Reliability Function Estimated for Generalized Exponential Rayleigh Distribution Under Type-I Censored Data and Fuzzy Data

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Abstract

In this paper, maximum likelihood estimation method (MLEM), one of the most well-liked and frequently applied classic methods, is used to estimate the two scale and one shape parameters of the Generalized Exponential-Rayleigh distribution for type-I censored data, which is one of the most Rights censored data. Based on an iterative process to get approximated values for these two scale parameters and one shape parameter using the Newton-Raphson method to locate estimate value for these parameters by using the simulation procedure utilizing monte-Carlo technique to find Reliability function underneath various sample sizes and the initial values are different for the parameters for all estimated parameters of Generalized Exponential-Rayleigh by implement the initial value in the MATLAP program, Subsequently, conducting a comparative analysis between the estimated reliability function and its non-estimated counterpart employing the mean squares error methodology. In the last finding the pdf function f (t), reliability function R (t) and hazard function h (t) for simulation data. Also, we provide some examples to clarify how can we apply our results on fuzzy data tables.

Keywords: Generalized Exponential Rayleigh distribution; censored sample; Reliability function; Simulation, fuzzy data.

1. Introduction

One of the most commonly used functions in lifetime data analysis is the reliability function, a time-dependent function when the probability of an item working for a certain period time. Censored samples are the most prevalent in life, when age data are analyzed, there must be a failure in some units that enter experiments or die and other units do not fail or do not die, and these units exceed the time of the experiment without field and death. Data that has been censored may be classified into three categories: left-censored, interval-censored and right-censored. Furthermore, the right-censored category includes three types of censoring: type one, type two, and progressively-censored. Now we show some estimation methods of new mixture Distribution with simulation, in 2021 Iden and Qesma submitted Estimating of Survival Function under Type One Censoring Sample for Mixture Distribution [1]. In 2023 alkinani and shalan [2] provided the mathematical and statistical characteristics for the suggested distribution, which is the generalized exponential Rayleigh distribution, using a three-parameter model.. We will also see some research on censored data that has been studied previously, In 2023 Riham and Iden Estimating the Parameters of Exponential-Rayleigh Distribution under Type-I Censored Data Abstract [3]. Ali T. M. and Iden H. in 2019 introduced non-parametric estimation of hazard function using father wavelet transformation $\{\varphi_{J,k}(x)\}, 0 \leq k \leq 2^J - 1, J > 0$ on subspase V_J for progressively right censoring data [4]. Ali T. M. and Iden H. in 2019 applied randomly right censored data to estimate nonparametric density function using semi-symmetric coiflet (coifN, N=1,2,...,9) wavelet procedure [5].

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The objective of this research is to estimate the Reliability function for Generalized exponential Rayleigh distribution with three parameters under type one censored samples by using maximum likelihood With the use of the Monte-Carlo technique [1] by performing mean squares error procedure for different samples sizes and various initial values, .the research contains many sections.

In the history of mathematics, there are many applications of fuzzy logic and its generalizations with fuzzy statistics in many areas of real life and scientific fields, for more details, see [10-14].

2. Some definition for (GERD) [7]

A. The probability density function of (GERD) is as follows :

$$f(t_i;\rho,\omega,\mu)=\left(\frac{\rho}{\mu}t_i^{\frac{1}{\mu}-1}+\frac{\omega}{\mu}t_i^{\frac{2}{\mu}-1}\right)e^{-\left(\rho t_i^{\frac{1}{\mu}}+\frac{\omega}{2}t_i^{\frac{2}{\mu}}\right)} \quad ;t_i\geq 0 \tag{1}$$

This new model has two scale and one shape parameters

B. The cumulative distribution function is as follows:

$$F(t)=1-\exp\left\{-\left(\rho t_r^{\frac{1}{\mu}}+\frac{\omega}{2}t_r^{\frac{2}{\mu}}\right)\right\} \tag{2}$$

C. The Reliability function of this distribution is as follows:

$$R(t_r;\rho,\omega,\mu)=\exp\left\{-\left(\rho t_r^{\frac{1}{\mu}}+\frac{\omega}{2}t_r^{\frac{2}{\mu}}\right)\right\} \qquad \mu,\ ,\rho>0,\ t_r>0 \tag{3}$$

3. Type – one censoring data

This type of right censoring is really significant through which the number of rejected samples (r) is determined at the conclusion of a specific study or experiment. While the researcher overseeing the experiment knows the sample size (n) from the start, these samples are not stated over the time period. As a result, the number of residual samples after the study will be n-r.

