



On Some Symbolic 2-Plithogenic and 3-Plithogenic Real Series

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Abstract

The objective of this paper is to study the expansion of symbolic 2-plithogenic and symbolic 3-plithogenic real functions in one variable with real series, where many famous expansions will be presented according to the Taylor series applied for symbolic plithogenic functions defined over symbolic plithogenic rings. Also, we provide many related examples to clarify and to explain the expansion method and properties.

Keywords: Taylor series; real function; symbolic 2-plithogenic function; symbolic 3-plithogenic function.

1. Introduction and preliminaries:

The concept of symbolic n-plithogenic set was proposed in [3], as a novel generalization of fuzzy sets with many interesting properties that are very similar to neutrosophic sets [4, 11-12]. These sets were used widely to study the algebraic generalized structures generated from them such as symbolic n-plithogenic rings [1,5,8,9], matrices [16-19], modules, and spaces [7].

In [21], symbolic 2-plithogenic and 3-plithogenic real functions were defined and formulated by using a general kind of AH-isometry, with many interesting applications through many different scientific fields [3, 14-15, 22-26]. Also, many generalizations of symbolic n-plithogenic numbers can be found in [27].

In this work, we use the formulas of some symbolic 2-plithogenic and 3-plithogenic real functions in one variable [21], and we present a method to expand those functions by symbolic w-plithogenic and 3-plithogenic version of Taylor's series. Also, we explain the efficiency of the novel expansion by many other examples applied to symbolic 3-plithogenic functions in one variable, and symbolic 2-plithogenic functions in one variable.

First, we recall some basic concepts and definitions:

Definition [1]

The symbolic 2-plithogenic ring of real numbers is defined as follows:

$$2 - SP_R = \{t_0 + t_1P_1 + t_2P_2; t_i \in R, P_1 \times P_2 = P_2 \times P_1 = P_2, P_1^2 = P_2^2 = P_2\}$$

The addition operation on $2 - SP_R$ is defined as follows:

$$(t_0 + t_1P_1 + t_2P_2) + (t'_0 + t'_1P_1 + t'_2P_2) = (t_0 + t'_0) + (t_1 + t'_1)P_1 + (t_2 + t'_2)P_2$$

The multiplication on $2 - SP_R$ is defined as follows:

$$(t_0 + t_1P_1 + t_2P_2)(t'_0 + t'_1P_1 + t'_2P_2) = t_0t'_0 + (t_0t'_1 + t_1t'_0 + t_1t'_1)P_1 + (t_0t'_2 + t_1t'_2 + t_2t'_2 + t_2t'_0 + t_2t'_1)P_2$$

Remark.

If $T = t_0 + t_1P_1 + t_2P_2 \in 2 - SP_R$, then:

$$T^{-1} = \frac{1}{T} = \frac{1}{t_0} + \left[\frac{1}{t_0+t_1} - \frac{1}{t_0} \right] P_1 + \left[\frac{1}{t_0+t_1+t_2} - \frac{1}{t_0+t_1} \right] P_2, \text{ with } t_0 \neq 0, t_0 + t_1 \neq 0, t_0 + t_1 + t_2 \neq 0.$$

Definition. [21]

Let $2 - SP_R = \{a + bP_1 + cP_2; a, b, c \in R\}$ be the 2-plithogenic field of real numbers, a function $f = f(X): 2 - SP_R \rightarrow 2 - SP_R$ is called one variable symbolic 2-plithogenic real function, with

$$X = x_0 + x_1P_1 + x_2P_2 \in 2 - SP_R.$$

Definition [21]

Let $2 - SP_R$ be the symbolic 2-plithogenic field of reals, we define its AH-isometry as follows:

$I: 2 - SP_R \rightarrow R \times R \times R$ such that:

$$I(x + yP_1 + zP_2) = (x, x + y, x + y + z).$$

It is easy to see that I is a ring isomorphism with the inverse:

$I^{-1}: R \times R \times R \rightarrow 2 - SP_R$ such that:

$$I^{-1}(x, y, z) = x + (y - x)P_1 + (z - y)P_2$$

Definition [21]

Let $f: 2 - SP_R \rightarrow 2 - SP_R$ be a symbolic 2-plithogenic real function with one variable, we define the canonical formula as follows:

$$I^{-1} \circ I(f): 2 - SP_R \rightarrow 2 - SP_R$$

Definition [21]

