On Certain Algebraic Properties of Symbolic 3-Plithogenic Real Square Matrices

P. Prabakaran[1], **Bilal Abdallah**[2,*], **Turayeva Dinara Tulkunovna**[3]

[1] Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam - 638401, Tamil Nadu, India
[2] College of Business Administration, American University of the Middle East,54200, Egaila, Kuwait
[3] Tashkent State University of Economics, Uzbekistan

Emails: prabakaranpvkr@gmail.com; Bilal.Abdallah@aum.edu.kw; d.turaeva@tsue.uz

Abstract

The main objective of this article is to study the inverse of invertible symbolic 3-plithogenic real square matrices using the concept of adjoints and characteristic polynomials. Also, the symbolic 3-plithogenic version of Cayley-Hamilton theorem was proved and provided enough examples to enhance understanding.

Keywords: Symbolic 3-plithogenic matrix; symbolic 3-plithogenic adjoint; symbolic 3-plithogenic determinant; symbolic 3-plithogenic inverse.

1 Introduction

The concept of refined neutrosophic structure was studied by many authors in.[1–4] Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation.[19]

In,[14] the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings are studied. Further, Taffach[21,22] studied the concepts of symbolic 2-plithogenic vector spaces and modules. In,[7] the concept of symbolic 2-plithogenic matrices, determinants, eigen values and vectors, exponents, and diagonalization are discussed.

Laterally, many authors defined and studied symbolic 3-plithogenic algebraic structures, such as symbolic 3-plithogenic rings, vectors spaces and modules.[8–10] Recently, Merkepci[16] introduced and studied symbolic 3-plithogenic and 4-plithogenic square matrices and its algebraic properties such as determinant, invertibility, Eigen values, diagonalization, etc.

As a continuation of the previous study of symbolic 3-plithogenic matrices, this work discusses the symbolic 3-plithogenic adjoint, where the inverse of symbolic 3-plithogenic matrices will be defined in terms of the symbolic 3-plithogenic adjoint and determinant. We present the symbolic 3-plithogenic characteristic polynomials and the symbolic 3-plithogenic version of the Cayley-Hamilton theorem. Also, we illustrate many examples to clarify the validity of our work.

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2 Preliminaries

Definition 2.1. [16] The ring of symbolic 3-plithogenic real numbers is defined as follows:

$$3-SP_R=\left\{x+yP_1+zP_2+tP_3; P_i^2=P_i, P_i\times P_j=P_{max(i,j)}\right\}$$

Addition on $3-SP_R$ is defined as follows:
$[x_0+x_1P_1+x_2P_2+x_3P_3]+[y_0+y_1P_1+y_2P_2+y_3P_3]=(x_0+y_0)+(x_1+y_1)P_1+(x_2+y_2)P_2+(x_3+y_3)P_3$.

Multiplication on $3-SP_R$ is defined as follows:
$[x_0+x_1P_1+x_2P_2+x_3P_3]\cdot[y_0+y_1P_1+y_2P_2+y_3P_3]=(x_0y_0)+(x_0y_1+x_1y_0+x_1y_1)P_1+(x_0y_2+x_2y_1+x_2y_2+x_2y_0+x_1y_2)P_2+(x_0y_3+x_1y_3+x_2y_3+x_3y_3+x_3y_0+x_3y_1+x_3y_2)P_3$.

Remark 2.2. [16] If we let $X=x_0+x_1P_1x_2P_2+x_3P_3\in 3-SP_R$, we have the following:
X is invertible if and only if $x_0\neq 0, x_0+x_1\neq 0, x_0+x_1+x_2\neq 0, x_0+x_1+x_2+x_3\neq 0$, and

$$X^{-1}=\frac{1}{X}=\frac{1}{x_0}+\left[\frac{1}{x_0+x_1}-\frac{1}{x_0}\right]P_1+\left[\frac{1}{x_0+x_1+x_2}-\frac{1}{x_0+x_1}\right]P_2+\left[\frac{1}{x_0+x_1+x_2+x_3}-\frac{1}{x_0+x_1+x_2}\right]P_3.$$

Definition 2.3. [16] A square symbolic 3-plithogenic matrix is a matrix with symbolic 3-plithogenic entries.

