



Fusion Data Analysis of the Monitoring Procedure among Ecuadorian Law Professionals using Indeterminate Likert Scales

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Abstract

The study provides a fusion data analysis to investigate the attitudes and perceptions of legal professionals in Ecuador regarding the effectiveness and fairness of the monitoring procedure, using a questionnaire based on indeterminate Likert scales. By employing Triple Refined Indeterminate Neutrosophic Sets and the Minimum Spanning Tree, responses were analyzed to reveal trends and groupings in opinions. The identification of response clusters suggested marked differences and homogeneous subgroups in perspectives, highlighting specific areas within legislation and judicial procedures that require attention. The threshold used for the Minimum Spanning Tree provided a quantitative view of cohesion and discrepancy, which has significant implications for legislative reform and judicial practice. This innovative approach offers a valuable model for future research, with the potential to influence policy-making and the promotion of legislative reforms based on empirical data.

Keywords: Fusion Data Analysis; monitoring procedure; legal perceptions; indeterminate Likert scales; cluster analysis; judicial reform

1. Introduction

The Monitoring Procedure, recently incorporated into the Ecuadorian legal system and codified in the General Organic Code of Processes (COGEP) in articles 356 to 361, represents a legal strategy for the recovery of unsecured monetary obligations not covered by executive titles. This procedure stands as a tool for the effective collection of overdue debts that do not exceed the threshold of fifty basic unified salaries, provided they have a predetermined and enforceable payment term. This legal mechanism aims to optimize and simplify disputes over smaller amounts, reducing procedural times and moving toward a more agile justice administration in accordance with the constitutional mandates of the Republic.

Currently, the delay in resolving legal conflicts emerges as a substantial issue within the judicial system. The power granted to the defendant to interpose exceptions or defenses, expressing disagreement with the fulfillment of the financial obligation, can significantly hinder the procedural progress of the claim. This phenomenon contradicts, to some extent, the conception of speed attributed to this procedure by various specialists, omitting that its design is intended to safeguard the creditor's rights expediently.

Within the spectrum of the Monitoring Procedure, two variants are distinguished: the undocumented process and the documented process. The former allows the creditor to initiate legal actions for debt compliance even in the absence of documentation that evidences the debtor's obligation, while the latter requires the presentation of documents or instruments that demonstrate the existence of the debt. It is crucial to emphasize that the documented

model of the monitoring trial fully satisfies fundamental legal principles, offering the counterparty the opportunity to examine the documents or invoices that support the claim for pending payment. Unlike the pure monitoring process, this documented approach ensures an evidentiary basis for the claim, meeting procedural requirements and strengthening the creditor's position.

In the context of the data analysis carried out on the monitoring procedure, it is crucial to focus on the main problem identified: the lack of a legally determined term that obliges to call a hearing for the involved parties after an objection is presented, as specified in Article 359 of the COGEP. This omission of a precise temporal delimitation leads to an infringement of the right to legal security, attributable to the lack of clear, defined, and specific guidelines that the judicial authority must follow. Such a situation grants judges the freedom to establish the timeframe for the hearing call, introducing an element of judicial discretion. Although each procedure is governed by a legally established term, the legislation and its subsequent amendments have omitted to include this specification for the monitoring procedure, which is crucial and generates legal uncertainty for the parties by not knowing the precise moment when the hearing for the claim of smaller amounts will be scheduled.

From a broader perspective, the constitutional principle of procedural economy seeks to prevent the redundancy of legal actions, avoiding the need to initiate a second procedure. By applying this principle, the optimization of resources is linked to the purpose of achieving effective judicial resolutions with the minimum required effort. This strategy favors an expedient and adequate administration of justice, facilitating access to justice for the parties without subjecting them to tedious and exhausting processes that previously prolonged litigation, mainly for economic motivations. Instead, the satisfaction and effective resolution of the interests of those involved are prioritized.

Therefore, the streamlining and efficiency of judicial processes contribute significantly to the protection of the legal security of the parties. The implementation of a single hearing within the monitoring procedure is fundamental for the speed of its resolution. However, legislative obstacles that compromise this objective are still faced. Thus, this study aims to evaluate the negative impact derived from the absence of a legally established term for scheduling such a hearing in the face of opposition in the monitoring procedure, in order to ensure the principles of celerity and procedural economy.