Let $3 - SP_R = \{a + bP_1 + cP_2 + dP_3; a, b, c, d \in R\}$ be the 3-plithogenic field of real numbers, a function $f = f(X): 3 - SP_R \rightarrow 3 - SP_R$ is called one variable symbolic 3-plithogenic real function, with

$$X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in 3 - SP_R.$$

Definition. [21]

Let $3 - SP_R$ be the symbolic 3-plithogenic field of reals, we define its AH-isometry as follows:

$I: 3 - SP_R \rightarrow R \times R \times R \times R$ such that:

$$I(x + yP_1 + zP_2 + tP_3) = (x, x + y, x + y + z, x + y + z + t).$$

It is easy to see that I is a ring isomorphism with the inverse:

$I^{-1}: R \times R \times R \times R \rightarrow 3 - SP_R$ such that:

$$I^{-1}(x, y, z, t) = x + (y - x)P_1 + (z - y)P_2 + (t - z)P_3$$

Definition [21]

Let $f: 3 - SP_R \rightarrow 3 - SP_R$ be a symbolic 3-plithogenic real function with one variable, we define the canonical formula as follows:

$$I^{-1} \circ I(f): 3 - SP_R \rightarrow 3 - SP_R$$

Example.

Consider $f(X) = X^2 - P_1 + P_3$, its canonical formula is:

$$\begin{aligned} I(f(X)) &= [I(X)]^2 + I(-P_1 + P_3) = (x_0, x_0 + x_1, x_0 + x_1 + x_2, x_0 + x_1 + x_2 + x_3)^2 + (0, -1, -1, 0) \\ &= (x_0^2, (x_0 + x_1)^2 - 1, (x_0 + x_1 + x_2)^2 - 1, (x_0 + x_1 + x_2 + x_3)^2) \end{aligned}$$

$$I^{-1} \circ I(f(X)) =$$

$$\begin{aligned} &x_0^2 + P_1[(x_0 + x_1)^2 - x_0^2 - 1] + P_2[(x_0 + x_1 + x_2)^2 - (x_0 + x_1)^2] \\ &+ P_3[(x_0 + x_1 + x_2 + x_3)^2 - (x_0 + x_1 + x_2)^2 + 1] \end{aligned}$$

For example:

$$f(1 + P_3) = (1 + P_3)^2 - P_1 + P_3 = 1 - P_1 + 4P_3.$$

If we put values $x_0 = 1, x_1 = 0, x_2 = 0, x_3 = 1$

in the canonical formula, then we get:

$$I^{-1} \circ I(f(X)) = (1)^2 + P_1[(1)^2 - (1)^2 - 1] + P_2[(1)^2 - (1)^2] + P_3[(2)^2 - (1)^2 + 1] = 1 - P_1 + 4P_3.$$

Main discussion**Definition:**

Let $2 - SP_{\mathbb{R}} = \{x + yp_1 + zp_2; x, y, z \in \mathbb{R}\}$ be the ring of symbolic 2-plithogenic real numbers, Consider the following sequence:

$U_n = x_n + y_nP_1 + z_nP_2$; x_n, y_n, z_n are three real sequences, we define:

$$\sum_{n=1}^j U_n = \sum_{n=1}^j x_n + \sum_{n=1}^j y_nP_1 + \sum_{n=1}^j z_nP_2. \quad (1)$$

It is called a finite series.

If $j = \infty$, then it is called an infinite series.

Example:

Consider: $U_n = \frac{1}{n} + \frac{1}{n^2}P_1 + \frac{3}{n}P_2$, then:

$$\sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} \frac{1}{n} + \sum_{n=1}^{\infty} \frac{1}{n^2}P_1 + \sum_{n=1}^{\infty} \frac{3}{n}P_2$$

Definition:

We say that $\sum_{n=1}^{\infty} U_n$ is convergent if and only if:

$\sum x_n, \sum y_n, \sum z_n$ are convergent.

Otherwise, it is called divergent.

Remark:

1] if $\sum x_n = a, \sum y_n = b, \sum z_n = c$, then $\sum U_n = a + bP_1 + cP_2$.

