



On Soft 2-metric spaces

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Abstract

The main purpose of this article is to introduce the concept of $soft_2$ – metric spaces that are new metric spaces based on the soft-elements and to study their properties, also to introduce some related concepts such as $soft_2$ – open ball, $soft_2$ closed ball, $soft_2$ – open sets, $soft_2$ closed sets, $soft_2$ – interior elements, $soft_2$ – limit elements, $soft_2$ – closure of sets, and study some of their properties. And that is proved that every $soft_2$ – open ball is a $soft_2$ – open set, every $soft_2$ – closed ball is a $soft_2$ – closed set. And reformulated and proved some theorems in $soft_2$ – metric space, such as Cantor's theorem and other theorems.

Keywords: $soft_2$ – elements; $soft_2$ – metric spaces; $soft_2$ – open sets; $soft_2$ – closed sets.

1. Introduction

In 1960, Introduced the notion of a 2-metric space by Gähler in a paper [1] as follows: Let X be a non-empty set and let $\partial: X \times X \times X \rightarrow \mathbb{R}$ be a map satisfying the following conditions:

1. For each pair $x, y \in X; x \neq y$ there is a point $z \in X$ such that $\partial(x, y, z) \neq 0$.
2. $\partial(x, y, z) = 0$ If at least $x = y$ or $x = z$ or $y = z$.
3. $\partial(x, y, z) = \partial(x, z, y) = \partial(y, z, x)$ for each triple $x, y, z \in X$. (symmetry)
4. $\partial(x, y, z) \leq \partial(x, y, v) + \partial(y, z, v) + \partial(z, x, v)$ for all $x, y, z, v \in X$.

Then ∂ is referred to as a 2-metric on X , and (X, ∂) is referred to as a 2-metric space, which is a generalization of a metric space. Geometrically this space was proven to have a distinct nonlinear structure, quite unlike a metric space, rather $\partial(x, y, z)$ represents the area of a triangle formed by the vertices x, y and z in X . Since then it has been studied by a number of mathematician researchers, such as Gähler in [1], [2], [3], Iseki in [4], and Lahiri and Das and Dey in [5], as well as several other researchers. On the other hand, in 1999 Introduced the notion of a soft set by Moldotsove [6] as follows: Let U be an initial universe set and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and \mathcal{A} be a non-empty subset of E . A pair $(\mathcal{F}, \mathcal{A})$ is called a soft set (over U) if and only if \mathcal{F} is a mapping defined by $\mathcal{F}: \mathcal{A} \rightarrow \mathcal{P}(U)$. For $\gamma \in \mathcal{A}$, $\mathcal{F}(\gamma)$ may be considered as the set of γ – approximate elements of the soft set $(\mathcal{F}, \mathcal{A})$. Later, in several fields, this hypothesis was expanded, such as soft topological space, soft group, soft ideal, soft linear space, soft metric space, soft Banach Algebra, and many others, that need not be mentioned. Also in 2010, the concept of soft element and the soft real number was introduced by S. Das and S. K. Samanta [7] which is a good help in our research and we depend on it a lot. The above motivated the introduction of the concepts of soft-2-metric space which we called a $soft_2$ – metric space which is a 2-metric space in terms of the soft element, and on the

concept $soft_2$ – metric space we defined the following concepts and studied their properties: $soft_2$ – open ball, $soft_2$ – closed ball, $soft_2$ – open set, $soft_2$ – closed set, $soft_2$ – interior element, $soft_2$ – limit element, $soft_2$ – closure. And also reformulated and proved some theorems in $soft_2$ – metric space, such as Cantor's theorem. It is worth noting that we named each element in the $soft_2$ – metric space by $soft_2$ – element and we named a subset of $soft_2$ – metric space by $soft_2$ – subset.

2. Preliminaries

In this section, some basic information is given on soft elements, soft sets, and 2-metric spaces.

Definition 2.1: [8] Let (F, A) and (G, B) are two soft sets over a common universe U then

1. If $A \subseteq B$, and $F(\gamma) \subseteq G(\gamma)$, For some $\gamma \in A$, then (F, A) is called a soft subset of (G, B) , this written by $(F, A) \subseteq (G, B)$.
2. If (G, B) is a soft subset of (F, A) , then (F, A) is called a soft super set of (G, B) , this is written by $(F, A) \supseteq (G, B)$.

Definition 2.2: [8] A soft set (F, A) is said to be soft equal to (G, B) if and only if $(F, A) \subseteq (G, B)$ and $(F, A) \supseteq (G, B)$, where (F, A) and (G, B) are two soft sets over a common universe U .

Definition 2.3: [5] A soft set (F, A) over U is called a null soft set, represented by ϕ , if $F(\gamma) = \emptyset$, for each $\gamma \in A$,

Definition 2.4: [8] a soft set (F, A) over U is called an absolute soft set, represented by \hat{U} , if $F(\gamma) = U$, for each $\gamma \in A$.

Definition 2.5:[9] (Complement of a soft set) A soft set $(F, A)^c$ is the complement of a soft set (F, A) , and it is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow \mathcal{P}(U)$ is a mapping provided by $F^c(\gamma) = U - F(\gamma), \forall \gamma \in A$.

Definition 2.6:[10] Let (F, A) and (G, B) be two soft sets over the common universe U , then

- (i) A soft set (\mathcal{H}, C) is said to be a union of (F, A) and (G, B) denoted by $(\mathcal{H}, C) = (F, A) \cup (G, B)$, which is defined as $C = A \cup B$, and for all $\gamma \in C$,

$$\mathcal{H}(\gamma) = \begin{cases} F(\gamma) & \gamma \in A - B \\ G(\gamma) & \gamma \in B - A \\ F(\gamma) \cup G(\gamma) & \gamma \in A \cap B \end{cases}$$
- (ii) A soft set (\mathcal{H}, C) is said to be an intersection of (F, A) and (G, B) , denoted by $(\mathcal{H}, C) = (F, A) \cap (G, B)$, is defined as $C = A \cap B$, and $\mathcal{H}(\gamma) = F(\gamma) \cap G(\gamma)$ for each $\gamma \in C$.

Proposition 2.7: (Distributive laws). [11] Let (F, A) , (G, B) , and (\mathcal{H}, C) are three soft sets over universe set U , we have the following statement:

1. $(F, A) \cap [(G, B) \cup (\mathcal{H}, C)] = [(F, A) \cap (G, B)] \cup [(F, A) \cap (\mathcal{H}, C)]$,
2. $(F, A) \cup [(G, B) \cap (\mathcal{H}, C)] = [(F, A) \cup (G, B)] \cap [(F, A) \cup (\mathcal{H}, C)]$.

Proposition 2.8.[11] Let $(F_1, A_1), (F_2, A_2)$ and $(G_1, B_1), (G_2, B_2)$ are soft sets over universe set U . If $(F_i, A_i) \subseteq (G_i, B_i), i = 1, 2$, then the following statements are true:

1. $(F_1, A_1) \cap (F_2, A_2) \subseteq (G_1, B_1) \cap (G_2, B_2)$.
2. $(F_1, A_1) \cup (F_2, A_2) \subseteq (G_1, B_1) \cup (G_2, B_2)$.

Theorem 2.9: [12] Let $(F, A), (G, B)$ be two non-null soft subsets over universe set U then: $(F, A) \cup (G, B) = \phi$ if and only if $(F, A) = \phi$ and $(G, B) = \phi$.

Definition 2.10:[7] (soft-element) Assume that X is a non-empty set and that E is a non-empty parameter set. Then a function $\varepsilon: E \rightarrow X$ is called a soft element of X . A soft element ε of X is said to belong to a soft set $A_{\tilde{}}$ of X , denoted by $\varepsilon \tilde{\in} A_{\tilde{}}$, if $\varepsilon(\gamma) \in A_{\tilde{}}(\gamma)$, for all $\gamma \in E$. Thus, a soft set $A_{\tilde{}}$ of X concerning the index set E can be expressed as $A_{\tilde{}}(\gamma) = \{\varepsilon(\gamma), \varepsilon \tilde{\in} A_{\tilde{}}\}, \gamma \in E$.

Definition 2.11: [7] Let \mathbb{R} represent the set of real numbers, $\mathcal{P}(\mathbb{R})$ represent the collection of all non-empty bounded subsets of \mathbb{R} , and $A_{\tilde{}}$ represent a set of parameters. The mapping $F: A_{\tilde{}} \rightarrow \mathcal{P}(\mathbb{R})$ is said to be a soft real set. It is represented by $(F, A_{\tilde{}})$. If $(F, A_{\tilde{}})$ is a singleton soft set, then identifying $(F, A_{\tilde{}})$ with the associated soft element, will be said as a soft real number.

Definition 2.12:[13] Let \tilde{r}, \tilde{s} be two soft real numbers, then for each $\gamma \in A_{\tilde{}}$

1. $\tilde{r} \tilde{\leq} \tilde{s}$ if $\tilde{r}(\gamma) \leq \tilde{s}(\gamma)$.
2. $\tilde{r} \tilde{\geq} \tilde{s}$ if $\tilde{r}(\gamma) \geq \tilde{s}(\gamma)$.
3. $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(\gamma) < \tilde{s}(\gamma)$.
4. $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(\gamma) > \tilde{s}(\gamma)$.

