The Mathematical Exploration of π value and its Approximation

Prem Kumar Singh

Department of Computer Science and Engineering,
Gandhi Institute of Technology and Management-Visakhapatnam,
Andhra Pradesh 530045, India
ORCID: 0000-0003-1465-6572
***Correspondence:** premsingh.csjm@gmail.com

Abstract

In this paper the author will illustrate some basic mathematics about π and its computation in comparison of both Indian Knowledge System as well as others. Same time the infinite series computation and its application is discussed with a pseudo code for decision making process. It is believed that this computation will be helpful for early career researcher for various applications of pie in knowledge processing tasks.

Keyword: Decision making; Indian Knowledge System; Knowledge representation; Mathematics; Turiyam.

1. Indian Knowledge System and Pie value computation.

The computation of π or infinite series was considered as one of the most interesting mathematical proof [1-3]. It is started by Baudhayana while measuring the area of circle using square as shown in Fig. 1 [4-5]. It means the area of circle is almost near to square of its radius which may be approx to 3 as per Baudhayana written in Sulbasutra [6-7]. Hence its mathematical exploration is required for Turiyam awareness for better understanding [8].

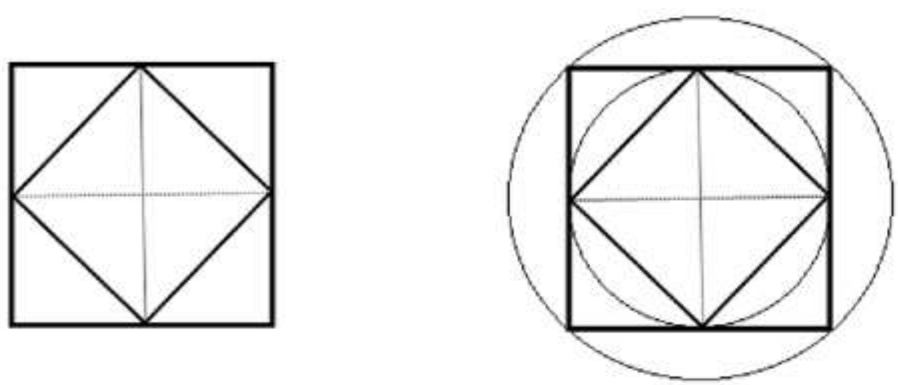

Figure 1: The Baudhayana circle area approximation.

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

It again started by Pingala as binary number system and its addition in term of Laghu(L) and Guru (G) in Sanskrit[9-11]. It is also depicted as Pingala's Chandaḥśāstra in Sanskrit literature [12-13]. Due to which the Sanskrit is considered as one of the scientific language for knowledge representation [14]. In other countries they known as Pascal Triangle as shown in Fig. 2.

