



Interval Valued Neutrosophic Subalgebra in INK-Algebra

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Abstract

This work presents the concept of interval-valued neutrosophic *INK*-subalgebras, also known as *IVN INK*-subalgebras, which are the level and strong level neutrosophic *INK*-subalgebras. Next, we establish and validate a few theorems that establish the connection between these concepts and neutrosophic *INK*-subalgebras. We define the images and inverse images of *IVN INK*-subalgebras and study the transformations of the homomorphic images and inverse images of interval valued neutrosophic (briefly, *IVN*) *INK*-subalgebra into *IVN INK*-subalgebras.

Keywords: interval valued neutrosophic set; neutrosophic *INK*-subalgebra and interval valued neutrosophic *INK*-subalgebra.

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1 Introduction

BCK-algebra and *BCI*-algebra are two classes of abstract algebras introduced by Imai and Iséki in 1966.¹ The *BCK*-algebra class is recognized as a legitimate subclass of the *BCI*-algebra class.^{2,7} A concept known as *d*-algebra was presented by Neggers et al.,⁸ It is a generalization of *BCH/BCI/BCK*-algebras and generalizes several of the theorems covered in *BCI*-algebra. The concept of *INK*-algebras was first developed by Kaviyarasu et al.,³ The notions of fuzzy *INK*-algebras were introduced and applied to *INK*-algebras by Kaviyarasu et al.,⁵ The notion of fuzzy sets was first presented by Zadeh.¹⁰ An *IVFS*, or fuzzy set with an interval-valued membership function, is Zadeh's expansion of the fuzzy set idea in.¹⁰ He created a technique for approximative inference using his *IVFS*s, which he refers to as a *IVFS*. Interval-valued fuzzy *INK*-subalgebras and neutrosophic set in *INK*-algebra were defined by Kaviyarasu et al.^{4,6} In the current work, we presented the idea of interval-valued neutrosophic *INK*-subalgebras (abbreviated *IVN INK*-subalgebras) of a *INK*-algebra and studied some of its features, utilizing the notion of interval-valued fuzzy set by Zadeh. We demonstrate that each *INK*-subalgebra of a *INK*-algebra U may be realized as a *IV* level *INK*-subalgebra of a *IVN INK*-subalgebra of U . From this, we derive several related findings that are specified in the abstract.

2 Preliminaries

Definition 2.1. ⁴ An algebra $(U, *, 0)$ is a non-null set U with a constant '0' and a single binary operation '*' is called *INK*-algebras if it satisfying the following conditions.

- (i) $\alpha * \alpha = 0$
- (ii) $\alpha * 0 = \alpha$ for $\alpha \in U$
- (iii) $0 * \alpha = \alpha$
- (iv) $(\beta * \alpha) * (\beta * \gamma) = (\alpha * \gamma)$ for all $\alpha, \beta, \gamma \in U$.

Definition 2.2.⁴ Let U be a INK -algebra and $T \subseteq U$. Then T is said to a INK -subalgebra (briefly, INK -subalg) of U , if $\alpha * \beta \in T$, for all $\alpha, \beta \in U$.

Definition 2.3.³ A mapping $f : U \rightarrow V$ of INK -algebras is called a INK -homomorphism if $f(\alpha * \beta) = f(\alpha) * f(\beta)$ for all $\alpha, \beta \in U$.

Now, we go over a few fuzzy logic concepts (see¹⁰).

Let U be a set. A fuzzy set τ_1 in U is characterized by a membership function $\mu_{\tau_1} : U \rightarrow [0, 1]$. Let f be a mapping from the set U to the set V and let τ_2 be a fuzzy set in V with membership function μ_{τ_2} . The inverse image of τ_2 , denoted $f^{-1}(\tau_2)$, is the fuzzy set in U with membership function $\mu_{f^{-1}(\tau_2)}$ defined by $\mu_{f^{-1}(\tau_2)}(\alpha) = \mu_{\tau_2}(f(\alpha))$ for all $\alpha \in U$. Conversely, let τ_1 be a fuzzy set in U with membership function μ_{τ_1} . Then the image of τ_1 , denoted by $f(\tau_1)$, is the fuzzy set in V such that:

$$\mu_{f(\tau_1)}(\beta) = \begin{cases} \sup_{\gamma \in f^{-1}(\beta)} \mu_{\tau_1}(\gamma) & \text{if } f^{-1}(\beta) = \{\alpha : f(\alpha) = \beta\} \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

A fuzzy set τ_1 in the INK -algebra U with the membership function μ_{τ_1} is said to be have the sup property if for any subset $T \subseteq U$ there exists $\alpha_0 \in T$ such that $\mu_{\tau_1}(\alpha_0) = \sup_{t \in T} \mu_{\tau_1}(t)$.

Definition 2.4.⁴ A fuzzy subset μ in a INK -algebra U is called a fuzzy INK -subalgebra (briefly, $FINK$ -subalg) of U if $\mu(\alpha * \beta) \geq \min\{\mu(\alpha), \mu(\beta)\}$, for all $\alpha, \beta \in U$.

Definition 2.5.⁹ A fuzzy subset μ in a INK -algebra U is called a anti fuzzy subalgebra⁹ (briefly, $AFINK$ -subalg) of U if $\mu(\alpha * \beta) \leq \max\{\mu(\alpha), \mu(\beta)\}$, for all $\alpha, \beta \in U$.

Definition 2.6.⁹ Let μ be a fuzzy set of a set U . For a fixed $t \in [0, 1]$, the set $\mu_t = \{\alpha \in U / \mu(\alpha) \geq t\}$ is called an upper level of μ .

Definition 2.7.⁹ Let U be a fuzzy subset of S . Then for $t \in [0, 1]$, the t -level cut of U is the set $U_t := \{x \in S / U(x) \geq t\}$.

Definition 2.8.⁹ Let U be a fuzzy subset of a INK -algebra S . Then for $t \in [0, 1]$, the lower t -level cut of U is the set $U^t := \{x \in S / U(x) \leq t\}$. Clearly $U^1 = X$ and $U_t \cup U^t = X$ for $t \in [0, 1]$. If $t_1 < t_2$, then $U^{t_1} \subseteq U^{t_2}$.

An interval valued neutrosophic set (briefly, $IVNS$)⁴ Λ in a non-empty set X is defined to be a structure $\Lambda = \{(\alpha, T_\Lambda(\alpha), I_\Lambda(\alpha), F_\Lambda(\alpha)) | \alpha \in X\}$, where $T_\Lambda : U \rightarrow \theta[0, 1]$, $I_\Lambda : U \rightarrow \theta[0, 1]$ and $F_\Lambda : U \rightarrow \theta[0, 1]$, which are called a truth membership function, an indeterminacy membership function and a falsity membership function respectively.

The intervals $T_\Lambda(\alpha)$, $I_\Lambda(\alpha)$ and $F_\Lambda(\alpha)$ denote the intervals of the degree of membership, indeterminacy and non-membership of the element α to the set $\theta[0, 1]$, respectively, where $T_\Lambda^-(\alpha) = [T_\Lambda^L(\alpha), T_\Lambda^U(\alpha)]$, $I_\Lambda^-(\alpha) = [I_\Lambda^L(\alpha), I_\Lambda^U(\alpha)]$ and $F_\Lambda^-(\alpha) = [F_\Lambda^L(\alpha), F_\Lambda^U(\alpha)]$ for all $\alpha \in X$. Also, note that $\overline{T_\Lambda(\alpha)} = 1 - T_\Lambda(\alpha) = [1 - T_\Lambda^U(\alpha), 1 - T_\Lambda^L(\alpha)]$,

$$\overline{I_\Lambda(\alpha)} = 1 - I_\Lambda(\alpha) = [1 - I_\Lambda^U(\alpha), 1 - I_\Lambda^L(\alpha)]$$

and $\overline{F_\Lambda(\alpha)} = 1 - F_\Lambda(\alpha) = [1 - F_\Lambda^U(\alpha), 1 - F_\Lambda^L(\alpha)]$ for all $\alpha \in X$, where $(\alpha, \overline{T_\Lambda(\alpha)}, \overline{I_\Lambda(\alpha)}, \overline{F_\Lambda(\alpha)})$ represents the complement of α in Λ .

We define $\overline{\Lambda} = (\overline{T_\Lambda}, \overline{I_\Lambda}, \overline{F_\Lambda})$ as the complement of $\Lambda = (T_\Lambda, I_\Lambda, F_\Lambda)$. For the sake of simplicity, we shall use the symbol $\Lambda = (T_\Lambda, I_\Lambda, F_\Lambda)$ for the $IVNS$ is given by $\Lambda = \{(\alpha, T_\Lambda(\alpha), I_\Lambda(\alpha), F_\Lambda(\alpha)) | \alpha \in U\}$.