3. $soft_2$ – metric space.

In this section, we introduce the concept of $soft_2$ – metric spaces, which are new metric spaces based on the soft-elements, also we defined the $soft_2$ – open ball, $soft_2$ – closed ball, $soft_2$ – open set, $soft_2$ – closed set, and we studied some relations between them.

Noting that, we use the notation $\tilde{x}, \tilde{y}, \tilde{z}$ to represent $soft_2$ – elements of a soft set; $\tilde{r}, \tilde{s}, \tilde{t}$ to represent soft real numbers; $\tilde{r}, \tilde{s}, \tilde{t}$ will represent a certain type of soft real numbers such that, $\tilde{r}(\gamma) = r$ for each $\gamma \in A_{\tilde{}}$, etc. Keep in mind that r generally has nothing to do with \tilde{r} ; $\mathbb{R}(A_{\tilde{}})^*$ to denote the collection of all non-negative soft real sets.

Definition 3.1: Assume that X is a non-empty set and that $A_{\tilde{}}$ is a non-empty set of parameters. Let \tilde{X} be an absolute soft set, which means that $F(\gamma) = \tilde{X}$ for all $\gamma \in A_{\tilde{}}$, where $(F, A_{\tilde{}}) = \tilde{X}$. Let $SE(\tilde{X})$ represent the collection of all \tilde{X} 's soft elements.

A mapping $\partial: SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \rightarrow \mathbb{R}(A_{\tilde{}})^*$ is called a $soft_2$ – metric on the soft set \tilde{X} if ∂ satisfying the following conditions:

1. For each pair of distinct soft-elements $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$, there is a soft-element $\tilde{z} \tilde{\in} \tilde{X}$ such that $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \neq \bar{0}$.
2. $\partial(\tilde{x}, \tilde{y}, \tilde{z}) = \bar{0}$ if at least two of the three $soft_2$ – elements $\tilde{x}, \tilde{y}, \tilde{z}$ are equal.
3. $\partial(\tilde{x}, \tilde{y}, \tilde{z}) = \partial(\tilde{x}, \tilde{z}, \tilde{y}) = \partial(\tilde{y}, \tilde{z}, \tilde{x})$ for all triple $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} \tilde{X}$.
4. $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \tilde{\leq} \partial(\tilde{x}, \tilde{y}, \tilde{v}) + \partial(\tilde{y}, \tilde{z}, \tilde{v}) + \partial(\tilde{z}, \tilde{x}, \tilde{v})$ for all $\tilde{x}, \tilde{y}, \tilde{z}, \tilde{v} \tilde{\in} \tilde{X}$.

(\tilde{X}, ∂) is called $soft_2$ – metric space.

Example 3.2: Assume that X is a non-empty set and that $A_{\tilde{}}$ is a non-empty set of parameters. Let \tilde{X} be an absolute soft set, which means that $F(\varepsilon) = \tilde{X}$ for all $\varepsilon \in A_{\tilde{}}$, where $(F, A_{\tilde{}}) = \tilde{X}$. Define $\partial: SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \rightarrow \mathbb{R}(A_{\tilde{}})^*$ as follows: for each $soft_2$ – elements $\tilde{x}, \tilde{y}, \tilde{z}$ in \tilde{X}

$$\partial(\tilde{x}, \tilde{y}, \tilde{z}) = \begin{cases} \bar{1} & \text{if } \tilde{x}, \tilde{y}, \tilde{z} \text{ are distinct} \\ \bar{0} & \text{if at least two of } \tilde{x}, \tilde{y}, \tilde{z} \text{ are equal} \end{cases}$$

Then ∂ it fulfills all the $soft_2$ – metric axioms.

Definition 3.3: Let (\tilde{X}, ∂) be a $soft_2$ – metric space and $(\tilde{Y}, A_{\tilde{}})$ be a non-null soft set of \tilde{X} , then the mapping $\partial_{\tilde{Y}}: SE(\tilde{Y}, A_{\tilde{}}) \times SE(\tilde{Y}, A_{\tilde{}}) \times SE(\tilde{Y}, A_{\tilde{}}) \rightarrow \mathbb{R}(A_{\tilde{}})^*$ given by $\partial_{\tilde{Y}}(\tilde{x}, \tilde{y}, \tilde{z}) = \partial(\tilde{x}, \tilde{y}, \tilde{z})$ for each $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} (\tilde{Y}, A_{\tilde{}})$ is a $soft_2$ – metric on $(\tilde{Y}, A_{\tilde{}})$. This metric $\partial_{\tilde{Y}}$ is referred to as the relative $soft_2$ –

metric induced on (\hat{Y}, \mathcal{A}) by ∂ . The $soft_2$ – metric space $(\hat{Y}, \partial_{\hat{Y}}, \mathcal{A})$ is called a $soft_2$ – metric subspace or simply a subspace of the $soft_2$ – metric space $(\hat{X}, \partial, \mathcal{A})$

Definition 3.4: Let (\hat{X}, ∂) be a $soft_2$ – metric space, $\tilde{x}, \tilde{y} \in \hat{X}$ and a soft real number $\tilde{r} \succ \bar{0}$ Then the subset $B_{\tilde{r}}(\tilde{x}, \tilde{y}) = \{\tilde{z} \in \hat{X}; \partial(\tilde{x}, \tilde{y}, \tilde{z}) \prec \tilde{r}\}$ of $SE(\hat{X})$ is said to be a $soft_2$ – open ball with centered at \tilde{x}, \tilde{y} and radius \tilde{r} .

Definition 3.5: Let (\hat{X}, ∂) be a $soft_2$ – metric space, $\tilde{x}, \tilde{y} \in \hat{X}$ and a soft real number $\tilde{r} \succ \bar{0}$ Then the subset $B_{\tilde{r}}[\tilde{x}, \tilde{y}] = \{\tilde{z} \in \hat{X}; \partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \tilde{r}\}$ of $SE(\hat{X})$ is said to be a $soft_2$ – closed ball with centered at \tilde{x}, \tilde{y} and radius \tilde{r} .

Example 3.6: Consider the discrete $soft_2$ – metric space (\hat{X}, ∂) .

$$\partial(\tilde{x}, \tilde{y}, \tilde{z}) = \begin{cases} \bar{1} & \text{if } \tilde{x}, \tilde{y}, \tilde{z} \text{ are distinct} \\ \bar{0} & \text{if at least two of } \tilde{x}, \tilde{y}, \tilde{z} \text{ are equal} \end{cases}$$

$$\begin{aligned} \text{Now, } B_{\bar{1}}(\tilde{x}, \tilde{y}) &= \{\tilde{z} \in \hat{X}; \partial(\tilde{x}, \tilde{y}, \tilde{z}) \prec \bar{1}; \tilde{x}, \tilde{y} \in \hat{X}\} \\ &= \{\tilde{z} \in \hat{X}; \partial(\tilde{x}, \tilde{y}, \tilde{z}) = \bar{0}; \tilde{x}, \tilde{y} \in \hat{X}\} = \{\tilde{x}, \tilde{y}\}. \end{aligned}$$

$$\begin{aligned} \text{And } B_{\bar{1}}[\tilde{x}, \tilde{y}] &= \{\tilde{z} \in \hat{X}; \partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \bar{1}; \tilde{x}, \tilde{y} \in \hat{X}\} \\ &= \{\tilde{z} \in \hat{X}; \partial(\tilde{x}, \tilde{y}, \tilde{z}) = \bar{0} \text{ or } \partial(\tilde{x}, \tilde{y}, \tilde{z}) = \bar{1}; \tilde{x}, \tilde{y} \in \hat{X}\} = SE(\hat{X}) \end{aligned}$$

Definition 3.7: (Soft₂ – open set) Suppose that (\hat{X}, ∂) be a $soft_2$ – metric space, then a $soft_2$ – subset (F, \mathcal{A}) of \hat{X} is said to be a $soft_2$ – open set if for each $soft_2$ element \tilde{x} in (F, \mathcal{A}) there are a finite number of $soft_2$ elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \hat{X}$, and soft real number $\tilde{r}_i \geq \bar{0}, i = 1, 2, \dots, n$. such that $\bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, \mathcal{A}))$.

Theorem 3.8: Every $soft_2$ – open ball in a $soft_2$ – metric space (\hat{X}, ∂) is a $soft_2$ – open set.