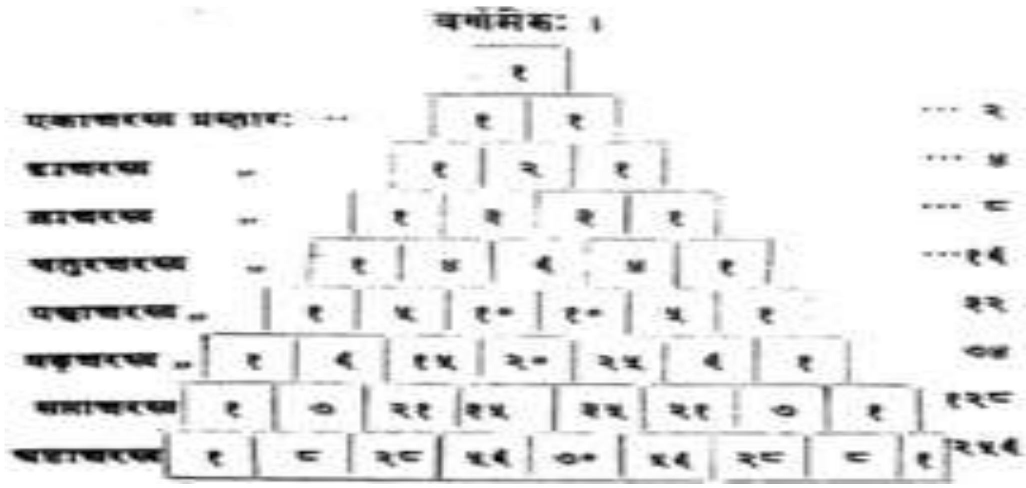

Figure 2: The Pingala's Chandaḥśāstra to generate the series of Number

It is believed that the Pingala series concept is used in Indian Knowledge system to build Jyotirlinga as shown in Fig. 3. Fig. 4 represents some other geometry build on Pingala Series.

Figure 3: The Pingala's Chandaḥśāstra to generate the series of Number

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

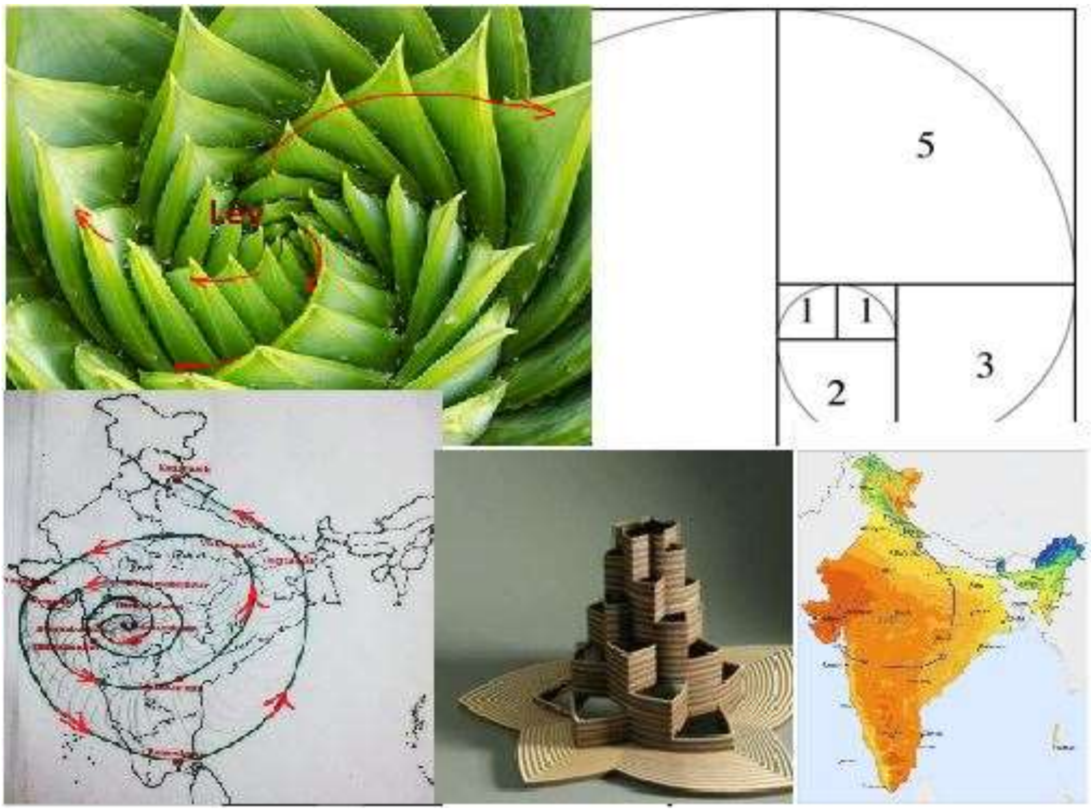

Figure 4: Some other geometry based on Pingala Series

The Aryabhata gave a formula to compute the Pie value as written in Sanskrit [15-16]:

caturadhikaṃ śatamaṣṭaguṇaṃ dvāṣaṣṭistathā sahasrāṇām ayutadvayaviṣkambhasyāsanno vṛttapariṇāhaḥ.