In this framework, a significant opportunity emerges for the use of empirical research methodologies aimed at deepening the understanding and resolution of this legal issue. In this sense, the use of indeterminate Likert scales is postulated as a valuable methodological tool for the detailed analysis of the perceptions and opinions of legal professionals involved in such processes. The application of the Likert scale in studies on legislation and the procedural field offers a structured and quantifiable way of collecting data, providing insights into the acceptance, effectiveness, and perception of justice in imposed sanctions. [1]

A traditional Likert scale typically consists of a series of statements that respondents rate on a scale of agreement or disagreement, usually with five or seven points [2]. However, the implementation of a Likert scale with indeterminate options adds an additional dimension to the analysis, more accurately reflecting the complexity of human opinions in the legislative context [3].

The use of indeterminate Likert scales, which allow for more nuanced responses than traditional scales, offers a way to capture the complexity of attitudes and perceptions of judges, lawyers, and prosecutors regarding the effectiveness, clarity, and fairness of the monitoring procedure as currently legislated [4]. By allowing degrees of agreement, disagreement, and indeterminate neutrality, these scales facilitate the collection of data that reflects the diversity of experiences and viewpoints among legal professionals, thus providing a more complete picture of the practical and theoretical implications of the current regulations.

Empirical research based on indeterminate Likert scales enables the conduct of quantitative and qualitative analyses that can reveal significant correlations and underlying trends in perceptions of the monitoring procedure. This, in turn, allows for more precise identification of areas of ambiguity, controversy, or consensus within the legal community, shedding light on aspects of the procedure that require clarification, reform, or strengthening to align practice with the principles of speed and procedural economy [5].

In the context of the problem presented, the quantitative analysis derived from the use of these scales could be complemented with qualitative interviews and content analysis to explore in depth the reasons behind indeterminate or ambivalent perceptions. Thus, the combination of quantitative and qualitative methods enriches the research, allowing not only the identification of perceived problems in legislation and its application but also the formulation of evidence-based recommendations for their resolution.

Ultimately, the use of indeterminate Likert scales in the study of perceptions of the monitoring procedure is presented as an innovative approach that promotes greater precision and analytical depth. This methodological

approach contributes to the construction of a solid empirical basis for informed legislative and judicial decision-making, aimed at improving the efficiency, transparency, and justice of the conflict resolution system in the Ecuadorian legal context [6].

In this context, the purpose of this research focuses on the use of indeterminate Likert scales to assess the perceptions and attitudes of legal professionals regarding the effectiveness, clarity, and fairness of the monitoring procedure within the framework of the COGEP. This study ultimately aims to contribute to the optimization of the monitoring procedure in Ecuador, providing an empirical basis for future legislative reforms or adjustments in judicial practice that facilitate a faster and fairer resolution of small monetary disputes.

2. Neutrosophy and Refined Neutrosophic Set

Introduced by Smarandache in 2000, Neutrosophy examines an entity or concept, denoted as "A", in relation to its opposition "Anti-A", its negation "Non-A", and the state of being neither "A" nor "Anti-A", referred to as "Neut-A". Within this framework, a Single-Valued Neutrosophic Set (SVNS) for an entity "A" within a domain "X" is defined through membership functions for truth $T_A(x)$, indeterminacy $I_A(x)$, and falsity $F_A(x)$. These functions, bound within the interval $[0, 1]$, satisfy the condition: $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for each element x in X , thus encapsulating the SVNS as $A = \{x, T_A(x), I_A(x), F_A(x) | x \in X\}$. [7]

According to the refined neutrosophic logic as formulated by Smarandache, the following is obtained:[8]

Refining this concept, the multifaceted nature of truth, indeterminacy, and falsity into subcategories ($T_1, T_2, \dots, T_p, I_1, I_2, \dots, I_r, F_1, F_2, \dots, F_s$ respectively) where, p, r, s are all positive integers such that $p + r + s = n$. This granular differentiation fosters enhanced precision and alignment with real-world data variance, such as that observed in Likert scales.