Proof: Let $B_{\tilde{r}}(\tilde{x}, \tilde{y})$ be a $soft_2$ – open ball where \tilde{x}, \tilde{y} are $soft_2$ – elements in \hat{X} and $\tilde{r} \succ \bar{0}$, let $\tilde{a} \in B_{\tilde{r}}(\tilde{x}, \tilde{y})$ then $\partial(\tilde{x}, \tilde{y}, \tilde{a}) \prec \tilde{r}$, which implies that

$$\tilde{r} - \partial(\tilde{x}, \tilde{y}, \tilde{a}) \succ \bar{0} \tag{1}$$

Let

$$\tilde{r}_1 = \frac{\tilde{r} - \partial(\tilde{x}, \tilde{y}, \tilde{a})}{2} \tag{2}$$

so if $\tilde{z} \in (B_{\tilde{r}_1}(\tilde{x}, \tilde{a}) \cap B_{\tilde{r}_1}(\tilde{y}, \tilde{a}))$ then $\tilde{z} \in B_{\tilde{r}_1}(\tilde{x}, \tilde{a})$ and $\tilde{z} \in B_{\tilde{r}_1}(\tilde{y}, \tilde{a})$; From (2) if $\tilde{z} \in B_{\tilde{r}_1}(\tilde{x}, \tilde{a})$ then

$$\partial(\tilde{x}, \tilde{a}, \tilde{z}) \prec \tilde{r}_1 = \frac{\tilde{r} - \partial(\tilde{x}, \tilde{y}, \tilde{a})}{2} \tag{3}$$

if $\tilde{z} \in B_{\tilde{r}_1}(\tilde{y}, \tilde{a})$ then

$$\partial(\tilde{y}, \tilde{a}, \tilde{z}) \prec \tilde{r}_1 = \frac{\tilde{r} - \partial(\tilde{x}, \tilde{y}, \tilde{a})}{2} \tag{4}$$

Now, from (3) and (4) we get

$$\partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \partial(\tilde{x}, \tilde{y}, \tilde{a}) + \partial(\tilde{x}, \tilde{a}, \tilde{z}) + \partial(\tilde{y}, \tilde{a}, \tilde{z}) \prec \partial(\tilde{x}, \tilde{y}, \tilde{a}) + \frac{\tilde{r} - \partial(\tilde{x}, \tilde{y}, \tilde{a})}{2} + \frac{\tilde{r} - \partial(\tilde{x}, \tilde{y}, \tilde{a})}{2} = \tilde{r}.$$

So that $B_{\tilde{r}_1}(\tilde{x}, \tilde{a}) \cap B_{\tilde{r}_1}(\tilde{y}, \tilde{a}) \subset B_{\tilde{r}}(\tilde{x}, \tilde{y})$, then $B_{\tilde{r}}(\tilde{x}, \tilde{y})$ is a $soft_2$ – open set.

Definition 3.9: (Soft₂ – closed set) Let (\hat{X}, ∂) be a $soft_2$ – metric space then a $soft_2$ – subset (F, \mathcal{A}) of \hat{X} is said to be a $soft_2$ – closed set if and only if $(F, \mathcal{A})^c$ is $soft_2$ – open.

Theorem 3.10: Every $soft_2$ – closed ball in a $soft_2$ – metric space (\hat{X}, ∂) is a $soft_2$ – closed set.

Proof: Let (\hat{X}, ∂) be a $soft_2$ – metric space, and let $B_{\tilde{r}}[\tilde{x}, \tilde{y}] = \{\tilde{z} \in \hat{X}; \partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \tilde{r}\}$ be a $soft_2$ – closed ball in \hat{X} , we must show that $B_{\tilde{r}}[\tilde{x}, \tilde{y}]$ is a $soft_2$ – closed set, that is $(B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c$ is $soft_2$ – open set.

If $(B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c = \phi$, then it is a $soft_2$ – open set, thus $B_{\tilde{r}}[\tilde{x}, \tilde{y}]$ is a $soft_2$ – closed set.

Assume that $(B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c \neq \phi$, let a $soft_2$ – element $\tilde{a} \in (B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c$, then $\tilde{a} \notin B_{\tilde{r}}[\tilde{x}, \tilde{y}]$, so from the definition of $B_{\tilde{r}}[\tilde{x}, \tilde{y}]$ implies that $\partial(\tilde{x}, \tilde{y}, \tilde{a}) \succ \tilde{r}$ then $\partial(\tilde{x}, \tilde{y}, \tilde{a}) - \tilde{r} \succ \tilde{0}$.

Now, suppose that

$$\tilde{s} = \frac{\partial(\tilde{x}, \tilde{y}, \tilde{a}) - \tilde{r}}{2} \tag{5}$$

we must show that $B_{\tilde{s}}(\tilde{x}, \tilde{a}) \tilde{\cap} B_{\tilde{s}}(\tilde{y}, \tilde{a}) \subseteq (B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c$. Let $\tilde{z} \in B_{\tilde{s}}(\tilde{x}, \tilde{a}) \tilde{\cap} B_{\tilde{s}}(\tilde{y}, \tilde{a})$ then $\tilde{z} \in B_{\tilde{s}}(\tilde{x}, \tilde{a})$, and $\tilde{z} \in B_{\tilde{s}}(\tilde{y}, \tilde{a})$ therefore

$$\partial(\tilde{x}, \tilde{z}, \tilde{a}) \lesssim \tilde{s} \tag{6}$$

and

$$\partial(\tilde{y}, \tilde{z}, \tilde{a}) \lesssim \tilde{s} \tag{7}$$

Now from (5), (6) and (7) we have

$$\begin{aligned} \partial(\tilde{x}, \tilde{y}, \tilde{a}) &\gtrsim \partial(\tilde{x}, \tilde{y}, \tilde{z}) + \partial(\tilde{x}, \tilde{z}, \tilde{a}) + \partial(\tilde{y}, \tilde{z}, \tilde{a}) \\ &\gtrsim \partial(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{s} + \tilde{s} \gtrsim \partial(\tilde{x}, \tilde{y}, \tilde{z}) + \partial(\tilde{x}, \tilde{y}, \tilde{a}) - \tilde{r}, \end{aligned}$$

Then $\partial(\tilde{x}, \tilde{y}, \tilde{a}) \succ \tilde{r}$, thus $\tilde{z} \notin B_{\tilde{r}}[\tilde{x}, \tilde{y}]$ then $\tilde{z} \in (B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c$. So that

$$B_{\tilde{s}}(\tilde{x}, \tilde{a}) \tilde{\cap} B_{\tilde{s}}(\tilde{y}, \tilde{a}) \subseteq (B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c \tag{8}$$

which implies that $(B_{\tilde{r}}[\tilde{x}, \tilde{y}])^c$ is a $soft_2$ – open set. Hence $B_{\tilde{r}}[\tilde{x}, \tilde{y}]$ is a $soft_2$ – closed set.

Definition 3.11: the topology generated in $soft_2$ – metric space \tilde{X} by taking the family of all $soft_2$ – open ball as s subbasis, is said to be $soft_2$ – metric topology and denoted by $(\tilde{X}, \tilde{\mathcal{T}})$.

Any member in $(\tilde{X}, \tilde{\mathcal{T}})$ is said to be $soft_2$ – open set and this complement is $soft_2$ – closed set.

Lemma 3.12: A $soft_2$ – subset (F, A) of $(\tilde{X}, \tilde{\mathcal{T}})$ is $soft_2$ – open set if and only if for any $soft_2$ – element $\tilde{x} \in (F, A)$ there exists a finite number of $soft_2$ – elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and soft real elements $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n \succ \tilde{0}$ such that $\tilde{x} \in B_{\tilde{r}_1}(\tilde{x}, \tilde{y}_1) \tilde{\cap} B_{\tilde{r}_2}(\tilde{x}, \tilde{y}_2) \tilde{\cap} \dots \tilde{\cap} B_{\tilde{r}_n}(\tilde{x}, \tilde{y}_n) \subset SE(F, A)$

Proof: Because each set of the form intersection of the $soft_2$ – open balls $B_{\tilde{r}_1}(\tilde{x}, \tilde{y}_1) \tilde{\cap} B_{\tilde{r}_2}(\tilde{x}, \tilde{y}_2) \tilde{\cap} \dots \tilde{\cap} B_{\tilde{r}_n}(\tilde{x}, \tilde{y}_n)$ is $soft_2$ – open by definition, the claim is satisfied immediately.