Remark 2.4. [16] Any symbolic 3-plithogenic matrix can be written as $A=A_0+A_1P_1+A_2P_2+A_3P_3$. We denote the set of all symbolic 3-plithogenic n-square matrices by $3-SP_M$.

Theorem 2.5. [16] *Let $S=S_0+S_1+S_2P_2+S_3P_3\in 3-SP_M$, then the following is true:*

1. *S is invertible if and only if $S_0, S_0+S_1, S_0+S_1+S_2, S_0+S_1+S_2+S_3$ are invertible.*
2. $S^{-1}=S_0^{-1}+\left[(S_0+S_1)^{-1}-S_0^{-1}\right]P_1+\left[(S_0+S_1+S_2)^{-1}-(S_0+S_1)^{-1}\right]P_2$
   $+\left[(S_0+S_1+S_2+S_3)^{-1}-(S_0+S_1+S_2)^{-1}\right]P_3$
3. *For $n\in N$,* $S^n=S_0^n+[(S_0+S_1)^n-S_0^n]P_1+[(S_0+S_1+S_2)^n-(S_0+S_1)^n]P_2$
   $+[(S_0+S_1+S_2+S_3)^n-(S_0+S_1+S_2)^n]P_3$

Definition 2.6. [16] Let $S=S_0+S_1P_1+S_2P_2+S_3P_3\in 3-SP_M$, then we define the following:

$$\begin{aligned}det(S)=det(S_0)+[det(S_0+S_1)-det(S_0)]P_1+[det(S_0+S_1+S_2)-det(S_0+S_1)]P_2\\+[det(S_0+S_1+S_2+S_3)-det(S_0+S_1+S_2)]P_3\end{aligned}$$

Theorem 2.7. [16] *S is invertible if and only if $det(S)$ is invertible in $3-SP_R$.*

3 Adjoint of Symbolic 3-Plithogenic Square Matrices

We begin this section with the following definition.

Definition 3.1. Let $S=S_0+S_1+S_2P_2+S_3P_3$ be a symbolic 3-plithogenic square matrix with real entries. The adjoint matrix of S is defined as

$$\begin{aligned}adjS=adjS_0+[adj(S_0+S_1)-adjS_0]P_1+[adj(S_0+S_1+S_2)-adj(S_0+S_1)]P_2\\+[adj(S_0+S_1+S_2+S_3)-adj(S_0+S_1+S_2)]P_3.\end{aligned}$$

Example 3.2. Consider the following symbolic 3-plithogenic 3×3 matrix:

$$S=\begin{pmatrix}-3+P_1-P_2+P_3 & 1+P_1 & 5\\ -P_1+P_2 & 3P_1+2P_3 & 4P_2-P_3\\ -1+2P_1-P_2 & 5+2P_2 & 7+P_1+10P_2+P_3\end{pmatrix}$$

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Here,

$$S_0 = \begin{pmatrix} -3 & 1 & 5 \\ 0 & 0 & 0 \\ -1 & 5 & 7 \end{pmatrix}, \quad S_0 + S_1 = \begin{pmatrix} -2 & 2 & 5 \\ -1 & 3 & 0 \\ 1 & 5 & 8 \end{pmatrix} \quad S_0 + S_1 + S_2 = \begin{pmatrix} -3 & 2 & 5 \\ 0 & 3 & 4 \\ 0 & 7 & 18 \end{pmatrix}, \text{ and}$$

$$S_0 + S_1 + S_2 + S_3 = \begin{pmatrix} -2 & 2 & 5 \\ 0 & 5 & 3 \\ 0 & 7 & 19 \end{pmatrix}$$