4. MLEM for type – one censoring sample

The greatest affinity is the primary benefit of the maximum likelihood partiality technique. that is to say, as data rises, the estimate converges with the parameters more effectively and quickly, maximizing the function's value. Let $t_1\leq t_2 \ldots \leq t_i$ be a random sample for Generalized Exponential Rayleigh Distribution, then the likelihood of this distribution is as follows:

n = the size of the sample

r = the number of failures

$$L=\frac{n!}{(n-r)!}[S(t_r;\rho,\omega,\mu)]^{n-r}\prod_{i=0}^{r}[f(t_i;\rho,\omega,\mu)]$$

$$L=\frac{n!}{(n-r)!}\left[e^{-\left(\rho t_r^{\frac{1}{\mu}}+\frac{\omega}{2}t_r^{\frac{2}{\mu}}\right)}\right]^{n-r}\prod_{i=0}^{r}\left[\left(\frac{\rho}{\mu}t_i^{\frac{1}{\lambda}-1}+\frac{\omega}{\mu}t_i^{\frac{2}{\lambda}-1}\right)e^{-\left(\rho t_i^{\frac{1}{\mu}}+\frac{\omega}{2}t_i^{\frac{2}{\mu}}\right)}\right]$$

$$\ln L=\ln K-(n-r)\left[\rho t_r^{\frac{1}{\mu}}+\frac{\omega}{2}t_r^{\frac{2}{\mu}}\right]+\sum_{i=1}^{r}\ln\left(\frac{\rho}{\mu}t_i^{\frac{1}{\mu}-1}+\frac{\omega}{\mu}t_i^{\frac{2}{\mu}-1}\right)-\sum_{i=1}^{r}\left(\rho t_i^{\frac{1}{\mu}}+\frac{\omega}{2}t_i^{\frac{2}{\mu}}\right)$$

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$$f(\rho) = \frac{\partial \ln L}{\partial \rho} = -(n-r)t_r^{\frac{1}{\mu}} + \sum_{i=1}^{r}\left(\frac{t_i^{\frac{1}{\mu}-1}}{\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}}\right) - \sum_{i=1}^{r} t_i^{\frac{1}{\mu}}$$

$$f(\omega) = -\frac{(n-r)}{2}t_r^{\frac{2}{\mu}} + \sum_{i=1}^{r}\left(\frac{t_i^{\frac{2}{\mu}-1}}{\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}}\right) - \frac{1}{2}\sum_{i=1}^{r} t_i^{\frac{2}{\mu}}$$

$$f(\mu) = (n-r)\left(\frac{\rho t_r^{\frac{1}{\mu}}\ln t_r + \omega t_r^{\frac{2}{\mu}}\ln t_r}{\mu^2}\right) - \frac{1}{\mu^2}\sum_{i=1}^{r}\frac{\rho t_i^{\frac{1}{\mu}-1}\ln t_i + \mu\rho t_i^{\frac{1}{\mu}-1} + 2\omega t_i^{\frac{2}{\mu}-1}\ln t_i + \mu\omega t_i^{\frac{2}{\mu}-1}}{\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}} + \frac{\sum_{i=1}^{r}\left(\rho t_i^{\frac{1}{\mu}}\ln t_i + \omega t_i^{\frac{2}{\mu}}\ln t_i\right)}{\mu^2}$$

$$\begin{bmatrix}\rho_{k+1}\\ \omega_{k+1}\\ \mu_{k+1}\end{bmatrix} = \begin{bmatrix}\rho_k\\ \omega_k\\ \mu_k\end{bmatrix} - J^{-1}\begin{bmatrix}f(\rho)\\ f(\omega)\\ f(\mu)\end{bmatrix} \quad , \quad J^{-1} = \begin{bmatrix}\frac{\partial f(\rho)}{\partial \rho} & \frac{\partial f(\rho)}{\partial \omega} & \frac{\partial f(\rho)}{\partial \mu}\\ \frac{\partial f(\omega)}{\partial \rho} & \frac{\partial f(\omega)}{\partial \omega} & \frac{\partial f(\omega)}{\partial \mu}\\ \frac{\partial f(\mu)}{\partial \rho} & \frac{\partial f(\mu)}{\partial \omega} & \frac{\partial f(\mu)}{\partial \mu}\end{bmatrix}^{-1}$$

$$\frac{\partial f(\rho)}{\partial \rho} = -\sum_{i=1}^{r}\frac{t_i^{\frac{2}{\mu}-2}}{\left(\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}\right)^2}$$