The advent of Triple Refined Indeterminate Neutrosophic Sets (TRINS) by Kandasamy and Smarandache in 2016 [9], and its counterpart, the Double-Valued Neutrosophic Set (DVNS), represents a significant leap in capturing nuanced feedback. TRINS, for instance, differentiates indeterminacy into three distinct memberships: positive indeterminacy $IP_A(x)$, neutral indeterminacy $I_A(x)$, and negative indeterminacy $IN_A(x)$, each associated with a weighting factor $w_m \in [0, 5]$.

Thus, for any element $x \in X$, the memberships are defined as $P_A(x), IP_A(x), I_A(x), IN_A(x), N_A(x) \in [0, 1]$, with their weighted values spanning $[0, 5]$, ensuring $0 \leq P_A(x) + IP_A(x) + I_A(x) + IN_A(x) + N_A(x) \leq 5$. This allows a TRINS representation as $A = \{x, P_A(x), IP_A(x), I_A(x), IN_A(x), N_A(x) | x \in X\}$.

Considering a practical example from a restaurant feedback scenario, where a customer rates various dishes, TRINS enables a detailed mapping beyond the conventional average/neutral feedback. For instance, the customer's experience can be captured through varied degrees of satisfaction across multiple dishes, highlighting TRINS' capability to represent complex consumer preferences comprehensively.

Furthermore, the theoretical underpinning of TRINS includes set-theoretical operations like associativity, distributivity, commutativity, idempotency, absorption, and adherence to DeMorgan's laws, underscoring its robust mathematical foundation and applicability in capturing and analyzing intricate feedback mechanisms.

Indeterminate Likert scaling

The traditional five-point Likert scale, encompassing options from "Strongly disagree" to "Strongly agree," is reinterpreted within the framework of an indeterminate Likert scale. This reinterpretation translates the conventional responses into a spectrum of memberships ranging from "Negative membership" to "Positive membership." This translation aligns the responses with an indeterminacy perspective, thereby offering a nuanced understanding of participant responses. [3]

In the context of a five-star rating system, this nuanced approach assigns "Negative membership" to the "Strongly disagree" equivalent, or a one-star rating, capturing a distinct level of dissatisfaction or disagreement. Conversely, "Positive membership" corresponds to "Strongly agree," or a five-star rating, indicating a high level of satisfaction or agreement. Intermediate responses, such as "Disagree," "Neither agree nor disagree," and "Agree," are respectively mapped to "Indeterminacy leaning towards negative membership," "Indeterminate membership," and "Indeterminacy leaning towards positive membership," reflecting varying degrees of ambivalence or certainty towards the subject matter.

This indeterminate scaling method allows for a more granular representation of feedback, illustrated by hypothetical scenarios where a restaurant customer evaluates service quality. For example, exemplary waiter service could elicit a response leaning towards "Very satisfied," potentially represented by a fraction such as 0.5, indicating a high degree of positive sentiment. However, if the customer experienced a significant wait time for their order, this might result in a partial attribution towards "Very unsatisfied," denoted by a value like 0.25, capturing a specific aspect of dissatisfaction. Similarly, ambivalence regarding staff politeness, neither distinctly positive nor negative, could be assigned a value indicating indeterminate sentiment, reflecting the customer's hesitation to categorize the experience as either good or bad. [10]

Through this indeterminate Likert scaling, responses are not merely categorized but are understood in terms of their position within a continuum of satisfaction, dissatisfaction, and uncertainty. This method offers a refined tool for capturing the complexity of human emotions and perceptions, providing deeper insights into the nuanced spectrums of agreement and satisfaction.

The concept of an indeterminate Likert scale presents a flexible and adaptive framework for capturing the nuances of human perceptions and responses. This adaptability allows the scale to be expanded beyond the conventional five-point format to include a seven-point scale, or indeed any other multipoint scale, depending on the specific requirements of a research study. The capability to adjust the scale caters to the diverse needs of researchers, who can tailor the degrees of agreement or disagreement—encompassing Truth, Indeterminacy, and Falsity memberships—to better align with the objectives of their investigations.

Such customization facilitates a more granular analysis of respondent attitudes, enabling the delineation of finer shades of opinion and sentiment that might otherwise be obscured within more rigid scaling frameworks. This approach, known as the multipoint indeterminate Likert scale, offers a broader spectrum for categorizing responses, thereby enhancing the precision and depth of data analysis. The potential applications and implications of these adaptable scaling mechanisms are vast, inviting further exploration and study within the field. The exploration of multipoint indeterminate Likert scales represents a promising avenue for future research, promising to unlock richer insights into the complex landscape of human attitudes and beliefs.

