Representing Symbolic 3-Plithogenic Matrices with Symbolic 3-Plithogenic Linear Transformations

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Abstract

The objective of this article is to study about the representation of symbolic 3-plithogenic matrices by linear transformations between symbolic 3-plithogenic vector spaces, where it proves that every symbolic 3-plithogenic matrix can be represented uniquely by a linear transformation between symbolic 3-plithogenic vector spaces. On the other hand, this work introduces an algorithm to compute a basis of any symbolic 3-plithogenic vector space depending on the classical basis of its corresponding classical vector space.

Keywords: Symbolic 3-plithogenic vector space; symbolic 3-plithogenic matrice; linear transformation.

1 Introduction

The concept of refined neutrosophic structure was studied by many authors in.[1–3,8] Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation.[23]

In,[14] the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings are studied. Further, Taffach[24,25] studied the concepts of symbolic 2-plithogenic vector spaces and modules. In,[7] the concept of symbolic 2-plithogenic matrices, determinants, eigen values and vectors, exponents, and diagonalization are discussed.

Laterally, many authors defined and studied symbolic 3-plithogenic algebraic structures, such as symbolic 3-plithogenic rings, vectors spaces and modules.[10,12,13] Recently, Merkepci[16] introduced and studied symbolic 3-plithogenic and 4-plithogenic square matrices and its algebraic properties such as determinant, invertibility, Eigen values, diagonalization, etc.

Moreover, Mohammad Abobala[4] presented the representation of neutrosophic matrices defined over a neutrosophic field by neutrosophic linear transformations between neutrosophic vector spaces, where he proved that every neutrosophic matrix can be represented uniquely by a neutrosophic linear transformation. As the continuation in[5] Mohammad Abobala et.al studied the neutrosophic linear transformations and their relationship with neutrosophic matrices to the case of refined neutrosophic matrices and refined neutrosophic linear transformations.

Through this work, we continue the previous efforts presented in,[22] the study of symbolic 2-plithogenic linear transformations and their relationship with symbolic 2-plithogenic matrices to the case of symbolic 3-plithogenic matrices and symbolic 3-plithogenic linear transformations.

All symbolic 3-plithogenic matrices are defined over the symbolic 3-plithogenic field of real numbers $3-SP_R$.

https://doi.org/10.54216/JNFS.070203
Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023

2 Preliminaries

Definition 2.1. [16] Let R e a ring. The symbolic 3-plithogenic ring is defined as follows:

$$3-SP_R = \{x_0 + x_1P_1 + x_2P_2 + x_3P_3; P_i^2 = P_i, P_i \times P_j = P_{max(i,j)}\}$$

Addition on $3-SP_R$ is defined as follows:
$[x_0+x_1P_1+x_2P_2+x_3P_3]+[y_0+y_1P_1+y_2P_2+y_3P_3] = (x_0+y_0)+(x_1+y_1)P_1+(x_2+y_2)P_2+(x_3+y_3)P_3$.

Multiplication on $3-SP_R$ is defined as follows:
$[x_0 + x_1P_1 + x_2P_2 + x_3P_3] \cdot [y_0 + y_1P_1 + y_2P_2 + y_3P_3] = (x_0y_0) + (x_0y_1 + x_1y_0 + x_1y_1)P_1 + (x_0y_2 + x_2y_1 + x_2y_2 + x_2y_0 + x_1y_2)P_2 + (x_0y_3 + x_1y_3 + x_2y_3 + x_3y_3 + x_3y_0 + x_3y_1 + x_3y_2)P_3$.
It is clear that $(3-SP_R)$ is a ring. If R is a field, then $3-SP_R$ is called a symbolic 3-plithogenic field.

