Fusion of Centrality Measures with D-OWA in Neutrosophic Cognitive Maps to Develop a Composite Centrality Indicator

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Abstract

This study utilized Neutrosophic Cognitive Maps (NCMs) integrated with the D-OWA operator to analyze the nutritional rights of pregnant women in Ecuador, with a focus on the crucial role of nutrition education. The innovative application of the D-OWA operator enabled the computation of a composite centrality measure by merging key centrality indicators—degree, closeness, and betweenness—each appropriately weighted according to its relevance to the analysis. This methodology provided a sophisticated evaluation of the factors impacting maternal nutrition, demonstrating how combining various centrality measures offers a deeper and more comprehensive insight into the dynamics of complex systems. The calculated composite centrality measures revealed the system's intricate structure, pinpointing critical nodes and pathways that could be targeted most effectively through interventions. The findings underscore the significant benefits of using composite centrality measures to enhance decision-making in public health and other sectors characterized by complexity and uncertainty. The potential for refining and expanding this approach in future research suggests that it could be further supported by technological advancements, enabling more efficient analysis and scalability across diverse complex systems.

Keywords: Neutrosophic Cognitive Maps; D-OWA Operator; Centrality Measures; Nutritional Rights

1. Introduction

In recent years, Neutrosophic Cognitive Maps (NCMs)[1] have emerged as a powerful tool for modeling complex systems where uncertainty and indeterminacy play critical roles. Developed from traditional cognitive maps, NCMs incorporate the principles of neutrosophy[2], a framework introduced by Florentin Smarandache in the late 1990s. Neutrosophy extends beyond the realms of fuzzy logic and intuitionistic set theory, offering a more sophisticated approach to handle situations where contradictions and uncertainties are prevalent. This makes NCMs particularly effective in areas where relationships among variables are not entirely understood or are inherently ambiguous.

An NCM defines each concept and causal connection not just in binary terms (true or false) or by a degree of membership, as seen in fuzzy cognitive maps, but also through degrees of truth, falsehood, and indeterminacy. Such a nuanced approach allows for a more accurate representation of real-world complexities, enabling the mapping of relationships that are not strictly deterministic. This capability is crucial in fields like healthcare, environmental science, and social policy, where variables often interact in unpredictable ways.

Centrality measures within NCMs, such as degree, closeness, and betweenness [3], provide distinct values based on different calculation methods, highlighting the varying significance of nodes within a network. These measures are essential for understanding the influence and role of individual nodes, and they can be particularly insightful in contexts where the importance of specific elements varies by location or scenario. The ability to quantify such differences is invaluable in tailoring interventions or policies to the unique characteristics of each system.

The application of the D-OWA (Dependent- OWA) [4] operator further enhances the functionality of NCMs. As a specialized variant of aggregation operators, D-OWA allows for the differentiated weighting of elements based on their importance, offering a more refined approach to data aggregation. This method is particularly useful for creating

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a composite centrality measure that reflects the composite influence of nodes in a network. By integrating D-OWA with NCMs, researchers and policymakers can achieve a deeper understanding of the dynamic interactions within complex systems, making informed decisions that consider the nuanced realities of the environments they are addressing. This study leverages the strengths of NCMs and the D-OWA operator to examine the factors influencing nutritional rights among pregnant women in Ecuador, shedding light on the pivotal role of nutrition education in this context.

2. Methodology

2.1 Neutrosophic Cognitive Maps

Neutrosophic cognitive maps (NCMs) are an extension of traditional cognitive maps, integrating the neutrosophy approach to handle uncertainty, indeterminacy, and contradiction. Neutrosophy, a theory proposed by Florentin Smarandache in the late 1990s, generalizes previous work in fuzzy logic, intuitionistic set theory, and fuzzy set theory, among others, to more effectively deal with uncertainty and indeterminacy [5].