Definition 2.9. ⁶ A $NS \Lambda$ in U is called a neutrosophic INK -subalgebra, (briefly, $N INK$ -subalg) of U if it satisfies the following condition, for all $\alpha, \beta \in U$.

- (i) $T_{\Lambda}(\alpha * \beta) \geq \min\{T_{\Lambda}(\alpha), T_{\Lambda}(\beta)\}$,
- (ii) $I_{\Lambda}(\alpha * \beta) \leq \max\{I_{\Lambda}(\alpha), I_{\Lambda}(\beta)\}$,
- (iii) $F_{\Lambda}(\alpha * \beta) \geq \min\{F_{\Lambda}(\alpha), F_{\Lambda}(\beta)\}$.

3 Interval-valued neutrosophic subalgebra in INK -algebra

In this section, we introduce the concept of $IVN INK$ -subalgebra and investigate some related properties.

Definition 3.1. An $IVNS \Lambda$ in U is called an interval valued neutrosophic INK -subalgebra, (briefly, $IVN INK$ -subalg) of U if satisfying the following condition, for all $\alpha, \beta \in U$

- (i) $T_{\Lambda}^+(\alpha * \beta) \geq r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}$
- (ii) $I_{\Lambda}^+(\alpha * \beta) \leq r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}$
- (iii) $F_{\Lambda}^+(\alpha * \beta) \geq r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}$.

Example 3.2. Assume that the set $U = \{0, \alpha, \beta\}$ has the following table:

*	0	α	β
0	0	α	β
α	α	0	β
β	β	α	0

Then U is a INK -subalgebra. We define an $IVNS \Lambda = (T_{\Lambda}, I_{\Lambda}, F_{\Lambda})$ as following:

$$\begin{aligned} T(0) &= [0.6, 0.8], T(\alpha) = [0.4, 0.5], T(\beta) = [0.3, 0.4], \\ I(0) &= [0.2, 0.3], I(\alpha) = [0.5, 0.6], I(\beta) = [0.4, 0.7], \\ F(0) &= [0.6, 0.8], F(\alpha) = [0.4, 0.5], F(\beta) = [0.3, 0.4]. \end{aligned}$$

Hence Λ is an $IVN INK$ -subalgebra of U . The existence of a $IVN INK$ -subalg of U for Λ is easily verified.

Proposition 3.3. Every $IVN INK$ -subalgebra of U , satisfies the conditions

- (i) $T_{\Lambda}^+(0) \geq T_{\Lambda}^+(\alpha)$
- (ii) $I_{\Lambda}^+(0) \leq I_{\Lambda}^+(\alpha)$ and
- (iii) $F_{\Lambda}^+(0) \geq F_{\Lambda}^+(\alpha)$ for all $\alpha \in U$.

Proof. Assume that Λ in $IVN INK$ -subalgebra of U . Then for all $\alpha \in U$, we have

$$\begin{aligned} T_{\Lambda}^+(0) &= T_{\Lambda}^+(\alpha * \alpha) \\ &\geq r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\alpha)\} \\ &= r \min\{[T_{\Lambda}^L(\alpha), T_{\Lambda}^U(\alpha)], [T_{\Lambda}^L(\alpha), T_{\Lambda}^U(\alpha)]\} \\ &= [T_{\Lambda}^L(\alpha), T_{\Lambda}^U(\alpha)] \\ &= T_{\Lambda}^+(\alpha). \end{aligned}$$

$$\begin{aligned}
 I_{\Lambda}^{+}(0) &= I_{\Lambda}^{+}(\alpha * \alpha) \\
 &\leq r \max\{I_{\Lambda}^{+}(\alpha), I_{\Lambda}^{+}(\alpha)\} \\
 &= r \max\{[I_{\Lambda}^L(\alpha), I_{\Lambda}^U(\alpha)], [I_{\Lambda}^L(\alpha), I_{\Lambda}^U(\alpha)]\} \\
 &= [I_{\Lambda}^L(\alpha), I_{\Lambda}^U(\alpha)] \\
 &= I_{\Lambda}^{+}(\alpha).
 \end{aligned}$$

$$\begin{aligned}
 F_{\Lambda}^{+}(0) &= F_{\Lambda}^{+}(\alpha * \alpha) \\
 &\geq r \min\{F_{\Lambda}^{+}(\alpha), F_{\Lambda}^{+}(\alpha)\} \\
 &= r \min\{[F_{\Lambda}^L(\alpha), F_{\Lambda}^U(\alpha)], [F_{\Lambda}^L(\alpha), F_{\Lambda}^U(\alpha)]\} \\
 &= [F_{\Lambda}^L(\alpha), F_{\Lambda}^U(\alpha)] \\
 &= F_{\Lambda}^{+}(\alpha).
 \end{aligned}$$

Hence Λ is a *IVN INK*-subalg of U . □

Theorem 3.4. Let Λ be a *IVN INK*-subalgebra of U . If there exists a sequence $\{\alpha_n\}$ in U such that

- (i) $\lim_{n \rightarrow \infty} T_{\Lambda}^{+}(\alpha_n) = [1, 1]$, then $T_{\Lambda}^{+}(0) = [1, 1]$
- (ii) $\lim_{n \rightarrow \infty} I_{\Lambda}^{+}(\alpha_n) = [1, 1]$, then $I_{\Lambda}^{+}(0) = [1, 1]$
- (iii) $\lim_{n \rightarrow \infty} F_{\Lambda}^{+}(\alpha_n) = [1, 1]$, then $F_{\Lambda}^{+}(0) = [1, 1]$

Proof. By Proposition 3.3, we have

$$T_{\Lambda}^{+}(0) \geq T_{\Lambda}^{+}(\alpha), \text{ for all } \alpha \in U.$$

Then $T_{\Lambda}^{+}(0) \geq T_{\Lambda}^{+}(\alpha_n)$, for every positive integer n .

$$\text{Consider } [1, 1] \geq T_{\Lambda}^{+}(0) \geq \lim_{n \rightarrow \infty} T_{\Lambda}^{+}(\alpha_n) \geq [1, 1].$$

Hence, $T_{\Lambda}^{+}(0) = [1, 1]$.

$$I_{\Lambda}^{+}(0) \leq I_{\Lambda}^{+}(\alpha), \text{ for all } \alpha \in U.$$

Then $I_{\Lambda}^{+}(0) \leq I_{\Lambda}^{+}(\alpha_n)$, for every positive integer n .

$$\text{Consider } [1, 1] \leq I_{\Lambda}^{+}(0) \leq \lim_{n \rightarrow \infty} I_{\Lambda}^{+}(\alpha_n) \leq [1, 1].$$

Hence $I_{\Lambda}^{+}(0) = [1, 1]$.

$$F_{\Lambda}^{+}(0) \geq F_{\Lambda}^{+}(\alpha), \forall \alpha \in U.$$

Then $F_{\Lambda}^{+}(0) \geq F_{\Lambda}^{+}(\alpha_n)$, for every positive integer n .

$$\text{Consider } [1, 1] \geq F_{\Lambda}^{+}(0) \geq \lim_{n \rightarrow \infty} F_{\Lambda}^{+}(\alpha_n) \geq [1, 1].$$

Hence $F_{\Lambda}^{+}(0) = [1, 1]$. □

Theorem 3.5. An *IVNS* $\Lambda = \{T_{\Lambda}, I_{\Lambda}, F_{\Lambda}\}$ in U is a *IVN INK*-subalg of U iff $T_{\Lambda}^L, T_{\Lambda}^U, I_{\Lambda}^L, I_{\Lambda}^U, F_{\Lambda}^L$ and F_{Λ}^U are *INK*-subalg of U .