$$\frac{\partial f(\rho)}{\partial \omega} = -\sum_{i=1}^{r}\frac{t_i^{\frac{3}{\mu}-2}}{\left(\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}\right)^2}$$

$$\frac{\partial f(\rho)}{\partial \mu} = \frac{1}{\mu^2}\left[(n-r)t_r^{\frac{1}{\mu}}\ln t_r + \sum_{i=1}^{r} t_i^{\frac{1}{\mu}}\ln t_i + \omega\sum_{i=1}^{r}\frac{t_i^{\frac{3}{\mu}-2}\ln t_i}{\left(\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}\right)^2}\right]$$

$$\frac{\partial f(\omega)}{\partial \rho} = -\sum_{i=1}^{r}\frac{t_i^{\frac{3}{\mu}-2}}{\left(\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}\right)^2}$$

$$\frac{\partial f(\omega)}{\partial \omega} = -\sum_{i=1}^{r}\frac{t_i^{\frac{4}{\mu}-2}}{\left(\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}\right)^2}$$

$$\frac{\partial f(\omega)}{\partial \mu} = \frac{1}{\lambda^2}\left[(n-r)t_r^{\frac{2}{\mu}}\ln t_r - \rho\sum_{i=1}^{r}\frac{t_i^{\frac{3}{\mu}-2}\ln t_i}{\left(\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}\right)^2} + \sum_{i=1}^{r} t_i^{\frac{2}{\mu}}\ln t_i\right]$$

$$f(\mu) = (n-r)\left(\frac{\rho t_r^{\frac{1}{\mu}}\ln t_r + \omega t_r^{\frac{2}{\mu}}\ln t_r}{\mu^2}\right) - \frac{1}{\mu^2}\sum_{i=1}^{r}\frac{\rho t_i^{\frac{1}{\mu}-1}\ln t_i + \mu\rho t_i^{\frac{1}{\mu}-1} + 2\omega t_i^{\frac{2}{\mu}-1}\ln t_i + \mu\omega t_i^{\frac{2}{\mu}-1}}{\rho t_i^{\frac{1}{\mu}-1} + \omega t_i^{\frac{2}{\mu}-1}} + \frac{\sum_{i=1}^{r}\left(\rho t_i^{\frac{1}{\mu}}\ln t_i + \omega t_i^{\frac{2}{\mu}}\ln t_i\right)}{\mu^2}$$

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$$\frac{\partial f(\mu)}{\partial \rho} = \frac{1}{\mu^2}\left[(n-r)t_r^{\frac{1}{\mu}}\ln t_r + \sum_{i=1}^{r} t_i^{\frac{1}{\mu}}\ln t_i + \omega\sum_{i=1}^{r}\frac{t_i^{\frac{3}{\mu}-2}\ln t_i}{\left(\rho t_i^{\frac{1}{\mu}-1}+\omega t_i^{\frac{2}{\mu}-1}\right)^2}\right]$$

$$\frac{\partial f(\mu)}{\partial \omega} = \frac{1}{\mu^2}\left[(n-r)t_r^{\frac{2}{\mu}}\ln t_r - \rho\sum_{i=1}^{r}\frac{t_i^{\frac{3}{\mu}-2}\ln t_i}{\left(\rho t_i^{\frac{1}{\mu}-1}+\omega t_i^{\frac{2}{\mu}-1}\right)^2} + \sum_{i=1}^{r} t_i^{\frac{2}{\mu}}\ln t_i\right]$$

$$\frac{\partial f(\mu)}{\partial \mu} = \frac{n-r}{\mu^4}\left[-\rho t_r^{\frac{1}{\mu}}(\ln t_r)^2 - 2\omega t_r^{\frac{2}{\mu}-1}(\ln t_r)^2 - 2\mu\rho t_r^{\frac{1}{\mu}}\ln t_r - 2\mu\omega t_r^{\frac{2}{\mu}}\ln t_r\right]$$

$$+\frac{1}{\mu^2}\sum_{i=1}^{r}\frac{\mu^2\rho^2 2t_i^{\frac{2}{\mu}-1}+\rho\omega t_i^{\frac{3}{\mu}-2}(\ln t_i)^2+6\rho\omega\mu t_i^{\frac{3}{\mu}-2}\ln t_i+\omega^2\mu^2 t_i^{\frac{4}{\mu}-2}+2\rho^2\mu t_i^{\frac{2}{\mu}-1}\ln t_i+4\omega^2\mu t_i^{\frac{4}{\mu}-2}\ln t_i+2\rho\omega\mu^2 t_i^{\frac{2}{\mu}-1}}{\left(\rho t_i^{\frac{1}{\mu}-1}+\omega t_i^{\frac{2}{\mu}-1}\right)^2}$$

