0' + a_1'p_1 + a_2'p_2)$,

$B = (b_0 + b_1p_1 + b_2p_2) + J(b_0' + b_1'p_1 + b_2'p_2)$,

$X = (x_0 + x_1p_1 + x_2p_2) + J(x_0' + x_1'p_1 + x_2'p_2), J^2 = t \in]0,1[$.

Remark:

The previous equation is solvable if and if A is invertible.

Algorithm for solution:

Let's assume that A is invertible, then:

$$AX = (a_0 + a_1p_1 + a_2p_2)(x_0 + x_1p_1 + x_2p_2) + t(a_0' + a_1'p_1 + a_2'p_2)(x_0' + x_1'p_1 + x_2'p_2) + J[(a_0 + a_1p_1 + a_2p_2)(x_0' + x_1'p_1 + x_2'p_2) + (a_0' + a_1'p_1 + a_2'p_2)(x_0 + x_1p_1 + x_2p_2)]$$

AX=B is equivalent to:

$a_0x_0 + ta_0'x_0' = b_0$ (I).

$(a_0 + a_1)(x_0 + x_1) + t(a_0' + a_1')(x_0' + x_1') = b_0 + b_1$ (II).

$(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + t(a_0' + a_1' + a_2')(x_0' + x_1' + x_2') = b_0 + b_1 + b_2$ (III).

$a_0x_0' + a_0'x_0 = b_0'$ (IV),

$(a_0 + a_1)(x_0' + x_1') + (a_0' + a_1')(x_0 + x_1) = b_0' + b_1'$ (IIV),

Doi: https://doi.org/10.54216/JNFS.080105
Received: October 09, 2023 Revised: December 27, 2023 Accepted: January 25, 2024

$(a_0+a_1+a_2)(x_0{}'+x_1{}'+x_2{}')+(a_0{}'+a_1{}'+a_2{}')(x_0+x_1+x_2)=b_0{}'+b_1{}'+b_2{}'$ (IIIV),

To solve the equation system (I) → (IIIV), we will follow these steps.
Step(1):
We multiply (IV) by $\sqrt{t}$, then compute: (I)+(IV) and (I)-(IV), to get:
$(a_0x_0+ta_0{}'x_0{}')+\sqrt{t}(a_0x_0{}'+a_0{}'x_0)=b_0+b_0{}'\sqrt{t}$, thus
$(a_0+a_0{}'\sqrt{t})(x_0+x_0{}'\sqrt{t})=b_0+b_0{}'\sqrt{t}$ (E1).
Also,
$(a_0x_0+ta_0{}'x_0{}')\mp\sqrt{t}(a_0x_0{}'+a_0{}'x_0)=b_0-b_0{}'\sqrt{t}$, thus
$(a_0-a_0{}'\sqrt{t})(x_0-x_0{}'\sqrt{t})=b_0-b_0{}'\sqrt{t}$ (E2).
From (E1) and (E2), we get:

$$\begin{cases} x_0+x_0{}'\sqrt{t}=\dfrac{b_0+b_0{}'\sqrt{t}}{a_0+a_0{}'\sqrt{t}} \\ x_0-x_0{}'\sqrt{t}=\dfrac{b_0-b_0{}'\sqrt{t}}{a_0-a_0{}'\sqrt{t}} \end{cases}$$

Hence:

$$\begin{cases} x_0=\dfrac{1}{2}[\dfrac{b_0+b_0{}'\sqrt{t}}{a_0+a_0{}'\sqrt{t}}+\dfrac{b_0-b_0{}'\sqrt{t}}{a_0-a_0{}'\sqrt{t}}] \\ x_0{}'=\dfrac{1}{2\sqrt{t}}[\dfrac{b_0+b_0{}'\sqrt{t}}{a_0+a_0{}'\sqrt{t}}-\dfrac{b_0-b_0{}'\sqrt{t}}{a_0-a_0{}'\sqrt{t}}] \end{cases}$$