Definition 2.2. [24] Let V be a vector space over the field F, let $3-SP_F$ be the corresponding symbolic 3-plithogenic field.

$$3-SP_F = \{x + yP_1 + zP_2 + tP_3; x, y, z, t \in F, P_i^2 = P_i, P_i \times P_j = P_{max(i,j)}\}.$$

We define the symbolic 3-plithogenic vector space as follows:

$$3-SP_V = V + VP_1 + VP_2 + VP_3 = \{a + bP_1 + cP_2 + dP_3; a, b, c, d \in V\}.$$

Operations on $3-SP_V$ can be defined as follows:
Addition:
$[x_0+x_1P_1+x_2P_2+x_3P_3]+[y_0+y_1P_1+y_2P_2+y_3P_3] = (x_0+y_0)+(x_1+y_1)P_1+(x_2+y_2)P_2+(x_3+y_3)P_3$

Multiplication:
$[a + bP_1 + cP_2 + dP_3].[x_0 + x_1P_1 + x_2P_2 + x_3P_3] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2 + (ax_3 + bx_3 + cx_3 + dx_0 + dx_1 + dx_2 + dx_3)P_3$,
where $x_i, y_i \in V, a, b, c, d \in F$.

Definition 2.3. [24] Let V, W be two vector spaces over the field F. Let $3-SP_V$, $3-SP_W$ be the corresponding symbolic 3-plithogenic vector spaces over $3-SP_F$. Let $L_0, L_1, L_2, L_3 : V \to W$ be four linear transformations, we define the AH-linear transformations $L : 3-SP_V \to 3-LP_W$ as follows:

$$L = L_0 + L_1P_1 + L_2P_2 + L_3P_3; L(x + yP_1 + zP_2 + dP_3) = L_0(x) + L_1(y)P_1 + L_2(z)P_2 + L_3(d)P_3.$$

If $L_0 = L_1 = L_2 = L_3$, then L is called AHS-Linear transformation.

Definition 2.4. [16] A square symbolic 3-plithogenic matrix is a matrix with symbolic 3-plithogenic entries.

Remark 2.5. [16] Any symbolic 3-plithogenic matrix can be written as $A = A_0 + A_1P_1 + A_2P_2 + A_3P_3$.

Theorem 2.6. [16] *Let $S = S_0 + S_1 + S_2P_2 + S_3P_3 \in 3-SP_M$, then the following is true:*

1. *S is invertible if and only if $S_0, S_0 + S_1, S_0 + S_1 + S_2, S_0 + S_1 + S_2 + S_3$ are invertible.*
2. $S^{-1} = S_0^{-1} + \left[(S_0 + S_1)^{-1} - S_0^{-1}\right] P_1 + \left[(S_0 + S_1 + S_2)^{-1} - (S_0 + S_1)^{-1}\right] P_2 + \left[(S_0 + S_1 + S_2 + S_3)^{-1} - (S_0 + S_1 + S_2)^{-1}\right] P_3$
3. *For $n \in N$,* $S^n = S_0^n + [(S_0 + S_1)^n - S_0^n] P_1 + [(S_0 + S_1 + S_2)^n - (S_0 + S_1)^n] P_2 + [(S_0 + S_1 + S_2 + S_3)^n - (S_0 + S_1 + S_2)^n] P_3$

https://doi.org/10.54216/JNFS.070203
Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023

3 Matrix Representation of Symbolic 3-Plithogenic Linear Transformations

We begin this section with the following definition.

Definition 3.1. Let V, W be two vector spaces over R and let g, h, q, r be any four linear transformations from V to W. We define the corresponding full AH-linear transformation $f : 3 - SP_V \to 3 - SP_W$ as follows:

$$f(x + yP_1 + zP_2 + dP_3) = g(x) + [(g + h)(x + y) - g(x)]P_1 + [(g + h + q)(x + y + z) - (g + h)(x + y)]P_2 + [(g + h + q + r)(x + y + z + d) - (g + h + q)(x + y + z)]P_3.$$

We denote it by $f = g + hP_1 + qP_2 + rP_3$.

Theorem 3.2. *Let $f = g + hP_1 + qP_2 + rP_3 : 3 - SP_V \to 3 - SP_W$ be a full AH-linear transformations, then f is linear by classical meaning.*