$$+\frac{1}{\mu^4}\left[\sum_{i=1}^{r} -\rho t_r^{\frac{1}{\mu}}(\ln t_r)^2 - 2\omega t_r^{\frac{2}{\mu}-1}(\ln t_r)^2 - 2\mu\rho t_r^{\frac{1}{\mu}}\ln t_r - 2\mu\omega t_r^{\frac{2}{\mu}}\ln t_r\right]$$

In the end, we stopped the Newton –Raphson method by using the following formula:

$$\begin{bmatrix}\rho_{k+1}-\rho_k\\ \omega_{k+1}-\omega_k\\ \mu_{k+1}-\mu_k\end{bmatrix} \leq \begin{bmatrix}\varepsilon_\rho\\ \varepsilon_\omega\\ \varepsilon_\mu\end{bmatrix}$$

Finally, we estimate the frailer density function $\hat{f}(t)$, reliability function $\hat{R}(t)$ where:

$$\hat{f}(t) = \left(\frac{\rho}{\mu}t_i^{\frac{1}{\mu}-1} + \frac{\omega}{\mu}t_i^{\frac{2}{\mu}-1}\right)e^{-\left(\rho t_i^{\frac{1}{\mu}}+\frac{\omega}{2}t_i^{\frac{2}{\mu}}\right)}$$

$$\hat{R}(t) = \exp\left\{-\left(\rho t_r^{\frac{1}{\mu}} + \frac{\omega}{2}t_r^{\frac{2}{\mu}}\right)\right\}$$

5. Simulation application: [2], [3],[8]

A simulation is defined as a numerical scientific method that uses logical mathematical methods to describe the behavior of a certain approved system. Currently, numerous statistical fields embrace simulation techniques. There is a variety of methods of simulation with this segment, the Monte Carlo simulation has been conducted [9] .

U=F(x) , X=$F^{-1}(u)$

$$u = 1 - e^{-\left(\rho x^{\frac{1}{\mu}}+\frac{\omega}{2}x^{\frac{2}{\mu}}\right)}$$

$$1 - u = e^{-\left(\rho x^{\frac{1}{\mu}}+\frac{\omega}{2}x^{\frac{2}{\mu}}\right)}$$

$$\ln(1-u) = -\left(\rho x^{\frac{1}{\mu}} + \frac{\omega}{2}x^{\frac{2}{\mu}}\right)$$

$$\rho x^{\frac{1}{\mu}} + \frac{\omega}{2}x^{\frac{2}{\mu}} + \ln(1-u) = 0$$

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$$x^{\frac{1}{\mu}}\left(\frac{\omega}{2}x^2+\rho x\right)+\ln(1-u)=0$$

$$x=\left|\frac{-\propto\mp\sqrt{\rho^2-2\ln\omega+2\ln v\omega}}{\beta}\right|^{\mu} \qquad \mu,\omega,\rho>0$$

6. Numerical results

To find the parameters estimation, reliability function and mean square error procedure by applying the simulation technique then put the results in following tables.

Table 1: symbolize the estimation of parameters with M.S.E

ρ	ω	μ	n	$\hat{\rho}$	$\hat{\omega}$	$\hat{\mu}$	min
0.5	0.5	0.5	25	0.622017	0.451951	0.507757	4.52E-01
			50	0.624197	0.54009	0.50416	5.04E-01
			100	0.745813	0.49599	0.50202	4.96E-01
		1	25	0.527952	0.578327	1.005561	5.28E-01
			50	0.603027	0.500819	1.004494	5.01E-01
			100	0.56348	0.523453	1.004785	5.23E-01
	0.75	0.5	25	0.661405	0.839366	0.503687	5.04E-01
			50	0.675953	0.710799	0.50411	5.04E-01
			100	0.622252	0.734892	0.505018	5.05E-01
		1	25	0.597533	0.741725	1.005655	5.98E-01
			50	0.54055	0.671271	1.004213	5.41E-01
			100	0.607346	0.804848	1.005748	6.07E-01
0.1	0.5	0.5	25	0.132848	0.540238	0.503677	1.33E-01
			50	0.118384	0.569071	0.503973	1.18E-01
			100	0.12066	0.513302	0.504609	1.21E-01
		1	25	0.107959	0.507625	1.004232	1.08E-01
			50	0.111518	0.596806	1.006329	1.12E-01
			100	0.103989	0.538978	1.00519	1.04E-01
	0.75	0.5	25	0.132927	0.768292	0.504444	1.33E-01
			50	0.105993	0.682939	0.505584	1.06E-01
			100	0.105768	0.830043	0.504298	1.06E-01
		1	25	0.109557	0.850362	1.004174	1.10E-01
			50	0.112506	0.722355	1.005779	1.13E-01
			100	0.105017	0.73614	1.005478	1.05E-01