Step (2):
We multiply (IIV) by $\sqrt{t}$, then complete $(II)\mp(IIV)$ to get:
$[(a_0+a_1)+\sqrt{t}(a_0{}'+a_1{}')][(x_0+x_1)+\sqrt{t}(x_0{}'+x_1{}')=(b_0+b_1)+(b_0{}'+b_1{}')\sqrt{t}$ (E3)
$[(a_0+a_1)-\sqrt{t}(a_0{}'+a_1{}')][(x_0+x_1)-\sqrt{t}(x_0{}'+x_1{}')=(b_0+b_1)-(b_0{}'+b_1{}')\sqrt{t}$ (E4)
From (E3), (E4), we get:

$$\begin{cases} x_0+x_1=\dfrac{1}{2}(\dfrac{(b_0+b_1)+\sqrt{t}(b_0{}'+b_1{}')}{(a_0+a_1)+\sqrt{t}(a_0{}'+a_1{}')}+\dfrac{(b_0+b_1)-\sqrt{t}(b_0{}'+b_1{}')}{(a_0+a_1)-\sqrt{t}(a_0{}'+a_1{}')}) \\ x_0{}'+x_1{}'=\dfrac{1}{2\sqrt{t}}(\dfrac{(b_0+b_1)+\sqrt{t}(b_0{}'+b_1{}')}{(a_0+a_1)+\sqrt{t}(a_0{}'+a_1{}')}-\dfrac{(b_0+b_1)-\sqrt{t}(b_0{}'+b_1{}')}{(a_0+a_1)-\sqrt{t}(a_0{}'+a_1{}')}) \end{cases}$$

By applying the same method to (III) and (IIIV), we get:

$$\begin{cases} x_0+x_1+x_2=\dfrac{1}{2}(\dfrac{(b_0+b_1+b_2)+\sqrt{t}(b_0{}'+b_1{}'+b_2{}')}{(a_0+a_1+a_2)+\sqrt{t}(a_0{}'+a_1{}'+a_2{}')}+\dfrac{(b_0+b_1+b_2)-\sqrt{t}(b_0{}'+b_1{}'+b_2{}')}{(a_0+a_1+a_2)-\sqrt{t}(a_0{}'+a_1{}'+a_2{}')}) \\ x_0{}'+x_1{}'+x_2{}'=\dfrac{1}{2\sqrt{t}}(\dfrac{(b_0+b_1+b_2)+\sqrt{t}(b_0{}'+b_1{}'+b_2{}')}{(a_0+a_1+a_2)+\sqrt{t}(a_0{}'+a_1{}'+a_2{}')}-\dfrac{(b_0+b_1+b_2)-\sqrt{t}(b_0{}'+b_1{}'+b_2{}')}{(a_0+a_1+a_2)-\sqrt{t}(a_0{}'+a_1{}'+a_2{}')}) \end{cases}$$

then, we get $x_i{}', x_i$ for all $0\le i\le 2$.

Example:

Consider the equation:
$[(1+2p_1+p_2)+J(p_1+p_2)]X=4+35p_1-5p_2+J(48p_1-16p_2)$,
With $J^2=t=\frac{1}{4}$
We have:
$a_0=1, a_1=2, a_2=1, a_0{}'=0, a_1{}'=1, a_2{}'=1, b_0=4, b_1=35, b_2=-5, b_0{}'=0, b_1{}'=48, b_2{}'=-16, \sqrt{t}=\frac{1}{2}$
According to our algorithm, we can see that:

$$\begin{cases} x_0 = \frac{1}{2}\left(\frac{4+0}{1+0} + \frac{4-0}{1-0}\right) = 4 \\ x_0' = \frac{1}{2}(4-4) = 0 \\ x_0 + x_1 = \frac{1}{2}\left(\frac{39+\frac{1}{2}(48)}{3+\frac{1}{2}(1)} + \frac{39-\frac{1}{2}(48)}{3-\frac{1}{2}(1)}\right) = \frac{1}{2}\left(\frac{63}{\frac{7}{2}} + \frac{15}{\frac{5}{2}}\right) = 12 \\ x_0' + x_1' = \frac{1}{1}\left(\frac{63}{\frac{7}{2}} - \frac{15}{\frac{5}{2}}\right) = 12 \\ x_0 + x_1 + x_2 = \frac{1}{2}\left(\frac{34+\frac{1}{2}(32)}{4+\frac{1}{2}(2)} + \frac{34-\frac{1}{2}(32)}{4-\frac{1}{2}(2)}\right) = \frac{1}{2}\left(\frac{50}{5} + \frac{18}{3}\right) = 8 \\ x_0' + x_1' + x_2' = \frac{1}{1}\left(\frac{50}{5} - \frac{18}{3}\right) = 4 \end{cases}$$

Hence: $x_0 = 4, x_0' = 0, x_1 = 8, x_1' = 12, x_2 = -4, x_2' = -8$.
So that:
$X = (4 + 8p_1 - 4p_2) + J(12p_1 - 8p_2)$ is the solution.

3. Conclusion

In this paper we studied and solved many different types of symbolic 2-plithogenic linear algebraic equations where dual, split-complex, and weak fuzzy linear equations of symbolic 2-plithogenic type are solved and discussed. On the other hand, we illustrated many numerical examples to clarify the validity of our work.

As a future research direction, we aim to extend our results to symbolic 3-plithogenic algebraic equations.

References

[1] Deckelman, S.; Robson, B. Split-Complex Numbers and Dirac Bra-kets. Communications In Information and Systems 2014, 14, 135-159.

[2] Hatip, A., "An Introduction To Weak Fuzzy Complex Numbers ", Galoitica Journal Of Mathematical Structures and Applications, Vol.3, 2023.

[3] Khaldi, A., " A Study On Split-Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.

[4] Ahmad, K., " On Some Split-Complex Diophantine Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[5] Ali, R., " On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.

[6] Merkepci, M., and Abobala, M., " On Some Novel Results About Split-Complex Numbers, The Diagonalization Problem And Applications To Public Key Asymmetric Cryptography", Journal of Mathematics, Hindawi, 2023.

[7] Mohammad Abobala. (2023). A Short Contribution to Split-Complex Linear Diophantine Equations in Two Variables. Galoitica: Journal of Mathematical Structures and Applications, 6 (2), 32-35 (**Doi :** https://doi.org/10.54216/GJMSA.060204).

[8] Hamiyet Merkepci, Mohammad Abobala. (2023). On the Symbolic 2-Plithogenic Rings. International Journal of Neutrosophic Science, 20 (3), 115-122 (**Doi :** https://doi.org/10.54216/IJNS.200311).

[9] Hamiyet Merkepci, Ammar Rawashdeh: On The Symbolic 2-Plithogenic Number Theory and Integers. *Neutrosophic Sets and Systems*, Vol. 54, 2023, pp. 113-123. DOI: 10.5281/zenodo.7817721.

[10] Mohamed Soueycatt. On the Symbolic 2-Plithogenic Weak Fuzzy Complex Numbers. Vol. 59, 2023, pp. 287-295. DOI: zenodo.10031252/10.5281

[11] Oliver Von Shtawzen, Ammar Rawashdeh, Karla Zayood. On the Symbolic 2-Plithogenic Split-Complex Real Square Matrices and Their Algebraic Properties. Vol. 59, 2023, pp. 211-222. DOI: zenodo.10031206/10.5281

[12]Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , International J.Mathcombin, Vol 4,pp 1-14. 2011.

[13] Sarkis, M., " On The Solutions Of Fermat's Diophantine Equation In 3-refined Neutrosophic Ring of Integers", Neoma Journal of Mathematics and Computer Science, 2023.

Doi: https://doi.org/10.54216/JNFS.080105
Received: October 09, 2023 Revised: December 27, 2023 Accepted: January 25, 2024