Proof. Let T_{Λ}^L and T_{Λ}^U are *NINK*-subalg's of U and $\alpha, \beta \in U$. Then

$$\begin{aligned}
 T_{\Lambda}^{+}(\alpha * \beta) &\geq [T_{\Lambda}^L(\alpha * \beta), T_{\Lambda}^U(\alpha * \beta)] \\
 &\geq [\min\{T_{\Lambda}^L(\alpha), T_{\Lambda}^L(\beta)\}, \min\{T_{\Lambda}^U(\alpha), T_{\Lambda}^U(\beta)\}] \\
 &= r \min\{[T_{\Lambda}^L(\alpha), T_{\Lambda}^U(\alpha)], [T_{\Lambda}^L(\beta), T_{\Lambda}^U(\beta)]\} \\
 &= r \min\{T_{\Lambda}^{+}(\alpha), T_{\Lambda}^{+}(\beta)\}.
 \end{aligned}$$

Again, let I_{Λ}^L and I_{Λ}^U are *NINK*-subalg's of U and $\alpha, \beta \in U$. Then

$$\begin{aligned} I_{\Lambda}^+(\alpha * \beta) &\leq [I_{\Lambda}^L(\alpha * \beta), I_{\Lambda}^U(\alpha * \beta)] \\ &\leq [\max\{I_{\Lambda}^L(\alpha), I_{\Lambda}^L(\beta)\}, \max\{I_{\Lambda}^U(\alpha), I_{\Lambda}^U(\beta)\}] \\ &= r \max\{[I_{\Lambda}^L(\alpha), I_{\Lambda}^U(\alpha)], [I_{\Lambda}^L(\beta), I_{\Lambda}^U(\beta)]\} \\ &= r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}. \end{aligned}$$

Also, let F_{Λ}^L and F_{Λ}^U are *NINK*-subalg's of U and $\alpha, \beta \in U$. Then

$$\begin{aligned} F_{\Lambda}^+(\alpha * \beta) &\geq [F_{\Lambda}^L(\alpha * \beta), F_{\Lambda}^U(\alpha * \beta)] \\ &\geq [\min\{F_{\Lambda}^L(\alpha), F_{\Lambda}^L(\beta)\}, \min\{F_{\Lambda}^U(\alpha), F_{\Lambda}^U(\beta)\}] \\ &= r \min\{[F_{\Lambda}^L(\alpha), F_{\Lambda}^U(\alpha)], [F_{\Lambda}^L(\beta), F_{\Lambda}^U(\beta)]\} \\ &= r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}. \end{aligned}$$

Hence, Λ is an *IVN-INK*-subalg of U . Conversely, assume that Λ is a *IVN INK*-subalg of U . For any $\alpha, \beta \in U$, we have

$$\begin{aligned} [T_{\Lambda}^L(\alpha * \beta), T_{\Lambda}^U(\alpha * \beta)] &= T_{\Lambda}^+(\alpha * \beta) \\ &\geq r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\} \\ &= r \min\{[T_{\Lambda}^L(\alpha), T_{\Lambda}^U(\alpha)], [T_{\Lambda}^L(\beta), T_{\Lambda}^U(\beta)]\} \\ &= [\min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}] \\ T_{\Lambda}^L(\alpha * \beta) &\geq [\min\{T_{\Lambda}^L(\alpha), T_{\Lambda}^L(\beta)\}] \\ T_{\Lambda}^U(\alpha * \beta) &\geq [\min\{T_{\Lambda}^U(\alpha), T_{\Lambda}^U(\beta)\}]. \end{aligned}$$

$$\begin{aligned} [I_{\Lambda}^L(\alpha * \beta), I_{\Lambda}^U(\alpha * \beta)] &= I_{\Lambda}^+(\alpha * \beta) \\ &\leq r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\} \\ &= r \max\{[I_{\Lambda}^L(\alpha), I_{\Lambda}^U(\alpha)], [I_{\Lambda}^L(\beta), I_{\Lambda}^U(\beta)]\} \\ &= [\max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}] \\ I_{\Lambda}^L(\alpha * \beta) &\leq [\max\{I_{\Lambda}^L(\alpha), I_{\Lambda}^L(\beta)\}] \\ I_{\Lambda}^U(\alpha * \beta) &\leq [\max\{I_{\Lambda}^U(\alpha), I_{\Lambda}^U(\beta)\}]. \end{aligned}$$

$$\begin{aligned} [F_{\Lambda}^L(\alpha * \beta), F_{\Lambda}^U(\alpha * \beta)] &= F_{\Lambda}^+(\alpha * \beta) \\ &\geq r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\} \\ &= r \min\{[F_{\Lambda}^L(\alpha), F_{\Lambda}^U(\alpha)], [F_{\Lambda}^L(\beta), F_{\Lambda}^U(\beta)]\} \\ &= [\min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}] \\ F_{\Lambda}^L(\alpha * \beta) &\geq [\min\{F_{\Lambda}^L(\alpha), F_{\Lambda}^L(\beta)\}] \\ F_{\Lambda}^U(\alpha * \beta) &\geq [\min\{F_{\Lambda}^U(\alpha), F_{\Lambda}^U(\beta)\}]. \end{aligned}$$

Hence $T_{\Lambda}^L, T_{\Lambda}^U, I_{\Lambda}^L, I_{\Lambda}^U, F_{\Lambda}^L$ and F_{Λ}^U are *NINK*-subalg's of U . □

Definition 3.6. Let $\Lambda_1^+ = \{T_{\Lambda_1}^+, I_{\Lambda_1}^+, F_{\Lambda_1}^+\}$ and $\Lambda_2^+ = \{T_{\Lambda_2}^+, I_{\Lambda_2}^+, F_{\Lambda_2}^+\}$ are two *IVNS*'s on a *INK*-algebra U , define the *IVNS* $(\Lambda_1 \cap \Lambda_2)^+ = \{T_{\Lambda_1 \cap \Lambda_2}^+, I_{\Lambda_1 \cap \Lambda_2}^+, F_{\Lambda_1 \cap \Lambda_2}^+\}$ on U by,

$$\begin{aligned} T_{\Lambda_1 \cap \Lambda_2}^+(\alpha) &= r \min\{T_{\Lambda_1}^+(\alpha), T_{\Lambda_2}^+(\alpha)\}, \\ I_{\Lambda_1 \cap \Lambda_2}^+(\alpha) &= r \max\{I_{\Lambda_1}^+(\alpha), I_{\Lambda_2}^+(\alpha)\} \\ F_{\Lambda_1 \cap \Lambda_2}^+(\alpha) &= r \min\{F_{\Lambda_1}^+(\alpha), F_{\Lambda_2}^+(\alpha)\}, \forall \alpha \in U. \end{aligned}$$

Then $(\Lambda_1 \cap \Lambda_2)^+ = \{T_{\Lambda_1 \cap \Lambda_2}^+, I_{\Lambda_1 \cap \Lambda_2}^+, F_{\Lambda_1 \cap \Lambda_2}^+\}$ is called the intersection of $\Lambda_1^+ = \{T_{\Lambda_1}^+, I_{\Lambda_1}^+, F_{\Lambda_1}^+\}$ and $\Lambda_2^+ = \{T_{\Lambda_2}^+, I_{\Lambda_2}^+, F_{\Lambda_2}^+\}$.

Theorem 3.7. Let Λ_1 and Λ_2 are two *IVNINK*-subalg's of U . Then $\Lambda_1 \cap \Lambda_2$ is an *IVNINK*-subalg of U .

Proof. Let $\alpha, \beta \in \Lambda_1 \cap \Lambda_2$, then $\alpha, \beta \in \Lambda_1$ and Λ_2 . Since, Λ_1 and Λ_2 are *IVNINK*-subalg's of U by Theorem 3.5, we have,

$$\begin{aligned} T_{\Lambda_1 \cap \Lambda_2}^+(\alpha * \beta) &= r \min\{T_{\Lambda_1}^+(\alpha * \beta), T_{\Lambda_2}^+(\alpha * \beta)\} \\ &\geq r \min\{r \min\{T_{\Lambda_1}^L(\alpha), T_{\Lambda_1}^U(\beta)\}, r \min\{T_{\Lambda_2}^L(\alpha), T_{\Lambda_2}^U(\beta)\}\} \\ &= r \min\{r \min\{T_{\Lambda_1}^L(\alpha), T_{\Lambda_2}^U(\alpha)\}, r \min\{T_{\Lambda_1}^L(\beta), T_{\Lambda_2}^U(\beta)\}\} \\ &= r \min\{T_{\Lambda_1 \cap \Lambda_2}^+(\alpha), T_{\Lambda_1 \cap \Lambda_2}^+(\beta)\} \end{aligned}$$

$$\begin{aligned} I_{\Lambda_1 \cap \Lambda_2}^+(\alpha * \beta) &= r \max\{I_{\Lambda_1}^+(\alpha * \beta), I_{\Lambda_2}^+(\alpha * \beta)\} \\ &\leq r \max\{r \max\{I_{\Lambda_1}^L(\alpha), I_{\Lambda_1}^U(\beta)\}, r \max\{I_{\Lambda_2}^L(\alpha), I_{\Lambda_2}^U(\beta)\}\} \\ &= r \max\{r \max\{I_{\Lambda_1}^L(\alpha), I_{\Lambda_2}^U(\alpha)\}, r \max\{I_{\Lambda_1}^L(\beta), I_{\Lambda_2}^U(\beta)\}\} \\ &= r \max\{I_{\Lambda_1 \cap \Lambda_2}^+(\alpha), I_{\Lambda_1 \cap \Lambda_2}^+(\beta)\} \end{aligned}$$

$$\begin{aligned} F_{\Lambda_1 \cap \Lambda_2}^+(\alpha * \beta) &= r \min\{F_{\Lambda_1}^+(\alpha * \beta), F_{\Lambda_2}^+(\alpha * \beta)\} \\ &\geq r \min\{r \min\{F_{\Lambda_1}^L(\alpha), F_{\Lambda_1}^U(\beta)\}, r \min\{F_{\Lambda_2}^L(\alpha), F_{\Lambda_2}^U(\beta)\}\} \\ &= r \min\{r \min\{F_{\Lambda_1}^L(\alpha), F_{\Lambda_2}^U(\alpha)\}, r \min\{F_{\Lambda_1}^L(\beta), F_{\Lambda_2}^U(\beta)\}\} \\ &= r \min\{F_{\Lambda_1 \cap \Lambda_2}^+(\alpha), F_{\Lambda_1 \cap \Lambda_2}^+(\beta)\}. \end{aligned}$$

