



Partner Sets for Generalizations of MultiNeutrosophic Sets

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Abstract

Fuzzy sets and their various generalizations, especially neutrosophic and multineutrosophic sets, have had an essential imprint in other scientific, engineering, and applied fields. This came from the characteristics of membership functions that determine the extent to which members belong to their sets, which is an important criterion. Hence, the idea of building the optimal membership function for fuzzy set using the arithmetic mean, we called this set by partner sets of a neutrosophic set of n -type.

Keywords: Fuzzy set; Neutrosophic set; partner set; multineutrosophic set; type-2 fuzzy set; type-n fuzzy set; partner cardinality set; partner cardinality topology; quasi-coincident; sticky set; packing set.

1. Introduction

The first to put the imprint of the fuzzy sets is Zadeh in his tagged research [5], fuzzy sets, inform and control in 1965. In the year 1975, Zadeh [6] introduced the second type of fuzzy sets and named it type-2 fuzzy sets. In 2002, Mendel and John [4] generalized the type-2 to n -times. It should be noted that Salama [1] generalized the fuzzy sets in 2012 identified by Cadeh in the space $X \times I \times I \times I$ with certain conditions and named them the neutrosophic topological space. Salama and some researchers also dealt with these sets and developed them at level space $P(X) \times P(X) \times P(X)$, and for more information you can refer to the book “Neutrosophic Crisp Set Theory” authored by Salama and Smarandache [2]. Then the researchers Ahmed, Said, Luay [3] came to generalize the neutrosophic crisp sets over $3n$ -times, i.e., in the space $P(X) \dots \times P(X)$ $3n$ -times, and binary operations have also been placed on these sets. Imran et al. [7] gave fresh insights into the concept of neutrosophic generalized semi generalized closed sets. Luay and Mustafa [9] also put a new sets imprint in the family of neutrosophic crisp sets that are defined in the space $P(X) \times P(X) \times P(X)$ n -times and with certain conditions with the placement of binary operations on them and they have been called “Axial Set Theory”. The important concept in our work is multineutrosophic set introduced by Smarandache in [10]. In this research, we will put an ideal solution to many questions and problems, especially engineering, which we think is an optimal solution for it, which talks about the process of choosing the priority of the membership function and what are its conditions and features that can be chosen.

Definition 1.1 [12]

Let X be nonempty universal set and A be any nonempty subset of X . Then $\{ \langle x, T, I, F \rangle : x \in X \}$ is called neutrosophic set and denoted briefly by A if $T, I, F: X \rightarrow [0,1]$ are degree of truth, indeterminacy, falsehood memberships of x corresponding with the subset A , respectively. It is clear that

$$0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3.$$

Moreover, if the values of T, I, F are numbers from $[0,1]$, then all of them have single-valued neutrosophic set (SVNS).

Definition 1.2 [10]

Let X be universal set and M be any nonempty subset of X . Then $\{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X \}$ is called multineutrosophic set and denoted briefly by M if $T_1, \dots, T_r : X \rightarrow [0,1]$ are the degree of truth membership functions of x corresponding with the subset M , $I_1, \dots, I_s : X \rightarrow [0,1]$ are the degree of indeterminacy membership functions of x corresponding with the subset M , and $F_1, \dots, F_t : X \rightarrow [0,1]$ are the degree of falsehood membership functions of x corresponding with the subset M where $r + s + t = n$. It is obvious that

$$0 \leq \sum_{i=1}^r \inf T_i + \sum_{j=1}^s \inf I_j + \sum_{k=1}^t \inf F_k \leq \sum_{i=1}^r \sup T_i + \sum_{j=1}^s \sup I_j + \sum_{k=1}^t \sup F_k \leq n.$$

Definition 1.3

Let \wedge_N, \wedge_F be the neutrosophic intersection and t-norm, respectively. Moreover, let \vee_N, \vee_F be the neutrosophic union and t-conorm, respectively. Let M_1, M_2 be multineutrosophic sets with same (r, s, t) -form in X where $M_1 = \{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X \}$ and $M_2 = \{ \langle x, T'_1, \dots, T'_r, I'_1, \dots, I'_s, F'_1, \dots, F'_t \rangle : x \in X \}$. Then

- (i) The multineutrosophic intersection
 $M_1 \wedge_N M_2 = \{ \langle x, T_1 \wedge_F T'_1, \dots, T_r \wedge_F T'_r, I_1 \vee_F I'_1, \dots, I_s \vee_F I'_s, F_1 \vee_F F'_1, \dots, F_t \vee_F F'_t \rangle : x \in X \}$
- (ii) The multineutrosophic union
 $M_1 \vee_N M_2 = \{ \langle x, T_1 \vee_F T'_1, \dots, T_r \vee_F T'_r, I_1 \wedge_F I'_1, \dots, I_s \wedge_F I'_s, F_1 \wedge_F F'_1, \dots, F_t \wedge_F F'_t \rangle : x \in X \}$