Then,

$$adjS_0 = \begin{pmatrix} 0 & 18 & 0 \\ 0 & -16 & 0 \\ 0 & 14 & 0 \end{pmatrix}, \quad adj(S_0 + S_1) = \begin{pmatrix} 24 & 9 & -15 \\ 8 & -11 & -5 \\ -2 & 8 & -4 \end{pmatrix}$$

$$adj(S_0 + S_1 + S_2) = \begin{pmatrix} 26 & -1 & -7 \\ 0 & -54 & 12 \\ 0 & 21 & -9 \end{pmatrix} \text{ and } adj(S_0 + S_1 + S_2 + S_3) = \begin{pmatrix} 74 & -3 & -19 \\ 0 & -38 & 6 \\ 0 & 14 & -10 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} adjS &= adjS_0 + [adj(S_0 + S_1) - adjS_0]P_1 + [adj(S_0 + S_1 + S_2) - adj(S_0 + S_1)]P_2 \\ &\qquad + [adj(S_0 + S_1 + S_2 + S_3) - adj(S_0 + S_1 + S_2)]P_3 \\ &= \begin{pmatrix} 0 & 18 & 0 \\ 0 & -16 & 0 \\ 0 & 14 & 0 \end{pmatrix} + \begin{pmatrix} 24 & -9 & -15 \\ 8 & -5 & -5 \\ -2 & -6 & -4 \end{pmatrix} P_1 + \begin{pmatrix} 2 & -10 & 8 \\ -8 & -43 & 17 \\ 2 & 13 & -5 \end{pmatrix} P_2 + \begin{pmatrix} 48 & -2 & -12 \\ 0 & 16 & -6 \\ 0 & -7 & -1 \end{pmatrix} P_3 \\ &= \begin{pmatrix} 24P_1 + 2P_2 + 48P_3 & 18 - 9P_1 - 10P_2 - 2P_3 & -15P_1 + 8P_2 - 12P_3 \\ 8P_1 - 8P_2 & -16 + 5P_1 + 43P_2 + 16P_3 & -5P_1 + 17P_2 - 6P_3 \\ -2P_1 + 2P_2 & 14 - 6P_1 + 13P_2 - 7P_3 & -4P_1 - 5P_2 - P_3 \end{pmatrix} \end{aligned}$$

Using the definition of adjoint of symbolic 3-plithogenic matrix we can modify the Theorem 2.5 as follows:

Theorem 3.3. *Let $S = S_0 + S_1P_1 + S_2P_2 + S_2P_3$ be a symbolic 3-plithogenic square matrix, then S is invertible if and only if $det(S)$ is invertible and*

$$S^{-1} = \frac{1}{detS}(adjS).$$

Proof. By Theorem 2.7, S is invertible if and only if $det(S)$ is invertible.

Also,

$$\begin{aligned} \frac{1}{detS}(adjS) &= \frac{adjS_0}{detS_0} + \left[\frac{adj(S_0 + S_1)}{det(S_0 + S_1)} - \frac{adjS_0}{detS_0}\right] P_1 + \left[\frac{adj(S_0 + S_1 + S_2)}{det(S_0 + S_1 + S_2)} - \frac{adj(S_0 + S_1)}{det(S_0 + S_1)}\right] P_2 \\ &\qquad + \left[\frac{adj(S_0 + S_1 + S_2 + S_3)}{det(S_0 + S_1 + S_2 + S_3)} - \frac{adj(S_0 + S_1 + S_2)}{det(S_0 + S_1 + S_2)}\right] P_3 \\ &= S_0^{-1} + \left[(S_0 + S_1)^{-1} - S_0^{-1}\right] P_1 + \left[(S_0 + S_1 + S_2)^{-1} - (S_0 + S_1)^{-1}\right] P_2 \\ &\qquad + \left[(S_0 + S_1 + S_2 + S_3)^{-1} - (S_0 + S_1 + S_2)^{-1}\right] P_3 \\ &= S^{-1} \end{aligned}$$

Hence the result holds by Theorem 2.5. □

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Example 3.4. Consider the symbolic 3-plithogenic 2×2 matrix

$$S = \begin{pmatrix} 1 + P_1 + P_2 + P_3 & 5 + P_1 - 2P_2 \\ 1 + 4P_1 - P_2 + 3P_3 & P_1 + P_3 \end{pmatrix}$$

Here, $detS = -5 - 23P_1 + 15P_2 - 7P_3$, and $adjS = \begin{pmatrix} P_1 + P_3 & -5 - P_1 + 2P_2 \\ -1 - 4P_1 + P_2 - 3P_3 & 1 + P_1 + P_2 + P_3 \end{pmatrix}$.