Conversely, Let (F, A) be a $soft_2$ – open set in \tilde{X} and \tilde{x} be a $soft_2$ – element in (F, A) , then there exists a finite number of $soft_2$ – open balls $B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)$, $i = 1, 2, \dots, n$ such that $\tilde{x} \in B_{\tilde{r}_1}(\tilde{x}, \tilde{y}_1) \tilde{\cap} B_{\tilde{r}_2}(\tilde{x}, \tilde{y}_2) \tilde{\cap} \dots \tilde{\cap} B_{\tilde{r}_n}(\tilde{x}, \tilde{y}_n) \subset SE(F, A)$, since $\tilde{x} \in B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)$ which implies that

$$\partial(\tilde{x}, \tilde{x}_i, \tilde{y}_i) = \tilde{s}_i \lesssim \tilde{r}_i \tag{9}$$

Choose

$$\tilde{t}_i \lesssim \frac{\tilde{r}_i - \tilde{s}_i}{2} \tag{10}$$

Then

$$B_{\tilde{t}_i}(\tilde{x}, \tilde{x}_i) \tilde{\cap} B_{\tilde{t}_i}(\tilde{x}, \tilde{y}_i) \subseteq B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \tag{11}$$

and this holds for $i = 1, 2, \dots, n$. Therefore,

$$\begin{aligned} \tilde{x} &\in [B_{\tilde{t}_1}(\tilde{x}, \tilde{x}_1) \tilde{\cap} B_{\tilde{t}_1}(\tilde{x}, \tilde{y}_1)] \tilde{\cap} [B_{\tilde{t}_2}(\tilde{x}, \tilde{x}_2) \tilde{\cap} B_{\tilde{t}_2}(\tilde{x}, \tilde{y}_2)] \tilde{\cap}, \dots, \tilde{\cap} [B_{\tilde{t}_n}(\tilde{x}, \tilde{x}_n) \tilde{\cap} B_{\tilde{t}_n}(\tilde{x}, \tilde{y}_n)] \\ &\subset \tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset (F, A). \end{aligned}$$

4. $soft_2$ – interior element, $soft_2$ – limit element, and $soft_2$ – closure.

In this section, we introduce the $soft_2$ – interior elements, $soft_2$ – limit elements, and $soft_2$ – closure of $soft_2$ – subsets in \tilde{X} , and studied some their properties, and also we formulated and proved some theorems about their properties.

Definition 4.1: ($soft_2$ – interior element) Let (\tilde{X}, ∂) be a $soft_2$ – metric space and $(F, A) \subseteq (\tilde{X}, \partial)$; Then a $soft_2$ – element $\tilde{x} \in \tilde{X}$ is said to be a $soft_2$ – interior element of (F, A) if there are a finite number of $soft_2$ – elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n \geq \bar{0}$ such that

$\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$, for all $i = 1, 2, \dots, n$.

Definition 4.2: The collection of all $soft_2$ – interior elements of (F, A) is called the $soft_2$ – interior of (F, A) and denoted by $(F, A)^{i_2}$. Thus

$(F, A)^{i_2} = \{\tilde{x} \in (F, A) : \tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE(F, A), \forall \tilde{y}_i \in \tilde{X} \text{ and } \forall \tilde{r}_i \geq \bar{0}; i = 1, 2, \dots, n\}$
 $SS((F, A)^{i_2})$ is said to be the $soft_2$ – interior of (F, A) .

Remark 4.3: $(F, A)^{i_2}$ is the union of all $soft_2$ – open set contained in (F, A) . i.e.

$$(F, A)^{i_2} = \bigcup \{(G, A) : (G, A) \text{ is } soft_2 \text{ – open set and } (G, A) \subseteq (F, A)\}$$

Theorem 4.4: Let $(F, A), (G, A)$ be two non-null $soft_2$ – subsets in a $soft_2$ – metric space (\tilde{X}, ∂) then:

(i) $SS((F, A)^{i_2}) \subseteq (F, A)$.

(ii) (F, A) is a $soft_2$ – open set if and only if $(F, A) = SS((F, A)^{i_2})$

(iii) $(F, A) \subseteq (G, A)$ then $SS((F, A)^{i_2}) \subseteq SS((G, A)^{i_2})$

(iv) $SS((F, A)^{i_2}) \cap SS((G, A)^{i_2}) = SS\left[\left((F, A) \cap (G, A)\right)^{i_2}\right]$

(v) $SS\left[\left((F, A) \cup (G, A)\right)^{i_2}\right] \supseteq SS((F, A)^{i_2}) \cup SS((G, A)^{i_2})$

Proof:

(i) Let $\tilde{x} \in SS((F, A)^{i_2})$ then there are a finite number of $soft_2$ – elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n \geq \bar{0}$ such that $\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$. So $\tilde{x} \in SE((F, A))$, which implies that

$$SS((F, A)^{i_2}) \subseteq (F, A) \tag{12}$$

(ii) Let (F, A) is $soft_2$ – open set, now by (12), $SS((F, A)^{i_2}) \subseteq (F, A)$. And since (F, A) is $soft_2$ – open set then for any $\tilde{x} \in (F, A)$ there are a finite number of $soft_2$ – elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and soft real number $\tilde{r}_i \geq \bar{0}, i = 1, 2, \dots, n$. such that

$\bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$, so $\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$, then

$\tilde{x} \in SS((F, A)^{i_2})$, therefore

$$(F, A) \subseteq SS((F, A)^{i_2}) \tag{13}$$

From (12) and (13), $(F, A) = SS((F, A)^{i_2})$

Conversely, suppose that $(F, A) = SS((F, A)^{i_2})$, then for any $\tilde{x} \in (F, A)$; $\tilde{x} \in SS((F, A)^{i_2})$, so there are a finite number of $soft_2$ – elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n \geq \bar{0}$ such that $\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$, therefore for any $\tilde{x} \in (F, A)$, then $\bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$.

Hence (F, A) is a $soft_2$ – open set

(iii) Let $\tilde{x} \in SS((F, A)^{i_2})$ then there are a finite number of $soft_2$ – elements

$\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n \geq \bar{0}$ such that

$\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$, for all $i = 1, 2, \dots, n$. But $(F, A) \subseteq (G, A)$ then

$\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A)) \subseteq SE((G, A))$; which implies that $\tilde{x} \in SS((G, A)^{i_2})$.

Hence

$$SS((F, A)^{i_2}) \subseteq SS((G, A)^{i_2}) \tag{14}$$

(iv) Let $\tilde{x} \in SS\left[\left((F, A) \cap (G, A)\right)^{i_2}\right]$. then there are a finite number of $soft_2$ – elements

$\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n \geq \bar{0}$ such that

$\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A) \cap (G, A))$, for all $i = 1, 2, \dots, n$. But $SE((F, A) \cap (G, A)) = SE((F, A)) \cap SE((G, A))$, therefore $\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$ and $\tilde{x} \in \bigcap_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((G, A))$, so that $\tilde{x} \in SS((F, A)^{i_2})$ and $\tilde{x} \in SS((G, A)^{i_2})$, then

$\tilde{x} \in [SS((F, A)^{i_2}) \cap SS((G, A)^{i_2})]$, Which implies that

$$SS\left[\left((F, A) \cap (G, A)\right)^{i_2}\right] \subseteq SS((F, A)^{i_2}) \cap SS((G, A)^{i_2}) \tag{15}$$

Conversely, let $\tilde{y} \in [SS((F, A)^{i_2}) \cap SS((G, A)^{i_2})]$ then $\tilde{y} \in SS((F, A)^{i_2})$ and $\tilde{y} \in SS((G, A)^{i_2})$ then, there are a finite number of $soft_2$ – elements $\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_n \geq \bar{0}$ such that $\tilde{y} \in \bigcap_{i=1}^n B_{\tilde{s}_i}(\tilde{y}, \tilde{z}_i) \subset SE((F, A))$, also there are a finite number of

$soft_2$ – elements $\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_n \geq \bar{0}$ such that $\tilde{y} \in \tilde{\cap}_{i=1}^n B_{\tilde{t}_i}(\tilde{y}, \tilde{u}_i) \subset SE((G, A))$. Now let;

$$\tilde{r}_i(\gamma) = \min\{\tilde{s}_i(\gamma), \tilde{t}_i(\gamma)\}; \forall \gamma \in A \text{ and } i = 1, 2, \dots, n.$$

$$\tilde{y}_i(\gamma) = \min\{\tilde{z}_i(\gamma), \tilde{u}_i(\gamma)\}; \forall \gamma \in A \text{ and } i = 1, 2, \dots, n.$$

So that, $\tilde{y} \in \tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{y}, \tilde{y}_i) \subset SE((F, A))$ and $\tilde{y} \in \tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{y}, \tilde{y}_i) \subset SE((G, A))$,

Then $\tilde{y} \in \tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{y}, \tilde{y}_i) \subset SE((F, A)) \cap SE((G, A)) = SE[(F, A) \dot{\cap} (G, A)]$ therefore

$\tilde{y} \in SS[(F, A) \dot{\cap} (G, A)]^{i_2}$, which implies that

$$SS((F, A)^{i_2}) \tilde{\cap} SS((G, A)^{i_2}) \cong SS[(F, A) \dot{\cap} (G, A)]^{i_2} \tag{16}$$

From (15) and (16) we get:

$$SS((F, A)^{i_2}) \tilde{\cap} SS((G, A)^{i_2}) = SS[(F, A) \tilde{\cap} (G, A)]^{i_2} \tag{17}$$

(v) Let $\tilde{x} \in SS[(F, A)^{i_2}] \cup SS((G, A)^{i_2})$ so that $\tilde{x} \in SS[(F, A)^{i_2}]$ or $\tilde{x} \in SS((G, A)^{i_2})$ then,

there are a finite number of $soft_2$ – elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \in \tilde{X}$, and a finite number of soft real numbers $\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n \geq \bar{0}$ such that

$\tilde{x} \in \tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A))$ or $\tilde{x} \in \tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((G, A))$, therefore $\tilde{x} \in \tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE(F, A) \cup SE(G, A) \subset SE((F, A) \cup (G, A))$, which implies that $\tilde{x} \in SS[(F, A) \cup (G, A)]^{i_2}$.