2. The partner set of multineutrosophic set

Definition 2.1

Let X be the non-empty universal set and let $M^n = \{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X, r + s + t = n \}$ be a multineutrosophic set in X . The fuzzy set M_n is called a **partner set to the multineutrosophic set** $M^n = \{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X, r + s + t = n \}$ and it has the following form

$$M_n = \{ \langle x, f_{M_n}(x) \rangle : x \in X \}$$

where

$$f_{M_n}(x) = \frac{1}{n} \left(\sum_{i=1}^r T_i + \sum_{j=1}^s I_j + \sum_{k=1}^t F_k \right), \forall x \in X.$$

In other words, the membership function of M_n is the mean or arithmetic mean (the sum of all $T_i, 1 \leq i \leq r, I_j, 1 \leq j \leq s$ and all $F_k, 1 \leq k \leq t$ divided by n). Here, we can take the mean (weighted mean, root mean square, cubic mean, geometric mean, weighted geometric mean, harmonic mean, weighted harmonic mean, generalized or power mean) or any tendency measure required to compare the scientific problems for T, I and F .

When we talk about the membership function and its specifications and which one is safer to choose, many scholars focus on these features of this function because it is considered a standard measure of the degree of belonging. We note that MATLAB has been fed with various forms of these functions. However, the question remains: Is any membership function safer determining the degree of belonging? Is it equivalent to the multineutrosophic with its partner? [11]

It is well known that to get an optimal solution for any linear programming problem using the direct simplex algorithm should be processed to be in standard form, the simplex method for solving an LP problem requires the problem to be expressed in the standard form. But not all LP problems appear in the standard form. In many cases, some of the constraints are expressed as inequalities rather than equations:

Definition 2.2

Let X be a non-empty universal set. For any multineutrosophic sets A^n and B^m , we defined the union and intersection of them concerning partners A_n, B_m .

- 1- $A_n \cap B_m = \{ \langle x, f_{A_n}(x) \wedge_F f_{B_m}(x) \rangle, x \in X \}$.
- 2- $A_n \cup B_m = \{ \langle x, f_{A_n}(x) \vee_F f_{B_m}(x) \rangle, x \in X \}$.

Example 2.3

Let $X = \{1,2,3,4\}$, be a set and $A = \{1,2\}, B = \{3,4\}$ be subsets of X . Assume that $T_1(x) = \frac{1}{x}, T_2(x) = \frac{1}{1+x}, I_1(x) = |T_1(x) - T_2(x)|, F_1(x) = \max\{0, T_1(x) + T_2(x) - 1\}$. Let us suppose that the multineutrosophic sets A^3 and B^4 are given as the following:

$$A^3 = \{ \langle x, T_1(x), I_1(x), F_1(x) \rangle : x \in X \}, B^4 = \{ \langle x, T_1(x), T_2(x), I_1(x), F_1(x) \rangle : x \in X \}.$$

Then the table below shows the process of union and intersection between their partners.

x	1	2	3	4
$f_{A_3}(x)$	0.67	0.28	0.28	0.283
$f_{B_4}(x)$	0.63	0.38	0.40	0.413
$A_3 \cap B_4$	0.63	0.28	0.28	0.283
$A_3 \cup B_4$	0.67	0.38	0.40	0.413

In general, we assign each multineutrosophic set with different membership function.

Definition 2.4

Let X be any non-empty set. Then

- (i) The **α -cut of the partner** A_n for a multineutrosophic set A^n is

$$A_{n,\alpha} = \{x \in X: f_{A_n}(x) \geq \alpha\}$$

and the set of all α -levels of A_n is

$$\Lambda(A_{n,\alpha}) = \{\alpha; f_{A_n}(x) = \alpha \text{ for some } x \in X\}$$

- (ii) The **plinth** of A_n is given as

$$P(A_n) = \inf_{x \in X} f_{A_n}(x)$$

- (iii) The **height** of A_n is given as

$$h(A_n) = \sup_{x \in X} f_{A_n}(x)$$

- (iv) A partner set A_n is called **norm** if $h(A_n) = 1$, and **subnormal** if $h(A_n) < 1$.

