Robots selection for sine trigonometric $(\wp_1, \wp_2, \wp_3)$ neutrosophic sets using different aggregation operators

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Abstract

This article discusses a new approach to multiple attribute decision-making (MADM) based on sine trigonometric(ST) $(\wp_1, \wp_2, \wp_3)$ neutrosophic sets (NS). We discuss the concept of ST $(\wp_1, \wp_2, \wp_3)$ neutrosophic weighted averaging (NWA), ST $(\wp_1, \wp_2, \wp_3)$ neutrosophic weighted geometric (NWG), ST $(\wp_1, \wp_2, \wp_3)$ generalized neutrosophic weighted averaging (GNWA) and ST $(\wp_1, \wp_2, \wp_3)$ generalized neutrosophic weighted geometric (GNWG). We presented during our discussion showed an algorithm that used these operators. Extensive Hamming distances are illustrated numerically. Also included in this communication are discussions of idempotency, boundness, commutativity, and monotonicity for ST $(\wp_1, \wp_2, \wp_3)$ NS. By using them, you can find the best option faster, easier and more conveniently. As a result, ST $(\wp_1, \wp_2, \wp_3)$ and more precise conclusions are more closely related. A comparison is made between some current models and those proposed to demonstrate the dependability and utility of the current models. Furthermore, fascinating findings were revealed in the study.

Keywords: Aggregating operator; decision making; STNWA; STNWG; STGNWA and STGNWG.

1 Introduction

Various uncertain theories have been developed to deal with the ambiguities throughout history, including fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS)[3] and spherical FS (SFS).[4] Atanassov later proposed the concept of an IFS divided into categories based on non-membership grade (NMD), which cannot exceed one; FS elements range from zero to one based on their membership grade (MD). If the MD and NMD grades combined are greater than one, the DM may receive a single problem. Yager[3] identifies PFS as the square sum of MD and NMD of an IFS with a value less than one. Cuong et al.[5] deals the concept of picture FS concepts are based on three points: positive MD, neutral MD, and negative MD. Additionally, it has more benefits than either PFS or IFS. Liu et al.[6] suggest a generalization of picture FS with AOs. A generalized PFS with application areas was proposed by Liu et al.[7] PFS and interval values were employed to analyze AOs.[8] It is rare for the positive, neutral and negative MDs to exceed one in the DM approach challenge. SFS is a

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concept introduced by Ashraf et al.,[4] in which positive, neutral and negative grades do not exceed one. The notion of SFS was investigated using TOPSIS models by Fatmaa et al.[9]

Aggregation operators (AOs) are essential for solving MADM issues. There are a number of IFS averaging operators that can be used to average IFS data discussed by Xu et al.[10] A weighted, ordered weighted, a hybrid and an IFS-derived geometric operator was also defined by Xu et al.[11] A generalized ordered weighted averaging operator (GOWA) was proposed by Li et al.[12] Zeng et al.[13] proposed distance measures and AOs for the calculation of ordered weighted distances. Based on PFS weighted, ordered weighted and weighted power conditions, Yager[3] developed averaging and geometric AOs. Peng et al.[14] discussed the concept of PFS can be derived from the properties of AOs. A generalized PFS was developed under AOs by Liu et al.[7] An spherical fuzzy Dombi AO has been developed by Ashraf et al.[15] Recently, Palanikumar et al. discussed the new aggregating operators[16]-.[20] The SFS and T-norm SFS can be learned more at.[21,22] The application of Muirhead power normal SFS to MADM was discussed by Temel et al.[23] Peng et al.[24] analyzed NSs with MADM with the help of MABAC and TOPSIS. Zhang et al.[25] deals that TOPSIS can be used as a generalization of PFS. In many applications and AOs and its algebraic structures are explained in Palanikumar et al.[26]-.[28] Throughout the remainder of this paper, an introduction is found in section 1. In section 2 discussion of STPFS and NS. Section 3 deals the concept of ST$(\wp_1, \wp_2, \wp_3)$ and its operations and distance measure between ST$(\wp_1, \wp_2, \wp_3)$ NNs. In section 4 an interaction between MADM and some AOs is described based on some NNs for ST$(\wp_1, \wp_2, \wp_3)$. A numerical example and an algorithm are discussed in section 5. Section 6 concludes with a conclusion. Based on the findings of the study, the following conclusions can be drawn:

1. To introduce a ED and HD for ST$(\wp_1, \wp_2, \wp_3)$ NSs.
2. By using ST$(\wp_1, \wp_2, \wp_3)$ NNs, ST$(\wp_1, \wp_2, \wp_3)$ NWA, ST$(\wp_1, \wp_2, \wp_3)$ NWG, G ST$(\wp_1, \wp_2, \wp_3)$ NWA and GST$(\wp_1, \wp_2, \wp_3)$ NWG operators are developed.
3. On the basis of AOs, MADM is explored using ST$(\wp_1, \wp_2, \wp_3)$ NSs.
4. To compute the different ideal values for ST$(\wp_1, \wp_2, \wp_3)$ NWA, ST$(\wp_1, \wp_2, \wp_3)$ NWG, GST$(\wp_1, \wp_2, \wp_3)$ NWA and G ST$(\wp_1, \wp_2, \wp_3)$ NWG.
5. DM results based on the positive integer $(\wp_1, \wp_2, \wp_3)$.

2 Background

This section contains a number of important definitions that we must review for our further learning. For our purpose $\pi/2 = \mathbb{k}$.

Definition 2.1. [8] Let Υ be an universal. The PIVFS $\hbar = \left\{\zeta, (\widetilde{\Xi^{\mathscr{T}}_{\hbar}}(\zeta), \widetilde{\Xi^{\mathscr{F}}_{\hbar}}(\zeta)) \middle| \zeta \in \Upsilon\right\}$, where $\widetilde{\Xi^{T}_{\hbar}}, \widetilde{\Xi^{F}_{\hbar}} : \Upsilon \to Int([0,1])$ denote the MD and NMD of $\zeta \in \Upsilon$ to $\hbar$, respectively, and $0 \leq (\Xi^{\mathscr{T}}_{\hbar}(\zeta))^2 + (\Xi^{\mathscr{F}}_{\hbar}(\zeta))^2 \leq 1$. For convenience, $\hbar = \left[\Xi^{\mathscr{T}}_{\hbar}, \Xi^{\mathscr{T}}_{\hbar}\right], \left[\Xi^{\mathscr{F}}_{\hbar}, \Xi^{\mathscr{F}}_{\hbar}\right]$ is called a Pythagorean interval-valued fuzzy number (PyIVFN).

Definition 2.2. The NS $\hbar = \left\{\zeta, (\Xi^{\mathscr{T}}_{\hbar}(\zeta), \Xi^{\mathscr{I}}_{\hbar}(\zeta), \Xi^{\mathscr{F}}_{\hbar}(\zeta)) | \zeta \in \Upsilon\right\}$, where $\Xi^{\mathscr{T}}_{\hbar}, \Xi^{\mathscr{I}}_{\hbar}, \Xi^{\mathscr{F}}_{\hbar} : \Upsilon \to [0,1]$ denote the positive MD, neutral MD and negative MD of $\zeta \in \Upsilon$, respectively and $0 \leq (\Xi^{\mathscr{T}}_{\hbar}(\zeta)) + (\Xi^{\mathscr{I}}_{\hbar}(\zeta)) + (\Xi^{\mathscr{F}}_{\hbar}(\zeta)) \leq 2$. For $M = \Xi^{\mathscr{T}}_{\hbar}, \Xi^{\mathscr{I}}_{\hbar}, \Xi^{\mathscr{F}}_{\hbar}$ is called a neutrosophic number (NN).