Hence,

$$\begin{aligned} S^{-1} &= \frac{1}{detS}(adjS) \\ &= \frac{1}{-5 - 23P_1 + 15P_2 - 7P_3} \begin{pmatrix} P_1 + P_3 & -5 - P_1 + 2P_2 \\ -1 - 4P_1 + P_2 - 3P_3 & 1 + P_1 + P_2 + P_3 \end{pmatrix} \\ &= \left(\frac{-1}{5} + \frac{23}{140}P_1 - \frac{15}{364}P_2 + \frac{7}{260}P_3 \right) \begin{pmatrix} P_1 + P_3 & -5 - P_1 + 2P_2 \\ -1 - 4P_1 + P_2 - 3P_3 & 1 + P_1 + P_2 + P_3 \end{pmatrix} \\ &= \begin{pmatrix} \frac{-1}{28}P_1 - \frac{15}{364}P_2 - \frac{6}{260}P_3 & 1 - \frac{22}{28}P_1 + \frac{34}{364}P_2 - \frac{28}{260}P_3 \\ \frac{1}{5} - \frac{3}{28}P_1 + \frac{47}{364}P_2 + \frac{11}{260} & \frac{-1}{5} + \frac{18}{28} + \frac{58}{364}P_2 + \frac{8}{260}P_3 \end{pmatrix} \end{aligned}$$

Remark 3.5. If X is a invertible symbolic 3-plithogenic square matrix and X^{-1} is its inverse, then $adjX = detX \cdot X^{-1}$.

Theorem 3.6. *Let $X = A + BP_1 + CP_2 + DP_3$ and $Y = M + NP_1 + SP_2 + RP_3$ be two symbolic 3-plithogenic invertible square matrices. Then XY is also invertible and $(XY)^{-1} = Y^{-1}X^{-1}$.*

Proof. By Theorem 3.3, if X is invertible then

$$det(A) \neq 0, det(A + B) \neq 0, det(A + B + C) \neq 0 \text{ and } det(A + B + C + D) \neq 0.$$

Similarly, if Y is invertible then

$$detM \neq 0\ , det(M + N) \neq 0, det(M + N + S) \neq 0 \text{ and } det(M + N + S + R) \neq 0.$$

This implies that,

$$\begin{aligned} det(AM) &= detA\ detM \neq 0 \\ det[(A + B)(M + N)] &= det(A + B)\ det(M + N) \neq 0 \\ det[(A + B + C)(M + N + S)] &= det(A + B + C)\ det(M + N + S) \neq 0 \\ det[(A + B + C + D)(M + N + S + R)] &= det(A + B + C + D)\ det(M + N + S + R) \neq 0 \end{aligned}$$

.

Now,

$$\begin{aligned} det(XY) = {} & det(AM) + [det((A + B)(M + N))]P_1 + [det((A + B + C)(M + N + S))]P_2 \\ & + [det((A + B + C + D)(M + N + S + R))]P_3 \neq 0 \end{aligned}$$

and hence XY is invertible. Also by associativity of matrix multiplication, we have

$$\begin{aligned} (XY)(Y^{-1}X^{-1}) = X(YY^{-1})X^{-1} = XX^{-1} = U_{n \times n} \\ (Y^{-1}X^{-1})(XY) = Y^{-1}(X^{-1}X)Y = Y^{-1}Y = U_{n \times n}. \end{aligned}$$

Thus, $(MN)^{-1} = N^{-1}M^{-1}$. □

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Theorem 3.7. *Let X and Y be two $m \times m$ symbolic 3-plithogenic invertible matrices. Then the following properties holds.*

(1) $det(adjX) = (detX)^{m-1}$.

(2) $adj(XY) = adjX\ adjY$.

(3) $adj(X^k) = (adjX)^k$ *for any positive integer* k.

(4) $adj(X^T) = (adjX)^T$.

(5) $adj(adjX) = (detX)^{m-2}X$

Proof. We can prove this results based on the properties adjoint of classical matrices. □

4 Characteristic Polynomial of Symbolic 3-Plithogenic Square Matrices

We begin this section with the following definition.