- (v) The **cardinality** of A_n is given as

$$|A_n| = \sum_{x \in X} f_{A_n}(x),$$

- (vi) The **relative cardinality** of A_n is given as

$$\|A_n\| = \frac{|A_n|}{|X|}$$

- (vii) The **partner cardinality set** of multineutrosophic set A^n is given

$$P_\alpha(A_n) = \{(|A_{n,\alpha}|, \alpha), \alpha \in \Lambda(A_{n,\alpha})\}$$

The **collection of all partner cardinality sets of multineutrosophic A^n** , for each natural number n is of the form

$$P_\alpha(X) = \{P_\alpha(A_n): \text{for each multineutrosophic set } A^n \text{ and natural number } n\}$$

Example 2.5

Under condition example 2.3, we will find $A_{3,\alpha}$ under $\Lambda(A_{3,\alpha}) = \{0.28, 0.283, 0.67\}$ as the following:

$$A_{3,0.28} = \{x \in X: f_{A_3}(x) \geq 0.28\} = \{1,2,3,4\} = X \Rightarrow |A_{3,0.28}| = |X| = 4$$

$$A_{3,0.283} = \{x \in X: f_{A_3}(x) \geq 0.283\} = \{1,4\} = A \Rightarrow |A_{3,0.283}| = 2$$

$$A_{3,0.67} = \{x \in X: f_{A_3}(x) \geq 0.67\} = \{1\} \Rightarrow |A_{3,0.67}| = |\{1\}| = 1$$

Then

$$P_\alpha(A_3) = \{ \langle |A_{3,\alpha}|, \alpha \rangle, \alpha \in \Lambda(A_{3,\alpha}) \} = \{ \langle |A_{3,0.28}|, 0.28 \rangle, \langle |A_{3,0.283}|, 0.283 \rangle, \langle |A_{3,0.67}|, 0.67 \rangle \}$$

$$= \{ \langle 4, 0.28 \rangle, \langle 2, 0.283 \rangle, \langle 1, 0.67 \rangle \}$$

we will find $B_{4,\alpha}$ under $\Lambda(B_{4,\alpha}) = \{0.38, 0.40, 0.413, 0.63\}$ as the following:

$$B_{4,0.38} = \{x \in X: f_{B_4}(x) \geq 0.38\} = \{1,2,3,4\} = X \Rightarrow |B_{4,0.38}| = |X| = 4$$

$$B_{4,0.40} = \{x \in X: f_{B_4}(x) \geq 0.40\} = \{1,3,4\} \Rightarrow |B_{4,0.40}| = 3$$

$$B_{4,0.413} = \{x \in X: f_{B_4}(x) \geq 0.413\} = \{1,4\} \Rightarrow |B_{4,0.413}| = 2$$

$$B_{4,0.63} = \{x \in X: f_{B_4}(x) \geq 0.63\} = \{1\} \Rightarrow |B_{4,0.63}| = 1$$

$$P_\alpha(B_4) = \{ \langle |B_{4,\alpha}|, \alpha \rangle, \alpha \in \Lambda(B_{4,\alpha}) \} = \{ \langle |B_{4,0.38}|, 0.38 \rangle, \langle |B_{4,0.40}|, 0.40 \rangle, \langle |B_{4,0.413}|, 0.413 \rangle, \langle |B_{4,0.63}|, 0.63 \rangle \}$$

$$= \{ \langle 4, 0.38 \rangle, \langle 3, 0.40 \rangle, \langle 2, 0.413 \rangle, \langle 1, 0.63 \rangle \}$$

Remark 2.6

From the above definition (i), we can conclude the following for any $0 < \alpha_1 < \alpha_2 < 1$ and $n \geq m$, whenever $n, m \in \mathbb{N}$

$$A_{n,\alpha_2} \subseteq A_{m,\alpha_1}, A_{n,\alpha_2} \cap A_{m,\alpha_1} = A_{n,\alpha_2}, \text{ and } A_{n,\alpha_2} \cup A_{m,\alpha_1} = A_{m,\alpha_1}$$

Definition 2.7

The subcollection $\tau_{p.c.}$ of $P_\alpha(X)$ is called **partner cardinality topology** if it satisfies the following:

- (i) $O_{p.c.}, 1_{p.c.} \in \tau_{p.c.}$ where $O_{p.c.} = \{ \langle k, 0 \rangle : \forall k \in \{1, 2, \dots, |X|\} \}$, $1_{p.c.} = \{ \langle k, 1 \rangle : \forall k \in \{1, 2, \dots, |X|\} \}$.
- (ii) For any $P_\alpha(A_n), P_\alpha(B_m) \in \tau_{p.c.}$, then there exists a non-zero $P_\alpha(C_k) \in \tau_{p.c.}$ such that $[P_\alpha(A_n) \wedge P_\alpha(B_m)] \not\subseteq P_\alpha(C_k)$

The symbol $\not\subseteq$ means **quasi-coincident**.