In an NCM, each causal relation and each concept is not only defined by a Boolean value (true or false) or a degree of membership (as in fuzzy cognitive maps) but also by degrees of truth, falsehood, and indeterminacy. This allows for modeling situations where the relationship between concepts is not completely known or is inherently uncertain.

The practical implementation of neutrosophic cognitive maps requires an interdisciplinary approach, combining knowledge from psychology, engineering, computer science, and mathematics, to effectively model the complexity of real systems under uncertainty.

To create an NCM, the following steps should be followed:

1. Concept Identification: Determine the concepts or variables that are relevant to the system or problem being modeled. These concepts can range from tangible elements to abstract ideas.

2. Definition of Relationships: Establish the causal relationships between these concepts. This involves determining how a change in one concept affects another. In the neutrosophic context, these relationships are not limited to being positive or negative; they must also be considered in terms of degrees of truth, falsehood, and indeterminacy.

3. Assignment of Neutrosophic Values: For each concept and each relationship, assign values that represent their degree of truth, falsehood, and indeterminacy. These values are usually expressed in the range [0, 1], where 0 represents the complete absence and 1 the complete presence of each property (truth, falsehood, indeterminacy).

4. Analysis and Simulation: Once the map is constructed, it can be used to simulate different scenarios and analyze how changes in certain concepts could affect the system. This requires a computational approach to handle the complexity of neutrosophic relationships.

Neutrosophic cognitive maps, by combining concepts from cognitive maps with neutrosophy theory, utilize a set of equations that help model and analyze causal relationships between concepts under conditions of uncertainty, indeterminacy, and contradiction. These equations are based on neutrosophic logic and the operation of neutrosophic sets, aiming to quantify how changes in one concept affect others within the map.

A neutrosophic set is defined by three components for each element: the degree of truth (T), the degree of falsehood (F), and the degree of indeterminacy (I), with each component ranging from 0 to 1. The basic definitions related to neutrosophy are:

Definition 1. A Neutrosophic Cognitive Map is a directed neutrosophic graph, whose vertices represent concepts and whose edges represent causal relationships between the vertices.

Vertex Representation: If there are k vertices $C_1, C_2, ..., C_k$, each can be represented by a vector $(x_1, x_2, ..., x_k)$ where $x_i \in \{0,1, I\}$ depending on the state of the vertex C_i at a specific time or situation:

- $x_i=0$: Vertex C_i is in an activated state.
- $x_i=1$: Vertex C_i is in a deactivated state.
- $x_i=I$: The state of vertex C_i is indeterminate.

Connections between vertices: A directed edge from C_m to C_n is called a connection and represents causality from C_m to C_n.

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Association of weights to each vertex: Each vertex in the NCM is associated with a weight from the set {0, 1, -1, I}. The weight of the edge C_mC_n, denoted as α_{mn}, indicates the influence of C_m on C_n and can be:

- α_{mn}= 0: C_m has no effect on C_n.
- α_{mn}= 1: An increase (decrease) in C_m produces an increase (decrease) in C_n.
- α_{mn}= -1: An increase (decrease) in C_m produces a decrease (increase) in C_n.
- α_{mn}= I: The effect of C_m on C_n is indeterminate.

Definition 2. An NCM with edges with weights in [6] is called a *Simple Neutrosophic Cognitive Map*.

Definition 3. If $C_1, C_2, \ldots, C_k$ are the vertices of an NCM. The neutrosophic matrix N(E) is defined as N(E) =α_{mn}), where α_{mn} denotes the weight of the directed edge C_mC_n, where $\alpha_{mn} \in [0.1]$ [6] N(E), is called the *neutrosophic adjacency matrix* of the NCM.

Definition 4. Let $C_1, C_2, \ldots, C_k$ be the vertices of an NCM. Let A = $(a_1, a_2, \ldots, a_k)$, where $a_m \in \{-1, 0, 1, I\}$. A is called the *instantaneous state neutrosophic vector* and means an indeterminate activated-deactivated state position of the vertex at a given instant.