Table 2: symbolize the estimation of Reliability function under time and MSe :

$\rho_{\circ}$	$\omega_{\circ}$	$\mu_{\circ}$	n	t	Reliability	$\hat{R}$	min R
0.5	0.5	0.5	25	0.1	0.994988	0.993311	0.993311
				0.2	0.979807	0.97381	0.97381
				0.3	0.954064	0.941809	0.941809
				0.4	0.917227	0.897565	0.897565
				0.5	0.868815	0.841527	0.841527
0.5	0.5	0.5	50	0.1	0.994988	0.99351	0.99351
				0.2	0.979807	0.974277	0.974277
				0.3	0.954064	0.942329	0.942329
				0.4	0.917227	0.897658	0.897658
				0.5	0.868815	0.840466	0.840466
0.5	0.5	0.5	100	0.1	0.994988	0.992487	0.992487
				0.2	0.979807	0.97029	0.97029
				0.3	0.954064	0.934173	0.934173

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				0.4	0.917227	0.885244	0.885244
				0.5	0.868815	0.824859	0.824859
0.5	0.5	1	25	0.1	0.948854	0.945139	0.945139
				0.2	0.895834	0.888505	0.888505
				0.3	0.841558	0.830624	0.830624
				0.4	0.786628	0.772184	0.772184
				0.5	0.731616	0.713853	0.713853
0.5	0.5	1	50	0.1	0.948854	0.938985	0.938985
				0.2	0.895834	0.87839	0.87839
				0.3	0.841558	0.818411	0.818411
				0.4	0.786628	0.759397	0.759397
				0.5	0.731616	0.701694	0.701694
0.5	0.5	1	100	0.1	0.948854	0.942139	0.942139
				0.2	0.895834	0.883476	0.883476
				0.3	0.841558	0.824425	0.824425
				0.4	0.786628	0.765556	0.765556
				0.5	0.731616	0.70742	0.70742
0.5	0.75	0.5	25	0.1	0.994975	0.99313	0.99313
				0.2	0.979611	0.972617	0.972617
				0.3	0.953098	0.938103	0.938103
				0.4	0.914297	0.889088	0.889088
				0.5	0.862054	0.825341	0.825341
0.5	0.75	0.5	50	0.1	0.994975	0.992945	0.992945
				0.2	0.979611	0.972035	0.972035
				0.3	0.953098	0.937209	0.937209
				0.4	0.914297	0.888335	0.888335
				0.5	0.862054	0.82553	0.82553
0.5	0.75	0.5	100	0.1	0.994975	0.993474	0.993474
				0.2	0.979611	0.974064	0.974064
				0.3	0.953098	0.941493	0.941493
				0.4	0.914297	0.895308	0.895308
				0.5	0.862054	0.835267	0.835267
0.5	0.75	1	25	0.1	0.947669	0.937875	0.937875
				0.2	0.891366	0.873733	0.873733
				0.3	0.832144	0.808359	0.808359
				0.4	0.771052	0.742705	0.742705
				0.5	0.709106	0.67768	0.67768
0.5	0.75	1	50	0.1	0.947669	0.943669	0.943669
				0.2	0.891366	0.884824	0.884824
				0.3	0.832144	0.824233	0.824233
				0.4	0.771052	0.762776	0.762776
				0.5	0.709106	0.701301	0.701301
0.5	0.75	1	100	0.1	0.947669	0.93649	0.93649
				0.2	0.891366	0.870511	0.870511
				0.3	0.832144	0.803003	0.803003
				0.4	0.771052	0.735079	0.735079
				0.5	0.709106	0.667787	0.667787
0.1	0.5	0.5	25	0.1	0.998976	0.998616	0.998616
				0.2	0.99561	0.994178	0.994178
				0.3	0.989036	0.985781	0.985781
				0.4	0.977849	0.971967	0.971967
				0.5	0.960189	0.950844	0.950844
0.1	0.5	0.5	50	0.1	0.998976	0.998736	0.998736
				0.2	0.99561	0.99466	0.99466
				0.3	0.989036	0.986803	0.986803
				0.4	0.977849	0.973597	0.973597
				0.5	0.960189	0.953004	0.953004