Definition 4.1. Let $S = S_0 + S_1P_1 + S_2P_2 + S_3P_3$ be a symbolic 3-plithogenic $n \times n$ square matrix with real entries and $\lambda = \lambda_0 + \lambda_1P_1 + \lambda_2P_2 + \lambda_3P_3$. The characteristic polynomial of S is defined as

$$\phi(\lambda) = \alpha(\lambda) + [\beta(\lambda) - \alpha(\lambda)]\,P_1 + [\gamma(\lambda) - \beta(\lambda)]\,P_2 + [\delta(\lambda) - \gamma(\lambda)]\,P_3$$

where,

$$\begin{aligned}
\alpha(\lambda) &= det(S_0 - \lambda U_{n\times n}) \\
\beta(\lambda) &= det(S_0 + S_1 - \lambda U_{n\times n}) \\
\gamma(\lambda) &= det(S_0 + S_1 + S_2 - \lambda U_{n\times n}) \\
\delta(\lambda) &= det(S_0 + S_1 + S_2 + S_3 - \lambda U_{n\times n}).
\end{aligned}$$

Example 4.2. Consider the following symbolic 3-plithogenic 2×2 matrix:

$$S = \begin{pmatrix} 2 + P_1 + 3P_2 - P_3 & 1 - P_1 - P_2 \\ 3 + 4P_1 + P_3 & 1 + P_2 \end{pmatrix}$$

with

$$S_0 = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix},\ \ S_0 + S_1 = \begin{pmatrix} 3 & 0 \\ 7 & 1 \end{pmatrix},\ \ S_0 + S_1 + S_2 = \begin{pmatrix} 6 & -1 \\ 7 & 2 \end{pmatrix} \text{ and } S_0 + S_1 + S_2 + S_3 = \begin{pmatrix} 5 & -1 \\ 8 & 2 \end{pmatrix}.$$

Here,

$$\begin{aligned}
\alpha(\lambda) &= det(S_0 - \lambda U_{n\times n}) = \begin{pmatrix} 2-\lambda & 1 \\ 3 & 1-\lambda \end{pmatrix} = \lambda^2 - 3\lambda - 1. \\
\beta(\lambda) &= det(S_0 + S_1 - \lambda U_{n\times n}) = \begin{pmatrix} 3-\lambda & 0 \\ 7 & 1-\lambda \end{pmatrix} = \lambda^2 - 4\lambda + 3. \\
\gamma(\lambda) &= det(S_0 + S_1 + S_2 - \lambda U_{n\times n}) = \begin{pmatrix} 6-\lambda & -1 \\ 7 & 2-\lambda \end{pmatrix} = \lambda^2 - 8\lambda + 19. \\
\delta(\lambda) &= det(S_0 + S_1 + S_2 + S_3 - \lambda U_{n\times n}) = \begin{pmatrix} 5-\lambda & -1 \\ 8 & 2-\lambda \end{pmatrix} = \lambda^2 - 7\lambda + 18.
\end{aligned}$$

Hence the characteristic polynomial of S is

$$\begin{aligned}
\phi(\lambda) &= \lambda^2 - 3\lambda - 1 + [(\lambda^2 - 4\lambda + 3) - (\lambda^2 - 3\lambda - 1)]P_1 + [(\lambda^2 - 8\lambda + 19) - (\lambda^2 - 4\lambda + 3)]P_2 \\
&\qquad + [(\lambda^2 - 7\lambda + 18) - (\lambda^2 - 8\lambda + 19)]P_3 \\
&= \lambda^2 - 3\lambda - 1 + (-\lambda + 4)P_1 + (-4\lambda + 16)P_2 + (\lambda - 1)P_3.
\end{aligned}$$

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Theorem 4.3 (Symbolic 3-plithogenic Cayely-Hamilton Theorem). *Every symbolic 3-plithogenic square matrix satisfies its characteristic polynomial.*

Proof. We can prove this result based on the Cayely-Hamilton theorem for classical matrices. □

Example 4.4. Consider the symbolic 3-plithogenic 2×2 matrix given in Example 4.2

$$S = \begin{pmatrix} 2+P_1+3P_2-P_3 & 1-P_1-P_2 \\ 3+4P_1+P_3 & 1+P_2 \end{pmatrix}$$