Definition 2.3. The NS $\hbar = \left\{\zeta, (\Xi^{\mathscr{T}}_{\hbar}(\zeta), \Xi^{\mathscr{I}}_{\hbar}(\zeta), \Xi^{\mathscr{F}}_{\hbar}(\zeta)) | \zeta \in \Upsilon\right\}$, where $\Xi^{\mathscr{T}}_{\hbar}, \Xi^{\mathscr{I}}_{\hbar}, \Xi^{\mathscr{F}}_{\hbar} : \Upsilon \to [0,1]$ denote the positive MD, neutral MD and negative MD of $\zeta \in \Upsilon$ to $\hbar$, respectively and $0 \leq (\Xi^{\mathscr{T}}_{\hbar}(\zeta))^2 + (\Xi^{\mathscr{I}}_{\hbar}(\zeta))^2 + (\Xi^{\mathscr{F}}_{\hbar}(\zeta))^2 \leq 2$. For all $\zeta \in \Upsilon$, $\sqrt{2 - \left((\Xi^{\mathscr{T}}_{\hbar}(\zeta))^2 + (\Xi^{\mathscr{I}}_{\hbar}(\zeta))^2 + (\Xi^{\mathscr{F}}_{\hbar}(\zeta))^2\right)}$ is called the grade of refusal of membership of ζ in $\hbar$. For convenience, $\hbar = \Xi^{\mathscr{T}}_{\hbar}, \Xi^{\mathscr{I}}_{\hbar}, \Xi^{\mathscr{F}}_{\hbar}$ is called a Pythagorean neutrosophic number (PyNN).

Definition 2.4. [10] Let $\hbar_1 = (a_1, b_1) \in N$and $\hbar_2 = (a_2, b_2) \in N$. Then the distance between $\hbar_1$ and $\hbar_2$ is defined as $\mathfrak{D}(\hbar_1, \hbar_2) = \sqrt{(a_1 - a_2)^2 + \frac{1}{2}(b_1 - b_2)^2}$, where N is a natural number.

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Definition 2.5. Let $\mathscr{L} = (\Xi^{\mathscr{M}}, \Xi^{\mathscr{N}\mathscr{M}})$ be a STPyFN. Then we define a STPyFN as:
$\sin\mathscr{L} = \left\{ \sin^2\left(\Bbbk\cdot(\Xi^{\mathscr{M}}_{\mathscr{L}}(\zeta))\right), 1-\sin^2\left(\Bbbk\cdot(1-\Xi^{\mathscr{N}\mathscr{M}}_{\mathscr{L}}(\zeta))\right) \right\}$. It is clear that the $\sin\mathscr{L}$ is also STPyFN, we define the function $\sin^2\left(\Bbbk\cdot\Xi^{\mathscr{M}}_{\mathscr{L}}(\zeta)\right) : \mathscr{U} \to [0,1]$ such that $0 \leq \sin^2\left(\Bbbk\cdot\Xi^{\mathscr{M}}_{\mathscr{L}}(\zeta)\right) \leq 1$ and $1 - \sin^2\left(\Bbbk\cdot\left(1-\Xi^{\mathscr{N}\mathscr{M}}_{\mathscr{L}}(\zeta)\right)\right) : \mathscr{U} \to [0,1]$ such that $0 \leq 1-\sin^2\left(\Bbbk\cdot\left(1-\Xi^{\mathscr{N}\mathscr{M}}_{\mathscr{L}}(\zeta)\right)\right) \leq 1$. Therefore, $\sin\mathscr{L} = \left\{ \sin^2\left(\Bbbk\cdot(\Xi^{\mathscr{M}}_{\mathscr{L}}(\zeta))\right), 1-\sin^2\left(\Bbbk\cdot(1-\Xi^{\mathscr{N}\mathscr{M}}_{\mathscr{L}}(\zeta))\right) \right\}$ is a STPyFN.

3 Find ST$(\wp_1, \wp_2, \wp_3)$ NN distance measure

We discuss the concept of ST$(\wp_1, \wp_2, \wp_3)$ neutrosophic number (NN). As a result, the ST$(\wp_1, \wp_2, \wp_3)$NN and its operations were defined.

Definition 3.1. The ST$(\wp_1, \wp_2, \wp_3)$ NS $\hbar = \left\{\zeta, \left(\Xi^{\mathscr{T}}_{\hbar}(\zeta), \Xi^{\mathscr{I}}_{\hbar}(\zeta), \Xi^{\mathscr{F}}_{\hbar}(\zeta)\right)\Big| \zeta \in \Upsilon\right\}$, where $\Xi^{T}_{\hbar}, \Xi^{I}_{\hbar}, \Xi^{F}_{\hbar} : \Upsilon \to [0,1]$ denote the positive MD, neutral MD and negative MD of $\zeta \in \Upsilon$ to $\hbar$, respectively and $0 \leq (\sin\Bbbk\cdot\Xi^{\mathscr{T}}_{\hbar}(\zeta))^{\wp_1} + (\sin\Bbbk\cdot\Xi^{\mathscr{I}}_{\hbar}(\zeta))^{\wp_2} + (\sin\Bbbk\cdot\Xi^{\mathscr{F}}_{\hbar}(\zeta))^{\wp_3} \leq 1$. For convenience, $\hbar = \left(\Xi^{\mathscr{T}}_{\hbar}, \Xi^{\mathscr{I}}_{\hbar}, \Xi^{\mathscr{F}}_{\hbar}\right)$ is represent a ST$(\wp_1, \wp_2, \wp_3)$ NN.

Definition 3.2. Let $\hbar = (\Xi^{\mathscr{T}}, \Xi^{\mathscr{I}}, \Xi^{\mathscr{F}})$, $\hbar_1 = (\Xi^{\mathscr{T}}_1, \Xi^{\mathscr{I}}_1, \Xi^{\mathscr{F}}_1)$ and $\hbar_2 = (\Xi^{\mathscr{T}}_2, \Xi^{\mathscr{I}}_2, \Xi^{\mathscr{F}}_2)$ be any three ST$(\wp_1, \wp_2, \wp_3)$ NNs, and $\mho > 0$. Then

1. $\sin\hbar_1 \diamondsuit \sin\hbar_2 =$
$$\begin{bmatrix} \sqrt[\wp_1]{(\sin^2\Bbbk\cdot\Xi^{\mathscr{T}}_1)^{\wp_1} + (\sin^2\Bbbk\cdot\Xi^{\mathscr{T}}_2)^{\wp_1} - (\sin^2\Bbbk\cdot\Xi^{\mathscr{T}}_1)^{\wp_1}\cdot(\sin^2\Bbbk\cdot\Xi^{\mathscr{T}}_2)^{\wp_1}}, \\ \sqrt[\wp_2]{(\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_1)^{\wp_2} + (\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_2)^{\wp_2} - (\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_1)^{\wp_2}\cdot(\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_2)^{\wp_2}}, \\ (\sin^2\Bbbk\cdot\Xi^{\mathscr{F}}_1)^{wp_3}\cdot(\sin^2\Bbbk\cdot\Xi^{\mathscr{F}}_2)^{wp_3} \end{bmatrix}$$
2. $\sin\hbar_1 \triangle \sin\hbar_2 =$
$$\begin{bmatrix} (\sin^2\Bbbk\cdot\Xi^{\mathscr{T}}_1)^{wp_1}\cdot(\sin^2\Bbbk\cdot\Xi^{\mathscr{T}}_2)^{wp_1}, \\ \sqrt[\wp_2]{(\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_1)^{\wp_2} + (\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_2)^{\wp_2} - (\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_1)^{\wp_2}\cdot(\sin^2\Bbbk\cdot\Xi^{\mathscr{I}}_2)^{\wp_2}}, \\ \sqrt[\wp_3]{(\sin^2\Bbbk\cdot\Xi^{\mathscr{F}}_1)^{\wp_3} + (\sin^2\Bbbk\cdot\Xi^{\mathscr{F}}_2)^{\wp_3} - (\sin^2\Bbbk\cdot\Xi^{\mathscr{F}}_1)^{\wp_3}\cdot(\sin^2\Bbbk\cdot\Xi^{\mathscr{F}}_2)^{\wp_3}} \end{bmatrix}$$
3. $$\mho\cdot\sin\hbar = \begin{bmatrix} \sqrt[\wp_1]{1-\left(1-(\sin^2\Bbbk\cdot\Xi^{\mathscr{T}})^{\wp_1}\right)^{\mho}}, \\ \sqrt[\wp_2]{1-\left(1-(\sin^2\Bbbk\cdot\Xi^{\mathscr{I}})^{\wp_2}\right)^{\mho}}, \\ ((\sin^2\Bbbk\cdot\Xi^{\mathscr{F}})^{\wp_3})^{\mho} \end{bmatrix}$$
4. $$\sin\hbar^{\mho} = \begin{bmatrix} ((\sin^2\Bbbk\cdot\Xi^{\mathscr{T}})^{\wp_1})^{\mho}, \\ \sqrt[\wp_2]{1-\left(1-(\sin^2\Bbbk\cdot\Xi^{\mathscr{I}})^{\wp_2}\right)^{\mho}}, \\ \sqrt[\wp_3]{1-\left(1-(\sin^2\Bbbk\cdot\Xi^{\mathscr{F}})^{\wp_3}\right)^{\mho}} \end{bmatrix}.$$

We introduce HD measures for ST$(\wp_1, \wp_2, \wp_3)$ NNs and study mathematical properties.