Hence, $SS[(F, A) \cup (G, A)]^{i_2} \cong SS((F, A)^{i_2}) \cup SS((G, A)^{i_2})$

Definition 4.5: ($soft_2$ – limit element) Suppose that (\tilde{X}, ∂) be a $soft_2$ – metric space. Then a $soft_2$ – element $\tilde{x} \in \tilde{X}$ is said to be $soft_2$ – limit element of a $soft_2$ – subset (F, A) if $((\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)) - \{\tilde{x}\}) \tilde{\cap} (F, A) \neq \phi$, where $\{\tilde{y}_i \in \tilde{X}, i = 1, 2, \dots, n\}$ are a $soft_2$ – elements in \tilde{X} . The collection of all $soft_2$ – limit elements of (F, A) is called a derived set of (F, A) and denoted by $(F, A)^\partial$.

Theorem 4.6: Let (\tilde{X}, ∂) be a $soft_2$ – metric space and $(F, A), (G, A)$ are two $soft_2$ – subsets in \tilde{X} . Then

- (i) $\phi^\partial = \phi$ and $\tilde{X}^\partial = \tilde{X}$.
- (ii) If $(F, A) \subset (G, A)$ then $(F, A)^\partial \subset (G, A)^\partial$.
- (iii) $((F, A) \tilde{\cap} (G, A))^\partial \subset (F, A)^\partial \tilde{\cap} (G, A)^\partial$.
- (iv) $((F, A) \cup (G, A))^\partial = (F, A)^\partial \cup (G, A)^\partial$.

Proof:

(ii) Let $(F, A) \subset (G, A)$ and $\tilde{x} \in (F, A)^\partial$ then \tilde{x} is a $soft_2$ – limit element of (F, A) thus for every finite $soft_2$ – open ball $\{B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i); \tilde{y}_i \in \tilde{X} \text{ where } i = 1, 2, \dots, n\}$, such that $((\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)) - \{\tilde{x}\}) \tilde{\cap} (F, A) \neq \phi$, but $(F, A) \subset (G, A)$ then $((\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)) - \{\tilde{x}\}) \tilde{\cap} (G, A) \neq \phi$, therefore, $\tilde{x} \in (G, A)^\partial$, which implies that

$$(F, A)^\partial \subset (G, A)^\partial \tag{18}$$

(iii) We prove this by contrapositive, suppose that $\tilde{x} \in [(F, A)^\partial \tilde{\cap} (G, A)^\partial]$ then $\tilde{x} \in (F, A)^\partial$ or $\tilde{x} \in (G, A)^\partial$ then there are a finite number of $soft_2$ – open balls $\{B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i); \tilde{y}_i \in \tilde{X} \text{ where } i = 1, 2, \dots, n\}$, and $\{B_{\tilde{s}_i}(\tilde{x}, \tilde{z}_i); \tilde{z}_i \in \tilde{X} \text{ where } i = 1, 2, \dots, n\}$, such that $(\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) - \{\tilde{x}\}) \tilde{\cap} (F, A) = \phi$ and $(\tilde{\cap}_{i=1}^n B_{\tilde{s}_i}(\tilde{x}, \tilde{z}_i) - \{\tilde{x}\}) \tilde{\cap} (G, A) = \phi$. So that

$$\begin{aligned} & (\tilde{\cap}_{i=1}^n [B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \tilde{\cap} B_{\tilde{s}_i}(\tilde{x}, \tilde{z}_i)] - \{\tilde{x}\}) \tilde{\cap} ((F, A) \tilde{\cap} (G, A)) \\ &= [(\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) - \{\tilde{x}\}) \tilde{\cap} (F, A)] \tilde{\cap} [(\tilde{\cap}_{i=1}^n B_{\tilde{s}_i}(\tilde{x}, \tilde{z}_i) - \{\tilde{x}\}) \tilde{\cap} (G, A)] \\ &= \phi \tilde{\cap} \phi = \phi, \end{aligned}$$

which implies that $\tilde{x} \notin ((F, A) \tilde{\cap} (G, A))^\partial$. Hence

$$((F, A) \tilde{\cap} (G, A))^\partial \subset (F, A)^\partial \tilde{\cap} (G, A)^\partial \tag{19}$$

(iv) Let $\tilde{x} \in ((F, A) \cup (G, A))^\partial$, then \tilde{x} is a $soft_2$ – limit element of $(F, A) \cup (G, A)$ then for every finite $soft_2$ – open ball $\{B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i); \tilde{y}_i \in \tilde{X} \text{ where } i = 1, 2, \dots, n\}$, such that $((\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)) - \{\tilde{x}\}) \tilde{\cap} ((F, A) \cup (G, A)) \neq \phi$, but from (Proposition 2.9) $((\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)) - \{\tilde{x}\}) \tilde{\cap} ((F, A) \cup (G, A)) =$

$[((\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)) - \{\tilde{x}\}) \tilde{\cap} (F, A)] \cup [((\tilde{\cap}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)) - \{\tilde{x}\}) \tilde{\cap} (G, A)]$, therefore

$\left(\left(\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)\right) - \{\tilde{x}\}\right) \tilde{\cap} (F, A) \neq \phi$ or $\left(\left(\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)\right) - \{\tilde{x}\}\right) \tilde{\cap} (G, A) \neq \phi$, since by (Theorem 2.9)

if $\left[\left(\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)\right) - \{\tilde{x}\}\right] \tilde{\cap} (F, A) = \phi$ and $\left[\left(\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)\right) - \{\tilde{x}\}\right] \tilde{\cap} (G, A) = \phi$ then $\left(\left(\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)\right) - \{\tilde{x}\}\right) \tilde{\cap} ((F, A) \tilde{\cup} (G, A)) = \phi$, which is a contradiction. So $\tilde{x} \tilde{\in} (F, A)^\partial$ or $\tilde{x} \tilde{\in} (G, A)^\partial$ then $\tilde{x} \tilde{\in} (F, A)^\partial \tilde{\cup} (G, A)^\partial$, we get

$$(F, A) \tilde{\cup} (G, A)^\partial \tilde{\subset} [(F, A)^\partial \tilde{\cup} (G, A)^\partial] \tag{20}$$

Conversely, we must show that $(F, A)^\partial \tilde{\cup} (G, A)^\partial \tilde{\subset} (F, A) \tilde{\cup} (G, A)^\partial$, since $(F, A) \tilde{\subset} [(F, A) \tilde{\cup} (G, A)]$ and $(G, A) \tilde{\subset} [(F, A) \tilde{\cup} (G, A)]$, then from (18)

$$(F, A)^\partial \tilde{\subset} ((F, A) \tilde{\cup} (G, A))^\partial \text{ and } (G, A)^\partial \tilde{\subset} ((F, A) \tilde{\cup} (G, A))^\partial, \text{ so that} \tag{21}$$

$$[(F, A)^\partial \tilde{\cup} (G, A)^\partial] \tilde{\subset} ((F, A) \tilde{\cup} (G, A))^\partial$$

As a result of (20) and (21), we get

$$(F, A) \tilde{\cup} (G, A)^\partial = (F, A)^\partial \tilde{\cup} (G, A)^\partial.$$

Theorem 4.7: Let (\hat{X}, ∂) be a *soft₂ – metric space* then a soft set $(F, A) \tilde{\subset} \hat{X}$ is *soft₂ – closed* if and only if $(F, A)^\partial \tilde{\subset} (F, A)$.

Proof: Let (F, A) be a *soft₂ – closed* set then $(F, A)^c$ is *soft₂ – open* set, thus for any *soft₂ – element* $\tilde{x} \tilde{\in} (F, A)^c$ there are a finite number of soft elements $\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n \tilde{\in} \hat{X}$, and soft real number $\tilde{r}_i \geq \bar{0}, i = 1, 2, \dots, n$. such that $\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset SE((F, A)^c)$. Now, $(F, A) \tilde{\cap} (F, A)^c = \phi$, which implies that $(F, A) \cap \left(\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)\right) = \phi$, thus $\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)$ contains no *soft₂ – element* of (F, A) and \tilde{x} is not *soft₂ – limit element* of (F, A) . Thus no *soft₂ – element* of $(F, A)^c$ can be a *soft₂ – limit element* of (F, A) then (F, A) contains all its *soft₂ – limit elements*. Hence $(F, A)^\partial \tilde{\subset} (F, A)$.

Conversely, Let $(F, A)^\partial \tilde{\subset} (F, A)$; show (F, A) is a *soft₂ – closed* it is enough to show $(F, A)^c$ is *soft₂ – open*.