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0.1	0.5	0.5	100	0.1	0.998976	0.998706	0.998706
				0.2	0.99561	0.994589	0.994589
				0.3	0.989036	0.986789	0.986789
				0.4	0.977849	0.973911	0.973911
				0.5	0.960189	0.95413	0.95413
0.1	0.5	1	25	0.1	0.987578	0.98661	0.98661
				0.2	0.970446	0.968506	0.968506
				0.3	0.948854	0.945973	0.945973
				0.4	0.923116	0.919373	0.919373
				0.5	0.893597	0.889124	0.889124
0.1	0.5	1	50	0.1	0.987578	0.985714	0.985714
				0.2	0.970446	0.965895	0.965895
				0.3	0.948854	0.940887	0.940887
				0.4	0.923116	0.911143	0.911143
				0.5	0.893597	0.87718	0.87718
0.1	0.5	1	100	0.1	0.987578	0.986809	0.986809
				0.2	0.970446	0.968588	0.968588
				0.3	0.948854	0.945622	0.945622
				0.4	0.923116	0.918281	0.918281
				0.5	0.893597	0.886995	0.886995
0.1	0.75	0.5	25	0.1	0.998963	0.99857	0.99857
				0.2	0.995411	0.993886	0.993886
				0.3	0.988035	0.984637	0.984637
				0.4	0.974725	0.968724	0.968724
				0.5	0.952717	0.943455	0.943455
0.1	0.75	0.5	50	0.1	0.998963	0.998849	0.998849
				0.2	0.995411	0.995036	0.995036
				0.3	0.988035	0.987372	0.987372
				0.4	0.974725	0.973944	0.973944
				0.5	0.952717	0.952281	0.952281
0.1	0.75	0.5	100	0.1	0.998963	0.998854	0.998854
				0.2	0.995411	0.994958	0.994958
				0.3	0.988035	0.98686	0.98686
				0.4	0.974725	0.972236	0.972236
				0.5	0.952717	0.948102	0.948102
0.1	0.75	1	25	0.1	0.986344	0.984732	0.984732
				0.2	0.965605	0.961497	0.961497
				0.3	0.93824	0.93089	0.93089
				0.4	0.904837	0.893685	0.893685
				0.5	0.866104	0.850789	0.850789
0.1	0.75	1	50	0.1	0.986344	0.985013	0.985013
				0.2	0.965605	0.963293	0.963293
				0.3	0.93824	0.935306	0.935306
				0.4	0.904837	0.901656	0.901656
				0.5	0.866104	0.863045	0.863045
0.1	0.75	1	100	0.1	0.986344	0.985706	0.985706
				0.2	0.965605	0.9645	0.9645
				0.3	0.93824	0.936859	0.936859
				0.4	0.904837	0.9034	0.9034
				0.5	0.866104	0.86484	0.86484

From table (1,2) we see that:

- The R(t) is decreasing when the size of sample is increasing for all initial value of parameters.
- Noting that the R(t) are decreasing when the initial values $\rho_\circ$, $\omega_\circ$, $\mu_\circ$ are increasing.
- The MSE are decreasing when the samples sizes are increasing for all initial value of parameters.

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- The MSE are decreasing when the initial values $\rho_\circ$, $\omega_\circ$, $\mu_\circ$ are increasing.

7. Numerical results on fuzzy data

To find the parameters estimation, reliability function and mean square error procedure by applying the simulation technique then put the results in following tables.