Definition 3.3. For any two ST$(\wp_1, \wp_2, \wp_3)$ NNs $\hbar_1 = (\Xi^{\mathscr{T}}_1, \Xi^{\mathscr{I}}_1, \Xi^{\mathscr{F}}_1)$ and $\hbar_2 = (\Xi^{\mathscr{T}}_2, \Xi^{\mathscr{I}}_2, \Xi^{\mathscr{F}}_2)$. Then

$$\mathfrak{D}_H(\hbar_1, \hbar_2) = \frac{1}{2}\left|1 + (\Xi^{\mathscr{T}}_1)^2 - (\Xi^{\mathscr{I}}_1)^2 - (\Xi^{\mathscr{F}}_1)^2 - \left(1 + (\Xi^{\mathscr{T}}_2)^2 - (\Xi^{\mathscr{I}}_2)^2 - (\Xi^{\mathscr{F}}_2)^2\right)\right|$$

where $\mathfrak{D}_H(\hbar_1, \hbar_2)$ is called the HD between $\hbar_1$ and $\hbar_2$.

Theorem 3.4. *Let $\hbar_1 = (\Xi^{\mathscr{T}}_1, \Xi^{\mathscr{I}}_1, \Xi^{\mathscr{F}}_1)$, $\hbar_2 = (\Xi^{\mathscr{T}}_2, \Xi^{\mathscr{I}}_2, \Xi^{\mathscr{F}}_2)$ and $\hbar_3 = (\Xi^{\mathscr{T}}_3, \Xi^{\mathscr{I}}_3, \Xi^{\mathscr{F}}_3)$ be the any three ST$(\wp_1, \wp_2, \wp_3)$ NNs. Then*

1. $\mathfrak{D}_H(\hbar_1, \hbar_2) = 0$ *if and only if* $\hbar_1 = \hbar_2$.
2. $\mathfrak{D}_H(\hbar_1, \hbar_2) = \mathfrak{D}_H(\hbar_2, \hbar_1)$.
3. $\mathfrak{D}_H(\hbar_1, \hbar_3) \leq \mathfrak{D}_H(\hbar_1, \hbar_2) + \mathfrak{D}_H(\hbar_2, \hbar_3)$.

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4 AOs based on ST$(\wp_1, \wp_2, \wp_3)$ NN

Here we describe the AOs using ST$(\wp_1, \wp_2, \wp_3)$ NWA, ST$(\wp_1, \wp_2, \wp_3)$ NWG, GST$(\wp_1, \wp_2, \wp_3)$ NWA, and GST$(\wp_1, \wp_2, \wp_3)$ NWG.

4.1 ST$(\wp_1, \wp_2, \wp_3)NWA$

Definition 4.1. Let $\hbar_i = (\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}})$ be the ST$(\wp_1, \wp_2, \wp_3)$NNs, $W = (\varrho_1, \varrho_2, ..., \varrho_n)$ be the weight of $\sin \hbar_i$, $\varrho_i \geq 0$ and $\diamond_{i=1}^{n} \varrho_i = 1$. Then ST$(\wp_1, \wp_2, \wp_3)$ NWA $(\hbar_1, \hbar_2, ..., \hbar_n) = \diamond_{i=1}^{n} \varrho_i \sin \hbar_i$.

Theorem 4.2. *Let* $\sin \hbar_i = \left(\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}}\right)$ *be the ST*$(\wp_1, \wp_2, \wp_3)$ *NNs. Then*

$$ST(\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2, ..., \hbar_n) = \begin{bmatrix} \sqrt[\wp_1]{1 - \bigodot_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}}, \\ \sqrt[\wp_2]{1 - \bigodot_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}}, \\ \bigodot_{i=1}^{n} ((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3})^{\varrho_i} \end{bmatrix}.$$

Proof. If $n = 2$, then ST$(\wp_1, \wp_2, \wp_3)$ NWA$(\hbar_1, \hbar_2) = \varrho_1 \sin \hbar_1 \diamond \varrho_2 \sin \hbar_2$, where

$$\varrho_1 \sin \hbar_1 = \begin{bmatrix} \sqrt[\wp_1]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\varrho_1}}, \\ \sqrt[\wp_2]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\varrho_1}}, \\ ((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{F}})^{\wp_3})^{\varrho_1} \end{bmatrix}$$

$$\varrho_2 \sin \hbar_2 = \begin{bmatrix} \sqrt[\wp_1]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{T}})^{\wp_1}\right)^{\varrho_2}}, \\ \sqrt[\wp_2]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{I}})^{\wp_2}\right)^{\varrho_2}}, \\ ((\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{F}})^{\wp_3})^{\varrho_2} \end{bmatrix}.$$

Now,

$$\varrho_1 \sin \hbar_1 \diamond \varrho_2 \sin \hbar_2 \quad = \begin{bmatrix} \sqrt[\wp_1]{\begin{array}{c} \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\varrho_1}\right) + \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{T}})^{\wp_1}\right)^{\varrho_2}\right) \\ -\left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\varrho_1}\right) \cdot \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{T}})^{\wp_1}\right)^{\varrho_2}\right) \end{array}}, \\ \sqrt[\wp_2]{\begin{array}{c} \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\varrho_1}\right) + \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{I}})^{\wp_2}\right)^{\varrho_2}\right) \\ -\left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\varrho_1}\right) \cdot \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{I}})^{\wp_2}\right)^{\varrho_2}\right) \end{array}}, \\ ((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{F}})^{\wp_3})^{\varrho_1} \cdot ((\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{F}})^{\wp_3})^{\varrho_2} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\wp_1]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\varrho_1} \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{T}})^{\wp_1}\right)^{\varrho_2}}, \\ \sqrt[\wp_1]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\varrho_1} \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{I}})^{\wp_2}\right)^{\varrho_2}}, \\ ((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{F}})^{\wp_3})^{\varrho_1} \cdot ((\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{F}})^{\wp_3})^{\varrho_2} \end{bmatrix}$$

Hence,

$$ST(\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2) = \begin{bmatrix} \sqrt[\wp_1]{1 - \bigodot_{i=1}^{2} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}}, \\ \sqrt[\wp_2]{1 - \bigodot_{i=1}^{2} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}}, \\ \bigodot_{i=1}^{2} ((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3})^{\varrho_i} \end{bmatrix}.$$

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It valid for $n \geq 3$,

$$(\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2, ..., \hbar_l) = \begin{bmatrix} \sqrt[\wp_1]{1 - \bigodot_{i=1}^{l} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}}, \\ \sqrt[\wp_2]{1 - \bigodot_{i=1}^{l} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}}, \\ \bigodot_{i=1}^{l}((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3})^{\varrho_i} \end{bmatrix}.$$

If $n = l + 1$, then ST$(\wp_1, \wp_2, \wp_3)$ NWA $(\hbar_1, \hbar_2, ..., \hbar_l, \hbar_{l+1})$

$$= \begin{bmatrix} \sqrt[\wp_1]{\begin{array}{c} \Diamond_{i=1}^{l}\left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}\right) + \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_{l+1}^{\mathscr{T}})^{\wp_1}\right)^{\varrho_{l+1}}\right) \\ - \bigodot\limits_{i=1}^{l} \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}\right) \cdot \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_{l+1}^{\mathscr{T}})^{\wp_1}\right)^{\varrho_{l+1}}\right), \end{array}} \\ \sqrt[\wp_2]{\begin{array}{c} \Diamond_{i=1}^{l}\left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}\right) + \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_{l+1}^{\mathscr{I}})^{\wp_2}\right)^{\varrho_{l+1}}\right) \\ - \bigodot\limits_{i=1}^{l} \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}\right) \cdot \left(1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_{l+1}^{\mathscr{I}})^{\wp_2}\right)^{\varrho_{l+1}}\right), \end{array}} \\ \bigodot_{i=1}^{l}((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3})^{\varrho_i} \cdot ((\sin^2 \Bbbk \cdot \Xi_{l+1}^{\mathscr{F}})^{\wp_3})^{\varrho_{l+1}}, \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\wp_1]{1 - \bigodot\limits_{i=1}^{l+1} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}}, \\ \sqrt[\wp_2]{1 - \bigodot\limits_{i=1}^{l+1} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}}, \\ \bigodot_{i=1}^{l+1}((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3})^{\varrho_i} \end{bmatrix}.$$