Hence $\Lambda_1 \cap \Lambda_2$ is *IVNINK*-subalg of U . □

Definition 3.8. Let $\Lambda_1^+ = \{T_{\Lambda_1}^+, I_{\Lambda_1}^+, F_{\Lambda_1}^+\}$ and $\Lambda_2^+ = \{T_{\Lambda_2}^+, I_{\Lambda_2}^+, F_{\Lambda_2}^+\}$ are two *IVNS*'s on a *INK*-algebra U , define the *IVNS* $(\Lambda_1 \cup \Lambda_2)^+ = \{T_{\Lambda_1 \cup \Lambda_2}^+, I_{\Lambda_1 \cup \Lambda_2}^+, F_{\Lambda_1 \cup \Lambda_2}^+\}$ on U by,

$$\begin{aligned} T_{\Lambda_1 \cup \Lambda_2}^+(\alpha) &= r \max\{T_{\Lambda_1}^+(\alpha), T_{\Lambda_2}^+(\alpha)\}, \\ I_{\Lambda_1 \cup \Lambda_2}^+(\alpha) &= r \min\{I_{\Lambda_1}^+(\alpha), I_{\Lambda_2}^+(\alpha)\} \\ F_{\Lambda_1 \cup \Lambda_2}^+(\alpha) &= r \max\{F_{\Lambda_1}^+(\alpha), F_{\Lambda_2}^+(\alpha)\} \end{aligned}$$

$\forall \alpha \in U$. Then $(\Lambda_1 \cup \Lambda_2)^+ = \{T_{\Lambda_1 \cup \Lambda_2}^+, I_{\Lambda_1 \cup \Lambda_2}^+, F_{\Lambda_1 \cup \Lambda_2}^+\}$ is called the union of $\Lambda_1^+ = \{T_{\Lambda_1}^+, I_{\Lambda_1}^+, F_{\Lambda_1}^+\}$ and $\Lambda_2^+ = \{T_{\Lambda_2}^+, I_{\Lambda_2}^+, F_{\Lambda_2}^+\}$.

Theorem 3.9. Let Λ_1 and Λ_2 are two *IVN INK*-subalg's of U . Then $\Lambda_1 \cup \Lambda_2$ is an *IVN INK*-subalg of U .

Proof. Let $\alpha, \beta \in \Lambda_1 \cup \Lambda_2$, then $\alpha, \beta \in \Lambda_1$ and Λ_2 . Since Λ_1 and Λ_2 are *IVN INK*-subalg's of U by Theorem 3.5, we have,

$$\begin{aligned} T_{\Lambda_1 \cup \Lambda_2}^+(\alpha * \beta) &= r \max\{T_{\Lambda_1}^+(\alpha * \beta), T_{\Lambda_2}^+(\alpha * \beta)\} \\ &\geq r \max\{r \min\{T_{\Lambda_1}^L(\alpha), T_{\Lambda_1}^U(\beta)\}, r \min\{T_{\Lambda_2}^L(\alpha), T_{\Lambda_2}^U(\beta)\}\} \\ &= r \min\{r \max\{T_{\Lambda_1}^L(\alpha), T_{\Lambda_2}^U(\alpha)\}, r \min\{T_{\Lambda_1}^L(\beta), T_{\Lambda_2}^U(\beta)\}\} \\ &= r \min\{T_{\Lambda_1 \cup \Lambda_2}^+(\alpha), T_{\Lambda_1 \cup \Lambda_2}^+(\beta)\}. \end{aligned}$$

$$\begin{aligned} I_{\Lambda_1 \cup \Lambda_2}^+(\alpha * \beta) &= r \min\{I_{\Lambda_1}^+(\alpha * \beta), I_{\Lambda_2}^+(\alpha * \beta)\} \\ &\leq r \min\{r \max\{I_{\Lambda_1}^L(\alpha), I_{\Lambda_1}^U(\beta)\}, r \max\{I_{\Lambda_2}^L(\alpha), I_{\Lambda_2}^U(\beta)\}\} \\ &= r \max\{r \min\{I_{\Lambda_1}^L(\alpha), I_{\Lambda_2}^U(\alpha)\}, r \min\{I_{\Lambda_1}^L(\beta), I_{\Lambda_2}^U(\beta)\}\} \\ &= r \max\{I_{\Lambda_1 \cup \Lambda_2}^+(\alpha), I_{\Lambda_1 \cup \Lambda_2}^+(\beta)\}. \end{aligned}$$

$$\begin{aligned}
F_{\Lambda_1 \cup \Lambda_2}^+(\alpha * \beta) &= r \max\{F_{\Lambda_1}^+(\alpha * \beta), F_{\Lambda_2}^+(\alpha * \beta)\} \\
&\geq r \max\{r \min\{F_{\Lambda_1}^L(\alpha), F_{\Lambda_1}^U(\beta)\}, r \min\{F_{\Lambda_2}^L(\alpha), F_{\Lambda_2}^U(\beta)\}\} \\
&= r \min\{r \max\{F_{\Lambda_1}^L(\alpha), F_{\Lambda_2}^L(\alpha)\}, r \min\{F_{\Lambda_1}^U(\beta), F_{\Lambda_2}^U(\beta)\}\} \\
&= r \min\{F_{\Lambda_1 \cup \Lambda_2}^+(\alpha), F_{\Lambda_1 \cup \Lambda_2}^+(\beta)\}.
\end{aligned}$$

Hence $\Lambda_1 \cup \Lambda_2$ is *IVN INK*-subalg of U . □

Corollary 3.10. Let $\{\Lambda_i/i \in \Lambda\}$ be a family of *IVN INK*-subalg's of U . Then $\bigcap_{i \in \Lambda} \Lambda_i$ is also an *IVN INK*-subalg of U .

Definition 3.11. Let Λ be an *IVN* in U and $[\alpha_1, \alpha_2] \in \theta[0, 1]$. Then the *IV* level neutrosophic *INK*-subalgebra (briefly, *IVN INK*-subalg) $U(\Lambda; [\alpha_1, \alpha_2])$ of Λ and strong *IVN INK*-subalgebra (briefly, *sIVN INK*-subalg) $U(\Lambda; < [\alpha_1, \alpha_2])$ of U are defined as following:

- (i) $U(\Lambda; [\alpha_1, \alpha_2]) := \{\alpha \in U | T_{\Lambda}^+(\alpha) \geq [\alpha_1, \alpha_2]\}$, $U(\Lambda; >, [\alpha_1, \alpha_2]) := \{\alpha \in U | T_{\Lambda}^+(\alpha) > [\alpha_1, \alpha_2]\}$.
- (ii) $U(\Lambda; [\alpha_1, \alpha_2]) := \{\alpha \in U | I_{\Lambda}^+(\alpha) \leq [\alpha_1, \alpha_2]\}$, $U(\Lambda; <, [\alpha_1, \alpha_2]) := \{\alpha \in U | I_{\Lambda}^+(\alpha) < [\alpha_1, \alpha_2]\}$.
- (iii) $U(\Lambda; [\alpha_1, \alpha_2]) := \{\alpha \in U | F_{\Lambda}^+(\alpha) \geq [\alpha_1, \alpha_2]\}$, $U(\Lambda; \geq, [\alpha_1, \alpha_2]) := \{\alpha \in U | F_{\Lambda}^+(\alpha) \geq [\alpha_1, \alpha_2]\}$.