It says first add 4 with 100 as (4+100=104). Try to multiply it (i.e. 104) via 8 i.e. (104 × 8=832). In this multiplication add 62,000 i.e. 62832. The obtain number consider as circumference of a circle and divide it by 20000 considering a circle having diameter 20,000. It provides the value of π=3.1416. This he used as Spherical geometry construction as shown in Fig. 5.

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

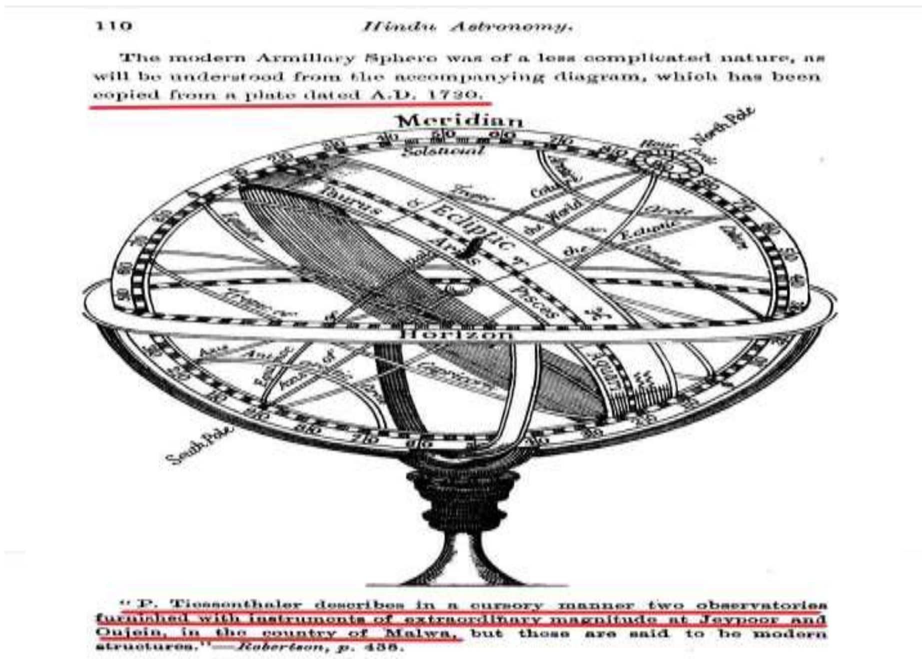

Figure 5: The Aryabhata Spherical geometry for data representation

It was extended by Narayana's as cows sequence motivated from Pingala series [17]. He formulated the problem as let us suppose one cow at starting of the year. The cow give birth one calf in each year from the age of 4 years old. In this case try to approximate the number of calves produced every year. It gave third order sequence as $C_n = C_{n-1} + C_{n-3}$ where n>=3, $C_0=1$, $C_1=1$, $C_2=1$, $C_3=1$.

The same concept used in Egyptian Papyrus as distinct way [18]. They used division of circumference and its division by diameter as shown in Fig. 6. It is used as a game where 64 ball used to represent the circle and 9 ball on the diameter of circle used to divide them for computation of pie shown in Fig. 7. Later it is used to represent as Square and its area computation equivalent to circle as shown in Fig. 8. Its computation is shown in Fig. 9 in a mathematical way.