Table 3: symbolize the estimation of parameters with M.S.E

ρ	ω	μ	n	$\hat{\rho}$	$\hat{\omega}$	$\hat{\mu}$	min
0.33	0.21	0.6	25	0.622012	0.451954	0.507753	4.52E-01
			50	0.624157	0.5406	0.50416	5.04E-01
			100	0.745813	0.49599	0.50202	4.96E-01
		1	25	0.527952	0.578327	1.005561	5.28E-01
			50	0.603027	0.500819	1.004494	5.01E-01
			100	0.56348	0.523453	1.004785	5.23E-01
	0.35	0.5	25	0.661405	0.839366	0.503687	5.04E-01
			50	0.675953	0.710799	0.50411	5.04E-01
			100	0.622252	0.734856	0.505018	5.05E-01
		1	25	0.597533	0.741725	1.005655	5.98E-01
			50	0.54055	0.671271	1.004213	5.41E-01
			100	0.607354	0.804848	1.005711	6.07E-01
0.1	0.13	0.13	25	0.132848	0.540238	0.503677	1.33E-01
			50	0.118354	0.569071	0.503973	1.18E-01
			100	0.12066	0.513302	0.504609	1.21E-01
		1	25	0.107959	0.507625	1.004232	1.08E-01
			50	0.111518	0.596806	1.006329	1.12E-01
			100	0.103954	0.538922	1.00719	1.04E-01
	0.65	0.15	25	0.132905	0.768292	0.504444	1.33E-01
			50	0.105993	0.792939	0.505584	1.06E-01
			100	0.105705	0.830043	0.504298	1.06E-01
		1	25	0.109557	0.850362	1.004174	1.10E-01
			50	0.112505	0.722355	1.005779	1.13E-01
			100	0.105017	0.73614	1.005478	1.05E-01

Table 4: symbolize the estimation of Reliability function under time and MSe :

$\rho_\circ$	$\omega_\circ$	$\mu_\circ$	n	t	Reliability	$\hat{R}$	min R
0.5	0.5	0.5	25	0.1	0.994988	0.993311	0.993311
				0.2	0.979807	0.97381	0.97381
				0.3	0.954064	0.941809	0.941809
				0.4	0.917227	0.897565	0.897565
				0.5	0.868815	0.841527	0.841527
0.5	0.5	0.5	50	0.1	0.994988	0.99351	0.99351
				0.2	0.979807	0.974277	0.974277
				0.3	0.954064	0.942329	0.942329
				0.4	0.917227	0.897658	0.897658
				0.5	0.868815	0.840466	0.840466
0.5	0.5	0.5	100	0.1	0.994988	0.992487	0.992487
				0.2	0.979807	0.97029	0.97029
				0.3	0.954064	0.934173	0.934173
				0.4	0.917227	0.885244	0.885244
				0.5	0.868815	0.824859	0.824859
0.5	0.5	1	25	0.1	0.948854	0.945139	0.945139
				0.2	0.895834	0.888505	0.888505
				0.3	0.841558	0.830624	0.830624
				0.4	0.786628	0.772184	0.772184
				0.5	0.731616	0.713853	0.713853

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0.5	0.5	1	50	0.1	0.948854	0.938985	0.938985
				0.2	0.895834	0.87839	0.87839
				0.3	0.841558	0.818411	0.818411
				0.4	0.786628	0.759397	0.759397
				0.5	0.731616	0.701694	0.701694
0.5	0.5	1	100	0.1	0.948854	0.942139	0.942139
				0.2	0.895834	0.883476	0.883476
				0.3	0.841558	0.824425	0.824425
				0.4	0.786628	0.765556	0.765556
				0.5	0.731616	0.70742	0.70742
0.5	0.75	0.5	25	0.1	0.994975	0.99313	0.99313
				0.2	0.979611	0.972617	0.972617
				0.3	0.953098	0.938103	0.938103
				0.4	0.914297	0.889088	0.889088
				0.5	0.862054	0.825341	0.825341
0.5	0.75	0.5	50	0.1	0.994975	0.992945	0.992945
				0.2	0.979611	0.972035	0.972035
				0.3	0.953098	0.937209	0.937209
				0.4	0.914297	0.888335	0.888335
				0.5	0.862054	0.82553	0.82553
0.5	0.75	0.5	100	0.1	0.994975	0.993474	0.993474
				0.2	0.979611	0.974064	0.974064
				0.3	0.953098	0.941493	0.941493
				0.4	0.914297	0.895308	0.895308
				0.5	0.862054	0.835267	0.835267
0.5	0.75	1	25	0.1	0.947669	0.937875	0.937875
				0.2	0.891366	0.873733	0.873733
				0.3	0.832144	0.808359	0.808359
				0.4	0.771052	0.742705	0.742705
				0.5	0.709106	0.67768	0.67768
0.5	0.75	1	50	0.1	0.947669	0.943669	0.943669
				0.2	0.891366	0.884824	0.884824
				0.3	0.832144	0.824233	0.824233
				0.4	0.771052	0.762776	0.762776
				0.5	0.709106	0.701301	0.701301
0.5	0.75	1	100	0.1	0.947669	0.93649	0.93649
				0.2	0.891366	0.870511	0.870511
				0.3	0.832144	0.803003	0.803003
				0.4	0.771052	0.735079	0.735079
				0.5	0.709106	0.667787	0.667787
0.1	0.5	0.5	25	0.1	0.998976	0.998616	0.998616
				0.2	0.99561	0.994178	0.994178
				0.3	0.989036	0.985781	0.985781
				0.4	0.977849	0.971967	0.971967
				0.5	0.960189	0.950844	0.950844
0.1	0.5	0.5	50	0.1	0.998976	0.998736	0.998736
				0.2	0.99561	0.99466	0.99466
				0.3	0.989036	0.986803	0.986803
				0.4	0.977849	0.973597	0.973597
				0.5	0.960189	0.953004	0.953004
0.1	0.5	0.5	100	0.1	0.998976	0.998706	0.998706
				0.2	0.99561	0.994589	0.994589
				0.3	0.989036	0.986789	0.986789
				0.4	0.977849	0.973911	0.973911
				0.5	0.960189	0.95413	0.95413
0.1	0.5	1	25	0.1	0.987578	0.98661	0.98661
				0.2	0.970446	0.968506	0.968506