Theorem 3.12. Let Λ_1 be an *IVFS* of U and Λ_2 be the closure of image of $\Lambda = \{T_{\Lambda}, I_{\Lambda}, F_{\Lambda}\}$. Then the subsequent conditions are equivalent:

- (i) Λ_1 is an *IVN INK*-subalg of U .
- (ii) For all $[\alpha_1, \alpha_2] \in Im(\Lambda = \{T_{\Lambda}, I_{\Lambda}, F_{\Lambda}\})$, the non empty level subset $U(\Lambda; [\alpha_1, \alpha_2])$ of Λ_1 is a *INK*-subalg of U .
- (iii) For all $[\alpha_1, \alpha_2] \in Im(\Lambda = \{T_{\Lambda}, I_{\Lambda}, F_{\Lambda}\})$, the non empty strong level subset $U(\Lambda; <, [\alpha_1, \alpha_2])$ of Λ_1 is a *INK*-subalg of U .
- (iv) For all $[\alpha_1, \alpha_2] \in \theta[0, 1]$, the non empty strong level subset $U(\Lambda_1; <, [\alpha_1, \alpha_2])$ of Λ_1 is a *INK*-subalg of U .
- (v) For all $[\alpha_1, \alpha_2] \in \theta[0, 1]$, the non empty level subset $U(\Lambda_1; [\alpha_1, \alpha_2])$ of Λ_1 is a *INK*-subalg of U .

Proof. (i) $\rightarrow$ (iv): Let Λ_1 be an *IVN INK*-subalg of U , $[\alpha_1, \alpha_2] \in \theta[0, 1]$ and $\alpha, \beta \in U(\Lambda_1; <, [\alpha_1, \alpha_2])$, then we have

$$\begin{aligned}
T_{\Lambda_1}^+(\alpha * \beta) &\geq r \min\{T_{\Lambda_1}^+(\alpha), T_{\Lambda_1}^+(\beta)\} \geq r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2], \\
I_{\Lambda_1}^+(\alpha * \beta) &\leq r \max\{I_{\Lambda_1}^+(\alpha), I_{\Lambda_1}^+(\beta)\} \leq r \max\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2], \\
F_{\Lambda_1}^+(\alpha * \beta) &\geq r \min\{F_{\Lambda_1}^+(\alpha), F_{\Lambda_1}^+(\beta)\} \geq r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2].
\end{aligned}$$

Thus $\alpha * \beta \in U(\Lambda_1; <, [\alpha_1, \alpha_2])$. Hence $U(\Lambda_1; <, [\alpha_1, \alpha_2])$ is a neutrosophic *INK*-subalg of U .

(iv) $\rightarrow$ (iii): It's evident.

(iii) $\rightarrow$ (ii): Let $[\alpha_1, \alpha_2] \in Im(\Lambda_1 = \{T_{\Lambda_1}, I_{\Lambda_1}, F_{\Lambda_1}\})$. Then $U(\Lambda_1; [\alpha_1, \alpha_2])$ is a non empty. Since $U(\Lambda_1; [\alpha_1, \alpha_2]) = \bigcap_{[\alpha_1, \alpha_2] < [\beta_1, \beta_2]} U(\Lambda_1; [\alpha_1, \alpha_2])$, where $[\beta_1, \beta_2] \in Im(\Lambda_1 = \{T_{\Lambda_1}, I_{\Lambda_1}, F_{\Lambda_1}\})|_{\Lambda_2}$. Then

by (iii) and Corollary 3.10, $U(\Lambda_1; [\alpha_1, \alpha_2])$ is a *IVN INK*-subalg of U .

(ii) $\rightarrow$ (v): Let $[\alpha_1, \alpha_2] \in \theta[0, 1]$ and $U(\Lambda_1; [\alpha_1, \alpha_2])$ be nonempty. Suppose $\alpha, \beta \in U(\Lambda_1; [\alpha_1, \alpha_2])$. Let (i) $[\gamma_1, \gamma_2] = \min\{T_{\Lambda_1}(\alpha), T_{\Lambda_1}(\beta)\}$, it is clear that $[\gamma_1, \gamma_2] = \min\{T_{\Lambda_1}(\alpha), T_{\Lambda_1}(\beta)\} \geq \{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\}$. Thus $\alpha, \beta \in U(\Lambda_1; [\gamma_1, \gamma_2])$ and $[\gamma_1, \gamma_2] \in Im(T_{\Lambda_1})$.

(ii) $[\gamma_1, \gamma_2] = \max\{I_{\Lambda_1}(\alpha), T_{\Lambda_1}(\beta)\}$, it is clear that $[\gamma_1, \gamma_2] = \max\{I_{\Lambda_1}(\alpha), I_{\Lambda_1}(\beta)\} \leq \{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\}$. Thus $\alpha, \beta \in U(\Lambda_1; [\gamma_1, \gamma_2])$ and $[\gamma_1, \gamma_2] \in Im(T_{\Lambda_1})$.

(iii) $[\gamma_1, \gamma_2] = \min\{F_{\Lambda_1}(\alpha), F_{\Lambda_1}(\beta)\}$, it is clear that $[\gamma_1, \gamma_2] = \min\{F_{\Lambda_1}(\alpha), F_{\Lambda_1}(\beta)\} \geq \{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\}$. Thus $\alpha, \beta \in U(\Lambda_1; [\gamma_1, \gamma_2])$ and $[\gamma_1, \gamma_2] \in Im(F_{\Lambda_1})$, by (ii) $U(\Lambda_1; [\gamma_1, \gamma_2])$ is a *N INK*-subalg's of U , hence $\alpha * \beta \in U(\Lambda_1; [\gamma_1, \gamma_2])$. Then we have,

$$(i) \quad T_{\Lambda_1}^+(\alpha * \beta) \geq r \min\{T_{\Lambda_1}^+(\alpha), T_{\Lambda_1}^+(\beta)\} \geq \{[\gamma_1, \gamma_2], [\gamma_1, \gamma_2]\} = [\gamma_1, \gamma_2] \geq [\alpha_1, \alpha_2].$$

$$(ii) \quad I_{\Lambda_1}^+(\alpha * \beta) \leq r \max\{I_{\Lambda_1}^+(\alpha), I_{\Lambda_1}^+(\beta)\} \leq \{[\gamma_1, \gamma_2], [\gamma_1, \gamma_2]\} = [\gamma_1, \gamma_2] \leq [\alpha_1, \alpha_2].$$

(iii) $F_{\Lambda_1}^+(\alpha * \beta) \geq r \min\{F_{\Lambda_1}^+(\alpha), F_{\Lambda_1}^+(\beta)\} \leq \{[\gamma_1, \gamma_2], [\gamma_1, \gamma_2]\} = [\gamma_1, \gamma_2] \leq [\alpha_1, \alpha_2]$. Therefore $\alpha * \beta \in U(\Lambda_1; [\alpha_1, \alpha_2])$. Then $\alpha * \beta \in U(\Lambda_1; [\alpha_1, \alpha_2])$ is a *N INK*-subalg of U .

(v) $\rightarrow$ (i): Assume that the non empty set $U(\Lambda_1; [\alpha_1, \alpha_2])$ is a *N INK*-subalg of U , for every $[\alpha_1, \alpha_2] \in \theta[0, 1]$. In contrary, let $\alpha_0, \beta_0 \in U$ be such that

(i) $T_{\Lambda_1}^+(\alpha_0 * \beta_0) < r \min\{T_{\Lambda_1}^+(\alpha_0), T_{\Lambda_1}^+(\beta_0)\}$. Let $T_{\Lambda_1}^+(\alpha_0) = [\delta_1, \delta_2]$, $T_{\Lambda_1}^+(\beta_0) = [\delta_3, \delta_4]$ and $T_{\Lambda_1}^+(\alpha_0 * \beta_0) = [\alpha_1, \alpha_2]$. Then $[\alpha_1, \alpha_2] < r \min\{[\delta_1, \delta_2], [\delta_3, \delta_4]\} = [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}]$. So $\alpha_1 < \min\{\delta_1, \delta_3\}$ and $\alpha_2 < \min\{\delta_2, \delta_4\}$.

$$\text{Consider, } [g_1, g_2] = \frac{1}{2} T_{\Lambda_1}^+(\alpha_0 * \beta_0) + r \min\{T_{\Lambda_1}^+(\alpha_0), T_{\Lambda_1}^+(\beta_0)\}.$$

$$\begin{aligned} \text{We get that } [g_1, g_2] &= \frac{1}{2} ([\alpha_1, \alpha_2] + [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}]) \\ &= [\frac{1}{2}(\alpha_1 + \min\{\delta_1, \delta_3\}), \frac{1}{2}(\alpha_2 + \min\{\delta_2, \delta_4\})]. \end{aligned}$$

$$\text{Therefore, } \min\{\delta_1, \delta_3\} > g_1 = \frac{1}{2}(\alpha_1 + \min\{\delta_1, \delta_3\}) > \alpha_1.$$

$$\min\{\delta_2, \delta_4\} > g_2 = \frac{1}{2}(\alpha_2 + \min\{\delta_2, \delta_4\}) > \alpha_2.$$

Hence, $[\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}] > [g_1, g_2] > [\alpha_1, \alpha_2] = T_{\Lambda_1}^+(\alpha_0 * \beta_0)$. So that $\alpha_0 * \beta_0 \notin U(\Lambda_1; [\alpha_1, \alpha_2])$.