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

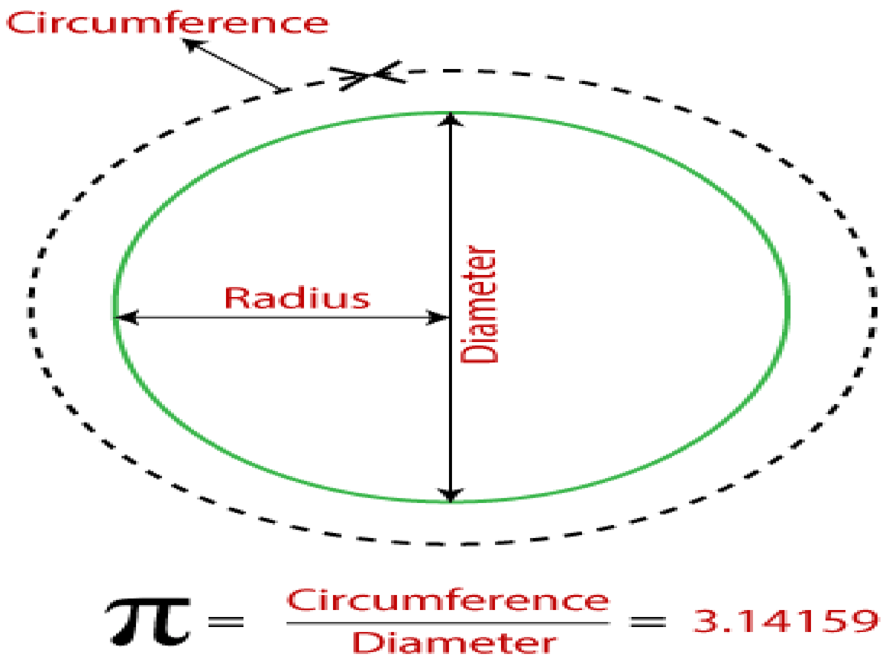

Fig 6: The Egyptian Papyrus to compute the Pie value

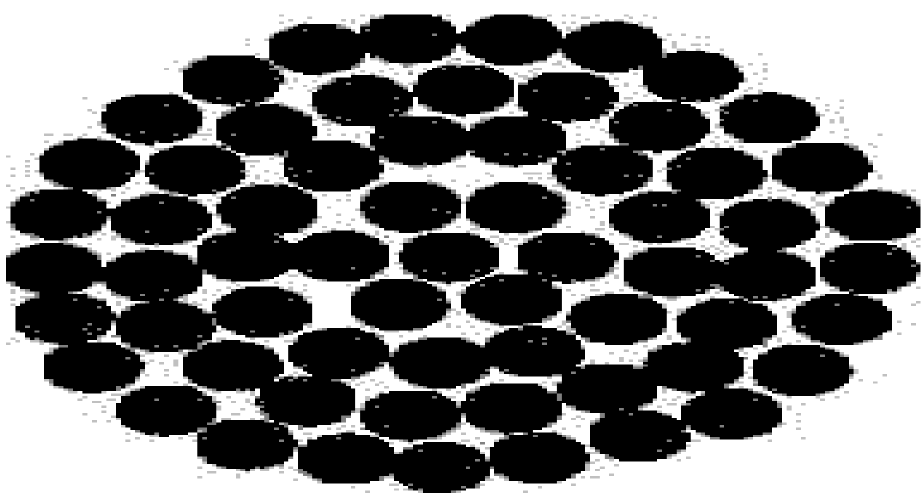

Figure 7: The Egyptian Papyrus as game of 64 Ball and its arrangement in the circle