Theorem 4.3. *Let* $\sin \hbar_i = \left(\sin \Bbbk \cdot \Xi_i^{\mathscr{T}}, \sin \Bbbk \cdot \Xi_i^{\mathscr{I}}, \sin \Bbbk \cdot \Xi_i^{\mathscr{F}}\right)$ *be the ST*$(\wp_1, \wp_2, \wp_3)$ *NNs. Then ST*$(\wp_1, \wp_2, \wp_3)$ *NWA* $(\hbar_1, \hbar_2, ..., \hbar_n) = \sin \hbar$ *(idempotency property).*

Proof. Since $\sin \Bbbk \cdot \Xi_i^{\mathscr{T}} = \sin \Bbbk \cdot \Xi^{\mathscr{T}}$, $\sin \Bbbk \cdot \Xi_i^{\mathscr{I}} = \sin \Bbbk \cdot \Xi^{\mathscr{I}}$ and $\sin \Bbbk \cdot \Xi_i^{\mathscr{F}} = \sin \Bbbk \cdot \Xi^{\mathscr{F}}$ and $\Diamond_{i=1}^{n} \varrho_i = 1$. Now, $(\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2, ..., \hbar_n)$

$$= \begin{bmatrix} \sqrt[\wp_1]{1 - \bigodot\limits_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}}, \\ \sqrt[\wp_2]{1 - \bigodot\limits_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}}, \\ \bigodot_{i=1}^{n}((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3})^{\varrho_i} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\wp_1]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}})^{\wp_1}\right)^{\Diamond_{i=1}^{n}\varrho_i}}, \\ \sqrt[\wp_2]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}})^{\wp_2}\right)^{\Diamond_{i=1}^{n}\varrho_i}}, \\ ((\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3})^{\Diamond_{i=1}^{n}\varrho_i}, \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} \sqrt[\wp_1]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}})^{\wp_1}\right)}, \\ \sqrt[\wp_2]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}})^{\wp_2}\right)}, \\ (\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3} \end{bmatrix} \\ &= \sin \hbar. \end{aligned}$$

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Theorem 4.4. *Let* $\hbar_i = \left(\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}}\right)$ *be the ST*$(\wp_1, \wp_2, \wp_3)$ *NNs. Then ST*$(\wp_1, \wp_2, \wp_3)$ *NWA*$(\hbar_1, \hbar_2, ..., \hbar_n)$ *where* $\underline{\Xi^{\mathscr{T}}} = \min \Xi_{ij}^{\mathscr{T}}$, $\overline{\Xi^{\mathscr{T}}} = \max \Xi_{ij}^{\mathscr{T}}$, $\underline{\Xi^{\mathscr{I}}} = \min \Xi_{ij}^{\mathscr{I}}$, $\overline{\Xi^{\mathscr{I}}} = \max \Xi_{ij}^{\mathscr{I}}$, $\underline{\Xi^{\mathscr{F}}} = \min \Xi_{ij}^{\mathscr{F}}$, $\overline{\Xi^{\mathscr{F}}} = \max \Xi_{ij}^{\mathscr{F}}$ *and where* $1 \leq i \leq n,\ j = 1, 2, ..., i_j$. *Then,* $\underline{\Xi^{\mathscr{T}}}, \underline{\Xi^{\mathscr{I}}}, \overline{\Xi^{\mathscr{F}}}$

$$
\begin{aligned}
&\leq (\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2, ..., \hbar_n) \\
&\leq \overline{\Xi^{\mathscr{T}}}, \overline{\Xi^{\mathscr{I}}}, \underline{\Xi^{\mathscr{F}}}].
\end{aligned}
$$

(Boundedness property).

Proof. Since, $\underline{\Xi^{\mathscr{T}}} = \min \Xi_{ij}^{\mathscr{T}}$, $\overline{\Xi^{\mathscr{T}}} = \max \Xi_{ij}^{\mathscr{T}}$ and $\underline{\Xi^{\mathscr{T}}} \leq \Xi_{ij}^{\mathscr{T}} \leq \overline{\Xi^{\mathscr{T}}}$.
Now,

$$
\begin{aligned}
\underline{\Xi^{\mathscr{T}}} &= \sqrt[\wp_1]{1 - \bigodot_{i=1}^{n} \left(1 - (\underline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}})^{\wp_1}\right)^{\varrho_i}} \\
&\leq \sqrt[\wp_1]{1 - \bigodot_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}} \\
&\leq \sqrt[\wp_1]{1 - \bigodot_{i=1}^{n} \left(1 - (\overline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}})^{\wp_1}\right)^{\varrho_i}}
\end{aligned}
$$

Since, $\underline{\Xi^{\mathscr{I}}} = \min \Xi_{ij}^{\mathscr{I}}$, $\overline{\Xi^{\mathscr{I}}} = \max \Xi_{ij}^{\mathscr{I}}$ and $\underline{\Xi^{\mathscr{I}}} \leq \Xi_{ij}^{\mathscr{I}} \leq \overline{\Xi^{\mathscr{I}}}$.
Now,

$$
\begin{aligned}
\underline{\Xi^{\mathscr{I}}} &= \sqrt[\wp_2]{1 - \bigodot_{i=1}^{n} \left(1 - (\underline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}})^{\wp_2}\right)^{\varrho_i}} \\
&\leq \sqrt[\wp_2]{1 - \bigodot_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}} \\
&\leq \sqrt[\wp_2]{1 - \bigodot_{i=1}^{n} \left(1 - (\overline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}})^{\wp_2}\right)^{\varrho_i}}.
\end{aligned}
$$

Since, $\underline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}} = \min(\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{F}})^{\wp_3}$, $\overline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}} = \max(\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{F}})^{\wp_3}$ and $\overline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}} \leq (\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{F}})^{\wp_3} \leq \underline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}}$. We have, $\overline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}} = \bigodot_{i=1}^{n}(\overline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}})^{\varrho_i} \leq \bigodot_{i=1}^{n}((\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{F}})^{\wp_3})^{\varrho_i} \leq \bigodot_{i=1}^{n}(\underline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}})^{\varrho_i} = \underline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}}$.
Therefore,

$$
\begin{aligned}
&\frac{1}{2} \times \left[\begin{array}{c} \left(\sqrt[\wp_1]{1 - \bigodot_{i=1}^{n} \left(1 - (\underline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}})^{\wp_1}\right)^{\varrho_i}}\right)^2 - \left(\sqrt[\wp_2]{1 - \bigodot_{i=1}^{n} \left(1 - (\underline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}})^{\wp_2}\right)^{\varrho_i}}\right)^2 \\ +1 - \left(\bigodot_{i=1}^{n}((\overline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}})^{\wp_3})^{\varrho_i}\right)^2 \end{array}\right] \\
\leq\ &\frac{1}{2} \times \left[\begin{array}{c} \left(\sqrt[\wp_1]{1 - \bigodot_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{T}})^{\wp_1}\right)^{\varrho_i}}\right)^2 - \left(\sqrt[\wp_2]{1 - \bigodot_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{I}})^{\wp_2}\right)^{\varrho_i}}\right)^2 + \\ +1 - \left(\bigodot_{i=1}^{n}((\sin^2 \Bbbk \cdot \Xi_{ij}^{\mathscr{F}})^{\wp_3})^{\varrho_i}\right)^2 \end{array}\right] \\
\leq\ &\frac{1}{2} \times \left[\begin{array}{c} \left(\sqrt[\wp_1]{1 - \bigodot_{i=1}^{n} \left(1 - (\overline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}})^{\wp_1}\right)^{\varrho_i}}\right)^2 - \left(\sqrt[\wp_2]{1 - \bigodot_{i=1}^{n} \left(1 - (\overline{\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}})^{\wp_2}\right)^{\varrho_i}}\right)^2 + \\ +1 - \left(\bigodot_{i=1}^{n}(\underline{(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}})^{\wp_3}})^{\varrho_i}\right)^2 \end{array}\right].
\end{aligned}
$$

Hence, $\underline{\Xi^{\mathscr{T}}}, \underline{\Xi^{\mathscr{I}}}, \overline{\Xi^{\mathscr{F}}} \leq ST(\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2, ..., \hbar_n) \leq \overline{\Xi^{\mathscr{T}}}, \overline{\Xi^{\mathscr{I}}}, \underline{\Xi^{\mathscr{F}}}$.