(ii) $I_{\Lambda_1}^+(\alpha_0 * \beta_0) > r \max\{I_{\Lambda_1}^+(\alpha_0), I_{\Lambda_1}^+(\beta_0)\}$. Let $I_{\Lambda_1}^+(\alpha_0) = [\delta_1, \delta_2]$, $I_{\Lambda_1}^+(\beta_0) = [\delta_3, \delta_4]$ and $I_{\Lambda_1}^+(\alpha_0 * \beta_0) = [\alpha_1, \alpha_2]$. Then, $[\alpha_1, \alpha_2] > r \max\{[\delta_1, \delta_2], [\delta_3, \delta_4]\} = [\max\{\delta_1, \delta_3\}, \max\{\delta_2, \delta_4\}]$. So $\alpha_1 < \max\{\delta_1, \delta_3\}$ and $\alpha_2 < \max\{\delta_2, \delta_4\}$.

$$\text{Consider } [g_1, g_2] = \frac{1}{2} I_{\Lambda_1}^+(\alpha_0 * \beta_0) + r \max\{I_{\Lambda_1}^+(\alpha_0), I_{\Lambda_1}^+(\beta_0)\}.$$

$$\begin{aligned} \text{We get that } [g_1, g_2] &= \frac{1}{2} ([\alpha_1, \alpha_2] + [\max\{\delta_1, \delta_3\}, \max\{\delta_2, \delta_4\}]) \\ &= [\frac{1}{2}(\alpha_1 + \max\{\delta_1, \delta_3\}), \frac{1}{2}(\alpha_2 + \max\{\delta_2, \delta_4\})]. \end{aligned}$$

$$\text{Therefore, } \max\{\delta_1, \delta_3\} < g_1 = \frac{1}{2}(\alpha_1 + \max\{\delta_1, \delta_3\}) < \alpha_1.$$

$$\max\{\delta_2, \delta_4\} < g_2 = \frac{1}{2}(\alpha_2 + \max\{\delta_2, \delta_4\}) < \alpha_2.$$

Hence, $[\max\{\delta_1, \delta_3\}, \max\{\delta_2, \delta_4\}] < [g_1, g_2] < [\alpha_1, \alpha_2] = I_{\Lambda_1}^+(\alpha_0 * \beta_0)$. So that $\alpha_0 * \beta_0 \notin U(\Lambda_1; [\alpha_1, \alpha_2])$.

(iii) $F_{\Lambda_1}^+(\alpha_0 * \beta_0) < r \min\{F_{\Lambda_1}^+(\alpha_0), F_{\Lambda_1}^+(\beta_0)\}$. Let $F_{\Lambda_1}^+(\alpha_0) = [\delta_1, \delta_2]$, $F_{\Lambda_1}^+(\beta_0) = [\delta_3, \delta_4]$ and $F_{\Lambda_1}^+(\alpha_0 * \beta_0) = [\alpha_1, \alpha_2]$. Then $[\alpha_1, \alpha_2] < r \min\{[\delta_1, \delta_2], [\delta_3, \delta_4]\} = [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}]$. So, $\alpha_1 < \min\{\delta_1, \delta_3\}$

and $\alpha_2 < \min\{\delta_2, \delta_4\}$.

$$\text{Consider } [g_1, g_2] = \frac{1}{2} F_{\Lambda_1}^+(\alpha_0 * \beta_0) + r \min\{F_{\Lambda_1}^+(\alpha_0), F_{\Lambda_1}^+(\beta_0)\}.$$

$$\begin{aligned} \text{We get that } [g_1, g_2] &= \frac{1}{2} ([\alpha_1, \alpha_2] + [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}]) \\ &= [\frac{1}{2}(\alpha_1 + \min\{\delta_1, \delta_3\}), \frac{1}{2}(\alpha_2 + \min\{\delta_2, \delta_4\})]. \end{aligned}$$

$$\text{Therefore, } \min\{\delta_1, \delta_3\} > g_1 = \frac{1}{2}(\alpha_1 + \min\{\delta_1, \delta_3\}) > \alpha_1.$$

$$\min\{\delta_2, \delta_4\} > g_2 = \frac{1}{2}(\alpha_2 + \min\{\delta_2, \delta_4\}) > \alpha_2.$$

Hence, $[\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}] > [g_1, g_2] > [\alpha_1, \alpha_2] = F_{\Lambda_1}^+(\alpha_0 \times \beta_0)$. So that $\alpha_0 \times \beta_0 \notin U(\Lambda_1; [\alpha_1, \alpha_2])$, which is a contraction. Since

- (i) $T_{\Lambda_1}^+(\alpha_0) = [\delta_1, \delta_2] \geq [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}] > [g_1, g_2]$
 $T_{\Lambda_1}^+(\beta_0) = [\delta_3, \delta_4] \geq [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}] > [g_1, g_2]$ imply that $\alpha_0, \beta_0 \in U(\Lambda_1; [\alpha_1, \alpha_2])$.
 Thus, $T_{\Lambda_1}^+(\alpha * \beta) \geq r \min\{T_{\Lambda_1}^+(\alpha), T_{\Lambda_1}^+(\beta)\}$ for all $\alpha, \beta \in U$.
- (ii) $I_{\Lambda_1}^+(\alpha_0) = [\delta_1, \delta_2] \leq [\max\{\delta_1, \delta_3\}, \max\{\delta_2, \delta_4\}] < [g_1, g_2]$
 $I_{\Lambda_1}^+(\beta_0) = [\delta_3, \delta_4] \leq [\max\{\delta_1, \delta_3\}, \max\{\delta_2, \delta_4\}] < [g_1, g_2]$ imply that $\alpha_0, \beta_0 \in U(\Lambda_1; [\alpha_1, \alpha_2])$.
 Thus $I_{\Lambda_1}^+(\alpha * \beta) \leq r \max\{I_{\Lambda_1}^+(\alpha), I_{\Lambda_1}^+(\beta)\}$ for all $\alpha, \beta \in U$.
- (iii) $F_{\Lambda_1}^+(\alpha_0) = [\delta_1, \delta_2] \geq [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}] > [g_1, g_2]$
 $F_{\Lambda_1}^+(\beta_0) = [\delta_3, \delta_4] \geq [\min\{\delta_1, \delta_3\}, \min\{\delta_2, \delta_4\}] > [g_1, g_2]$ imply that $\alpha_0, \beta_0 \in U(\Lambda_1; [\alpha_1, \alpha_2])$.
 Thus $F_{\Lambda_1}^+(\alpha * \beta) \geq r \min\{F_{\Lambda_1}^+(\alpha), F_{\Lambda_1}^+(\beta)\}$ for all $\alpha, \beta \in U$.

Which completes the proof. □

Theorem 3.13. Each N INK-subalg of U is an IV level N INK-subalg of an IVN INK-subalg of U .

Proof. Let V be a N INK-subalg of U , and Λ be an IVN set on U defined by

$$(i) T_{\Lambda}^+(\alpha) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } \alpha \in V \\ [0, 0] & \text{otherwise} \end{cases}$$

$$(ii) I_{\Lambda}^+(\alpha) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } \alpha \in V \\ [0, 0] & \text{otherwise} \end{cases}$$

$$(iii) F_{\Lambda}^+(\alpha) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } \alpha \in V \\ [0, 0] & \text{otherwise} \end{cases}$$

where $\alpha_1, \alpha_2 \in [0, 1]$ with $\alpha_1 < \alpha_2$. It is clear that $U(\Lambda; [\alpha_1, \alpha_2]) = V$. Let $\alpha, \beta \in U$. We examine the subsequent cases:

Case (i): If $\alpha, \beta \in V$, then $\alpha * \beta \in V$. Therefore,

- (i) $T_{\Lambda}^+(\alpha * \beta) = [\alpha_1, \alpha_2] = r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}$
- (ii) $I_{\Lambda}^+(\alpha * \beta) = [\alpha_1, \alpha_2] = r \max\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}$
- (iii) $F_{\Lambda}^+(\alpha * \beta) = [\alpha_1, \alpha_2] = r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}.$

Case (ii): If $\alpha, \beta \notin V$, then

- (i) $T_{\Lambda}^+(\alpha) = [0, 0] = T_{\Lambda}^+(\beta)$ and so, $T_{\Lambda}^+(\alpha * \beta) \geq [0, 0] = r \min\{[0, 0], [0, 0]\} = r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}$.
- (ii) $I_{\Lambda}^+(\alpha) = [0, 0] = I_{\Lambda}^+(\beta)$ and so, $I_{\Lambda}^+(\alpha * \beta) \leq [0, 0] = r \max\{[0, 0], [0, 0]\} = r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}$.
- (iii) $F_{\Lambda}^+(\alpha) = [0, 0] = F_{\Lambda}^+(\beta)$ and so, $F_{\Lambda}^+(\alpha * \beta) \geq [0, 0] = r \min\{[0, 0], [0, 0]\} = r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}$.