Now, let \tilde{x} be any *soft₂ – element* in $(F, A)^c$, then $\tilde{x} \tilde{\notin} (F, A)$ and since $(F, A)^\partial \tilde{\subset} (F, A)$, so $\tilde{x} \tilde{\notin} (F, A)^\partial$ then \tilde{x} is not a *soft₂ – limit element* of (F, A) then there are a finite *soft₂ – open ball* $\{B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i); \tilde{y}_i \tilde{\in} \hat{X} \text{ where } i = 1, 2, \dots, n\}$, such that $\left(\left(\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i)\right) - \{\tilde{x}\}\right) \tilde{\cap} (F, A) = \phi$ thus $\tilde{\alpha}_{i=1}^n B_{\tilde{r}_i}(\tilde{x}, \tilde{y}_i) \subset (F, A)^c$, so that $(F, A)^c$ is *soft₂ – open* set. Hence (F, A) *soft₂ – closed*.

Definition 4.8: (soft₂ – closure) Let (\hat{X}, ∂) be a *soft₂ – metric space* then a soft set $(F, A) \tilde{\subset} \hat{X}$ is a *soft₂ – closure* of (F, A) , denoted by $cl_2(F, A)$ and defined by $cl_2(F, A) = \tilde{\cap} \{(G, A); (G, A) \text{ is } \textit{soft}_2\textit{ – closed and } (F, A) \tilde{\subset} (G, A)\}$.

Theorem 4.9: Let (\hat{X}, ∂) be a *soft₂ – metric space* and a soft set $(F, A) \tilde{\subset} \hat{X}$ then

- (i) (F, A) is *soft₂ – closed* if and only if $(F, A) = cl_2(F, A)$.
- (ii) $cl_2(F, A) = (F, A) \tilde{\cup} (F, A)^\partial$.

Proof: by definition of *soft₂ – closure* and (Theorem 4.7).

Theorem 4.10: Let (\hat{X}, ∂) be a *soft₂ – metric space* and $(F, A), (G, A)$ be a *soft₂ – subset* of \hat{X} , then:

1. $cl_2 \phi = \phi$.
2. $(F, A) \tilde{\subset} cl_2(F, A)$.
3. If $(F, A) \tilde{\subset} (G, A)$ then $cl_2(F, A) \tilde{\subset} cl_2(G, A)$.
4. $cl_2[(F, A) \tilde{\cup} (G, A)] = cl_2(F, A) \tilde{\cup} cl_2(G, A)$.
5. $cl_2[(F, A) \tilde{\cap} (G, A)] \tilde{\subset} cl_2(F, A) \tilde{\cup} cl_2(G, A)$.
6. $cl_2(cl_2(F, A)) = cl_2(F, A)$.

Proof: by (Theorem 4.6), (Definition 4.8), and (Theorem 4.9).

5. The sequence in *soft₂ – metric space*.

In this section, we define the convergent sequences and Cauchy sequences in *soft₂ – metric spaces*, and studied the complete *soft₂ – metric spaces* which is a *soft₂ – metric spaces* that every Cauchy sequence's is converge. And also we reformulated and proved some theorems in *soft₂ – metric spaces*, such as Cantor's Intersection theorem.

Definition 5.1: A sequence $\{\tilde{x}_n\}$ of the soft elements in *soft₂ – metric space* (\hat{X}, ∂) is said to be converge to the *soft₂ – element* $\tilde{x} \tilde{\in} \hat{X}$ if for all $\tilde{z} \tilde{\in} \hat{X}$, $\partial(\tilde{x}_n, \tilde{x}, \tilde{z}) \rightarrow 0$ as $n \rightarrow \infty$. This statement is written as $\tilde{x}_n \rightarrow \tilde{x}$ as $n \rightarrow \infty$ or $\lim_{n \rightarrow \infty} \tilde{x}_n = \tilde{x}$.

Definition 5.2: A sequence $\{\tilde{x}_n\}$ of the soft elements in $soft_2$ – metric space (\tilde{X}, ∂) is said to be Cauchy sequence in \tilde{X} if for all $\tilde{\varepsilon} \succeq \bar{0}$ and for all $\tilde{z} \in \tilde{X}$ there exist $k \in \mathbb{N}$ such that $\partial(\tilde{x}_n, \tilde{x}_m, \tilde{z}) \preceq \tilde{\varepsilon}$, i. e. $\partial(\tilde{x}_n, \tilde{x}_m, \tilde{z}) \rightarrow \bar{0}$ as $n, m \rightarrow \infty$.

Definition 5.3: (Complete $soft_2$ – metric space) A $soft_2$ – metric space (\tilde{X}, ∂) is said to be complete if every Cauchy sequence in \tilde{X} converges to a $soft_2$ – element of \tilde{X} .

Lemma 5.4: Let (\tilde{X}, ∂) be a $soft_2$ – metric space then a sequence $\{\tilde{x}_n\}$ is convergent to a $soft_2$ – element \tilde{x} in \tilde{X} if and only if for any $soft_2$ – open set (G, A) containing \tilde{x} there exists $m \in \mathbb{N}$ such that $\tilde{x}_n \tilde{\in} (G, A)$ for each $n \geq m$.

Proof: Let $\tilde{a} \tilde{\in} \tilde{X}$ and $\tilde{\varepsilon} \succeq \bar{0}$. But from (Theorem 3.8) the $soft_2$ – openball $B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{a})$ is a $soft_2$ – open set containing a $soft_2$ – element \tilde{x} , there exists $m \in \mathbb{N}$ such that $\tilde{x}_n \tilde{\in} B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{a})$ for any $n \geq m$. i. e. $\partial(\tilde{x}_n, \tilde{x}, \tilde{a}) \rightarrow \bar{0}$ as $n \rightarrow \infty$. which implies that $\{\tilde{x}_n\}$ is convergent to \tilde{x} .

Conversely, Suppose $\{\tilde{x}_n\}$ is convergent to a $soft_2$ – element \tilde{x} in \tilde{X} . Let (G, A) be any $soft_2$ – open set such that $\tilde{x} \tilde{\in} (G, A)$ then there are a finite number of $soft_2$ – elements $\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_k \tilde{\in} \tilde{X}$, and soft real numbers $\tilde{r}_i \succeq \bar{0}, i = 1, 2, \dots, k$. such that $\tilde{x} \tilde{\in} \tilde{\cap}_{i=1}^k B_{\tilde{r}_i}(\tilde{x}, \tilde{a}_i) \subset SE((G, A))$. But $\partial(\tilde{x}_n, \tilde{x}, \tilde{a}_i) \rightarrow \bar{0}$ as $n \rightarrow \infty$, there exists $m_i \in \mathbb{N}$ such that $\partial(\tilde{x}_n, \tilde{x}, \tilde{a}_i) \preceq \tilde{r}_i$ for each $n \geq m_i$ (i.e. $\tilde{x}_n \tilde{\in} B_{\tilde{r}_i}(\tilde{x}, \tilde{a}_i)$ for each $n \geq m_i; i = 1, 2, \dots, k$). Now let $m = \max\{m_1, m_2, \dots, m_k\}$, we get $\tilde{x}_n \tilde{\in} \tilde{\cap}_{i=1}^k B_{\tilde{r}_i}(\tilde{x}, \tilde{a}_i) \subset SE((G, A))$ for each $n \geq m$.

Definition 5.5: Let (\tilde{X}, ∂) be a $soft_2$ – metric space and a non-null soft set $(F, A) \tilde{\subset} \tilde{X}$ then: $\delta_{\tilde{z}}((F, A))(\gamma) = \sup\{\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma); \tilde{x}, \tilde{y} \tilde{\in} (F, A)\}; \forall \gamma \in A$ where \tilde{z} is a $soft_2$ – element in \tilde{X} .

Definition 5.6: Let (\tilde{X}, ∂) be a $soft_2$ – metric space and a non-null soft set $(F, A) \tilde{\subset} \tilde{X}$ then for any two $soft_2$ – elements $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$,

$$\delta(\tilde{x}, \tilde{y}, (F, A))(\gamma) = \inf\{\partial(\tilde{x}, \tilde{y}, \tilde{a})(\gamma); \tilde{a} \tilde{\in} (F, A)\} \forall \gamma \in A$$

Theorem 5.7: Let (\tilde{X}, ∂) be a $soft_2$ – metric space and $(F, A), (G, A)$ are non-null $soft_2$ – subsets of \tilde{X} , then if $(F, A) \tilde{\subset} (G, A)$, then $\delta_{\tilde{z}}((F, A)) \preceq \delta_{\tilde{z}}((G, A))$ where $soft_2$ – element $\tilde{z} \tilde{\in} \tilde{X}$.