- (iii) For any index Γ , $P_\alpha(A_{n_\gamma}) \in \tau_{p.c.}, \forall \gamma \in \Gamma$, then there exists a non-zero $P_\alpha(C_k) \in \tau_{p.c.}$ such that

$$\bigvee_{\gamma \in \Gamma} P_\alpha(A_{n_\gamma}) \not\subseteq P_\alpha(C_k)$$

Remark 2.8

- (i) Each $P_\alpha(A_n) \in \tau_{p.c.}$ is called **P_α -open set** and its complement is **P_α -closed set**.
- (ii) From example 2.4, we have $\tau_{p.c.} = \{O_{p.c.}, 1_{p.c.}, P_\alpha(A_3), P_\alpha(B_4)\}$ is partner cardinality topology. In general, and by this definition, it turns out that it is fuzzy topology.
- (iii) The indiscrete partner cardinality topology is $\{O_{p.c.}, 1_{p.c.}\}$ and the discrete partner cardinality topology is denoted by $P_{v\alpha}(X)$ with the standard intersection and standard union.

Definition 2.9

- (i) A **sticky set** for $P_\alpha(C_n)$ is given as $S(P_\alpha(C_n)) = \begin{cases} \wedge_F \{P_\alpha(F_m) \text{ is } P_\alpha\text{-closed} : \wedge_F P_\alpha(F_m) \not\subseteq P_\alpha(C_n)\} \\ O_{p.c.}, \text{ otherwise} \end{cases}$
- (ii) We called the partner cardinality set $P_\alpha(C_n)$ is **P_α -dense** if $S(P_\alpha(C_n)) = 1_{p.c.}$.

Remark 2.10

By the above definition, it is not necessary that $S(1_{p.c.}) = 1_{p.c.}$ and $S(O_{p.c.}) = O_{p.c.}$ as the following example.

Example 2.11

Let $X = \{1, 2, 3, 4\}$ and $\tau_{p.c.} = \{O_{p.c.}, 1_{p.c.}, P_\alpha(A_4), P_\alpha(B_4)\}$ where

$$P_\alpha(A_4) = \{ \langle 1, 0.84 \rangle, \langle 2, 0.8025 \rangle, \langle 3, 0.7975 \rangle, \langle 4, 0.775 \rangle \}$$

$$1_{p.c.} - P_\alpha(A_4) = \{ \langle 1, 0.16 \rangle, \langle 2, 0.1975 \rangle, \langle 3, 0.2025 \rangle, \langle 4, 0.225 \rangle \}$$

$$P_\alpha(B_4) = \{ \langle 1, 0.8275 \rangle, \langle 2, 0.815 \rangle, \langle 3, 0.8075 \rangle, \langle 4, 0.7425 \rangle \}$$

$$1_{p.c.} - P_\alpha(B_4) = \{ \langle 1, 0.1725 \rangle, \langle 2, 0.185 \rangle, \langle 3, 0.1925 \rangle, \langle 4, 0.2575 \rangle \}$$

Then $S(O_{p.c.}) = O_{p.c.}, S(1_{p.c.}) = O_{p.c.}, S(1_{p.c.} - P_\alpha(A_4)) = 1_{p.c.}, S(P_\alpha(A_4)) = 1_{p.c.} - P_\alpha(B_4),$

$S(P_\alpha(B_4)) = 1_{p.c.} - P_\alpha(A_4),$ and $S(1_{p.c.} - P_\alpha(B_4)) = 1_{p.c.}.$

Remark 2.10

- (i) The intersection of two quasi P_α -dense is quasi P_α -dense because if we take $S(P_\alpha(A_n)) = S(P_\alpha(B_m)) = 1_{p.c.}$ with $S(P_\alpha(A_n) \wedge_F P_\alpha(B_m)) \neq 1_{p.c.}$, then there exists $P_k^\alpha \notin S(P_\alpha(A_n) \wedge_F P_\alpha(B_m))$ and P_α -closed

$P_\alpha(F_n)$ such that $P_k^\alpha \in P_\alpha(F_n)$ and $P_\alpha(F_n) \not\subseteq (P_\alpha(A_n) \wedge_F P_\alpha(B_m)) \Rightarrow P_\alpha(F_n) \not\subseteq P_\alpha(A_n) \Rightarrow P_k^\alpha \notin S(P_\alpha(A_n))$. This contradiction.

(ii) If $P_\alpha(A_n)$ is a quasi P_α -dense and $P_\alpha(A_n) \subseteq P_\alpha(B_m)$, then $P_\alpha(B_m)$ is also quasi P_α -dense.