Case (iii): If $\alpha \in V$ and $\beta \notin V$, then

- (i) $T_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2]$ and $T_{\Lambda}^+(\beta) = [0, 0]$. Thus, $T_{\Lambda}^+(\alpha * \beta) \geq [0, 0] = r \min\{[\alpha_1, \alpha_2], [0, 0]\} = r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}$.
- (ii) $I_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2]$ and $I_{\Lambda}^+(\beta) = [0, 0]$. Thus, $I_{\Lambda}^+(\alpha * \beta) \leq [0, 0] = r \max\{[\alpha_1, \alpha_2], [0, 0]\} = r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}$.
- (iii) $F_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2]$ and $F_{\Lambda}^+(\beta) = [0, 0]$. Thus, $F_{\Lambda}^+(\alpha * \beta) \geq [0, 0] = r \min\{[\alpha_1, \alpha_2], [0, 0]\} = r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}$.

Case (iv): If $\beta \in V$ and $\alpha \notin V$, then by the same arguments as in case (iii), we can conclude that

- (i) $T_{\Lambda}^+(\alpha * \beta) \geq r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}$.
- (ii) $I_{\Lambda}^+(\alpha * \beta) \leq r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}$.
- (iii) $F_{\Lambda}^+(\alpha * \beta) \geq r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}$.

Therefore Λ is an *IVN INK*-subalg of U . □

Theorem 3.14. Let V be a subset of U and Λ be an *IVN* set on U which is given in the proof of Theorem 3.12. If Λ is an *IVN INK*-subalg of U , then V is a *N INK*-subalg of U .

Proof. Let Λ be an *IVN INK*-subalg of U , and $\alpha, \beta \in V$. Then,

- (i) $T_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2] = T_{\Lambda}^+(\beta)$, thus $T_{\Lambda}^+(\alpha * \beta) \geq r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\} = r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2]$.
- (ii) $I_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2] = I_{\Lambda}^+(\beta)$, thus $I_{\Lambda}^+(\alpha * \beta) \leq r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\} = r \max\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2]$.
- (iii) $F_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2] = F_{\Lambda}^+(\beta)$, thus $F_{\Lambda}^+(\alpha * \beta) \geq r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\} = r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2]$.

which implies that $\alpha * \beta \in V$. □

Theorem 3.15. If Λ is an *IVN INK*-subalg of U , then the set

$$(i) U_{T_{\Lambda}^+} := \{\alpha \in U | T_{\Lambda}^+(\alpha) = T_{\Lambda}^+(0)\}$$

$$(ii) U_{I_{\Lambda}^+} := \{\alpha \in U | I_{\Lambda}^+(\alpha) = I_{\Lambda}^+(0)\}$$

$$(iii) U_{F_{\Lambda}^+} := \{\alpha \in U | F_{\Lambda}^+(\alpha) = F_{\Lambda}^+(0)\}$$

is a *N INK*-subalg of U .

Proof. (i) Let $\alpha, \beta \in U_{T_{\Lambda}^+}$ then $T_{\Lambda}^+(\alpha) = T_{\Lambda}^+(0) = T_{\Lambda}^+(\beta)$, and so $T_{\Lambda}^+(\alpha * \beta) \geq r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\} = r \min\{T_{\Lambda}^+(0), T_{\Lambda}^+(0)\} = T_{\Lambda}^+(0)$, by Proposition 3.3 we get that $T_{\Lambda}^+(\alpha * \beta) = T_{\Lambda}^+(0)$ which means that $\alpha * \beta \in U_{T_{\Lambda}^+}$.

(ii) Let $\alpha, \beta \in U_{I_{\Lambda}^+}$ then $I_{\Lambda}^+(\alpha) = I_{\Lambda}^+(0) = I_{\Lambda}^+(\beta)$, and so $I_{\Lambda}^+(\alpha * \beta) \leq r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\} = r \max\{I_{\Lambda}^+(0), I_{\Lambda}^+(0)\} = I_{\Lambda}^+(0)$, by Proposition 3.3 we get that $I_{\Lambda}^+(\alpha * \beta) = I_{\Lambda}^+(0)$ which means that $\alpha * \beta \in U_{I_{\Lambda}^+}$.

(iii) Let $\alpha, \beta \in U_{F_{\Lambda}^+}$ then $F_{\Lambda}^+(\alpha) = F_{\Lambda}^+(0) = F_{\Lambda}^+(\beta)$, and so $F_{\Lambda}^+(\alpha * \beta) \geq r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\} = r \min\{F_{\Lambda}^+(0), F_{\Lambda}^+(0)\} = F_{\Lambda}^+(0)$, by Proposition 3.3 we get that $F_{\Lambda}^+(\alpha * \beta) = F_{\Lambda}^+(0)$ which means that $\alpha * \beta \in U_{F_{\Lambda}^+}$. $\square$

Theorem 3.16. Let Λ be an IVNS defined by:

$$(i) T_{\Lambda}^+(\alpha) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } a \in \Lambda \\ [\beta_1, \beta_2], & \text{otherwise} \end{cases}$$

for all $[\alpha_1, \alpha_2], [\beta_1, \beta_2] \in \theta[0, 1]$ with $[\alpha_1, \alpha_2] \geq [\beta_1, \beta_2]$.

$$(ii) I_{\Lambda}^+(\alpha) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } a \in \Lambda \\ [\beta_1, \beta_2], & \text{otherwise} \end{cases}$$

for all $[\alpha_1, \alpha_2], [\beta_1, \beta_2] \in \theta[0, 1]$ with $[\alpha_1, \alpha_2] \leq [\beta_1, \beta_2]$.

$$(iii) F_{\Lambda}^+(\alpha) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } a \in \Lambda \\ [\beta_1, \beta_2], & \text{otherwise} \end{cases}$$

for all $[\alpha_1, \alpha_2], [\beta_1, \beta_2] \in \theta[0, 1]$ with $[\alpha_1, \alpha_2] \geq [\beta_1, \beta_2]$. Then Λ is an an IVN INK-subalg if and only if Λ is a N INK-subalg of U . Moreover, in this case

$$(i) U_{T_{\Lambda}^+} = \Lambda,$$

$$(ii) U_{I_{\Lambda}^+} = \Lambda,$$

$$(iii) U_{F_{\Lambda}^+} = \Lambda.$$

Proof. Let Λ be an an IVN INK-subalg. Let $\alpha, \beta \in U$ be such that $\alpha * \beta \in \Lambda$. Then,

$$(i) T_{\Lambda}^+(\alpha * \beta) \geq r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\} = r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2] \text{ and so } \alpha * \beta \in \Lambda.$$

$$(ii) I_{\Lambda}^+(\alpha * \beta) \leq r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\} = r \max\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2] \text{ and so } \alpha * \beta \in \Lambda.$$

$$(iii) F_{\Lambda}^+(\alpha * \beta) \geq r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\} = r \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2] \text{ and so } \alpha * \beta \in \Lambda.$$

Conversely, suppose that Λ is a N INK-subalg of U , let $\alpha, \beta \in U$.