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

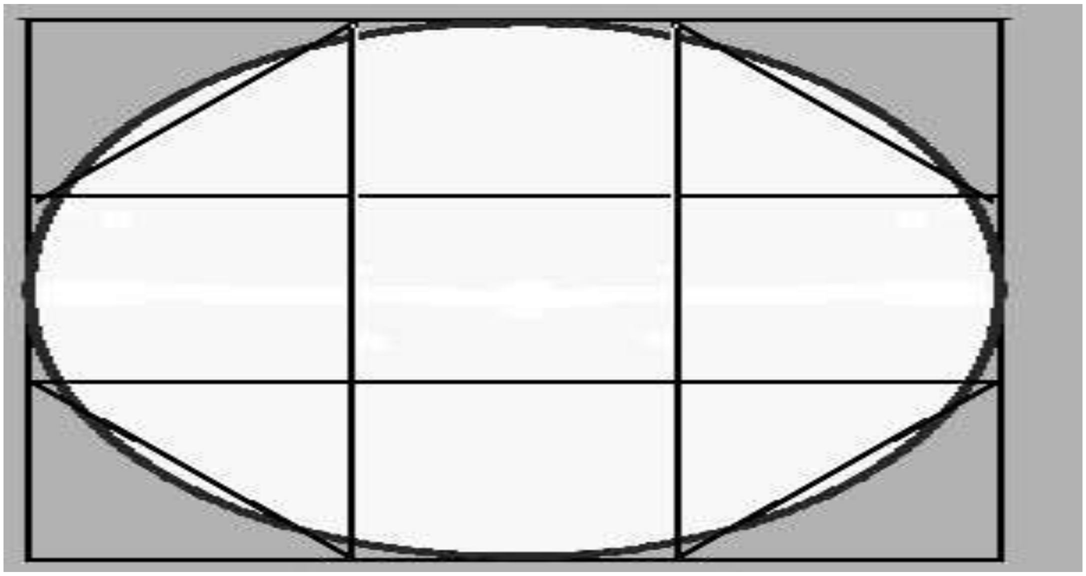

Figure 8: The area of Square equal to area of circle and four triangle area shown in Fig. 9

(2r)X(2r) = Area of Circle+4 X Area of Triangle;

4r2=Area of circle+4X1/2X(2r/3)X(2r/3);

Пr2= 4r2-8r2/9;

П= 28/9=3.11

Figure 9: The area of Square equal to area of circle and four triangle area shown in Fig. 8

The Madhava gave sine, cosine and arc tan(*x*) series and its computation at early 14th century as shown in Fig 10 [19].

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}\theta^{2k+1},$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\theta^{2k},$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}x^{2k+1} \quad \text{where } |x| \le 1.$$

Figure 10: The Madhava series for Sin, Cos and arc tan(*x*)

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

The Madhava wrote a formula computation of Pie in Sanskrit as shown in Fig. 11. It is illustrated mathematically in Fig, 12.

vyāse vāridhinihate rūpahṛte vyāsasāgarābhihate /

triśarādiviṣamasaṃkhyābhaktamṛṇaṃ svaṃ pṛthak kramāt kuryāt //

Figure 11: The Madhava formula for computation of Pie

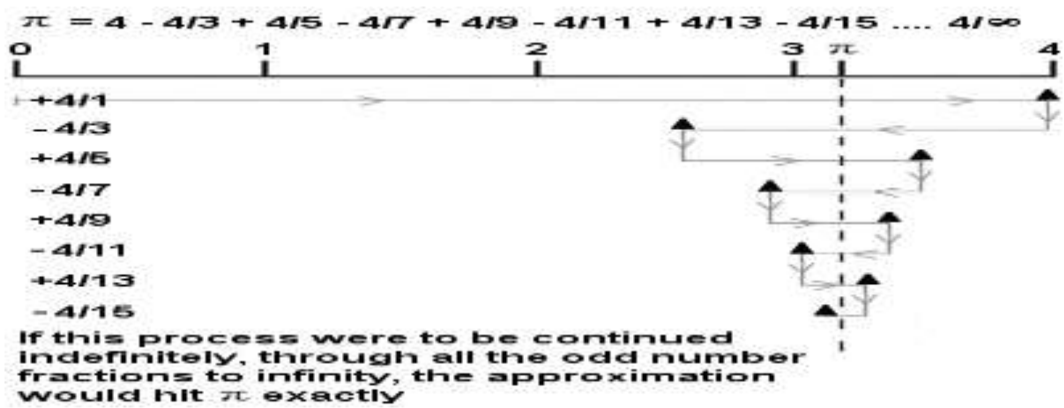

Figure 12: The mathematical exploration of Formula Shown in Fig. 11

Just put arctan(x)=1 in Madhava series shown in Fig. 10. It will provide Leibniz formula for Pie as shown in Fig. 13. This series and its square multiplication provides approximate of Euler Pie value approximation as shown in Fig, 14 [20].