- a_m = 0 if C_m is deactivated (has no effect),
- a_m = 1 if C_m is activated (takes effect),
- a_m = I if C_m is indeterminate (its effect cannot be determined).

Definition 5. Let $C_1, C_2, \ldots, C_k$ be the vertices of an NCM. Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_mC_n}$ be the edges of the NCM, then the edges constitute a directed cycle.

The NCM is said to be *cyclical* if it has a directed cycle. It is said to be *acyclic* if it does not have any *directed cycle*.

Definition 6. An NCM that contains cycles is said to have *feedback*. When feedback exists in the NCM, it is said to be a *dynamic system*.

Definition 7. Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_{k-1}C_k}$ be a cycle. When C_m is activated and its causality flows through the edges of the cycle and is then the cause of C_m itself, then the dynamic system circulates. This is true for each vertex C_m with m = 1, 2, …, k. The equilibrium state for this dynamic system is called the *hidden pattern*.

Definition 8. If the equilibrium state of a dynamic system is a unique state, then it is called a *fixed point*. An example of a fixed point is when a dynamic system is started by being activated by C_1. If it is assumed that the NCM settles on C_1 and C_k, that is, the state remains as (1, 0, …, 0, 1), then this neutrosophic state vector is called a fixed point.

Definition 9. If the NCM is established with a repeating neutrosophic state vector of the form:

$A_1 \rightarrow A_2 \rightarrow \cdots \rightarrow A_m \rightarrow A_1$, then the equilibrium is called the NCM *limit cycle*.

2.2 OWA Operators

Aggregation operators constitute a class of mathematical functions used to consolidate information. These operators take n values within a certain domain D and produce a single return value in the same domain. They are applied in a wide range of areas, playing a crucial role in the evaluation of options and the generation of alternatives, especially in contexts where decisions must be made based on multiple criteria. [7], [8]

The D-OWA Dependent- OWA) operator represents a specific variant of these aggregation operators, which extends the functionality of traditional operators such as the average, median, and sum. This is achieved by allowing a differentiated weighting of elements according to their importance in the process of combining information, facilitating a more nuanced consideration of data during aggregation [9], [10]. The operations performed by a D-OWA operator include:

An OWA operator is a function F: R_n of dimension n if it has an associated vector W of dimension n with [11]:

$$w_{ij} \in [0,1] \text{ y } \sum_{j=1}^{n} w_j=1 \tag{1}$$

such that

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$$F(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j\, b_j \tag{2}$$

where b_j is the jth largest of the a_j

The vector W, on the other hand, is used to indicate the level of compensation between criteria and the decision-maker's level of optimism [12]. Aggregation operators can be employed to obtain composite indicators that summarize in a single value the outcomes of other indicators. In the present work, the use of the dependent OWA operator (D-OWA) is proposed, where the weight vector of the OWA operator:

$$w = (w_1, w_2, \dots, w_n) \tag{3}$$

$$w_j = \frac{s(a_{\sigma(j),}\mu)}{\sum_{j=1}^{n} s(a_{\sigma(j),}\mu)}, j = 1,2, \dots, n \tag{4}$$

where: $s(a_{\sigma(j),}\mu)$ is the degree of similarity between the jth argument and the arithmetic mean (μ).

Note that in this case, the weights of the vector w are determined based on the input arguments that one wishes to aggregate, therefore, it is a neat type OWA operator. The level of compensation of the aggregation (orness) can be calculated using equation (5):

$$\text{orness}(w) = \frac{1}{n-1} \frac{\sum_{j=1}^{n} (n-j) s(a_{j,}\mu)}{\sum_{j=1}^{n} s(a_{j,}\mu)} \tag{5}$$

The "orness" of the OWA operator quantifies the degree of optimism reflected in the aggregation process, indicating how much the aggregation leans towards the most favorable outcomes among the evaluated criteria.