(i) If $\alpha, \beta \in \Lambda$ then $\alpha * \beta \in \Lambda$, thus

$$T_{\Lambda}^+(\alpha * \beta) = [\alpha_1, \alpha_2] = r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}.$$

$$I_{\Lambda}^+(\alpha * \beta) = [\alpha_1, \alpha_2] = r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}.$$

$$F_{\Lambda}^+(\alpha * \beta) = [\alpha_1, \alpha_2] = r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}.$$

(ii) If $a \in \Lambda$ or $b \notin \Lambda$, then

$$T_{\Lambda}^+(\alpha * \beta) \geq [\beta_1, \beta_2] = r \min\{T_{\Lambda}^+(\alpha), T_{\Lambda}^+(\beta)\}.$$

$$I_{\Lambda}^+(\alpha * \beta) \leq [\beta_1, \beta_2] = r \max\{I_{\Lambda}^+(\alpha), I_{\Lambda}^+(\beta)\}.$$

$$F_{\Lambda}^+(\alpha * \beta) \geq [\beta_1, \beta_2] = r \min\{F_{\Lambda}^+(\alpha), F_{\Lambda}^+(\beta)\}.$$

This show that Λ is an *IVN INK*-subalg. Moreover, we have

$$(i) U_{T_{\Lambda}^+} := \{\alpha \in U | T_{\Lambda}^+(\alpha) = T_{\Lambda}^+(0)\} = \{\alpha \in U | T_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2]\} = \Lambda.$$

$$(ii) U_{I_{\Lambda}^+} := \{\alpha \in U | I_{\Lambda}^+(\alpha) = I_{\Lambda}^+(0)\} = \{\alpha \in U | I_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2]\} = \Lambda.$$

$$(iii) U_{F_{\Lambda}^+} := \{\alpha \in U | F_{\Lambda}^+(\alpha) = F_{\Lambda}^+(0)\} = \{\alpha \in U | F_{\Lambda}^+(\alpha) = [\alpha_1, \alpha_2]\} = \Lambda.$$

□

Definition 3.17. Let f be a mapping from the set U into a set V . Let $\Lambda_2 = \{T_{\Lambda_2}, I_{\Lambda_2}, F_{\Lambda_2}\}$ be an *IVN* set in V . Then the inverse image of Λ_2 , denoted by $f^{-1}(\Lambda_2)$, is the *IVN* set in U with the membership function given by

$$(i) T_{\Lambda_2}^+ f^{-1}(\Lambda_2)(\alpha) = T_{\Lambda_2}^+(f(\alpha)), \text{ for all } \alpha \in U.$$

$$(ii) I_{\Lambda_2}^+ f^{-1}(\Lambda_2)(\alpha) = I_{\Lambda_2}^+(f(\alpha)), \text{ for all } \alpha \in U.$$

$$(iii) F_{\Lambda_2}^+ f^{-1}(\Lambda_2)(\alpha) = F_{\Lambda_2}^+(f(\alpha)), \text{ for all } \alpha \in U.$$

Lemma 3.18. Let f be a mapping from the set U into a set V . Let $m = [m^L, m^U]$ and $n = [n^L, n^U]$ be *IVFS*'s in U and V respectively. Then

$$(i) f^{-1}(n) = [f^{-1}(n^L), f^{-1}(n^U)],$$

$$(ii) f(m) = [f(m^L), f(m^U)].$$

Proposition 3.19. ⁶ Let f be a *N INK*-homomorphism from U into V and G be a *N INK*-subalg of V with the membership function μ_G . Then the inverse image $f^{-1}(G)$ of G is a *N INK*-subalg of U .

Proposition 3.20. ⁶ Let f be a *N INK*-homomorphism from U onto V and D be a *N INK*-subalg of U with the sup property. Then the image $f(D)$ of D is a *N INK*-subalg of V .

Proposition 3.21. Let f be a $\Lambda = \{T_{\Lambda}, I_{\Lambda}, F_{\Lambda}\}$ *N INK* homomorphism from U into V and G be an *IVN INK*-subalg of V with the membership function μ_G . Then the inverse image $f^{-1}(G)$ of G is an *IVN INK*-subalg of U .

Proof. Since,

$$(i) T_{\Lambda_2} = [T_{\Lambda_2}^L, T_{\Lambda_2}^U]$$

$$(ii) I_{\Lambda_2} = [I_{\Lambda_2}^L, I_{\Lambda_2}^U]$$

$$(iii) F_{\Lambda_2} = [F_{\Lambda_2}^L, F_{\Lambda_2}^U]$$

is an *IVN INK*-subalg of V , by Theorem 3.5, we get that $T_{\Lambda_2}^L, T_{\Lambda_2}^U, I_{\Lambda_2}^L, I_{\Lambda_2}^U, F_{\Lambda_2}^L$ and $F_{\Lambda_2}^U$ are *N INK*-subalg of V . By Proposition 3.20, $f^{-1}[T_{\Lambda_2}^L], f^{-1}[T_{\Lambda_2}^U], f^{-1}[I_{\Lambda_2}^L], f^{-1}[I_{\Lambda_2}^U], f^{-1}[F_{\Lambda_2}^L]$ and $f^{-1}[F_{\Lambda_2}^U]$ are *N INK*-subalg of U , by above Lemma 3.18 and Theorem 3.5, we can conclude that,

$$(i) f^{-1}(\Lambda_2) = [f^{-1}[T_{\Lambda_2}^L], f^{-1}[T_{\Lambda_2}^U]],$$

$$(ii) f^{-1}(\Lambda_2) = [f^{-1}[I_{\Lambda_2}^L], f^{-1}[I_{\Lambda_2}^U]],$$

$$(iii) f^{-1}(\Lambda_2) = [f^{-1}[F_{\Lambda_2}^L], f^{-1}[F_{\Lambda_2}^U]].$$

are *IVN INK*-subalg of U . □

Definition 3.22. Let f be a mapping from the set U into a set V , and Λ_1 be an *IVNS* in U with membership functions T_{Λ_1} , I_{Λ_1} and F_{Λ_1} . Then the image of Λ_1 , denoted by $f(\Lambda_1)$, is the *IVNS* in V with membership function defined by:

$$T_{f(\Lambda_1)}^+(\beta) = \begin{cases} \sup_{\gamma \in f^{-1}(\beta)} T_{\Lambda_1}^+(\gamma) & \text{if } f^{-1}(\beta) \neq \emptyset, \forall \beta \in B \\ [0, 0], & \text{otherwise} \end{cases}$$

$$I_{f(\Lambda_1)}^+(\beta) = \begin{cases} \inf_{\gamma \in f^{-1}(\beta)} I_{\Lambda_1}^+(\gamma) & \text{if } f^{-1}(\beta) \neq \emptyset, \forall \beta \in B \\ [0, 0], & \text{otherwise} \end{cases}$$

$$F_{f(\Lambda_1)}^+(\beta) = \begin{cases} \sup_{\gamma \in f^{-1}(\beta)} F_{\Lambda_1}^+(\gamma) & \text{if } f^{-1}(\beta) \neq \emptyset, \forall \beta \in B \\ [0, 0], & \text{otherwise} \end{cases}$$

where $f^{-1}(\beta) = \{a | f(a) = \beta\}$.

Theorem 3.23. Let f be a *N INK*-homomorphism from U onto V . If Λ_1 is an *IVN INK*-subalg of U with the sup property, then the image $f(\Lambda_1)$ of Λ_1 is an *IVN INK*-subalg of V .

Proof. Assume that Λ_1 is an *IVN INK*-subalg of U , then

$$(i) \Lambda_1 = [T_{\Lambda_1}^L, T_{\Lambda_1}^U]$$

$$(ii) \Lambda_1 = [I_{\Lambda_1}^L, I_{\Lambda_1}^U]$$

$$(iii) \Lambda_1 = [F_{\Lambda_1}^L, F_{\Lambda_1}^U]$$

are *IVN INK*-subalg of U if and only if $T_{\Lambda_2}^L, T_{\Lambda_2}^U, I_{\Lambda_2}^L, I_{\Lambda_2}^U, F_{\Lambda_2}^L$ and $F_{\Lambda_2}^U$ are *N INK*-subalg of U . By Proposition 3.21 $f[T_{\Lambda_1}^L], f[T_{\Lambda_1}^U], f[I_{\Lambda_1}^L], f[I_{\Lambda_1}^U], f[F_{\Lambda_1}^L]$ and $f[F_{\Lambda_1}^U]$ are *N INK*-subalg of V , by Lemma 3.18 and Theorem 3.12 we can conclude that,

$$(i) f(\Lambda_1) = [f(T_{\Lambda_1}^L), f(T_{\Lambda_1}^U)],$$

$$(ii) f(\Lambda_1) = [f(I_{\Lambda_1}^L), f(I_{\Lambda_1}^U)],$$

$$(iii) f(\Lambda_1) = [f(F_{\Lambda_1}^L), f(F_{\Lambda_1}^U)].$$

is an *IVN INK*-subalg of V . □

4 Conclusions

In this paper, *IVN INK*-subalgebra is defined and studied some properties of it. Furthermore, several features of the homomorphism of *IVN INK*-subalgebra of *INK*-algebra are defined.

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