(iii) The union of two quasi P_α -dense is not necessary to be quasi P_α -dense if we have

$$P_\alpha(C_4) = \{ \langle 1, 0.8 \rangle, \langle 2, 0.7 \rangle, \langle 3, 0.9 \rangle, \langle 4, 0.89 \rangle \}$$

$$P_\alpha(F_4) = \{ \langle 1, 0.7 \rangle, \langle 2, 0.67 \rangle, \langle 3, 0.3 \rangle, \langle 4, 0.6 \rangle \}$$

$$S(P_\alpha(C_4)) = S(P_\alpha(F_4)) - 1_{p.c.} \text{ but } S(P_\alpha(C_4) \vee_F P_\alpha(F_4)) = 1_{p.c.} - S(P_\alpha(B_4)) \neq 1_{p.c.}$$

(iv) The relation $S(S(P_\alpha(A_n))) \subseteq S(P_\alpha(A_n))$ holds true but the converse may not true. See above example,

$$S(1_{p.c.} - P_\alpha(A_4)) = 1_{p.c.} \text{ but } S(1_{p.c.}) = 0_{p.c.}$$

(v) The following holds true $S(P_\alpha(A_n) \vee_F P_\alpha(B_m)) = S(P_\alpha(A_n)) \vee_F S(P_\alpha(B_m))$ because there exists $A \not\subseteq B \vee C$ iff $A \not\subseteq B \vee A \not\subseteq C$.

(vi) The following holds true $S(P_\alpha(A_n) \wedge_F P_\alpha(B_m)) = S(P_\alpha(A_n)) \wedge_F S(P_\alpha(B_m))$ but the converse may not true. See the above example, $1_{p.c.} - P_\alpha(A_4) \wedge_F 1_{p.c.} - P_\alpha(B_4) = 0_{p.c.}$ and $S(0_{p.c.}) = 0_{p.c.}$ but $1_{p.c.} \wedge_B 1_{p.c.} = 1_{p.c.}$.

(vii) The following holds true, $P_\alpha(A_n) \wedge_F S(P_\alpha(B_m)) \subseteq S(P_\alpha(A_n) \wedge_F P_\alpha(B_m))$. Since for any fuzzy point $P_k^\alpha \in P_\alpha(A_n) \wedge_F S(P_\alpha(B_m)) \Rightarrow P_k^\alpha \in P_\alpha(A_n) \wedge_F P_k^\alpha \in S(P_\alpha(B_m))$. if it is possible that $P_k^\alpha \notin S(P_\alpha(A_n) \wedge_F P_\alpha(B_m)) \Rightarrow P_k^\alpha \notin S(P_\alpha(B_m))$. This is contradiction.

Definition 2.11

(i) Packing set for any partner cardinality set $P_\alpha(A^n)$, according to the formula

$$P(P_\alpha(A_n)) = 1 - S(1 - P_\alpha(A_n)).$$

(ii) There are same facts that we can get from this definition which are

$$S(1 - P_\alpha(A_n)) = 1 - P(P_\alpha(A_n))$$

$$S(P_\alpha(A_n)) = 1 - P(1 - P_\alpha(A_n))$$

$$P(1 - P_\alpha(A_n)) = 1 - S(P_\alpha(A_n))$$

$$P(P_\alpha(A_n)) \wedge P_\alpha(B_m) = P(P_\alpha(A_n)) \wedge P(P_\alpha(B_m))$$

$$P(P_\alpha(A_n)) \vee P(P_\alpha(B_m)) \subseteq P(P_\alpha(A_n)) \wedge P_\alpha(B_m)$$

$$P(P_\alpha(A_n)) \subseteq P(P(P_\alpha(A_n)))$$

3. Conclusion

The set $S(P_\alpha(A_n))$ does not have to be P_α -closed for P_α -open set and according to the type of intersection adopted for example in the case of taking the standard intersection $S(P_\alpha(D_4)) = \{ \langle 1, 0.16 \rangle, \langle 2, 0.1925 \rangle, \langle 3, 0.2025 \rangle, \langle 4, 0.225 \rangle \}$ where $P_\alpha(D_4) = \{ \langle 1, 0.9 \rangle, \langle 2, 0.84 \rangle, \langle 3, 0.83 \rangle, \langle 4, 0.84 \rangle \}$. Moreover, the set $P(P_\alpha(A_n))$ does not have to be P_α -closed for P_α -open corresponding any partner set $P_\alpha(A_n)$. A new definition can be built to a topology space that is synonymous with what was built in Definition 2.7.

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