Proof. For all $X = x + yP_1 + zP_2 + dP_3, Y = x' + y'P_1 + z'P_2 + d'P_3 \in 3 - SP_V$, we have:

$$\begin{aligned}
f(X+Y) &= f[(x+x') + (y+y')P_1 + (z+z')P_2 + (d+d')P_2] \\
&= g(x+x') + [(g+h)(x+x'+y+y') - g(x+x')]P_1 \\
&\quad +[(g+h+q)(x+x'+y+y'+z+z') - (g+h)(x+x'+y+y')]P_2 \\
&\quad +[(g+h+q+r)(x+x'+y+y'+z+z'+d+d') - (g+h+q)(x+x'+y+y'+z+z')]P_3 \\
&= g(x) + [(g+h)(x+y) - g(x)]P_1 + [(g+h+q)(x+y+z) - (g+h)(x+y)]P_2 \\
&\quad +[(g+h+q+r)(x+y+z+d) - (g+h+q)(x+y+z)]P_3 \\
&\quad +g(x') + [(g+h)(x'+y') - g(x')]P_1 + [(g+h+q)(x'+y'+z') - (g+h)(x'+y')]P_2 \\
&\quad +[(g+h+q+r)(x'+y'+z'+d') - (g+h+q)(x'+y'+z')]P_3 \\
&= f(X) + f(Y).
\end{aligned}$$

For all $\alpha = \alpha_0 + \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3$, we have:

$$\begin{aligned}
\alpha \cdot X &= \alpha_0 x + [(\alpha_0+\alpha_1)(x+y) - \alpha_0 x]P_1 + [(\alpha_0+\alpha_1+\alpha_2)(x+y+z) - (\alpha_0+\alpha_1)(x+y)]P_2 \\
&\quad +[(\alpha_0+\alpha_1+\alpha_2+\alpha_3)(x+y+z+d) - (\alpha_0+\alpha_1)(x+y+z)]P_3
\end{aligned}$$

Now,

$$\begin{aligned}
f(\alpha \cdot X) &= g(\alpha_0 x) + [(g+h)[(\alpha_0+\alpha_1)(x+y)] - g(\alpha_0 x)]P_1 \\
&\quad +[(g+h+q)[(\alpha_0+\alpha_1+\alpha_2)(x+y+z)] - (g+h)[(\alpha_0+\alpha_1)(x+y)]]P_2 \\
&\quad +[(g+h+q+r)[(\alpha_0+\alpha_1+\alpha_2+\alpha_3)(x+y+z)] - (g+h+q)[(\alpha_0+\alpha_1+\alpha_3)(x+y+z)]]P_3 \\
&= \alpha_0 g(x) + [(\alpha_0+\alpha_1)(g+h)(x+y) - \alpha_0 g(x)]P_1 \\
&\quad +[(\alpha_0+\alpha_1+\alpha_2)(g+h+q)(x+y+z) - (\alpha_0+\alpha_1)(g+h)(x+y)]P_2 \\
&\quad +[(\alpha_0+\alpha_1+\alpha_2+\alpha_3)(g+h+q+r)(x+y+z+d) - (\alpha_0+\alpha_1+\alpha_2)(g+h+q)(x+y+z)]P_3 \\
&= (\alpha_0+\alpha_1 P_1+\alpha_2 P_2+\alpha_3 P_3)[g(x) + [(g+h)(x+y) - g(x)]P_1 \\
&\quad +[(g+h+q)(x+y+z) - (g+h)(x+y)]P_2 \\
&\quad +[(g+h+q+r)(x+y+z+d) - (g+h+q)(x+y+z)]P_3] \\
&= \alpha \cdot f(X).
\end{aligned}$$

□

Definition 3.3. Let $M = M_0 + M_1P_1 + M_2P_2 + M_3P_3$ be a symbolic 3-plithogenic matrix. We say that M is the matrix representation of the full AH-linear transformation $f = g + hP_1 + qP_2 + rP_3$ if and only if $f(x + yP_1 + zP_2 + dP_3) = M \cdot (x + yP_1 + zP_2 + dP_3)$.

Theorem 3.4. *If $f = g + hP_1 + qP_2 + rP_3 : 3 - SP_V \to 3 - SP_W$ is a full AH-linear transformation, then $M = M_0 + M_1P_1 + M_2P_2 + M_3P_3$ is the matrix representation of $f = g + hP_1 + qP_2 + rP_3$ if and only if M_0 is the matrix representation g, M_1 is the matrix representation of h, M_2 is the matrix representation of q and M_3 is the matrix representation of r.*

https://doi.org/10.54216/JNFS.070203
Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023

Proof. Assume that $M = M_0+M_1P_1+M_2P_2+M_3P_3$ is the matrix representation of $f = g+hP_1+qP_2+rP_3$. We have to prove that M_0 is the matrix representation g, M_1 is the matrix representation of h, M_2 is the matrix representation of q and M_3 is the matrix representation of r.