Results

From an analysis carried out by experts, the factors that directly influence the fulfillment of the right of Ecuadorian pregnant women to adequate nutrition during pregnancy were evaluated. Table 1 lists the legal issues that are the subject of study in this research.

Table 1: Legal issues affecting the right to nutrition for pregnant women. Source: Own elaboration

Coding	Factor	Description
F1	Unequal access to health services	There is a disparity in access to quality health services, which affects the ability of pregnant women to receive the necessary medical care, including guidance on proper nutrition during pregnancy.
F2	Lack of supervision and control of food quality	The lack of adequate supervision by authorities can lead to the availability of low-quality or contaminated foods, which poses a risk to the health of pregnant women and their unborn children.
F3	Gaps in the implementation of public policies	Although Ecuador may have policies and programs on paper to ensure access to adequate nutrition during pregnancy, the effective implementation of these measures can be deficient due to budgetary constraints, lack of coordination between governmental entities, and other factors.

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F4	Limitations in access to economic resources	Many pregnant women in Ecuador may face economic limitations that prevent them from accessing a balanced and nutritious diet during pregnancy, negatively affecting their health and that of their unborn children.
F5	Lack of education and awareness about nutrition during pregnancy	Many pregnant women in Ecuador may face economic limitations that prevent them from accessing a balanced and nutritious diet during pregnancy, negatively affecting their health and that of their unborn children.
F6	Gender inequality and discrimination	In some communities, pregnant women may face discrimination based on their gender, limiting their access to adequate nutritional resources and health services during pregnancy.

The procedure for the development of the research is described in Figure 1 and Tables 2, 3, 4, and 5.

Table 2: Adjacency Matrix associated with the NCM. Note: Python software was used for the execution of the method. Source: own elaboration.

	F1	F2	F3	F4	F5	F6
F1	(0, 1, 0)	(0, 1, 0)	(0.5, 0.5, 0)	(0, 1, 0)	(1, 0, 0)	(0.6, 0.4, 0)
F2	(0, 1, 0)	(0, 1, 0)	(0.4, 0.6, 0)	(0, 1, 0)	(0.7, 0.3, 0)	(0.3, 0.7, 0)
F3	(1, 0, 0)	(0, 1, 0)	(0, 1, 0)	(0.9, 0.1, 0)	(1, 0, 0)	(0, 1, 0)
F4	(0.3, 0.7, 0)	(0, 1, 0)	(0.9, 0.1, 0)	(0, 1, 0)	(1, 0, 0)	(0.8, 0.2, 0)
F5	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(0, 1, 0)	(1, 0, 0)
F6	(0.8, 0.2, 0)	(0, 1, 0)	(0.8, 0.2, 0)	(0, 1, 0)	(0.9, 0.1, 0)	(0, 1, 0)

This matrix shows how each factor influences the others according to the assigned degrees of truth, falsehood, and indeterminacy. For example, the entry F1→F3 with values (0.5, 0.5, 0) suggests an equitable influence between being true and false, reflecting total uncertainty about how unequal access to health services impacts the implementation of public policies, without indeterminacy.

Entries with (1, 0, 0) indicate a total belief in the influence of one factor over another, like F3→F5, suggesting that gaps in the implementation of public policies fully influence the lack of education and awareness about nutrition during pregnancy. Figure 1 shows a representation of the NCM, illustrating the causal relationships between the nodes.

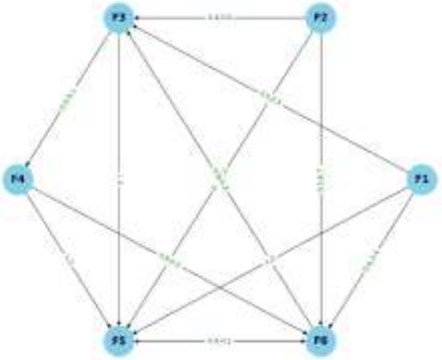

Figure 1: Neutrosophic Cognitive Map. Source: Own elaboration

The D-OWA operator requires defining weights based on the relative importance that the decision-maker assigns to each measure of centrality. Subsequently, the centrality values obtained in the NCM are normalized, and then a simplified approximation of the D-OWA calculation is carried out

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Table 3: Centralized Analysis. Note: Source: Own elaboration.