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				0.3	0.948854	0.945973	0.945973
				0.4	0.923116	0.919373	0.919373
				0.5	0.893597	0.889124	0.889124
0.1	0.5	1	50	0.1	0.987578	0.985714	0.985714
				0.2	0.970446	0.965895	0.965895
				0.3	0.948854	0.940887	0.940887
				0.4	0.923116	0.911143	0.911143
				0.5	0.893597	0.87718	0.87718
0.1	0.5	1	100	0.1	0.987578	0.986809	0.986809
				0.2	0.970446	0.968588	0.968588
				0.3	0.948854	0.945622	0.945622
				0.4	0.923116	0.918281	0.918281
				0.5	0.893597	0.886995	0.886995
0.1	0.75	0.5	25	0.1	0.998963	0.99857	0.99857
				0.2	0.995411	0.993886	0.993886
				0.3	0.988035	0.984637	0.984637
				0.4	0.974725	0.968724	0.968724
				0.5	0.952717	0.943455	0.943455
0.1	0.75	0.5	50	0.1	0.998963	0.998849	0.998849
				0.2	0.995411	0.995036	0.995036
				0.3	0.988035	0.987372	0.987372
				0.4	0.974725	0.973944	0.973944
				0.5	0.952717	0.952281	0.952281
0.1	0.75	0.5	100	0.1	0.998963	0.998854	0.998854
				0.2	0.995411	0.994958	0.994958
				0.3	0.988035	0.98686	0.98686
				0.4	0.974725	0.972236	0.972236
				0.5	0.952717	0.948102	0.948102
0.1	0.75	1	25	0.1	0.986344	0.984732	0.984732
				0.2	0.965605	0.961497	0.961497
				0.3	0.93824	0.93089	0.93089
				0.4	0.904837	0.893685	0.893685
				0.5	0.866104	0.850789	0.850789
0.1	0.75	1	50	0.1	0.986344	0.985013	0.985013
				0.2	0.965605	0.963293	0.963293
				0.3	0.93824	0.935306	0.935306
				0.4	0.904837	0.901656	0.901656
				0.5	0.866104	0.863045	0.863045
0.1	0.75	1	100	0.1	0.986344	0.985706	0.985706
				0.2	0.965605	0.9645	0.9645
				0.3	0.93824	0.936859	0.936859
				0.4	0.904837	0.9034	0.9034
				0.5	0.866104	0.86484	0.86484

From table (3,4) we see that:

- The R(t) is decreasing when the size of sample is increasing for all initial value of parameters.
- Noting that the R(t) are decreasing when the initial values $\rho_\circ$, $\omega_\circ$, $\mu_\circ$ are increasing.
- The MSE are decreasing when the samples sizes are increasing for all initial value of parameters.
- The MSE are decreasing when the initial values $\rho_\circ$, $\omega_\circ$, $\mu_\circ$ are increasing.

8. Conclusion

Three factors comprise the GERD, which may be derived in this paper: two scales and one shape for the type 1 censoring sample. Next, we use the maximum likelihood estimator method to estimate the values of these parameters. Subsequently, we utilize simulation to determine and approximate the values of Pdf and R(t). It is seen

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that both f(t) and R(t) decrease as lifetimes and sample size increase, for all initial parameter values as in the (3,4) tables.

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