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Theorem 4.5. *Let* $\hbar_i = (\Xi^{\mathscr{T}}_{t_{ij}}, \Xi^{\mathscr{I}}_{t_{ij}}, \Xi^{\mathscr{F}}_{t_{ij}})$ *and* $W_i = (\Xi^{\mathscr{T}}_{h_{ij}}, \Xi^{\mathscr{I}}_{h_{ij}}, \Xi^{\mathscr{F}}_{h_{ij}})$ *be the ST*$(\wp_1, \wp_2, \wp_3)$ *NWAs. For any* i, *if there is* $(\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{t_{ij}})^2 \leq (\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{h_{ij}})^2$ *and* $(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{t_{ij}})^2 \leq (\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{h_{ij}})^2$ *and* $(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{t_{ij}})^2 \geq (\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{h_{ij}})^2$ *or* $\hbar_i \leq W_i$. *Prove that* $(\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2, ..., \hbar_n) \leq ST(\wp_1, \wp_2, \wp_3)NWA(W_1, W_2, ..., W_n)$, *where* $(i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *(monotonicity property).*

Proof. For any i, $(\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{t_{ij}})^2 \leq \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{h_{ij}}\right)^2$.
Therefore, $1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{t_i})^2 \geq 1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{h_i})^2$.
Hence, $\bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{t_i}\right)^2\right)^{\varrho_i} \geq \bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{h_i}\right)^2\right)^{\varrho_i}$
and $\sqrt[\wp_1]{1 - \bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{t_i}\right)^{\wp_1}\right)^{\varrho_i}} \leq \sqrt[\wp_1]{1 - \bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{h_i}\right)^{\wp_1}\right)^{\varrho_i}}$.
For any i, $(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{t_{ij}})^{\wp_2} \leq \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{h_{ij}}\right)^{\wp_2}$.
Therefore, $1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{t_i}\right)^{\wp_2} \geq 1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{h_i}\right)^{\wp_2}$.
Hence, $\bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{t_i}\right)^{\wp_2}\right)^{\varrho_i} \geq \bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{h_i}\right)^{\wp_2}\right)^{\varrho_i}$.
This implies that $\sqrt[\wp_2]{1 - \bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{t_i}\right)^{\wp_2}\right)^{\varrho_i}} \leq \sqrt[\wp_2]{1 - \bigodot_{i=1}^n \left(1 - \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{h_i}\right)^{\wp_2}\right)^{\varrho_i}}$.
For any i, $\left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{t_{ij}}\right)^2 \geq \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{h_{ij}}\right)^2$ and $\left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{t_{ij}}\right)^{\wp_3} \geq \left(\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{h_{ij}}\right)^{\wp_3}$.
Therefore, $1 - \left(\bigodot_{i=1}^n \sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{t_{ij}}\right)^{\wp_3} \leq 1 - \left(\bigodot_{i=1}^n \sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{h_{ij}}\right)^{\wp_3}$.

$$
\begin{aligned}
& \frac{1}{2} \times \left[\begin{array}{c} \left(\sqrt[\wp_1]{1 - \bigodot\limits_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{ti})^{\wp_1}\right)^{\varrho_i}}\right)^2 - \left(\sqrt[\wp_2]{1 - \bigodot\limits_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_{ti})^{\wp_2}\right)^{\varrho_i}}\right)^2 \\ +1 - \left(\bigodot_{i=1}^n (\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{tij})^{\wp_3}\right)^2 \end{array} \right] \\
\leq \; & \frac{1}{2} \times \left[\begin{array}{c} \left(\sqrt[\wp_1]{1 - \bigodot\limits_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_{hi})^{\wp_1}\right)^{\varrho_i}}\right)^2 - \left(\sqrt[\wp_2]{1 - \bigodot\limits_{i=1}^{n} \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}^{\sin \hbar}}_{hi})^{\wp_2}\right)^{\varrho_i}}\right)^2 \\ +1 - \left(\bigodot_{i=1}^n (\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_{hij})^{\wp_3}\right)^2 \end{array} \right].
\end{aligned}
$$

Hence, $ST(\wp_1, \wp_2, \wp_3)NWA\,(\hbar_1, \hbar_2, ..., \hbar_n) \leq ST(\wp_1, \wp_2, \wp_3)NWA\,(W_1, W_2, ..., W_n)$.

4.2 ST$(\wp_1, \wp_2, \wp_3)$ NWG

Definition 4.6. Let $\hbar_i = \left(\Xi^{\mathscr{T}}_i, \Xi^{\mathscr{I}}_i, \Xi^{\mathscr{F}}_i\right)$ be the ST$(\wp_1, \wp_2, \wp_3)$NNs. Then ST$(\wp_1, \wp_2, \wp_3)$NWG $(\hbar_1, \hbar_2, ..., \hbar_n) = \bigodot_{i=1}^n \sin \hbar_i^{\varrho_i}$.

Theorem 4.7. *Let* $\hbar_i = \left(\Xi^{\mathscr{T}}_i, \Xi^{\mathscr{I}}_i, \Xi^{\mathscr{F}}_i\right)$ *be the ST*$(\wp_1, \wp_2, \wp_3)$ *NNs. Then*

$$
ST(\wp_1, \wp_2, \wp_3)NWG(\hbar_1, \hbar_2, ..., \hbar_n) = \left[\begin{array}{c} \bigodot_{i=1}^n ((\sin^2 \Bbbk \cdot \Xi^{\mathscr{T}}_i)^{\wp_1})^{\varrho_i}; \\ \sqrt[\wp_2]{1 - \bigodot_{i=1}^n \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{I}}_i)^{\wp_2}\right)^{\varrho_i}}, \\ \sqrt[\wp_3]{1 - \bigodot_{i=1}^n \left(1 - (\sin^2 \Bbbk \cdot \Xi^{\mathscr{F}}_i)^{\wp_3}\right)^{\varrho_i}} \end{array} \right].
$$

Proof. It follows from Theorem 4.2.

Theorem 4.8. *Let* $\sin \hbar_i = \left(\sin \Bbbk \cdot \Xi^{\mathscr{T}}_i, \sin \Bbbk \cdot \Xi^{\mathscr{I}}_i, \sin \Bbbk \cdot \Xi^{\mathscr{F}}_i\right)$ *be the ST*$(\wp_1, \wp_2, \wp_3)$*NNs and all are equal. Then ST*$(\wp_1, \wp_2, \wp_3)$*NWG*$(\hbar_1, \hbar_2, ..., \hbar_n) = \sin \hbar$.

Proof. It follows from Theorem 4.3.

Remark 4.9. It has other properties, including boundedness and monotonicity, as well as having ST$(\wp_1, \wp_2, \wp_3)$NWG.

Proof. It follows from Theorem 4.4 and Theorem 4.5.

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4.3 Generalized ST$(\wp_1, \wp_2, \wp_3)$ NWA (GST$(\wp_1, \wp_2, \wp_3)$NWA)

Definition 4.10. Let $\hbar_i = \left(\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}}\right)$ be the ST$(\wp_1, \wp_2, \wp_3)$NN. Then GST$(\wp_1, \wp_2, \wp_3)$ NWA $(\hbar_1, \hbar_2, ..., \hbar_n) = \left(\diamond_{i=1}^{n} \varrho_i (\sin \hbar_i)^{\mho}\right)^{1/\mho}$.