Node	Degree	Closeness	Betweenness
F1	1.4	0.833	0.017
F2	0.8	0.556	0.0
F3	1.6	1.0	0.092
F4	1.2	0.625	0.017
F5	2.0	1.0	0.308
F6	1.4	0.833	0.017

A simplified aggregated value for each node is calculated based on the average values of the three centrality measures, and then it is discussed how they adjust to incorporate divergence. The simplified aggregated values for each node, based on the average of the centrality measures (Degree, Closeness, Betweenness), ordered from highest to lowest, are (Table 4):

Table 4: Centrality values. Source: Own elaboration.

Node	Average Centrality
F5	1,103
F3	0.897
F1	0.75
F6	0.75
F4	0.614
F2	0.452

This simplified analysis suggests that node F5 (Lack of education and awareness about nutrition during pregnancy) has the highest average centrality, indicating its central importance in the network. This is followed by node F3 (Gaps in the implementation of public policies) and then F1 and F6 with equal values.

To incorporate divergence and adjust weights in a D-OWA approach, it is necessary to calculate the disparity between the centrality measures of each node and adjust the aggregation weights accordingly. This additional step ensures that both the relative importance of each measure and the variability among them are taken into account, promoting an aggregation that reflects both consensus and diversity in the centrality evaluations.

To incorporate divergence and adjust weights in a D-OWA approach, the disparity between the centrality measures for each node is calculated. A common way to do this is by using the variability or standard deviation of the centrality measures for each node. Then, this disparity is used to adjust the weights of each centrality measure in the aggregation, giving more weight to the measures with greater variability.

The D-OWA adjusted centrality values, which incorporate divergence through weight adjustment based on the standard deviation of centrality measures for each node, ordered from highest to lowest, are (Table 5):

Table 5: D-OWA adjusted centrality values. Source: Own elaboration.

Node	D-OWA Centrality
F5	0.703
F3	0.511
F1	0.391
F6	0.391
F4	0.272
F2	0.139

This divergence-adjusted approach reflects how the aggregated centrality of each node is influenced not only by the average importance of its centrality measures but also by the variability among these measures. By giving greater

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weight to measures with higher variability, the D-OWA operator allows for a more nuanced representation of centrality that can highlight subtle differences between nodes that might not be evident with a simple average.

Again, node F5 (Lack of education and awareness about nutrition during pregnancy) is shown to be the most central and significant within the network, followed by node F3 (Gaps in the implementation of public policies), underscoring its critical importance in the network considered. The application of D-OWA offers a valuable perspective on how the diversity of roles and relationships can affect the relative importance of nodes in the network.

3. Conclusion

This study successfully employed Neutrosophic Cognitive Maps (NCMs) integrated with the D-OWA operator to delve into the complexities of nutritional rights among pregnant women in Ecuador, highlighting the crucial role of nutrition education. The use of the D-OWA operator was particularly innovative in calculating a composite centrality measure that combines different centrality metrics (degree, closeness, and betweenness) with weighted importance according to their relevance. This methodological approach allowed for a nuanced analysis of the relative importance of various factors influencing maternal nutrition, demonstrating how differentiated weighting can provide deeper insights into the influence and roles of nodes within complex systems. Future research should consider extending the use of NCMs integrated with D-OWA to other areas of public health, where uncertainty often complicates decision-making. Further exploration into refining the assignment of neutrosophic values and expanding the use of sophisticated aggregation operators could enhance the modeling accuracy and applicability of NCMs. Additionally, incorporating advancements in technology, such as artificial intelligence, could automate and optimize the processing of neutrosophic data, potentially transforming policy development and intervention strategies in complex environments.

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