Since M is the matrix representation of f, we have $f(x + yP_1 + zP_2 + dP_3) = M \cdot (x + yP_1 + zP_2 + dP_3)$. This implies that,

$g(x) + [(g+h)(x+y) - g(x)]P_1 + [(g+h+q)(x+y+z) - (g+h)(x+y)]P_2 + [(g+h+q+r)(x+y+z+d) - (g+h+q)(x+y+z)]P_3 = M_0x + [(M_0 + M_1)(x+y) - M_0x]P_1 + [(M_0 + M_1 + M_2)(x+y+z) - (M_0 + M_1)(x+y)]P_2 + [(M_0 + M_1 + M_2 + M_3)(x+y+z+d) - (M_0 + M_1 + M_2)(x+y+z)]P_3$.

Hence, we have

$$\begin{gathered} g(x) = M_0 \cdot x, \\ (g+h)(x+y) = (M_0 + M_1)(x+y), \\ (g+h+q)(x+y+z) = (M_0 + M_1 + M_2)(x+y+z), \\ (g+h+q+r)(x+y+z+d) = (M_0 + M_1 + M_2 + M_3)(x+y+z+d). \end{gathered}$$

This implies that, M_0 is the matrix representation of g, $M_0 + M_1$ is the matrix representation of $g + h$, $M_0 + M_1 + M_2$ is the matrix representation of $g+h+q$ and $M_0+M_1+M_2+M_3$ is the matrix representation of $g + h + q + r$.

Hence, M_0 is the matrix representation g, M_1 is the matrix representation of h, M_2 is the matrix representation of q and M_3 is the matrix representation of r.

Conversely, assume that M_0 is the matrix representation g, M_1 is the matrix representation of h, M_2 is the matrix representation of q and M_3 is the matrix representation of r. We have to show that, $M = M_0+M_1P_1+M_2P_2 + M_3P_3$ is the matrix representation of $f = g + hP_1 + qP_2 + rP_3$.
For all $x + yP_1 + zP_2 + dP_3 \in 3 - SP_V$, we have

$$\begin{aligned} M(x + yP_1 + zP_2 + dP_3) &= [M_0 + M_1P_1 + M_2P_2 + M_3P_3](x + yP_1 + zP_2 + dP_3) \\ &= M_0x + [(M_0 + M_1)(x+y) - M_0x]P_1 \\ &\quad +[(M_0 + M_1 + M_2)(x+y+z) - (M_0 + M_1)(x+y)]P_2 \\ &\quad +[(M_0 + M_1 + M_2 + M_3)(x+y+z+d) - (M_0 + M_1 + M_3)(x+y+z)]P_3 \\ &= g(x) + [(g+h)(x+y) - g(x)]P_1 \\ &\quad +[(g+h+q)(x+y+z) - (g+h)(x+y)]P_2 \\ &\quad +[(g+h+q+r)(x+y+z+d) - (g+h+q)(x+y+z)]P_3 \\ &= f(x + yP_1 + zP_2 + dP_3). \end{aligned}$$

Hence M is the matrix representation of f. □

In order to prove that every symbolic 3-plithogenic matrix can be represented by a unique AH-linear transformation, we introduce the following algorithm to derive a basis of $3 - SP_V$ from any classical basis of V.

Theorem 3.5. *Let $T = \{t_1, t_2, \cdots, t_n\}$ be a basis of the vector space V over the field F, then the set:*

$$T_P = \{t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2 + (t_l - t_k)P_3; 1 \le i, j, k, l \le n\}$$

is a basis of $3 - SP_V$.