2] if $\sum U_n$ is divergent, we say that $\sum_{n=1}^{\infty} U_n = \infty$

Example:

Consider the function: $f: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$f(x_0 + x_1P_1 + x_2P_2) = e^{x_0} + e^{x_1}P_1 + e^{x_2}P_2$, then:

$$\begin{cases} e^{x_0} = \sum_{n=0}^{\infty} \frac{(x_0)^n}{n!} \\ e^{x_1} = \sum_{n=0}^{\infty} \frac{(x_1)^n}{n!} \\ e^{x_2} = \sum_{n=0}^{\infty} \frac{(x_2)^n}{n!} \end{cases}$$

So that:

$$f(X) = \sum_{n=0}^{\infty} \frac{x_0^n}{n!} + \sum_{n=0}^{\infty} \frac{x_1^n}{n!} P_1 + \sum_{n=0}^{\infty} \frac{x_2^n}{n!} P_2 = \sum_{n=0}^{\infty} \frac{x_0^n + x_1^n P_1 + x_2^n P_2}{n!}$$

Example:

Consider the functions: $f: 2 - SP_{\mathbb{R}} \rightarrow 2SP_{\mathbb{R}}, g: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}, h: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$$f(x_0 + x_1P_1 + x_2P_2) = \sin(x_0) + \sin(x_1)P_1 + \sin(x_2)P_2,$$

$$g(x_0 + x_1P_1 + x_2P_2) = \cos(x_0) + \cos(x_1)P_1 + \cos(x_2)P_2,$$

$$h(x_0 + x_1P_1 + x_2P_2) = \sin(x_0) + \cos(x_1)P_1 + \sin(x_2)P_2,$$

we have:

$$\begin{cases} \sin(x_0) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x_0^{2n+1}}{(2n+1)!} \\ \sin(x_1) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x_1^{2n+1}}{(2n+1)!} \\ \sin(x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x_2^{2n+1}}{(2n+1)!} \\ \cos(x_0) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x_0^{2n}}{(2n)!} \\ \cos(x_1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x_1^{2n} \\ \cos(x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x_2^{2n} \end{cases}$$

$$\text{Hence, } f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [x_0^{2n+1} + x_1^{2n+1}P_1 + x_2^{2n+1}P_2]$$

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [x_0^{2n} + x_1^{2n}P_1 + x_2^{2n}P_2]$$

$$h(x) = \sum_{n=0}^{\infty} \left[\frac{(-1)^n}{(2n+1)!} (x_0^{2n+1} + x_2^{2n+1}P_2) + \frac{(-1)^n}{(2n)!} x_1^{2n}P_1 \right]$$

Example:

Consider $f: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$$f(x_0 + x_1P_1 + x_2P_2) = \ln(x_0) + \ln(x_1)P_1 + \ln(x_2)P_2,$$

$g: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$$g(x_0 + x_1P_1 + x_2P_2) = e^{x_0} + \ln(x_2)P_1 + \ln(x_1)P_2,$$

$h: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$$h(x_0 + x_1P_1 + x_2P_2) = \ln(x_0) + e^{x_0}P_1 + e^{x_2+x_1}P_2,$$

we have:

$$\begin{cases} \ln(x_0) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x_0 - 1)^n \\ \ln(x_1) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x_1 - 1)^n \\ \ln(x_2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x_2 - 1)^n \end{cases}$$

And:
$$\begin{cases} e^{x_0} = \sum_{n=0}^{\infty} \frac{x_0^n}{n!} \\ e^{x_2+x_1} = \sum_{n=0}^{\infty} \frac{(x_1+x_2)^n}{n!} \end{cases}$$

So that:

$$\begin{aligned} f(x) &= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} [(x_0 - 1)^n + (x_1 - 1)^n P_1 + (x_2 - 1)^n P_2] \\ g(x) &= \sum_{n=0}^{\infty} \frac{x_0^n}{n!} + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} [(x_2 - 1)^n P_1 + (x_1 - 1)^n P_2] \\ h(x) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - 1)^n + \sum_{n=0}^{\infty} \frac{1}{n!} (x_0^n P_1 + (x_1 + x_2)^n P_2) \end{aligned}$$

Example:

Consider the functions:

$$f: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$$

$$g: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$$

$$h: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$$

such that:

$$f(x_0 + x_1 P_1 + x_2 P_2) = e^{x_1 - x_0} + \sin(x_0 - 2x_2) P_1 + \cos(x_2) P_2,$$

$$g(x_0 + x_1 P_1 + x_2 P_2) = \sin(3x_1) + \ln(x_0 + x_1 + x_2) P_1 + \cos(2x_0) P_2,$$

$$h(x_0 + x_1 P_1 + x_2 P_2) = \cos(x_1 + x_0) + e^{x_1 + x_2 - x_0} P_1 + \ln(2x_0 - x_1) P_2,$$

we have:

$$\left\{ \begin{aligned} e^{x_1 - x_0} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_1 - x_0)^n \\ \sin(x_0 - 2x_2) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x_0 - 2x_2)^{2n+1} \\ \cos(x_2) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_2)^{2n} \\ \sin(3x_1) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (3x_1)^{2n+1} \\ \ln(x_0 + x_1 + x_2) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 + x_1 + x_2 - 1)^n \\ \cos(2x_0) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (2x_0)^{2n} \\ \cos(x_1 + x_0) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_1 + x_0)^{2n} \\ e^{x_1 + x_2 - x_0} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_1 + x_2 - x_0)^n \\ \ln(2x_0 - x_1) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (2x_0 - x_1 - 1)^n \end{aligned} \right.$$

So that:

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_1 - x_0)^n + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x_0 - 2x_1)^{2n+1} P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_2)^{2n} P_2 \\ g(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (3x_1)^{2n+1} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 + x_1 + x_2 - 1)^n P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (2x_0)^{2n} P_2 \\ h(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_1 + x_0)^{2n} + \sum_{n=0}^{\infty} \frac{1}{n!} (x_1 + x_2 - x_0)^n P_1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (2x_0 - x_1 - 1)^n P_2 \end{aligned}$$

Definition:

Let $3 - SP_{\mathbb{R}} = \{x + yP_2 + zP_3; x, y, z, t \in \mathbb{R}\}$ be the ring of symbolic 3-plithogenic real numbers, consider the following sequence: $U_n = x_n + y_nP_1 + z_nP_2 + t_nP_3$; x_n, y_n, z_n, t_n are four real sequences, we define:

$$\sum_{n=1}^j U_n = \sum_{n=1}^j x_n + \sum_{n=1}^j y_n P_1 + \sum_{n=1}^j z_n P_2 + \sum_{n=1}^j t_n P_3 \quad (2)$$

It is a symbolic 3-plithogenic finite series.

For $j = \infty$, we get the infinite case.

The convergent and divergent symbolic 3-plithogenic series is defined as the case of symbolic 2-plithogenic case.

Example:

Consider the following functions:

$$\begin{aligned} f: 3 - SP_{\mathbb{R}} &\rightarrow 3 - SP_{\mathbb{R}} \\ g: 3 - SP_{\mathbb{R}} &\rightarrow 3 - SP_{\mathbb{R}} \\ h: 3 - SP_{\mathbb{R}} &\rightarrow 3 - SP_{\mathbb{R}} \\ k: 3 - SP_{\mathbb{R}} &\rightarrow 3 - SP_{\mathbb{R}} \\ l: 3 - SP_{\mathbb{R}} &\rightarrow 3 - SP_{\mathbb{R}} \\ s: 3 - SP_{\mathbb{R}} &\rightarrow 3 - SP_{\mathbb{R}} \end{aligned}$$

Such that:

$$\begin{aligned} f(x_0 + x_1P_1 + x_2P_2 + x_3P_3) &= e^{x_0+x_1} + \ln(x_1)P_1 + \sin(x_2)P_2 + \cos(x_3)P_3, \\ g(x_0 + x_1P_1 + x_2P_2 + x_3P_3) &= \ln(x_0 - x_2) + e^{x_2}P_1 + \cos(x_0 + x_3)P_2 + \sin(x_3)P_3, \\ h(x_0 + x_1P_1 + x_2P_2 + x_3P_3) &= e^{x_0} + \cos(x_3 - x_2)P_1 + \sin(x_2 + 2x_3)P_2 + \ln(x_0)P_3, \\ k(x_0 + x_1P_1 + x_2P_2 + x_3P_3) &= \ln(x_0) + \ln(x_1)P_1 + e^{x_2}P_2 + e^{x_3}P_3, \\ l(x_0 + x_1P_1 + x_2P_2 + x_3P_3) &= \sin(x_0) + \sin(x_1)P_1 + \cos(x_2)P_2 + \cos(x_3)P_3, \\ s(x_0 + x_1P_1 + x_2P_2 + x_3P_3) &= e^{x_3+x_1} + e^{x_0-x_1}P_1 + \ln(2x_2 - 5x_3)P_2 + \cos(x_0 + 4x_2)P_3, \end{aligned}$$