Indeterminate Minimum Spanning Tree (MST) clustering algorithm using distance measures

The methodology for calculating distance measures within the framework of TRINS involves an intricate algorithm designed to quantify the dissimilarities between pairs of TRINS. These sets, residing within a defined universe of discourse $X = x_1, x_2, \dots, x_n$, are comprehensively represented as A and B , with each set comprising elements x_i and their associated membership functions across five categories: Positive $PA(x_i)$, Positive Indeterminacy $IPA(x_i)$, Indeterminacy $IA(x_i)$, Negative Indeterminacy $INA(x_i)$, and Negative $NA(x_i)$, all of which assume values within the interval $[0,5]$.

The concept introduces a weighting system for each element x_i in the universe, with weights w_i adhering to the conditions $w_i \geq 0$ for all i and the sum $\sum_{i=1}^n w_i = 1$, ensuring a normalized distribution of importance across the elements.

The generalized weighted distance between any two TRINS A and B , denoted $d_{\lambda(A,B)}$, is formulated as: [11]

$$d_{\lambda}(A, B) = \left\{ \frac{1}{5} \sum_{i=1}^n w_i \left[|P_A(x_i) - P_B(x_i)|^{\lambda} + |I_{PA}(x_i) - I_{PB}(x_i)|^{\lambda} + |I_A(x_i) - I_B(x_i)|^{\lambda} |I_{NA}(x_i) - I_{NB}(x_i)|^{\lambda} + |N_A(x_i) - N_B(x_i)|^{\lambda} \right] \right\}^{1/\lambda} \quad (1)$$

where $\lambda > 0$ is a parameter determining the nature of the distance measure.

For $\lambda=1$, the equation simplifies to the TRINS weighted Hamming distance, encapsulating a linear disparity across the membership functions. Conversely, when $\lambda=2$, it embodies the TRINS weighted Euclidean distance, offering a quadratic perspective of the distance between the sets, and is expressed as:

$$d_2(A, B) = \left\{ \frac{1}{5} \sum_{i=1}^n w_i \left[|P_A(x_i) - P_B(x_i)|^2 + |I_{PA}(x_i) - I_{PB}(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 |I_{NA}(x_i) - I_{NB}(x_i)|^2 + |N_A(x_i) - N_B(x_i)|^2 \right] \right\}^{1/2}. \quad (2)$$

To compute the TRINS distance matrix D , which encapsulates the distances between multiple TRINS A_j ($j = 1, 2, \dots, m$), the following properties are adhered to:

1. Each element d_{ij} within the matrix D , representing the distance between A_i and A_j , falls within the range $[0,5]$.

2. $d_{ij} = 0$ if and only if $A_i = A_j$, signifying no dissimilarity between identical sets.
3. The distance is symmetric, i.e., $d_{ij} = d_{ji}$ for all i, j .

The necessary steps for constructing the matrix D are outlined as follows:

Initialization: The process begins with a set of TVNS $A_1, A_2, \dots, A_m$.

Creation of the Distance Matrix D: The distance matrix D is a square matrix of size $m \times n$, where m is the number of TVNS in

Calculation of Distances

- For each pair of TVNS A_i and A_j in the input set, the distance d_{ij} is calculated and assigned to the matrix D.
- Iteration is conducted over each element A_i with i ranging from 1 to m , and for each A_i , the process iterates again over each element A_j with j ranging from 1 to m .

Conditions:

- If $i = j$: This means we are comparing a TVNS with itself. In this case, the distance d_{ij} is 0 because the distance of any element to itself is always 0.
- If $i \neq j$: The distance d_{ij} between A_i and A_j is calculated using Equation 2.

Assignment of the Calculated Distance to the Matrix D: The calculated distance d_{ij} is assigned to the corresponding position in the matrix D.

The final result is a distance matrix D that represents the distances or dissimilarities between all pairs of TVNS in the input set. This matrix can be useful for subsequent analysis, such as clustering, and similarity/difference analysis, among others.