Proof. Let $X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in 3 - SP_V, x_0, x_1, x_2, x_3 \in V$. Then

https://doi.org/10.54216/JNFS.070203
Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023

$x_0=\sum_{i=1}^{n}\alpha_i t_i, x_0+x_1=\sum_{j=1}^{n}\beta_j t_j, x_0+x_1+x_2=\sum_{k=1}^{n}\gamma_k t_k, x_0+x_1+x_2+x_3=\sum_{l=1}^{n}\delta_l t_l,\ \alpha_i,\beta_j,\gamma_k,\delta_l\in F.$

Take, $A_{i,j,k,l}=\alpha_i+(\beta_j-\alpha_i)P_1+(\gamma_k-\beta_j)P_2+(\delta_l-\gamma_k)P_3,\ 1\le i,j,k,l\le n$ and $T_{i,j,k,l}=t_i+(t_j-t_i)P_1+(t_k-t_j)P_2+(t_l-t_k)P_3,\ 1\le i,j,k,l\le n.$

Now,

$$\begin{aligned}\sum_{i,j,k,l=1}^{n}A_{i,j,k,l}T_{i,j,k,l} &= \sum_{i=1}^{n}\alpha_i t_i+\left[\sum_{j=1}^{n}\beta_j t_j-\sum_{i=1}^{n}\alpha_i t_i\right]P_1+\left[\sum_{k=1}^{n}\gamma_k t_k-\sum_{j=1}^{n}\beta_j t_j\right]P_2\\ &\quad +\left[\sum_{l=1}^{n}\delta_l t_l-\sum_{k=1}^{n}\gamma_k t_k\right]P_3.\\ &= x_0+[x_0+x_1-x_0]P_1+[x_0+x_1+x_2-(x_0+x_1)]P_2\\ &\quad +[x_0+x_1+x_2+x_3-(x_0+x_1+x_2)]P_3\\ &= x_0+x_1P_1+x_2P_2+x_3P_3\\ &= X\end{aligned}$$

Thus, T_P generates $3-SP_V$.
Next, we have to show that T_P is linearly independent.
If $\sum_{i,j,k,l=1}^{n}A_{i,j,k,l}X=0$, then $\sum_{i=1}^{n}\alpha_i t_i=0$, $\sum_{j=1}^{n}\beta_j t_j=0$, $\sum_{k=1}^{n}\gamma_k t_k=0$ and $\sum_{l=1}^{n}\delta_l t_l=0$. This implies that, $\alpha_i=\beta_j=\gamma_k=\delta_l=0$ for all i,j,k,l and hence $A_{i,j,k,l}=0$.
Therefore, T_P is linearly independent. Thus, T_P is a basis of $3-SP_V$. □

Example 3.6. Consider the basis $T=\{u_1=(1,0),u_2=(0,1)\}$ of R^2. The corresponding basis of $3-SP_{R^2}$ is $T_P=\{T_1,T_2,\cdots,T_{16}\}$, where

$$\begin{aligned}
T_1 &= u_1+(u_1-u_1)P_1+(u_1-u_1)P_2+(u_1-u_1)P_3=(1,0)\\
T_2 &= u_2+(u_2-u_2)P_1+(u_2-u_2)P_2+(u_2-u_2)P_3=(0,1)\\
T_3 &= u_1+(u_1-u_1)P_1+(u_1-u_1)P_2+(u_2-u_1)P_3=(1,0)+(-1,1)P_3\\
T_4 &= u_1+(u_1-u_1)P_1+(u_2-u_1)P_2+(u_1-u_2)P_3=(1,0)+(-1,1)P_2+(1,-1)P_3\\
T_5 &= u_1+(u_2-u_1)P_1+(u_1-u_2)P_2+(u_1-u_1)P_3=(1,0)+(-1,1)P_1+(1,-1)P_2\\
T_6 &= u_1+(u_1-u_1)P_1+(u_2-u_1)P_2+(u_2-u_2)P_3=(1,0)+(-1,1)P_2\\
T_7 &= u_1+(u_2-u_1)P_1+(u_1-u_2)P_2+(u_2-u_1)P_3=(1,0)+(-1,1)P_1+(1,-1)P_2+(-1,1)P_3\\
T_8 &= u_1+(u_2-u_1)P_1+(u_2-u_2)P_2+(u_1-u_2)P_3=(1,0)+(-1,1)P_1+(1,-1)P_3\\
T_9 &= u_1+(u_2-u_1)P_1+(u_2-u_2)P_2+(u_2-u_2)P_3=(1,0)+(-1,1)P_1\\
T_{10} &= u_2+(u_1-u_2)P_1+(u_1-u_1)P_2+(u_1-u_1)P_3=(0,1)+(1,-1)P_1\\
T_{11} &= u_2+(u_1-u_2)P_1+(u_1-u_1)P_2+(u_2-u_1)P_3=(0,1)+(1,-1)P_1+(-1,1)P_3\\
T_{12} &= u_2+(u_1-u_2)P_1+(u_2-u_1)P_2+(u_1-u_2)P_3=(0,1)+(1,-1)P_1+(-1,1)P_2+(1,-1)P_3\\
T_{13} &= u_2+(u_2-u_2)P_1+(u_1-u_2)P_2+(u_1-u_1)P_3=(0,1)+(1,-1)P_2\\
T_{14} &= u_2+(u_1-u_2)P_1+(u_2-u_1)P_2+(u_2-u_2)P_3=(0,1)+(1,-1)P_1+(-1,1)P_2\\
T_{15} &= u_2+(u_2-u_2)P_1+(u_1-u_2)P_2+(u_2-u_1)P_3=(0,1)+(1,-1)P_2+(-1,1)P_3\\
T_{16} &= u_2+(u_2-u_2)P_1+(u_2-u_2)P_2+(u_1-u_2)P_3=(0,1)+(1,-1)P_3.
\end{aligned}$$