Theorem 4.11. *Let $\hbar_i = \left(\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}}\right)$ be the ST$(\wp_1, \wp_2, \wp_3)$ NNs. Then*

$$GST(\wp_1, \wp_2, \wp_3)NWA(\hbar_1, \hbar_2, ..., \hbar_n) = \begin{bmatrix} \left(\sqrt[\wp_1]{1 - \bigodot\limits_{i=1}^{n} \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}}\right)^{1/\wp_1}, \\ \left(\sqrt[\wp_2]{1 - \bigodot\limits_{i=1}^{n} \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}}\right)^{1/\wp_2}, \\ \sqrt[\wp_3]{1 - \left(1 - \left(\bigodot\limits_{i=1}^{n} \left(\sqrt[\wp_3]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_i}\right)^{\wp_3}\right)^{1/\wp_3}} \end{bmatrix}.$$

Proof. We can prove this first by demonstrating that,

$$\diamond_{i=1}^{n} \varrho_i (\sin \hbar_i)^{\mho} = \begin{bmatrix} \sqrt[\wp_1]{1 - \bigodot\limits_{i=1}^{n} \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}}, \\ \sqrt[\wp_2]{1 - \bigodot\limits_{i=1}^{n} \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}}, \\ \bigodot\limits_{i=1}^{n} \left(\sqrt[\wp_3]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_i} \end{bmatrix}.$$

Put $n = 2$,

$\varrho_1 (\sin \hbar_1)^{\mho} \diamond \varrho_2 (\sin \hbar_2)^{\mho}$

$$= \begin{bmatrix} \sqrt[\wp_1]{\begin{matrix} \left(\sqrt[\wp_1]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_1}}\right)^{\wp_1} + \left(\sqrt[\wp_1]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_1}}\right)^{\wp_1}, \\ -\left(\sqrt[\wp_1]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_1}}\right)^{\wp_1} \cdot \left(\sqrt[\wp_1]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_1}}\right)^{\wp_1} \end{matrix}} \\ , \\ \sqrt[\wp_2]{\begin{matrix} \left(\sqrt[\wp_2]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_1}}\right)^{\wp_2} + \left(\sqrt[\wp_2]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_1}}\right)^{\wp_2}, \\ -\left(\sqrt[\wp_2]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_1}}\right)^{\wp_2} \cdot \left(\sqrt[\wp_2]{1 - \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_1}}\right)^{\wp_2} \end{matrix}} \\ \left(\sqrt[\wp_3]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_1} \cdot \left(\sqrt[\wp_3]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_2^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_1}, \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\wp_1]{1 - \bigodot_{i=1}^{2} \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}}, \\ \sqrt[\wp_2]{1 - \bigodot_{i=1}^{2} \left(1 - \left((\sin^2 \Bbbk \cdot \Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}}, \\ \bigodot_{i=1}^{2} \left(\sqrt[\wp_2]{1 - \left(1 - (\sin^2 \Bbbk \cdot \Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_i} \end{bmatrix}.$$

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Hence, $\diamond_{i=1}^{l}\varrho_i(\sin\hbar_i)^{\mho}=\begin{bmatrix}\sqrt[\wp_1]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\Bbbk\cdot\Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}},\\ \sqrt[\wp_2]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\Bbbk\cdot\Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}},\\ \bigodot_{i=1}^{l}\left(\sqrt[\wp_3]{1-\left(1-(\sin^2\Bbbk\cdot\Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_i}\end{bmatrix}.$

If $n=l+1$, then $\diamond_{i=1}^{l}\varrho_i(\sin\hbar_i)^{\mho}+\varrho_{l+1}(\sin\hbar_{l+1})^{\mho}=\diamond_{i=1}^{l+1}\varrho_i(\sin\hbar_i)^{\mho}$.
Now,$\diamond_{i=1}^{l}\varrho_i(\sin\hbar_i)^{\mho}+\varrho_{l+1}(\sin\hbar_{l+1})^{\mho}=\varrho_1(\sin\hbar_1)^{\mho}\diamond\varrho_2(\sin\hbar_2)^{\mho}\diamond...\diamond\varrho_l(\sin\hbar_l)^{\mho}\diamond\varrho_{l+1}(\sin\hbar_{l+1})^{\mho}$

$$=\begin{bmatrix}\sqrt[\wp_1]{\begin{array}{c}\left(\sqrt[\wp_1]{1-\bigodot\limits_{i=1}^{l}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}}\right)^{\wp_1}+\left(\sqrt[\wp_1]{1-\left(1-\left((\sin^2\Bbbk\cdot\Xi_{l+1}^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_1}}\right)^{\wp_1}\\ -\left(\sqrt[\wp_1]{1-\bigodot\limits_{i=1}^{l}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}}\right)^{\wp_1}\cdot\left(\sqrt[\wp_1]{1-\left(1-\left((\sin^2\Bbbk\cdot\Xi_{l+1}^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_1}}\right)^{\wp_1}\end{array}},\\ \sqrt[\wp_2]{\begin{array}{c}\left(\sqrt[\wp_2]{1-\bigodot\limits_{i=1}^{l}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}}\right)^{\wp_2}+\left(\sqrt[\wp_2]{1-\left(1-\left((\sin^2\Bbbk\cdot\Xi_{l+1}^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_1}}\right)^{\wp_2}\\ -\left(\sqrt[\wp_2]{1-\bigodot\limits_{i=1}^{l}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}}\right)^{\wp_2}\cdot\left(\sqrt[\wp_2]{1-\left(1-\left((\sin^2\Bbbk\cdot\Xi_{l+1}^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_1}}\right)^{\wp_2}\end{array}},\\ \bigodot\limits_{i=1}^{l}\left(\sqrt[\wp_3]{1-\left(1-(\sin^2\Bbbk\cdot\Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_i}\cdot\left(\sqrt[\wp_3]{1-\left(1-(\sin^2\Bbbk\cdot\Xi_{l+1}^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_1}\end{bmatrix}$$

$$\diamond_{i=1}^{l+1}\varrho_i(\sin\hbar_i)^{\mho}=\begin{bmatrix}\sqrt[\wp_1]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\Bbbk\cdot\Xi_1^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}},\\ \sqrt[\wp_2]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\Bbbk\cdot\Xi_1^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}},\\ \bigodot_{i=1}^{l+1}\left(\sqrt[\wp_3]{1-\left(1-(\sin^2\Bbbk\cdot\Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_i}\end{bmatrix}.$$

$$\left(\diamond_{i=1}^{l+1}\varrho_i(\sin\hbar_i)^{\mho}\right)^{1/\mho}=\begin{bmatrix}\left(\sqrt[\mho]{1-\bigodot\limits_{i=1}^{l+1}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}\right)^{\varrho_i}}\right)^{1/\wp_1},\\ \left(\sqrt[\wp_2]{1-\bigodot\limits_{i=1}^{l+1}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}}\right)^{1/\wp_2},\\ \sqrt[\wp_3]{1-\left(1-\left(\bigodot\limits_{i=1}^{l+1}\left(\sqrt[\wp_3]{1-\left(1-(\sin^2\Bbbk\cdot\Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}}\right)^{\varrho_i}\right)^2\right)}^{1/\wp_3}\end{bmatrix}$$

Remark 4.12. An operator modified from the GST$(\wp_1,\wp_2,\wp_3)$ NWA operator to the ST$(\wp_1,\wp_2,\wp_3)$ NWA operator is performed if $\mho=1$.

Theorem 4.13. *If all* $\sin\hbar_i=\left(\sin\Xi_i^{\mathscr{T}},\sin\Xi_i^{\mathscr{I}},\sin\Xi_i^{\mathscr{F}}\right)$ *and all are equal. Then GST*$(\wp_1,\wp_2,\wp_3)$ *NWA*$(\hbar_1,\hbar_2,...,\hbar_n)=\sin\hbar$.

Proof. There is a proof based on the Theorem 4.3.

Remark 4.14. In the GST$(\wp_1,\wp_2,\wp_3)$ NWA operator, boundedness and monotonicity are satisfied.

Proof. There is a proof based on the Theorem 4.4 and Theorem 4.5.