We have:

$$\left\{ \begin{aligned} e^{x_0+x_1} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_0 + x_1)^n \\ \ln x_1 &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_1 - 1)^n \\ \sin x_2 &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x_2^{2n+1} \\ \cos x_3 &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x_3^{2n} \\ \ln(x_0 - x_2) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - x_2 - 1)^n \\ e^{x_2} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_2)^n \\ \cos(x_0 + x_2) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_0 + x_2)^{2n} \\ \sin x_3 &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x_3)^{2n+1} \end{aligned} \right.$$

And

$$\left\{ \begin{array}{l} e^{x_0} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_0)^n \\ \cos(x_3 - x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_3 - x_2)^{2n} \\ \sin(x_2 + 2x_3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x_2 + 2x_3)^{2n+1} \\ \ln x_0 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - 1)^n \end{array} \right.$$

And:

$$\left\{ \begin{array}{l} \ln x_0 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - 1)^n \\ \ln x_1 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_1 - 1)^n \\ e^{x_2} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_2)^n \\ e^{x_3} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_3)^n \end{array} \right.$$

And:

$$\left\{ \begin{array}{l} \sin x_0 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x_0)^{2n+1} \\ \sin x_1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x_1)^{2n+1} \\ \cos x_2 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_2)^{2n} \\ \cos x_3 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_3)^{2n} \end{array} \right.$$

And:

$$\left\{ \begin{array}{l} e^{x_3+x_1} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_3 + x_1)^n \\ e^{x_0-x_1} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_0 - x_1)^n \\ \ln(2x_2 - 5x_3) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (2x_2 - 5x_3 - 1)^n \\ \cos(x_0 + 4x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_0 + 4x_2)^{2n} \end{array} \right.$$

Thus:

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_0 + x_1)^n + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_1 - 1)^n P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x_2^{2n+1} P_2 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x_3^{2n} P_3 \\ g(x) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - x_2 - 1)^n + \sum_{n=0}^{\infty} \frac{1}{n!} x_2^n P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_0 + x_2)^{2n} P_2 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x_3^{2n+1} P_3 \\ h(x) &= \sum_{n=0}^{\infty} \frac{1}{n!} x_0^n + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_3 - x_2)^n P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x_2 + 2x_3)^{2n+1} P_2 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - 1)^n P_3 \\ k(x) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - 1)^n + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_1 - 1)^n P_1 + \sum_{n=0}^{\infty} \frac{1}{n!} [x_2^n P_2 + x_3^n P_3] \end{aligned}$$

$$l(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [x_0^{2n+1} + x_1^{2n+1} P_1] + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [x_2^{2n} P_2 + x_3^{2n} P_3]$$

$$s(x) = \sum_{n=0}^{\infty} \frac{1}{n!} [(x_3 + x_1)^n + (x_0 - x_1)^n P_1] + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (2x_2 - 5x_3 - 1)^n P_2 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_0 + 4x_2)^{2n}.$$

Definition:

Let $4 - SP_{\mathbb{R}} = \{x + yP_2 + zP_2 + tP_3; x, y, z, t, s \in \mathbb{R}\}$ be the ring of symbolic 4-plithogenic real numbers, consider the following sequence: $U_n = x_n + y_n P_1 + z_n P_2 + t_n P_3 + s_n P_4$; x_n, y_n, z_n, t_n, s_n are five real sequences, we define:

$$\sum_{n=1}^j U_n = \sum_{n=1}^j x_n + \sum_{n=1}^j y_n P_1 + \sum_{n=1}^j z_n P_2 + \sum_{n=1}^j t_n P_3 + \sum_{n=1}^j s_n P_4 \quad (3)$$

It is a symbolic 3-plithogenic finite series.

For $j = \infty$, we get the infinite case.

The convergent and divergent symbolic 4-plithogenic series is defined as the case of symbolic 2-plithogenic case.