Remark 3.7. $dim(3-SP_V)=(dimV)^4$

Theorem 3.8. *Let $f:3-SP_V\to 3-SP_W$ be a linear function, then f must be full AH-linear transformation.*

Proof. We have to prove that there exists four classical linear transformations $g,h,q,r:V\to W$ with the property that $f=g+hP_1+qP_2+rP_3$. We know that the image of a basis $v_{i,j,k,l}$ under a linear transformation must be a a basis, thus we have:

Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023

$$f(v_{i,j,k,l}) = \{v_i + (v_j - v_i)P_1 + (v_k - v_j)P_2 + (v_l - v_k)P_3\}) = \\ \{w_i + (w_j - w_i)P_1 + (w_k - w_j)P_2 + (w_l - w_k)P_3\}$$

is a basis of $3 - SP_W$, where $w_i, w_j, w_k, w_l \in W$.
Consider the functions:

$$g : V \to W; g(v_i) = w_i \\ h' : V \to W; h'(v_j) = w_j \\ q' : V \to W; q'(v_k) = w_k \\ r' : V \to W; r'(v_l) = w_l$$

$f(v_{i,j,k,l}) = g(v_1) + (h'(v_j) - g(v_i))P_1 + (q'(v_k) - h'(v_j))P_2 + (r'(v_l) - q'(v_k))P_3$.
By taking $h = h' - g$, $q = q' - g - h$ and $r = r' - g - h - q$, we get

$$\begin{aligned} f(v_{i,j,k,l}) &= g(v_1) + [(g+h)(v_j) - g(v_i)]P_1 + [(g+h+q)(v_k) - (g+h)(v_j)]P_2 + \\ &\quad +[(g+h+q+r)(v_l) - (g+h+q)(v_k)]P_3 \end{aligned}$$

Thus $f = g + hP_1 + qP_2 + rP_3$. It is clear that g, h', q' and r' are linear hence, h, q and r are also linear. Hence f is a full AH-linear transformation. □

Remark 3.9. Every linear transformation $f : 3 - SP_V \to 3 - SP_W$ can be represented by a unique symbolic 3-plithogenic matrix, which is equal to the corresponding matrix of the corresponding full AH-linear transformation.

In the following Example 3.10, we illustrate the representation of a AH-linear transformation by symbolic 3-plithogenic matrix .

Example 3.10. Consider the linear transformation $f : 3 - SP_{R^2} \to 3 - SP_{R^2}$ defined by

$$\begin{aligned} f[(x,y) + (s,t)P_1 + (m,n)P_2 + (u,v)P_3] &= f[(x + sP_1 + mP_2 + uP_3, y + tP_1 + nP_2 + vP_3)] \\ &= (2(x + sP_1 + mP_2 + uP_3), -(y + tP_1 + nP_2 + vP_3)) \\ &= (2x, -y) + (2s, -t)P_1 + (2m, -n)P_2 + (2u, -v)P_3. \end{aligned}$$

Now, we have to covert f into a full AH-linear transformation. By Theorem 3.8, f must be equal to $g + hP_1 + qP_2 + rP_3$, where $g, h, q, r : R^2 \to R^2$.