The characteristic polynomial of S is $\lambda^2 - 3\lambda - 1 + (-\lambda+4)P_1 + (-4\lambda+16)P_2 + (\lambda-1)P_3$. This implies that,

$$\begin{aligned} \phi(S) &= (S^2-3S-1)+(-S+4)P_1+(-4S+16)P_2+(S-1)P_3. \\ &= \begin{pmatrix} -P_1+11P_2-9P_3 & -5P_2+P_3 \\ 7P_1+28P_2-3P_3 & 1-3P_1-7P_2-P_3 \end{pmatrix} + \begin{pmatrix} P_1-3P_2+P_3 & P_2 \\ -7P_1-P_3 & 3P_1-P_3 \end{pmatrix} \\ &\quad + \begin{pmatrix} -8P_2+4P_3 & 4P_2 \\ -28P_2-4P_3 & 8P_2 \end{pmatrix} + \begin{pmatrix} 4P_3 & -P_3 \\ 8P_3 & P_3 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

Hence, $\phi(S) = 0$.

Remark 4.5. If S is a invertible symbolic 3-plithogenic matrix, then using Cayely-Hamilton theorem we can compute the inverse of S. See the following example.

Example 4.6. Consider the symbolic 3-plithogenic 2×2 matrix

$$S = \begin{pmatrix} 1+P_1+P_2-P_3 & -1+P_1 \\ 1-P_2 & 1+2P_3 \end{pmatrix}$$

with

$$S_0 = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, S_0+S_1 = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}, \ S_0+S_1+S_2 = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } S_0+S_1+S_2+S_3 = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}.$$

Here,

$$\begin{aligned} \alpha(\lambda) &= det(S_0 - \lambda U_{n\times n}) = \begin{pmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{pmatrix} = \lambda^2 - 2\lambda + 2. \\ \beta(\lambda) &= det(S_0 + S_1 - \lambda U_{n\times n}) = \begin{pmatrix} 2-\lambda & 0 \\ 1 & 1-\lambda \end{pmatrix} = \lambda^2 - 3\lambda + 2. \\ \gamma(\lambda) &= det(S_0 + S_1 + S_2 - \lambda U_{n\times n}) = \begin{pmatrix} 3-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix} = \lambda^2 - 4\lambda + 3. \\ \delta(\lambda) &= det(S_0 + S_1 + S_2 + S_3 - \lambda U_{n\times n}) = \begin{pmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{pmatrix} = \lambda^2 - 5\lambda + 6. \end{aligned}$$

Hence the characteristic polynomial of S is

$$\begin{aligned} \phi(\lambda) &= \alpha(\lambda) + [\beta(\lambda) - \alpha(\lambda)]\, P_1 + [\gamma(\lambda) - \beta(\lambda)]\, P_2 \\ &= (\lambda^2 - 2\lambda + 2 - \lambda) + (-\lambda)P_1 + (-\lambda+1)P_2 + (-\lambda+3)P_3. \end{aligned}$$

Now, by Cayely-Hamilton theorem we have $\phi(\lambda) = 0$, we have,

$$\begin{aligned} & (S^2 - 2S + 2)S + (-S)P_1 + (-S+1)P_2 + (-S+3)P_3 = 0 \\ \Rightarrow \quad & 2 + P_2 + 3P_3 = -S^2 + 2S + SP_1 + SP_2 + SP_3 \\ \Rightarrow \quad & (2 + P_2 + 3P_3)SS^{-1} = -S^2 + 2S + SP_1 + SP_2 + SP_3. \end{aligned}$$

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This implies that,

$$\begin{aligned} S^{-1} &= \frac{1}{2+P_2+3P_3}\left[-S+(2+P_1+P_2+P_3)U_{n\times n}\right] \\ &= \frac{1}{2+P_2+3P_3}\begin{pmatrix} 1+2P_3 & 1-P_1 \\ -1+P_2 & 1+P_1+P_2-P_3 \end{pmatrix} \\ &= \left(\frac{1}{2}-\frac{1}{6}P_2-\frac{1}{6}P_3\right)\begin{pmatrix} 1+2P_3 & 1-P_1 \\ -1+P_2 & 1+P_1+P_2-P_3 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}-\frac{1}{6}P_2+\frac{1}{6}P_3 & \frac{1}{2}-\frac{1}{2}P_1 \\ -\frac{1}{2}+\frac{1}{2}P_2 & \frac{1}{2}+\frac{1}{2}P1-\frac{2}{3}P_3 \end{pmatrix} \end{aligned}$$