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4.4 Generalized ST$(\wp_1, \wp_2, \wp_3)$NWG (GST$(\wp_1, \wp_2, \wp_3)$NWG)

Definition 4.15. Let $\hbar_i = \left(\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}}\right)$ be the ST$(\wp_1, \wp_2, \wp_3)$NNs. Then GST$(\wp_1, \wp_2, \wp_3)$NWG $(\hbar_1, \hbar_2, ..., \hbar_n) = \frac{1}{\mho}\left(\bigodot_{i=1}^{n}(\mho(\sin\hbar_i))^{\varrho_i}\right)$.

Theorem 4.16. *Let $\hbar_i = \left(\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}}\right)$ be the ST$(\wp_1, \wp_2, \wp_3)$NNs. Then GST$(\wp_1, \wp_2, \wp_3)$NWG$(\hbar_1, \hbar_2, ..., \hbar_n)$*

$$= \left[\begin{array}{c} \sqrt[\wp_1]{1-\left(1-\left(\bigodot_{i=1}^{n}\left(\sqrt[\wp_1]{1-\left(1-(\sin^2\Bbbk\cdot\Xi_i^{\mathscr{T}})^{\wp_1}\right)^{\wp_1}}\right)^{\varrho_i}\right)^{\wp_1}\right)^{1/\wp_1}}, \\ \left(\sqrt[\wp_2]{1-\bigodot_{i=1}^{n}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{I}})^{\wp_2}\right)^{\wp_2}\right)^{\varrho_i}}\right)^{1/\wp_2}, \\ \left(\sqrt[\wp_3]{1-\bigodot_{i=1}^{n}\left(1-\left((\sin^2\Bbbk\cdot\Xi_i^{\mathscr{F}})^{\wp_3}\right)^{\wp_3}\right)^{\varrho_i}}\right)^{1/\wp_3} \end{array}\right].$$

Proof. As a conclusion, we can say that Theorem 4.11 is based on this proof.

Remark 4.17. There is a conversion that takes place when $\mho = 1$, which converts the GST$(\wp_1, \wp_2, \wp_3)$ NWG into the ST$(\wp_1, \wp_2, \wp_3)$NWG.

Remark 4.18. Boundness and monotonicity properties that are satisfied by GST$(\wp_1, \wp_2, \wp_3)$NWG operators.

Proof. The following proof builds on Theorem 4.4 and Theorem 4.5.

Theorem 4.19. *If all $\sin\hbar_i = \left(\sin\Xi_i^{\mathscr{T}}, \sin\Xi_i^{\mathscr{I}}, \sin\Xi_i^{\mathscr{F}}\right)$ are equal. Then GST$(\wp_1, \wp_2, \wp_3)$NWG$(\hbar_1, \hbar_2, ..., \hbar_n) = \sin\hbar$.*

5 MADM approach based on ST$(\wp_1, \wp_2, \wp_3)$ NN

Let $\hbar = \{\hbar_1, \hbar_2, ..., \hbar_n\}$ be the set of n-alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the set of m-attributes, $w = \{w_1, w_2, ..., w_m\}$ be the weights of attributes, for $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$ $\sin\hbar_{ij} = \left(\Xi_{ij}^{\mathscr{T}}, \Xi_{ij}^{\mathscr{I}}, \Xi_{ij}^{\mathscr{F}}\right)$ denote ST$(\wp_1, \wp_2, \wp_3)$ NN of alternative $\sin\hbar_i$ in attribute C_j. Since $\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}} \in [0, 1]$ and $0 \le (\sin^2\Bbbk\cdot\Xi_i^{\mathscr{T}})^{\wp_1} + (\sin^2\Bbbk\cdot\Xi_i^{\mathscr{I}})^{\wp_2} + (\sin^2\Bbbk\cdot\Xi_i^{\mathscr{F}})^{\wp_3} \le 1$, where $\wp_1, \wp_2, \wp_3$ are a positive integers. A decision is reached using the following algorithm.

5.1 Algorithm for ST$(\wp_1, \wp_2, \wp_3)$ NN

Step-1: The decision values for ST$(\wp_1, \wp_2, \wp_3)$ NN should be entered.
Step-2: The aggregate value of each alternative should be found. On the basis of ST$(\wp_1, \wp_2, \wp_3)$ NN information aggregation operators, attribute C_j in $\widehat{\hbar}_i$, $\hbar_{ij} = \left(\Xi_{ij}^{\mathscr{T}}, \Xi_{ij}^{\mathscr{I}}, \Xi_{ij}^{\mathscr{F}}\right)$ is aggregated into $\hbar_i = \left(\Xi_i^{\mathscr{T}}, \Xi_i^{\mathscr{I}}, \Xi_i^{\mathscr{F}}\right)$.
Step-3: To calculate the positive and negative ideal values for each alternative, follow these steps: $\overrightarrow{\hbar}^{+} = (1, 1, 0)$ $\overrightarrow{\hbar}^{-} = (0, 0, 1)$.
Step-4: To obtain the EDs between each alternative with two ideal values, follow the steps below:

$$\mathfrak{D}_i^{+} = \mathfrak{D}_E\left(\overrightarrow{\hbar_i}, \overrightarrow{\hbar}^{+}\right);\ \mathfrak{D}_i^{-} = \mathfrak{D}_E\left(\overrightarrow{\hbar_i}, \overrightarrow{\hbar}^{-}\right).$$

Step-5: The following formula is used to compute relative closeness:

$$\mathfrak{D}_i^{*} = \frac{\mathfrak{D}_i^{-}}{\mathfrak{D}_i^{+} + \mathfrak{D}_i^{-}}.$$

Step-6: Based on the output, we can determine that a value of $\max\mathfrak{D}_i^{*}$ is optimal, and therefore we should choose that value as our optimal solution.

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5.2 Selection on robot sensors

A robot's operating conditions and environment are monitored by a variety of sensors. Transmitting these signals with a controller. Humans have sensory organs similar to those of robots. Robots must be able to understand their environment thoroughly in order to function effectively. Measurements can be made by sensors. These inputs can be used to generate and record electrical signals. Sensors for robots should be chosen according to their use and the environment in which they operate. The accuracy, precision, and sensitivity of sensors that measure the same input may differ as well. Keeping this in mind is important when designing robots. Sensors can measure inputs in many ways. It contains a variety of features such as environmental monitoring, force sensors, speed sensors, pressure sensors, temperature sensors, vibration sensors, rotation sensors, imaging sensors, light sensors, biometrics, acceleration sensors, current sensors, orientation sensors, gravity sensors, and speech recognition. The automation of robotics requires sensors. A robot cannot function without them. As robots interact with their environments, they obtain information from them using sensors. An application determines what type of sensor is appropriate based on the characteristics of a sensor. Light sensor $(\hbar_a)$, Proximity sensor $(\hbar_b)$, Sound sensor $(\hbar_c)$, Temperature sensor $(\hbar_d)$, Acceleration sensor $(\hbar_e)$ are the five-type of robotic sensors (alternative), can choose the best one. The selection of sensors using a set of criteria. It is important to consider the following factors when comparing sensors: Scores are assigned to sensors based on the following attributes: $C = \{C_1$: Resolution, C_2: Sensitivity, C_3 : Error, C_4 : Environment$\}$ and their weights are $w = \{0.4, 0.3, 0.2, 0.1\}$. The following are the DM details:

	C_1	C_2	C_3	C_4
$\hbar_a$	$(0.8, 0.6, 0.85)$	$(0.85, 0.9, 0.85)$	$(0.8, 0.85, 0.75)$	$(0.9, 0.75, 0.8)$
$\hbar_b$	$(0.85, 0.7, 0.8)$	$(0.8, 0.85, 0.75)$	$(0.85, 0.75, 0.8)$	$(0.85, 0.8, 0.85)$
$\hbar_c$	$((0.7, 0.8, 0.85)$	$(0.8, 0.85, 0.7)$	$(0.7, 0.8, 0.75)$	$(0.8, 0.7, 0.75)$
$\hbar_d$	$(0.8, 0.85, 0.9)$	$(0.85, 0.8, 0.75)$	$(0.8, 0.85, 0.8)$	$(0.85, 0.9, 0.8)$
$\hbar_e$	$(0.7, 0.9, 0.85)$	$(0.8, 0.85, 0.9)$	$(0.95, 0.7, 0.75)$	$(0.9, 0.75, 0.85)$