Here, we have $g(x, y) = (2x, -y)$ and $(g + h)[(x, y) + (s, t)] - g(x, y) = (2s, -t)$. This implies that $g(x+s, y+t)+h(x+s, y+t)-(2x, -y) = (2s, -t)$, so that $(2x+2s, -y-t)+h(x+s, y+t)-(2x, -y) = (2s, -t)$, hence $h(x + s, y + t) = (0, 0)$ for all x, y, s, t. This implies that $h(s, t) = 0$ is a zero map.

Also, $(g + h + q)[(x, y) + (s, t) + (m, n)] - (g + h)[(x, y) + (s, t)] = (2m, -n)$. Since $h = 0$, we have $g(x+s+m, y+t+n)+q(x+s+m, y+t+n)-g(x+s, y+t) = (2m, -n)$ so that $(2x+2s+2m, -y-t-n)+q(x+s+m, y+t+n)-(2x+2s, -y-t) = (2m, -n)$, hence $q(x+s+m, y+t+n) = (0, 0)$ for all x, y, s, t, m, n. This implies that q is a zero map.

Moreover, $(g+h+q+r)[(x, y)+(s, t)+(m, n)+(u, v)]-(g+h+q)[(x, y)+(s, t)+(u, v)] = (2u, -v)$. Since $h = 0, q = 0$, we have $g(x+s+m+u, y+t+n+v)+r(x+s+m+u, y+t+n+v)-g(x+s+m, y+t+n) = (2u, -v)$ so that $(2x+2s+2m+2u, -y-t-n-v)+r(x+s+m+u, y+t+n+v)-(2x+2s+2u, -y-t-v) = (2u, -v)$, hence $r(x + s + m + u, y + t + n + v) = (0, 0)$ for all x, y, s, t, m, n, u, v. This implies that r is a zero map.

Therefore, $f = g + 0P_1 + 0P_2 + 0P_3$.

Here, the matrix of g is $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$, the matrix of h is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, the matrix of q is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and the matrix

https://doi.org/10.54216/JNFS.070203

Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023

of r is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Thus the symbolic 3-plithogenic matrix of f is $M = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$ by Theorem 3.4.

Now, we verify that M is the symbolic 3-plithogenic matrix of f by the following computing:

$$\begin{aligned} M \cdot (x + sP_1 + mP_2 + uP_3, y + tP_1 + nP_2 + vP_3) &= \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x + sP_1 + mP_2 + uP_3 \\ y + tP_1 + nP_2 + vP_3 \end{pmatrix} \\ &= (2(x + sP_1 + mP_2 + uP_3), -(y + tP_1 + nP_2 + vP_3)) \\ &= f[(x, y) + (s, t)P_1 + (m, n)P_2 + (u, v)P_3]. \end{aligned}$$

In the following Example 3.11, we illustrate how a symbolic 3-plithogenic matrix can be turned into a full AH-linear transformation.

Example 3.11. Consider the symbolic 3-plithogenic matrix

$$M = \begin{pmatrix} 1 - P_3 & 1 + P_3 \\ 3 + P_2 & 2P_1 + P_3 \end{pmatrix}$$

Then M can be represented by a unique symbolic 3-plithogenic linear transformation $f : 3 - SP_{R^2} \rightarrow 3 - SP_{R^2}$ as follows:

$f(x + sP_1 + mP_2 + uP_3,\ y + tP_1 + nP_2 + vP_3) = M \cdot (x + sP_1 + mP_2 + uP_3,\ y + tP_1 + nP_2 + vP_3) = ((x + y) + (s + t)P_1 + (m + n)P_2 + (-x - s - m + y + t + n + 2v)P_3,\ 3x + (3s + 2y + 2t)P_1 + (x + s + 4m + 2n)P_2 + (4u + 3v + y + t + n)P_3)$.

4 Conclusion

In this paper, we have studied the solution to the problem of representing symbolic 3-plithogenic matrices by linear transformations between two symbolic 3-plithogenic vector spaces, where we proved that every symbolic 3-plithogenic matrix can be represented by a unique linear transformation between two symbolic 3-plithogenic vector spaces. Also, we have introduced an algorithm to obtain a basis for any symbolic 3-plithogenic vector space based on any basis of its corresponding classical vector space.