Example:

Consider the following functions:

$$\begin{aligned} f: 4 - SP_{\mathbb{R}} &\rightarrow 4 - SP_{\mathbb{R}} \\ g: 4 - SP_{\mathbb{R}} &\rightarrow 4 - SP_{\mathbb{R}} \\ h: 4 - SP_{\mathbb{R}} &\rightarrow 4 - SP_{\mathbb{R}} \\ k: 4 - SP_{\mathbb{R}} &\rightarrow 4 - SP_{\mathbb{R}} \\ l: 4 - SP_{\mathbb{R}} &\rightarrow 4 - SP_{\mathbb{R}} \end{aligned}$$

Such that:

$$f(x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3 + x_4 P_4) = e^{x_0 - x_4} + e^{x_1 + x_4} P_1 + e^{x_2 - x_3} P_2 + e^{x_3} P_3 + e^{x_4} P_4,$$

We have:

$$\left\{ \begin{aligned} e^{x_0 - x_4} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_0 - x_4)^n \\ e^{x_1 + x_4} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_1 + x_4)^n \\ e^{x_2 - x_3} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_2 - x_3)^n \\ e^{x_3} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_3)^n \\ e^{x_4} &= \sum_{n=0}^{\infty} \frac{1}{n!} (x_4)^n \end{aligned} \right.$$

$$\text{Thus, } g(x) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} (2x_1 + x_2)^{2n} + \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{(2n+1)!} (x_3)^{2n+1} P_1 + \dots$$

$$h(x) = \cos(x_4 - 5x_0) + \cos(x_1 + x_2) P_1 + \sin(x_2 + x_4) P_2 + \ln(x_0) P_3 + \ln(x_0 + 3x_1) P_4,$$

$$\left\{ \begin{aligned} \cos(x_4 - 5x_0) &= \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} (x_4 - 5x_0)^{2n} \\ \cos(x_1 + x_2) &= \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} (x_1 + x_2)^{2n} \\ \sin(x_2 + x_3) &= \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{(2n+1)!} (x_2 + x_3)^{2n+1} \\ \ln x_0 &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - 1)^n \\ \ln(x_0 + 3x_1) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 + 3x_1 - 1)^n \end{aligned} \right.$$

$$k(x) = e^{x_0 + x_1 - x_4} P_2 + e^{x_4 - 3x_1} P_4, \text{ we have:}$$

$$\begin{cases} e^{x_0+x_1-x_4} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_0+x_1-x_4)^n \\ e^{x_4-3x_1} = \sum_{n=0}^{\infty} \frac{1}{n!} (e^{x_4-3x_1})^n \end{cases}$$

Thus, $k(x) = \sum_{n=0}^{\infty} \frac{1}{n!} [(x_0+x_1-x_4)^n P_2 + (x_4-3x_1)^n P_4]$

$l(x) = \ln(2x_1+x_2) P_1 + \ln(x_3-5x_4) P_3 + e^{x_3} P_4$, we have:

$$\begin{cases} \ln(2x_1+x_2) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (2x_1+x_2-1)^n \\ \ln(x_3-5x_4) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_3-5x_4-1)^n \\ e^{x_3} = \sum_{n=0}^{\infty} \frac{1}{n!} x_3^n \end{cases}$$

Thus, $l(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} [(2x_1+x_2-1)^n P_1 + (x_3-5x_4-1)^n P_3] + \sum_{n=0}^{\infty} \frac{1}{n!} (x_3)^n$

The expansion of some famous plithogenic real functions.

The following formulas are proved in [21].