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}$$

such that:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots = \frac{\pi}{4}$$

Figure 13: The Leibniz formula for Pie put arctan(x)=1

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots$$

Fig. 14: The Euler approximation of Pie via Square of Leibniz formula

$$\frac{\pi - 3}{4} = \frac{1}{2 \times 3 \times 4} - \frac{1}{4 \times 5 \times 6} + \frac{1}{6 \times 7 \times 8} - \cdots$$

Figure 15: The Nilkantha approximation of Pie

The Nilkantha gave a infinite series to compute the Pie value using the mathematical expression as shown in Fig. 15. It can be observed that the Nilkantha series contains multiplication of 3-consecuting numbers. The value of variable increases by two or every iteration for the denominator with sign changes. Hence it gave a pseudo code for pie and its implementation. First create three variable n=2, pie=3, and sign=1. We can create a infinite series as for loop based on our choice from [0, 1000000]. Every iteration change the sign as sign*(-1). Compute the Pie value as = Pie + sign*(4/(n) * (n+1) * (n+2))**.** Increase the n value as 2 after the each iteration and print its value. In this way this code may take maximum O(*n*log(n)*(loglog(n))*. Hence the exploration of Pie value and its computation is totally based on Turiyam consciousness rather than only Indigenous or non-indigenous. In future the author will focus on exploring the pie value based on Buffon Needle Problem as shown in Fig. 16.

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

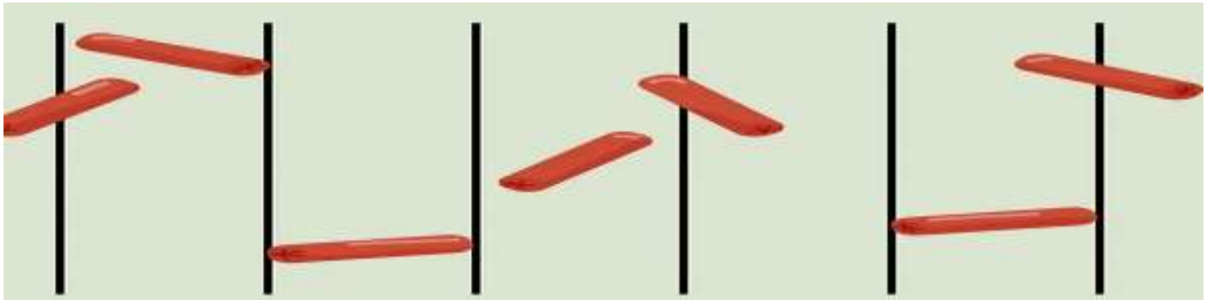

Figure 16: The Buffon's Needle Problem of Pie

In this paper the computation of Pie and infinite series computation based on Indian Knowledge system is explored with an example. Same time the comparative study of pie value computation by Indian Knowledge System with Egypt Papyrus and other mathematician is also compared. It is believed that the current paper will help to early career researcher and data visualization student. In near future the author will focus on other mathematical concept and its exploration with Indian Knowledge system.

2. Conclusion

This paper provide a mathematical exploration of Pie value and its computation in context of Indian Knowledge System as well as comparison with Egypt papyrus and others. It is also discussed that the computation of pie value and its implementation takes O(*n*log(n)*(loglog(n))* time as per Nilkantha work. In this way this paper will be helpful for various decision making process and applications of pie.

Acknowledgements: Author thanks the editorial team for their valuable time.

Funding: Author declares that, there is no funding for this paper.

Conflicts of Interest: Author declares that, there is no conflict of interest for the given paper.