The Indeterminate Minimum Spanning Tree (MST) Clustering algorithm, is designed to produce a Minimum Spanning Tree (S) and identify clusters within a given set of data points. This process leverages the concept of TRINS to manage data points and their relationships. Here's a step-by-step explanation of how the algorithm operates:[12]

Step 1: Calculation of the Distance Matrix

- The distance matrix D details the distances between all pairs of points $F_1, F_2, \dots, F_m$.

Step 2: Creation of the TRINS Graph

- A graph $G(V, E)$ is constructed with vertices V representing the data points and edges E representing the distances between these points.
- For each pair of points F_i and F_j , if $i \neq j$, an edge is created from F_i to F_j , with weight d_{ij} indicating their distance.

Step 3: Computation of the MST

- Kruskal's algorithm is employed to compute the MST of graph G . This involves sorting all the edges in E by their weights in increasing order.
- Iteratively, the algorithm selects the edge with the minimum weight and adds it to a subgraph S of G , provided it does not create a cycle within S . This process continues until the number of edges in S is one less than the number of vertices ($V-1$), ensuring that S spans all vertices without forming any loops.

Step 4: Clustering Using Threshold r

- To form clusters, the algorithm iterates through the edges of S , removing any edge whose weight d_{ij} is greater than or equal to a predefined threshold r .
- The removal of these edges results in the automatic formation of clusters, as the remaining components of graph S represent disconnected subgraphs, each encompassing a cluster of closely related points.

3. Materials and Methods

The current study employs a quantitative fusion data analysis approach supplemented with qualitative techniques to explore the perceptions and attitudes of legal professionals regarding the efficiency, clarity, and fairness of the monitoring procedure in the Ecuadorian legal system. This research is based on the design and application of a structured questionnaire, the transformation and quantitative analysis of responses using indeterminate Likert scales, and the grouping of these responses through a cluster analysis algorithm, specifically the TRIN-MST Algorithm.

The central instrument of this research is a questionnaire designed to capture the complexity of legal professionals' perceptions. This questionnaire includes items that evaluate the efficiency of the monitoring procedure, its consonance with the principles of speed and procedural economy, and collects suggestions for its optimization. The development of the questions is aimed at allowing responses on an indeterminate Likert scale, providing a broader spectrum of possibilities ranging from total agreement to total disagreement, including levels of positive and negative indeterminacy.

The distribution of the questionnaire is carried out among a sample of judges, prosecutors, and lawyers, who are invited to participate voluntarily. The selection of these participants is based on inclusion criteria that ensure a broad representation of experiences and perspectives within the legal field, for a total sample of 13 participants.

The responses obtained undergo a transformation process, in which each selection on the indeterminate Likert scale is assigned specific numerical values. This step facilitates the application of statistical and quantitative analyses, allowing an objective interpretation of trends and discrepancies in the participants' perceptions.

Using the TRINS distance algorithm, a distance matrix between the respondents' answers is calculated. This mathematical approach quantifies the perceptual differences among legal professionals, providing a solid basis for subsequent cluster analysis.

Finally, the TRIN-MST algorithm is applied to group the responses into clusters based on their similarity. This process involves constructing a minimum spanning tree from the distance matrix and identifying clusters by applying a specific distance threshold. The resulting clusters reveal patterns of shared perception among legal professionals, allowing for the identification of areas of consensus and divergence regarding the monitoring procedure.

This study adheres to methodological principles, ensuring the validity and reliability of the findings. Through this mixed approach, the aim is not only to describe existing perceptions but also to provide evidence-based recommendations for improving the monitoring procedure in the Ecuadorian legal context, thus contributing to more effective and fair legal practice.

4. Results

The implementation of the meticulously designed questionnaire facilitated the collection of participants' appraisals, which were encoded using the TRINS format. Table 1 displays the results compiled for this purpose.