Funding: This research received no external funding.

References

1. Abobala, M., “On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations”, Journal Of Mathematics, Hindawi, 2021.

2. Abobala, M., “On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations”, Mathematical Problems in Engineering, Hindawi, 2021.

3. Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., “The Algebraic Creativity In The Neutrosophic Square Matrices”, Neutrosophic Sets and Systems, Vol. 40, pp.1-11, 2021.

4. Abobala, M., “On the Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations”, Journal of Mathematics, Hindawi, 2021.

5. Abobala, M.; Ziena, B.M.; Doewes, R.; Hussein, Z., “The Representation of Refined Neutrosophic Matrices by Refined Neutrosophic Linear Functions”, International Journal of Neutrosophic Science, Vol. 19, 2022.

Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023

6. Abobala, M., "A Study of Nil Ideals and Kothe's Conjecture in Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, Hindawi, 2021.
7. Alfahal, A., Alhasan, Y., Abdulfatah, R., Mehmood, A., Kadhim, M, "On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties", International Journal of Neutrosophic Science, Vol. 21, PP. 96-104, 2023.
8. Adeleke, E. O., Agboola, A. A. A., and Smarandache, F., "Refined neutrosophic rings II", International Journal of Neutrosophic Science, Vol. 2, pp. 89-94, 2020.
9. Albasheer, O., Hajjari., A., and Dalla., R., "On The Symbolic 2-Plithogenic Rings and Their Algebraic Properties", Neutrosophic Sets and Systems, Vol. 54, 2023.
10. Albasheer, O., Hajjari., A., and Dalla., R., "On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.
11. Alfahal, A., Alhasan, Y., Abdulfatah, R., Mehmood, A., and Kadhim, M. "On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties", International Journal of Neutrosophic Science, Vol. 23, 2023.
12. Ali, R., and Hasan, Z., "An Introduction To The Symbolic 3-Plithogenic Vector Spaces", Galoitica Journal Of Mathematical Structures and Applications, Vol. 6, 2023.
13. Ali, R.; and Hasan, Z., "An Introduction to the Symbolic 3-Plithogenic Modules", Galoitica Journal Of Mathematical Structures and Applications, Vol. 6, 2023.
14. Merkepci, H., and Abobala, M., "On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.
15. Merkepci, H., and Rawashdeh, A., "On The Symbolic 2-Plithogenic Number Theory and Integers", Neutrosophic Sets and Systems, Vol. 54, 2023.
16. Merkepci, H., "On Novel Results about the Algebraic Properties of Symbolic 3-Plithogenic and 4-Plithogenic Real Square Matrices", Symmetry, 2023.
17. Othman, B.K., "On Some Algorithms for Solving Symbolic 3-Plithogenic Equations", Neoma Journal of Mathematics Computer Science 2023
18. Rawashdeh, A, "An Introduction to the Symbolic 3-plithogenic Number Theory", Neoma Journal of Mathematics Computer Science, 2023.
19. Prabakaran, P., and Smarandache, F., "On Some Algebraic Properties of Symbolic 2-Plithogenic Square Matrice", Neutrosophic Sets and Systems, Vol. 59, 2023.
20. Prabakaran, P., and Smarandache, F., "On Clean and Nil-Clean Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol. 59, 2023.
21. Prabakaran, P., Bol?var V. Jadan, Rita Azucena D. Vasquez, Marcos Lalama Flores, "Representation of Symbolic 2-Plithogenic Matrices by Symbolic 2-Plithogenic Linear Transformations", International Journal of Neutrosophic Science, Volume 23 , Issue 2, PP: 263-269, 2024.
22. Prabakaran, P., Gomez, G.A., Diaz Vasquez, R.A., and Yacelga, A.L., "A Note on Invertible Neutrosophic Square Matrices", International Journal of Neutrosophic Science", Vol. 22, 2023.
23. Smarandache, F., "Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, Vol. 53, 2023.
24. Taffach, N., "An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", Neutrosophic Sets and Systems, Vol. 54, 2023.
25. Taffach, N., and Ben Othman, K., "An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol. 54, 2023.

https://doi.org/10.54216/JNFS.070203
Received: May 11, 2023 Revised: September 22, 2023 Accepted: November 17, 2023