Ethics approval: This article does not contain any studies with human or animals participants.

References

[1] R. Roy (1990) The discovery of the series formula for π by Leibniz, Gregory and Nilakantha, Math. Mag. 63.
[2] Singh, P. K. (2022). A note on basic proof of some famous mathematical theorem and its illustration. J Neutrosophic Fuzzy Syst, 3(1), 39-53.
[3] Bag, Amulya Kumar (1976).Madhava's sine and cosine series. Indian Journal of History of Science. 11 (1): 54–57
[4] Raju CK. (2007). Cultural Foundations of Mathematics: The nature of mathematical proof and the transmission of the calculus from India to Europe in the 16th c. CE. Pearson Longman. ISBN 978-81-317-0871-2.
[5] Burgess, E. (1860). The Surya Siddhanta: A Text-Book Of Hindu Astronomy, Motilal Banarsidass Publishers Private Limited, New Delhi, Reprinted 1997.
[6] Sen, S.N. and Bag, A.K. (1983). The Sulbasutras, Indian National Science Academy, New Delhi
[7] Henderson, D.W. (2000). Square roots in the Sulba Sutra, In C.A. Gorini (ed.) Geometry at Work: Papers in Applied Geometry, MAA Notes Number 53, pp. 39–45.
[8] Singh P. K. (2023) Mathematical Concept Exploration Using Turiyam Cognition, Galoitica: Journal of Mathematical Structures and Applications, Vol. 9 , No. 1 , Oct 2023, pp. 08-22
[9] Playfair, J.J. (1789). Remarks on the astronomy of the brahmins, Transactions of the Royal Society of Edinburgh, 2, Part 1, pp. 135–192
[10] Burrow, R. (1790). A proof that the Hindoos had the binomial theorem, Asiatik Researches, 2, pp. 487–497.
[11] Tavernier, J.-B. (1889). Travels in India, Vol. 2, English Translation, Cambridge University Press, Cambridge.
[12] Datta, B. (1927). Early history of the arithmetic of zero and infinity in India, Bulletin of the Calcutta

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024

Mathematical Society, 18, pp. 165–176. Datta, B. (1932). The Science of the Sulbas. A Study in Early Hindu Geometry, Calcutta University Press, Calcutta.
[13] Datta, B. and Singh, A.N. (1938). History of Hindu Mathematics, Vol. 2: Algebra, Lahore; Reprint 1962, Asia Publishing House, Mumbai.
[14] Briggs, R. (1985). Knowledge representation in sanskrit and artificial intelligence, AI Magazine, 6, pp. 22–38.
[15] Joseph, G. G. (2010). The crest of the peacock: Non-European roots of mathematics. Princeton University Press.
[16] Rosenfeld, B.A. (1990). Al-Khw¯arizm¯ı and Indian Science, In W.H. Abdi et al. (ed.) Interaction between Indian and Central Asian Science and Technology in Medieval Times, Indian National Science Academy, New Delhi, Vol. I, pp. 132–139.
[17] Rajagopal, C.T. and Aiyar, T.V.V. (1951). On the Hindu Proof of Gregory's Series, Scripta Mathematica, 17, pp. 65–74.
[18] Frederic G. Kenyon (1899) Palaeography of Greek papyri (Oxford, Clarendon Press, 1899).
[19] Mallayya, V.M. (2004). An interesting algorithm for computation of sine tables of sine tables from the Golasara of Nilakantha, Ganita Bharati, 26, pp. 40–55.
[20] Jesseph, D.M. (1999). Squaring the Circle: The War Between Hobbes And Wallis, University of Chicago Press, Chicago
[21] Mantel, N. (1953). An extension of the Buffon needle problem. The Annals of Mathematical Statistics, 24(4), 674-677.

Doi: https://doi.org/10.54216/JNFS.080101
Received: October 22, 2023 Revised: December 19, 2023 Accepted: January 22, 2024