Case-(i): Aggregate information with ST$(\wp_1, \wp_2, \wp_3)$ NWA operators are as follows:

	ST$(\wp_1, \wp_2, \wp_3)$ NWA operator $(1, 1, 1)$
$\overrightarrow{\hbar_a}$	$(0.4327, 0.7114, 0.6475)$
$\overrightarrow{\hbar_b}$	$(0.6775, 0.8248, 0.1885)$
$\overrightarrow{\hbar_c}$	$(0.0427, 0.4050, 0.3121)$
$\overrightarrow{\hbar_d}$	$(0.4909, 0.6406, 0.3033)$
$\overrightarrow{\hbar_e}$	$(0.1660, 0.7296, 0.6757)$

To determine the ideal values of positive and negative alternatives, follow these steps:

$$\overrightarrow{\hbar}^{+} = (1, 1, 0) \text{ and } \overrightarrow{\hbar}^{-} = (0, 0, 1)$$

Following are the HD values between alternative options with both ideal values:

$\mathfrak{D}_1^+$	$\mathfrak{D}_2^+$	$\mathfrak{D}_3^+$	$\mathfrak{D}_4^+$	$\mathfrak{D}_5^+$
0.5536	0.1926	0.1947	0.1960	0.7210

$\mathfrak{D}_1^-$	$\mathfrak{D}_2^-$	$\mathfrak{D}_3^-$	$\mathfrak{D}_4^-$	$\mathfrak{D}_5^-$
0.1964	0.5574	0.5553	0.5540	0.0290

As a result, the closeness values should be calculated as follows:

$\mathfrak{D}_1^*$	$\mathfrak{D}_2^*$	$\mathfrak{D}_3^*$	$\mathfrak{D}_4^*$	$\mathfrak{D}_5^*$
0.2619	0.7432	0.7404	0.7386	0.0386

The following alternative rankings are provided:

$$\hbar_b > \hbar_c > \hbar_d > \hbar_a > \hbar_e.$$

Consequently, $\hbar_b$ is the best option.

Case-(ii): Aggregate information with ST$(\wp_1, \wp_2, \wp_3)$ NWG operators are as follows:

	ST$(\wp_1, \wp_2, \wp_3)$ NWA operator $(1,1,1)$
$\overrightarrow{\hbar_a}$	$(0.1643, 0.7114, 0.9061)$
$\overrightarrow{\hbar_b}$	$(0.3743, 0.8248, 0.8738)$
$\overrightarrow{\hbar_c}$	$(0.0391, 0.4050, 0.9191)$
$\overrightarrow{\hbar_d}$	$(0.1761, 0.6406, 0.8767)$
$\overrightarrow{\hbar_e}$	$(0.0795, 0.7296, 0.8884)$

To determine the ideal values of positive and negative alternatives, follow these steps:

$$\overrightarrow{\hbar}^+ = (1,1,0) \text{ and } \overrightarrow{\hbar}^- = (0,0,1)$$

Following are the HD values between alternative options with both ideal values:

$\mathfrak{D}_1^+$	$\mathfrak{D}_2^+$	$\mathfrak{D}_3^+$	$\mathfrak{D}_4^+$	$\mathfrak{D}_5^+$
0.9751	0.9778	0.7555	0.8610	0.9865

$\mathfrak{D}_1^-$	$\mathfrak{D}_2^-$	$\mathfrak{D}_3^-$	$\mathfrak{D}_4^-$	$\mathfrak{D}_5^-$
0.2251	0.2278	0.0055	0.1110	0.2365

As a result, the closeness values should be calculated as follows:

$\mathfrak{D}_1^*$	$\mathfrak{D}_2^*$	$\mathfrak{D}_3^*$	$\mathfrak{D}_4^*$	$\mathfrak{D}_5^*$
0.1876	0.1889	0.0072	0.1142	0.1934

The following alternative rankings are provided:

$$\hbar_e > \hbar_b > \hbar_a > \hbar_d > \hbar_c.$$

Consequently, $\hbar_e$ is the best option.

5.3 Analysis and discussion

As a result of the above information, we propose to apply the following strategies: ST$(\wp_1, \wp_2, \wp_3)$ NWA, NWG, GNWA and GNWG based on HD. Distances can be categorized as follows:

	$Weighted\ averaging$	$Weighted\ geometric$
$TOPSIS - Hamming\ distance^{28}$	$\hbar_a > \hbar_b > \hbar_e > \hbar_c > \hbar_d$	$\hbar_a > \hbar_b > \hbar_d > \hbar_e > \hbar_c$

Comparing the characteristics of several existing algorithms with those of the proposed algorithm illustrates the advantages of the proposed algorithm. ✓ and × are symbols used to indicate whether the operator's property is met or not. A further problem is that existing approaches refer only to MADM problems without discussing GDM problems. The Hamming distances can be used to compare the proposed and existing models and determine their usefulness and validity based on the Table.

$Hamming\ distance$	$weighted\ averaging$	$weighted\ geometric$
Palanikumar et al.[28]	×	×

Change the ST$(\wp_1, \wp_2, \wp_3)$ values from NWA approach. As a result, we obtain the following closeness values and orders:

	Relative closeness values					
(r)2-6 ST$(\wp_1, \wp_2, \wp_3)$	$\mathfrak{D}_1^*$	$\mathfrak{D}_2^*$	$\mathfrak{D}_3^*$	$\mathfrak{D}_4^*$	$\mathfrak{D}_5^*$	$Order$
$(1, 1, 1)$	0.2619	0.7432	0.7404	0.7386	0.0386	$\hbar_b > \hbar_c > \hbar_d > \hbar_a > \hbar_e$
$(2, 1, 1)$	0.3488	0.7892	0.7407	0.8297	0.0619	$\hbar_d > \hbar_b > \hbar_c > \hbar_a > \hbar_e$

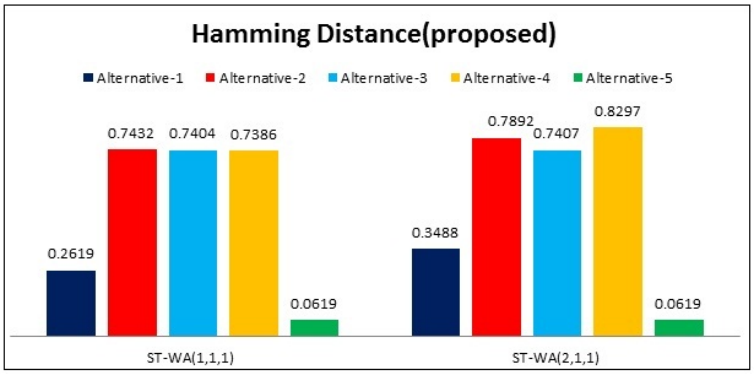

Figure-1. Different values.

A new order of alternative ranking is $\hbar_d > \hbar_b > \hbar_c > \hbar_a > \hbar_e$ if $(\wp_1 = 2, \wp_2 = 1, \wp_3 = 1)$. Therefore, $\hbar_b$ should be changed to $\hbar_d$ as the optimal alternative. As with the NWG operator, alternative rankings are determined according to ST$(\wp_1, \wp_2, \wp_3)$.

6 Conclusion:

In this study, we established HD for ST$(\wp_1, \wp_2, \wp_3)$NSs, which also have the advantage of being mathematically simple. The ED and HD are shown to be superior by employing appropriate. For ST$(\wp_1, \wp_2, \wp_3)$ NWA, ST$(\wp_1, \wp_2, \wp_3)$NWG, GST$(\wp_1, \wp_2, \wp_3)$ NWA and GST$(\wp_1, \wp_2, \wp_3)$NWG, we have proposed improved AO rules. Several algebraic operations were also provided as part of the creation of these operators. The research contained in this article will provide a major benefit to future researchers in this field, even though it is still at

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a very early stage of development. The ideas presented here will be beneficial to future academics interested in this field, as it is a large field. More detail will be provided on the following topics:
(1) The complex cubic FS and the IVPFS based on AOs.
(2) There are three types of complex normal vague set, complex normal vague spherical set, and complex normal vague NS.
(3) Complex NWAs, complex NWGs, complex GNWAs and complex GNWGs may be used to solve the problem.

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