5 Conclusion

In this work, the adjoint of symbolic 3-plithogenic square matrices was defined and the inverse of invertible symbolic 3-plithogenic square matrices was studied in terms of symbolic 3-plithogenic adjoint and symbolic 3-plithogenic determinant. Also, we have presented the concept of the characteristic polynomial of symbolic 3-plithogenic matrices and we have proved the symbolic 3-plithogenic version of of Cayley-Hamilton theorem with many examples that clarify the validity of this work.

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References

[1] Adeleke, E. O., Agboola, A. A. A., and Smarandache, F., "Refined neutrosophic rings II", International Journal of Neutrosophic Science, vol. 2, pp. 89-94, 2020.

[2] Abobala, M., "On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations", Journal Of Mathematics, Hindawi, 2021.

[3] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021.

[4] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., "The Algebraic Creativity In The Neutrosophic Square Matrices", Neutrosophic Sets and Systems, Vol. 40, pp.1-11, 2021.

[5] Abobala, M., "On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations", Journal of Mathematics, Hindawi, 2021.

[6] Abobala, M., "A Study of Nil Ideals and Kothe's Conjecture in Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, Hindawi, 2021.

[7] Abuobida Mohammed A. Alfahal, Yaser Ahmad Alhasan, Raja Abdullah Abdulfatah, Arif Mehmood, Mustafa Talal Kadhim, "On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties", International Journal of Neutrosophic Science, Vol. 21, PP. 96-104, 2023.

[8] Albasheer, O., Hajjari., A., and Dalla., R., "On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.

[9] Ali, R., Hasan, Z. "An Introduction to the Symbolic 3-Plithogenic Vector Spaces", Galoitica: Journal of Mathematical Structures and Applications, 2023, 6, 8-12.

[10] Ali, R.; Hasan, Z., "An Introduction to the Symbolic 3-Plithogenic Modules", Galoitica: Journal of Mathematical Structures and Applications, 2023, Vol 6, 13-17.

[11] Kandasamy, W. B. V., and Smarandache, F., "Some Neutrosophic Algebraic Structures and Neutrosophic n-Algebraic Structures"', (Arizona: Hexis Phoenix), 2006.

https://doi.org/10.54216/IJNS.230427
Received: June 17, 2023 Revised: January 15, 2024 Accepted: February 27, 2024

[12] Khaldi, A., Ben Othman, K., Von Shtawzen, O., Ali, R., and Mosa, S., "On Some Algorithms for Solving Different Types of Symbolic 2-Plithogenic Algebraic Equations", Neutrosophic Sets and Systems, Vol 54, 2023.

[13] Malath F. Alaswad, "On the Neutrosophic Characteristic Polynomials and Neutrosophic Cayley-Hamilton Theorem", International Journal of Neutrosophic Science, Vol. 18, 2022, PP. 59-71.

[14] Merkepci, H., and Abobala, M., "On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[15] Merkepci, H., and Rawashdeh, A., "On The Symbolic 2-Plithogenic Number Theory and Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.

[16] Merkepci, H., "On Novel Results about the Algebraic Properties of Symbolic 3-Plithogenic and 4-Plithogenic Real Square Matrices", Symmetry, 2023.

[17] Othman, B.K., "On Some Algorithms for Solving Symbolic 3-Plithogenic Equations", Neoma Journal of Mathematics Computer Science 2023

[18] Rawashdeh, A, "An Introduction to the Symbolic 3-plithogenic Number Theory", Neoma Journal of Mathematics Computer Science, 2023.

[19] Smarandache, F., "Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.

[20] Smarandache, F., "Introduction to Neutrosophic Statistics", USA: Sitech and Education Publishing, 2014.

[21] Taffach, N., "An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", Neutrosophic Sets and Systems, Vol 54, 2023.

[22] Taffach, N., and Ben Othman, K., "An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.

https://doi.org/10.54216/IJNS.230427

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