Table 1: Approval rate in each section of the questionnaire

F	The implementation of the single hearing streamlines the monitoring procedure	Current legislative provisions facilitate rapid and effective resolution of cases through the monitoring procedure, complying with the principles of speed and procedural economy.	The single hearing, as currently regulated and practiced, effectively contributes to the speed and economy of conflict resolution.	The deadlines established for the single hearing are adequate to prepare the defense
1	(0.1; 0.09; 0.05; 0.3; 0.46)	(0.05; 0.1; 0.2; 0.3; 0.35)	(0.1; 0.1; 0.1; 0.3; 0.4)	(0.15; 0.1; 0.1; 0.25; 0.4)
2	(0.15; 0.1; 0.1; 0.25; 0.4)	(0.15; 0.1; 0.1; 0.25; 0.4)	(0.05; 0.25; 0.5; 0.1; 0.1)	(0.05; 0.25; 0.5; 0.1; 0.1)

3	(0.1; 0.2; 0.5; 0.1; 0.1)	(0.05; 0.25; 0.5; 0.1; 0.1)	(0.2; 0.1; 0.1; 0.3; 0.3)	(0; 0; 0.2; 0.3; 0.5)
4	(0.1; 0.1; 0.1; 0.3; 0.4)	(0.2; 0.1; 0.1; 0.3; 0.3)	(0; 0; 0.2; 0.3; 0.5)	(0.15; 0.1; 0.1; 0.25; 0.4)
5	(0; 0.1; 0.2; 0.15; 0.55)	(0; 0; 0.2; 0.3; 0.5)	(0.15; 0.1; 0.1; 0.25; 0.4)	(0.15; 0.1; 0.1; 0.25; 0.4)
6	(0.05; 0.1; 0.2; 0.3; 0.35)	(0.1; 0.09; 0.05; 0.3; 0.46)	(0.1; 0.2; 0.5; 0.1; 0.1)	(0.1; 0.2; 0.5; 0.1; 0.1)
7	(0.15; 0.1; 0.1; 0.25; 0.4)	(0.1; 0.09; 0.05; 0.3; 0.46)	(0; 0; 0.2; 0.3; 0.5)	(0.1; 0.1; 0.1; 0.3; 0.4)
8	(0.05; 0.25; 0.5; 0.1; 0.1)	(0.15; 0.1; 0.1; 0.25; 0.4)	(0.1; 0.09; 0.05; 0.3; 0.46)	(0.05; 0.25; 0.5; 0.1; 0.1)
9	(0.2; 0.1; 0.1; 0.3; 0.3)	(0.1; 0.2; 0.5; 0.1; 0.1)	(0.1; 0.09; 0.05; 0.3; 0.46)	(0.15; 0.1; 0.1; 0.25; 0.4)
10	(0; 0; 0.2; 0.3; 0.5)	(0.1; 0.1; 0.1; 0.3; 0.4)	(0.15; 0.1; 0.1; 0.25; 0.4)	(0; 0.1; 0.2; 0.15; 0.55)
eleven	(0.1; 0.09; 0.05; 0.3; 0.46)	(0.05; 0.25; 0.5; 0.1; 0.1)	(0; 0.1; 0.2; 0.15; 0.55)	(0.15; 0.1; 0.1; 0.25; 0.4)
12	(0.1; 0.1; 0.1; 0.3; 0.4)	(0.2; 0.1; 0.1; 0.3; 0.3)	(0.15; 0.1; 0.1; 0.25; 0.4)	(0; 0.1; 0.2; 0.15; 0.55)
13	(0; 0.15; 0.2; 0.2; 0.45)	(0; 0; 0.2; 0.3; 0.5)	(0; 0.1; 0.2; 0.15; 0.55)	(0.15; 0.1; 0.1; 0.25; 0.4)

From this data, it is possible to obtain the distance matrix considering that a value of $\lambda=2$ is used for the distance calculation, for a value approximating the Euclidean, an equal value is assumed for all weights w_i so that $w_i = 0,25$ for each of the four variables evaluated. The resulting matrix D presents the distances calculated between each pair of variables, reflecting the differences in their membership profiles according to the perception of the unique hearing in the monitoring procedure.