Proof: Let $(F, A), (G, A)$ be a $soft_2$ – subsets of \tilde{X} , with $(F, A) \tilde{\subset} (G, A)$ for each $F(\gamma), G(\gamma)$ where $\gamma \in A$. Since $(F, A) \tilde{\subset} (G, A)$, thus for any two $soft_2$ – elements \tilde{x}, \tilde{y} such that $\tilde{x}, \tilde{y} \tilde{\in} (F, A)$ then $\tilde{x}, \tilde{y} \tilde{\in} (G, A)$, which implies that

$$\{\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma); \tilde{x}, \tilde{y} \tilde{\in} (F, A)\} \tilde{\subset} \{\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma); \tilde{x}, \tilde{y} \tilde{\in} (G, A)\},$$

where $\tilde{z} \tilde{\in} \tilde{X}$ and $\gamma \in A$. So that

$$\sup\{\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma); \tilde{x}, \tilde{y} \tilde{\in} (F, A)\} \preceq \sup\{\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma); \tilde{x}, \tilde{y} \tilde{\in} (G, A)\},$$

where $\tilde{z} \tilde{\in} \tilde{X}$ and $\gamma \in A$. Hence

$$\delta_{\tilde{z}}((F, A))(\gamma) \leq \delta_{\tilde{z}}((G, A))(\gamma) \text{ where } \tilde{z} \tilde{\in} \tilde{X} \text{ and } \gamma \in A. \text{ Thus}$$

$$\delta_{\tilde{z}}((F, A)) \preceq \delta_{\tilde{z}}((G, A)).$$

Proposition 5.8: Let (\tilde{X}, ∂) be a $soft_2$ – metric space and (F, A) be any $soft_2$ – sub set of \tilde{X} , then

$$\delta_{\tilde{z}}((F, A)) = \delta_{\tilde{z}}(cl_2(F, A)) \tag{22}$$

where \tilde{z} is a $soft_2$ – element in \tilde{X} .

Proof: Since $(F, A) \tilde{\subset} cl_2(F, A)$, we get $\delta_{\tilde{z}}((F, A)) \preceq \delta_{\tilde{z}}(cl_2(F, A))$. To show the converse inclusion, let $\tilde{x}, \tilde{y} \tilde{\in} cl_2(F, A)$ be arbitrary. If $\tilde{x}, \tilde{y} \tilde{\in} (F, A)$ then clearly $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \delta_{\tilde{z}}((F, A))$. Now, Suppose $\tilde{x} \notin (F, A)$ but $\tilde{y} \tilde{\in} (F, A)$. Let $\tilde{\varepsilon} \succ \bar{0}$ be arbitrary. Since $\tilde{x} \tilde{\in} cl_2(F, A)$, and $B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{y}) \tilde{\cap} B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{z})$ is a $soft_2$ – open set containing \tilde{x} , there exists $\tilde{u} \tilde{\in} [B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{y}) \tilde{\cap} B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{z})] \tilde{\cap} SE(F, A)$. Then $\tilde{y}, \tilde{u} \tilde{\in} (F, A)$ and $\partial(\tilde{x}, \tilde{y}, \tilde{u}) \preceq \tilde{\varepsilon}$ and $\partial(\tilde{x}, \tilde{u}, \tilde{z}) \preceq \tilde{\varepsilon}$. Thus $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \partial(\tilde{u}, \tilde{y}, \tilde{z}) + \partial(\tilde{x}, \tilde{u}, \tilde{z}) + \partial(\tilde{x}, \tilde{y}, \tilde{u}) \preceq \partial(\tilde{u}, \tilde{y}, \tilde{z}) + \tilde{\varepsilon} + \tilde{\varepsilon} \preceq \delta_{\tilde{z}}((F, A)) + 2\tilde{\varepsilon}$.

Because this is true for all $\tilde{\varepsilon} \succ \bar{0}$, we may deduce that $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \delta_{\tilde{z}}((F, A))$ for $\tilde{y} \tilde{\in} (F, A)$ and $\tilde{x} \tilde{\in} cl_2(F, A)$.

Finally, if $\tilde{x}, \tilde{y} \tilde{\in} cl_2(F, A) - (F, A)$ Then, by using the same argument, we can prove that in this case as well $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \preceq \delta_{\tilde{z}}((F, A))$. Thus

$$\delta_{\tilde{z}}(cl_2(F, A)) = \sup\{\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma); \tilde{x}, \tilde{y} \tilde{\in} cl_2(F, A)\}; \forall \gamma \in A \preceq \delta_{\tilde{z}}((F, A)).$$

So that $\delta_{\tilde{z}}((F, A)) = \delta_{\tilde{z}}(cl_2(F, A))$.

Theorem 5.9: (Cantor's Intersection Theorem for $soft_2$ – metric spaces)

A $soft_2$ – metric space (\tilde{X}, ∂) is complete if and only if for all decreasing sequence $\{(F, A)_n\}$ of non-null $soft_2$ – closed sets (i.e. $(F, A)_1 \supset (F, A)_2 \supset \dots \supset (F, A)_n \supset \dots$) such that $\delta_{\tilde{z}}((F, A)_n) \rightarrow \bar{0}$ as $n \rightarrow \infty, \forall \tilde{z} \tilde{\in} \tilde{X}$, then $(F, A) = \tilde{\cap}_{n=1}^{\infty} (F, A)_n$ consists of only one $soft_2$ – element.

Proof: Suppose that (\tilde{X}, ∂) is a complete $soft_2$ – metric space and $\{(F, A)_n\}$ be any decreasing sequence $\{(F, A)_n\}$ of non-null $soft_2$ – closed sets with $\delta_{\tilde{z}}((F, A)_n) \rightarrow \bar{0}$ as $n \rightarrow \infty, \forall \tilde{z} \tilde{\in} \tilde{X}$. Since $(F, A)_n(\gamma) \neq \phi$, for all $\gamma \in A$ and all $n \in \mathbb{N}$, choose a $soft_2$ – element $\tilde{x}_n \tilde{\in} (F, A)_n$, for all $n \in \mathbb{N}$.

We prove that $\{\tilde{x}_n\}$ is Cauchy in (\tilde{X}, ∂) . Since $\{(F, A)_n\}$ is decreasing, $\tilde{x}_m \tilde{\in} (F, A)_n \forall m \geq n$. Now, for any $soft_2$ - element $\tilde{z} \tilde{\in} \tilde{X}$, $m \geq n$,

$$\partial(\tilde{x}_m, \tilde{x}_n, \tilde{z}) \leq \delta_{\tilde{z}}((F, A)_n) \rightarrow \bar{0} \text{ as } n \rightarrow \infty. \text{ i. e. } \partial(\tilde{x}_m, \tilde{x}_n, \tilde{z}) \rightarrow \bar{0} \text{ as } m, n \rightarrow \infty.$$

This implies that $\{\tilde{x}_n\}$ is Cauchy in (\tilde{X}, ∂) .

Since (\tilde{X}, ∂) is a complete $soft_2$ - metric space then there exists a $soft_2$ - element $\tilde{x} \tilde{\in} \tilde{X}$ such that $\lim_{n \rightarrow \infty} \tilde{x}_n = \tilde{x}$. We claim that $\tilde{x} \tilde{\in} \tilde{\cap}_{n=1}^{\infty} (F, A)_n$. Suppose that $\tilde{x}_k \neq \tilde{x}$ for some k and up, otherwise

nothing to prove. Let $n \in \mathbb{N}$ be fixed. Let (G, A) be any $soft_2$ - open set containing \tilde{x} , then by (Lemma 5.4) there exists an appositive integer n_1 such that $\tilde{x}_k \tilde{\in} (G, A)$ for all $k \geq n_1$. Take $m = \max\{n, n_1\}$, we get $\tilde{x}_k \tilde{\in} [(G, A) - \{\tilde{x}\}] \cap (F, A)_n$ for all $k \geq m$. This implies that $\tilde{x} \tilde{\in} cl_2(F, A)_n$. But $\{(F, A)_n\}$ is a $soft_2$ - closed then $cl_2(F, A)_n = (F, A)_n$, thus $\tilde{x} \tilde{\in} (F, A)_n \forall n \in \mathbb{N}$. So that $\tilde{x} \tilde{\in} \tilde{\cap}_{n=1}^{\infty} (F, A)_n$. Now we must prove that $\tilde{\cap}_{n=1}^{\infty} (F, A)_n$ contains the of only one $soft_2$ - element. Suppose that $\tilde{\cap}_{n=1}^{\infty} (F, A)_n$ contains the two distinct $soft_2$ - elements \tilde{x} and \tilde{y} , choose the $soft_2$ - element $\tilde{z} \tilde{\in} \tilde{X}$ such that $\tilde{z} \neq \tilde{x}, \tilde{y}$, then

$$\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma) \leq \delta_{\tilde{z}}((F, A)_n(\gamma)) \forall \gamma \in A \text{ and } \forall n \in \mathbb{N}, \text{ by definition of } \delta_{\tilde{z}}((F, A)_n).$$

But $\delta_{\tilde{z}}((F, A)_n) \rightarrow \bar{0}$ as $n \rightarrow \infty$, then $\partial(\tilde{x}, \tilde{y}, \tilde{z})(\gamma) = \bar{0}$, which is a contradiction. Thus $\tilde{\cap}_{n=1}^{\infty} (F, A)_n$ contains the of only one $soft_2$ - element.