- 1] $e^{x_0+x_1P_1+x_2P_2} = e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}]$.
- 2] $e^{x_0+x_1P_1+x_2P_2+x_3P_3} = e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}] + P_3[e^{x_0+x_1+x_2+x_3} - e^{x_0+x_1+x_2}]$
- 3] $\ln(x_0+x_1P_1+x_2P_2) = \ln(x_0) + P_1[\ln(x_0+x_1) - \ln(x_0)] + P_2[\ln(x_0+x_1+x_2) - \ln(x_0+x_1)]$
- 4] $\ln(x_0+x_1P_1+x_2P_2+x_3P_3) = \ln(x_0) + P_1[\ln(x_0+x_1) - \ln(x_0)] + P_2[\ln(x_0+x_1+x_2) - \ln(x_0+x_1)] + P_3[\ln(x_0+x_1+x_2+x_3) - \ln(x_0+x_1+x_2)]$
- 5] $\sin(x_0+x_1P_1+x_2P_2) = \sin(x_0) + P_1[\sin(x_0+x_1) - \sin(x_0)] + P_2[\sin(x_0+x_1+x_2) - \sin(x_0+x_1)]$
- 6] $\sin(x_0+x_1P_1+x_2P_2+x_3P_3) = \sin(x_0) + P_1[\sin(x_0+x_1) - \sin(x_0)] + P_2[\sin(x_0+x_1+x_2) - \sin(x_0+x_1)] + P_3[\sin(x_0+x_1+x_2+x_3) - \sin(x_0+x_1+x_2)]$
- 7] $\cos(x_0+x_1P_1+x_2P_2) = \cos(x_0) + P_1[\cos(x_0+x_1) - \cos(x_0)] + P_2[\cos(x_0+x_1+x_2) - \cos(x_0+x_1)]$
- 8] $\cos(x_0+x_1P_1+x_2P_2+x_3P_3) = \cos(x_0) + P_1[\cos(x_0+x_1) - \cos(x_0)] + P_2[\cos(x_0+x_1+x_2) - \cos(x_0+x_1)] + P_3[\cos(x_0+x_1+x_2+x_3) - \cos(x_0+x_1+x_2)]$

The series formulas are:

- 1] $e^{x_0+x_1P_1+x_2P_2} = \sum_{n=0}^{\infty} \frac{1}{n!} [x_0^n + [(x_0+x_1)^n - x_0^n]P_1 + P_2[(x_0+x_1+x_2)^n - (x_0+x_1)^n]$.
- 2] $e^{x_0+x_1P_1+x_2P_2+x_3P_3} = \sum_{n=0}^{\infty} \frac{1}{n!} [x_0^n + [(x_0+x_1)^n - x_0^n]P_1 + [(x_0+x_1+x_2)^n - (x_0+x_1)^n]P_2 + [(x_0+x_1+x_2+x_3)^n - (x_0+x_1+x_2)^n]P_3]$
- 3] $\ln(x_0+x_1P_1+x_2P_2) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} [(x_0-1)^n + [(x_0+x_1-1)^n - (x_0-1)^n]P_1 + [(x_0+x_1+x_2-1)^n - (x_0+x_1-1)^n]P_2]$
- 4] $\ln(x_0+x_1P_1+x_2P_2+x_3P_3) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} [(x_0-1)^n + [(x_0+x_1-1)^n - (x_0-1)^n]P_1 + [(x_0+x_1+x_2-1)^n - (x_0+x_1-1)^n]P_2 + [(x_0+x_1+x_2+x_3-1)^n - (x_0+x_1+x_2-1)^n]P_3]$
- 5] $\sin(x_0+x_1P_1+x_2P_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [x_0^{2n+1} + [(x_0+x_1)^{2n+1} - x_0^{2n+1}]P_1 + [(x_0+x_1+x_2)^{2n+1} - (x_0+x_1)^{2n+1}]P_2]$
- 6] $\sin(x_0+x_1P_1+x_2P_2+x_3P_3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [x_0^{2n+1} + [(x_0+x_1)^{2n+1} - x_0^{2n+1}]P_1 + [(x_0+x_1+x_2)^{2n+1} - (x_0+x_1)^{2n+1}]P_2 + [(x_0+x_1+x_2+x_3)^{2n+1} - (x_0+x_1+x_2)^{2n+1}]P_3]$
- 7] $\cos(x_0+x_1P_1+x_2P_2) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} [x_0^{2n} + [(x_0+x_1)^{2n} - x_0^{2n}]P_1 + [(x_0+x_1+x_2)^{2n} - (x_0+x_1)^{2n}]P_2]$
- 8] $\cos(x_0+x_1P_1+x_2P_2+x_3P_3) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} [x_0^{2n} + [(x_0+x_1)^{2n} - x_0^{2n}]P_1 + [(x_0+x_1+x_2)^{2n} - (x_0+x_1)^{2n}]P_2 + [(x_0+x_1+x_2+x_3)^{2n} - (x_0+x_1+x_2)^{2n}]P_3]$.

5. Conclusion

In this work, we have studied the expansion of symbolic 2-plithogenic and symbolic 3-plithogenic real functions in one variable with real series, where we provided many related examples to clarify and to explain the expansion

method and properties.

In the future, we aim to generalize our study to symbolic n-plithogenic functions and symbolic n-plithogenic series.

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