D =		F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13
	F1	0.00	0.18	0.18	0.06	0.07	0.18	0.06	0.19	0.11	0.08	0.11	0.07	0.08
			1	4	4	2	2	8	2	2	0.08	8	5	5
	F2	0.18	0.00	0.24	0.18	0.19	0.04	0.18	0.18	0.22	0.18	0.21	0.18	0.19
		1		8	4	1	8	5	8	4	5	7		
	F3	0.18	0.24	0	0.19	0.19	0.24	0.20	0.18	0.13	0.18	0.16	0.17	0.19
		4	8		2	2	5	3	8	4	8	9	8	7
	F4	0.06	0.18	0.19	0	0.10	0.18	0.04	0.18	0.12	0.09	0.12	0.07	0.09
		4	4	2		5	4	9	4	7	5	8	7	2
	F5	0.07	0.19	0.19	0.18	0	0.18	0.09	0.18	0.15	0.08	0.15	0.10	0.06
		2	1	2	4		4	1	6	6	4	5	7	3
	F6	0.18	0.04	0.24	0.18	0.18	0	0.18	0.17	0.23	0.17	0.22	0.18	0.18
		2	8	5	4	4		2	5	5	9	8	1	3
	F7	0.06	0.18	0.20	0.04	0.09	0.18	0	0.18	0.14	0.09	0.15	0.09	0.07
		8	5	3	9	1	2		3	9	5			8
	F8	0.19	0.18	0.18	0.18	0.18	0.17	0.18	0	0.20	0.18	0.23	0.18	0.18
		2	8	8	4	6	5	3		9	6	2	2	1
	F9	0.11	0.22	0.13	0.12	0.15	0.23	0.14	0.20	0	0.15	0.07	0.13	0.15
		2	4	4	7	6	5	9	9		4	3	3	6
	F10	0.08	0.18	0.18	0.09	0.08	0.17	0.09	0.18	0.15	0	0.15	0.05	0.10
			5	8	5	4	9	5	6	4		6	5	1
	F11	0.11	0.21	0.16	0.12	0.15	0.22	0.15	0.23	0.07	0.15	0	0.14	0.14
		8	7	9	8	5	8		2	3	6		5	2
	F12	0.07	0.18	0.17	0.07	0.10	0.18	0.09	0.18	0.13	0.05	0.14	0	0.11
		5		8	7	7	1		2	3	5	5		5
	F13	0.08	0.19	0.19	0.09	0.06	0.18	0.07	0.18	0.15	0.10	0.14	0.11	0
		5		7	2	3	3	8	1	6	1	2	5	

In the context of the structural analysis provided by the minimum spanning tree (MST) derived from the graph $G(V, E)$, the representation of the most efficient connections between the nodes $F_1, F_2, \dots, F_{13}$, without

incurring in the formation of cycles, is highlighted. The edges that make up the MST are characterized by their weights, which symbolize the minimum distances required to interconnect all the nodes, thus optimizing the total cost of the system.

During the clustering process using a threshold r , this threshold is set to discern which edges present lengths considered excessive and, therefore, need to be excluded to facilitate the formation of clusters. The calculation of r , determined as the average of the non-zero weights, yields an approximate value of 0.092. This parameter is used as a cut-off criterion to identify those edges that exceed the established threshold, requiring their elimination to proceed with clustering.

Subsequently, by applying the threshold r , the edges whose weights are equal to or greater than this value are removed. The edges that remain within the MST, presenting weights below the threshold, denote the presence of relatively more robust links (i.e., shorter distances) between certain nodes.

The resulting configuration of the MST, once adjusted under the threshold r criterion, reveals the emergence of clusters composed of those nodes that maintain direct connections, that is, edges not excluded by the filtering process. Such an adjusted structure allows visualizing the grouping of nodes around the relative proximities they share, thus outlining the clusters formed based on the spatial cohesion of the points in question. See Figure 1.

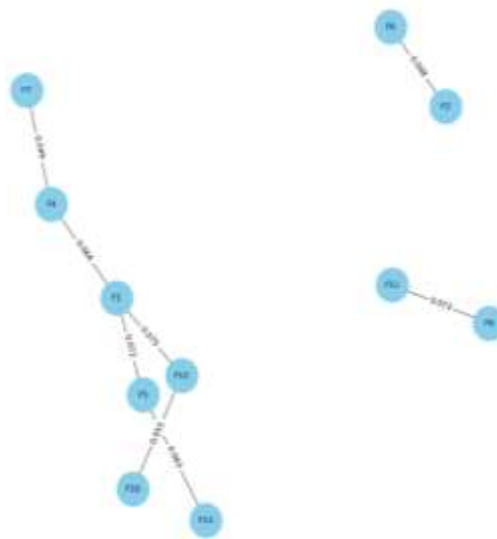


Figure1: MST graph

The resulting graph revealed direct connections between certain nodes, such as F_1 and F_5 , as well as F_4 and F_7 , which evidenced a significant alignment in the responses among these respondents. These direct links suggest that the pairs of points share similar properties or characteristics in their negative perceptions of evaluated aspects of the legislative provisions, implementation, and established timelines for the single hearing.