Conversely. Suppose that for any decreasing sequence $\{(F, A)_n\}$ of non-null $soft_2$ - closed sets such that $\delta_{\tilde{z}}((F, A)_n) \rightarrow \bar{0}$ as $n \rightarrow \infty, \forall \tilde{z} \tilde{\in} \tilde{X}$, then $(F, A) = \tilde{\cap}_{n=1}^{\infty} (F, A)_n$ consists of only one $soft_2$ - element, and $\{\tilde{x}_n\}$ be a Cauchy sequence of $soft_2$ - elements in (\tilde{X}, ∂) . Let $(F, A)_n = \{\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+2}, \dots\}$ for all $n \in \mathbb{N}$. Then $(F, A)_n \supset (F, A)_{n+1}$ and therefore $cl_2(F, A)_n \supset cl_2(F, A)_{n+1}$ for all $n \in \mathbb{N}$. So that $\{cl_2(F, A)_n\}$ is decreasing sequence of $soft_2$ - closed sets. But $\{\tilde{x}_n\}$ is a Cauchy then for a $soft_2$ - element $\tilde{z} \tilde{\in} \tilde{X}$ and $\tilde{\varepsilon} \tilde{>} \bar{0}$ arbitrary, there exists $n_1 \in \mathbb{N}$ such that $\partial(\tilde{x}_m, \tilde{x}_n, \tilde{z}) \tilde{<} \tilde{\varepsilon}$ for all $n, m > n_1$.

This prove that $\delta_{\tilde{z}}((F, A)_{n_1}) \tilde{\leq} \tilde{\varepsilon}$ then by (22) $\delta_{\tilde{z}}(cl_2(F, A)_{n_1}) \tilde{\leq} \tilde{\varepsilon}$. Since $\{(F, A)_n\}$ is decreasing then $\delta_{\tilde{z}}(cl_2(F, A)_n) \tilde{\leq} \delta_{\tilde{z}}(cl_2(F, A)_{n_1}) \tilde{\leq} \tilde{\varepsilon}$ for $n \geq n_1$. Thus $\delta_{\tilde{z}}(cl_2(F, A)_n) \rightarrow \bar{0}$ as $n \rightarrow \infty$.

Therefore, given the condition, $\tilde{\cap}_{n=1}^{\infty} (cl_2(F, A)_n) = \{\tilde{x}_0\}$ (say). Thus for any $soft_2$ - element $\tilde{z} \tilde{\in} \tilde{X}$,

$$\partial(\tilde{x}_n, \tilde{x}_0, \tilde{z}) \tilde{\leq} \delta_{\tilde{z}}(cl_2(F, A)_n) \rightarrow \bar{0} \text{ as } n \rightarrow \infty, \text{ i. e. } \partial(\tilde{x}_n, \tilde{x}_0, \tilde{z}) \rightarrow \bar{0} \text{ as } n \rightarrow \infty.$$

So that $\{\tilde{x}_n\}$ is converge to \tilde{x}_0 in (\tilde{X}, ∂) . Hence (\tilde{X}, ∂) is complete.

Theorem 5.10: Let (\tilde{X}, ∂) be a $soft_2$ - metric space and (F, A) be a non-null $soft_2$ - subsets of \tilde{X} , then for any two distinct $soft_2$ - elements $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$, then $\delta(\tilde{x}, \tilde{y}, (F, A))(\gamma) = \bar{0}$ if and only if at least one of the $soft_2$ - elements \tilde{x}, \tilde{y} belong to $cl_2((F, A))$.

Proof: Let $\delta(\tilde{x}, \tilde{y}, (F, A))(\gamma) = \bar{0}$ and $\tilde{y} \tilde{\notin} cl_2(F, A)$ It is necessary to demonstrate that $\tilde{x} \tilde{\in} cl_2(F, A)$. If $\tilde{x} \tilde{\in} (F, A)$ then $\tilde{x} \tilde{\in} (F, A) \cup (F, A)^\partial = cl_2(F, A)$. Suppose that $\tilde{x} \tilde{\notin} (F, A)$. And since $\delta(\tilde{x}, \tilde{y}, (F, A))(\gamma) = \inf\{\partial(\tilde{x}, \tilde{y}, \tilde{a})(\gamma); \tilde{a} \tilde{\in} (F, A)\} \forall \gamma \in A$ Then for any $\tilde{\varepsilon} \tilde{\geq} \bar{0}$ there is $\tilde{z} \tilde{\in} (F, A)$ such that $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \tilde{<} \tilde{\varepsilon}$, i.e. $\tilde{z} \tilde{\in} B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{y}) \cap (F, A)$. Since $\tilde{x} \tilde{\notin} (F, A)$ then $\tilde{z} \neq \tilde{x}$. Thus \tilde{x} is a $soft_2$ - limit element of (F, A) therefore $\tilde{x} \tilde{\in} (F, A)^\partial$. So that $\tilde{x} \tilde{\in} cl_2((F, A))$.

Conversely. $cl_2((F, A))$ contains at least one of the $soft_2$ - elements \tilde{x}, \tilde{y} , say \tilde{x} . But $cl_2((F, A)) = (F, A) \cup (F, A)^\partial$ then $\tilde{x} \tilde{\in} (F, A)$ or $\tilde{x} \tilde{\in} (F, A)^\partial$. If $\tilde{x} \tilde{\in} (F, A)$ then $\delta(\tilde{x}, \tilde{y}, (F, A))(\gamma) = \bar{0}; \forall \gamma \in A$. If $\tilde{x} \tilde{\in} (F, A)^\partial$ then \tilde{x} is a $soft_2$ - limit element of (F, A) , and since for $\tilde{\varepsilon} \tilde{\geq} \bar{0}$, $B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{y})$ is a $soft_2$ - open set containing \tilde{x} , then there is $soft_2$ - element $\tilde{z} \tilde{\in} (B_{\tilde{\varepsilon}}(\tilde{x}, \tilde{y})) \cap (F, A)$. Then $\partial(\tilde{x}, \tilde{y}, \tilde{z}) \tilde{\leq} \tilde{\varepsilon}$. Thus $\partial(\tilde{x}, \tilde{y}, (F, A)) \tilde{\leq} \tilde{\varepsilon}, \forall \tilde{\varepsilon} \tilde{\geq} \bar{0}$. So that $\delta(\tilde{x}, \tilde{y}, (F, A)) = \bar{0}$.

6. Conclusion

In this article we using the concept of soft-elements in soft-sets and concept of 2-metric spaces for define of $soft_2$ - metric space. We studied the some properties of the $soft_2$ - open sets and $soft_2$ - closed sets. $soft_2$ - metric topology is induced, which is a topology generated in a $soft_2$ - metric spaces with basis the set of all $soft_2$ - open balls. A notion of convergence in such spaces is investigated and reformulated of Cantor's Theorem in the $soft_2$ - metric space setting is furnished. There is a wide scope of further research on $soft_2$ - metric space and also on norm and inner product structures in soft set settings.

References

- [1] S. Gähler, "2-metrische Räume und ihre topologische Struktur". Math. Nachr. 26, 115-118 (1963).

- [2] S. Gahler, “Lineare 2-normierte Räume”, *Math. Nachr.*, 28 (1965), 1-43.
- [3] S. Gahler, “Über die uniformisierbarkeit 2-metrischer Räume”, *Math. Nachr.*, 28 (1965), 235-244.
- [4] K. Iseki, “Fixed point theorems in 2-metric spaces”, *Math. Seminar Notes, Kobe Univ.*, 3 (1975), 133-136.
- [5] B. K. Lahiri, P. Das and L. K. Dey, “Cantor’s theorem in 2-metric spaces and its applications to fixed point problems”. *Taiwanese J Math* 15:337–352 (2011).
- [6] D. Molodtsov, “Soft Set Theory First Results” *Computers and Mathematics with Applications* 37 (1999) 19-31.
- [7] S. Das and S. K. Samanta. “Soft Real Sets, Soft Real Numbers and Their Properties”. *The Journal of Fuzzy Mathematics* Vol. 20 No. 3, 2012.
- [8] P. K. Maji, R. Biswas, and A. R. Roy. “Soft Set Theory “Computers and Mathematics with Applications 45 (2003) 555-562.
- [9] P. Majumdar, S. K. Samanta “Similarity measure of soft sets” *New Mathematics and Natural Computation* · Vol. 4, No. 1 (2008).
- [10] D. Pei, D. Miao. “From Soft Sets to Information Systems” *Granular Computing, 2005, IEEE International Conference, Volume - 2.* 617 – 622.
- [11] P. Zhu and Q. Wen, “Operation of Soft Sets Revisited” *Hindawi Publishing Corporation Journal Applied Mathematics.* (2013).
- [12] A. Sezgin and A. O. Atagun, “On operations of soft sets,” *Computers & Mathematics with Applications*, vol. 61, no. 5, pp. 1457–1467, (2011).
- [13] S. Das and S. K. Samanta, “One soft metric space”. *The Journal of Fuzzy Mathematics.* Vol. 21, No. 3 (2013).