By applying a specific threshold for clustering, a main group composed of $F_1, F_4, F_5, F_7, F_{10}$, and F_{12} was observed. The members of this cluster consistently expressed negative perceptions of the analyzed variables, indicating a general trend of dissatisfaction or concern regarding the efficacy and speed of the monitoring procedure.

More isolatedly, the nodes F_2 and F_6 constituted a subgroup characterized by marked indeterminacy in their responses related to economy and efficiency in conflict resolution and deadlines for the single hearing. On the other hand, F_9 and F_{11} formed a small group that, although also showed indeterminacy regarding current legislation, tends to positively value the implementation of the single hearing as a factor that speeds up the monitoring procedure.

The existence of absent nodes reveals that there are respondents who might possess unique or extremely divergent perceptions compared to the majority collective, such that their responses exceed the established proximity threshold. This can be interpreted as their opinions or situations being different from the predominant trends detected in the study.

5. Discussion

In the Ecuadorian legislative sphere, the quantitative evaluation of perceptions and attitudes towards legislation and judicial procedures represents a significant challenge due to the inherent complexity in interpreting legal texts and the diversity of opinions among citizens. The application of indeterminate Likert scales together with the analysis using the TRINS Algorithm and the minimum spanning tree emerges as a robust methodology to address this complexity, facilitating the understanding of attitudes and problem-solving in this area.

Indeterminate Likert scales are characterized by allowing respondents to express uncertainty, more faithfully reflecting the spectrum of responses that can be presented to legislative statements. This indeterminacy is crucial in the legal context, where ambiguity and subjectivity are common. The inclusion of response options that capture indecision allows for a more accurate representation of perceptions that are not strictly positive or negative, thus offering a more nuanced image of reactions toward legislation.

The use of the TRINS algorithm, which considers both deterministic and stochastic elements, is presented as a particularly relevant data analysis tool to deal with the variability and uncertainty of responses. By identifying the intrinsic relationships and underlying structures in the data, TRINS enables the discovery of complex patterns and underlying trends among respondents' opinions regarding legislation and judicial procedures.

The MST, derived from the application of the TRINS algorithm, simplifies this complexity by offering a visual and analytical representation of the strongest and most representative connections among respondents. By focusing on the minimum connections necessary to maintain the network of responses, the MST highlights the most significant relationships and discards redundancies, providing a clear view of the groups of consensus and dissent within the surveyed population.

The importance of this methodology lies in its ability to inform and guide legislative decision-making. In this case, it was possible to observe the existence of population groups with specific needs or particular concerns that might not be evident through more traditional analysis methods.

6. Conclusion

The study in question provided an exploration of the use of indeterminate Likert scales to assess the perceptions and attitudes of legal professionals regarding the effectiveness, clarity, and fairness of the monitoring procedure. The implementation of indeterminate Likert scales proved to be fundamental in capturing the full range of attitudes and opinions, beyond traditional binary responses, thereby allowing a more sophisticated and accurate evaluation of the complex stances inherent in this area. The use of TRINS and the minimum spanning tree has allowed the unveiling of underlying patterns of agreement and discrepancy among participants, providing a deep understanding of the consensuses and divisions within the legal community. The application of a specific threshold in the MST has been instrumental in discerning clusters of perceptions, highlighting the presence of subgroups with homogeneous viewpoints and isolating those with atypical perspectives.

The conclusions derived from the study point to the existence of notable differences and defined segmentations among the opinions of legal professionals, with direct implications for legislative reform processes and judicial practice. The identification of these groups suggests specific areas of legislation and procedure that may require clarification, simplification, or improvement to align more closely with the needs and expectations of legal professionals. The methodology applied in this study is presented as a replicable and powerful model for future research in this field. These methods and tools are not only capable of distilling the complexities of legal opinion into comprehensible patterns but also offer an innovative approach to influencing policy-making and legislative reforms based on empirical data.

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