On Developing He Method with Mohand Transform to Find Numerical and Exact Solutions of Some Neutrosophic Partial Differential Equations

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Abstract

In this paper, we propose a novel combination of the (He) method with (Mohand transform), and this combination is summarized by representing the nonlinear part (or the residue of the operator) with (He) polynomials after applying the Mohand transform. To prove the accuracy of the proposed method, we applied it to several neutrosophic partial differential equations such as neutrosophic version of Helmholtz equation, neutrosophic version of non-linear oscillator, neutrosophic version of Burger's equation, and the neutrosophic version of telegraph equation. The accuracy and effectiveness of the application of the proposed method was verified by comparing the results obtained with other methods using the Maple software package.

Keywords: neutrosophic partial differential equation; Mohand transform; numerical solution; exact solution

1. Introduction:

In (2006) Shaher Monani used the method of (VIM) [2] to solve the Helmholtz equation, and in (2008) the researchers Syed Tauseef Mohyud-Din and Mohammad Noor [3] proposed to solve the same equation using the method of disorder homotopy (HPM)

Researchers wu y and He JH solved the nonlinear oscillator equation using the perturbation homotopy method in (2018) [4], and researchers Lu and Li Ha in (2018) [5] used the vim-Padé method to solve this problem.

The burger's equation was first introduced by Bateman in 1915 [6], and then studied by the researchers Mahgoub, M.A.M. and Alshikh in 2017 [7], solved using Mahgoub Transform and the work of researchers Hradyesh Kumar Mishra and Atulya K. Nagar solved this problem using the hi-LaPlace method [8] in (2012), and the researchers Mohamed Zebir and Mohand also solved it using the Mohand transform [9] in 2021.

In the work [10] the equation (telegraph equation) was solved using the He-Laplace method by the researchers (2019) Muhammad Nadeem and Fengquan Li.

Neutrosophic logic was presented by Smarandache [1] as a novel generalization of fuzzy logic, and his ideas were used by many authors to study algebra, analysis, and applied mathematics [12-18].

In our current work, we will apply the He-Mohand method in solving some versions of neutrosophic partial differential equations represented by neutrosophic real functions and numbers.

2. Main Discussion:

Definition:

We define the neutrosophic version of the Mohand Transform as follows:

$$M[f(t+gI)] = R(v+uI) = (v+uI)^2 \int_0^\infty f(t+gI)e^{-(v+uI)t} d(t+gI)\,, t+gI \geq 0, k_1+s_1I \leq v+uI \leq k_2+s_2I \quad (1)$$

$$f(t+gI) \in A\left\{ f(t+gI) : \exists M, k_1+s_1I, k_2+s_2I > 0; |f(t+gI)| < Me^{\frac{|t|}{k_j}}, if\ t+gI \in (-1)^j \times [0,\infty[\right\} \quad (2)$$

3. Combining He method with Mohand transform:

The novel combination can be described as applying He method and substituting the Laplace transform with the

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Neutrosophic Mohand transform. Then we extend the non-linear part with He polynomials, as follows:

$$H_n(u_0,u_1,\ldots,u_n)=\frac{1}{n!}\frac{\partial^n}{\partial p^n}[N(\sum_{i=0}^{\infty}p^i u_i)]_{p=0}; n=0,1,2,\ldots$$

We explain the novel technique on the following equation:

$A(u(x+yI))=L(u(x+yI))+N(u(x+yI))=g(x+yI)$ (3)

Where L is a linear differential operator, N is non-linear, and g(x) is analytical function.

By applying neutrosophic Mohand transform on (3), we get:

$M[L(u(x+yI))+N(u(x+yI))-g(x+yI)]=0$ (4)

Multiply (4) by Lagrange coefficient :

$\lambda\{M[L(u(x+yI))+M[N(u(x+yI))-g(x+yI)]\}=0$ (5)

By using He method, we get:

$u_{n+1}(v+uI)=u_n(v+uI)+\lambda\{M[L(u_n(x+yI))]+M[N(\tilde{u}_n(x+yI))-g(x+yI)]\}$ (6)

By applying the inverse Mohand transform, we get:

$u_{n+1}(x+yI)=u_n(x+yI)+M^{-1}\{\lambda(v+uI)\{M[L(u_n(x+yI))]+M[N(\tilde{u}_n(x+yI))-g(x+yI)]\}\}$ (7)

Thus

$N(x+yI)=\sum_{n=0}^{\infty}p^n H_n(u)$

$$H_n(u_0,u_1,\ldots,u_n)=\frac{1}{n!}\frac{\partial^n}{\partial p^n}[N(\sum_{i=0}^{\infty}p^i u_i)]_{p=0}; n=0,1,2,\ldots$$

This implies that:

$u_0+pu_1+\ldots=u_n(x+yI)+pM^{-1}\{\lambda(v+uI)\{M[L(u_n(x+yI))]+M[N(\tilde{u}_n(x+yI))-g(x+yI)]\}\}$
(8)

Example 1: Solve the neutrosophic Helmholtz problem:

$\frac{\partial^2 u(x+zI,y+rI)}{\partial(x+zI)^2}+\frac{\partial^2 u(x+zI,y+rI)}{\partial(y+rI)^2}+8u(x+zI,y+rI)=0$ (9)

With conditions:

$u(0,y+rI)=\sin(2y+2rI)\quad u_{x+zI}(0,y+rI)=0$

Applying neutrosophic Mohand transform generates:

$M\{\frac{\partial^2 u(x+zI,y+rI)}{\partial(x+zI)^2}+\frac{\partial^2 u(x+zI,y+rI)}{\partial(y+rI)^2}+8u(x+zI,y+rI)\}=0$ (10)

So that

$$\lambda\, M\{\frac{\partial^2 u(x+zI,y+rI)}{\partial(x+zI)^2}+\frac{\partial^2 u(x+zI,y+rI)}{\partial(y+rI)^2}+8u(x+zI,y+rI)\}=0$$

hence

$u_{n+1}=u_n+\lambda M\{\frac{\partial^2 u(x+zI,y+rI)}{\partial(x+zI)^2}+\frac{\partial^2 u(x+zI,y+rI)}{\partial(y+rI)^2}+8u_n(x+zI,y+rI)\}$ (11)

Also, we take: $\delta u_{n+1}(v+uI)=0$

$\delta u_{n+1}=\delta u_n+\delta\lambda\{(v^2u_n(x,v+uI)-(v+uI)^3u_n(x+zI,0)-(v+uI)^2u_n(x+zI,0))\}+\lambda\delta M\{\frac{\partial^2 u_n(x+zI,y+rI)}{\partial(y+rI)^2}+8u_n(x+zI,y+rI)\}$

$$\delta u_{n+1}=\delta u_n+\lambda(v+uI)^2\delta u_n$$
$$0=1+\lambda(v+uI)^2$$
$$\lambda=\frac{-1}{(v+uI)^2}$$

By using (11), we get:

$u_{n+1}=u_n-\frac{1}{(v+uI)^2}M\{\frac{\partial^2 u_n}{\partial(x+zI)^2}+\frac{\partial^2 u_n}{\partial(y+rI)^2}+8u_n\}$ (12)

Applying inverse Mohand transform, gives

$u_{n+1}(x+zI,y+rI)=u_n(x+zI,y+rI)-M^{-1}\{\frac{1}{v^2}M\{\frac{\partial^2 u_n(x+zI,y+rI)}{\partial(x+zI)^2}+\frac{\partial^2 u_n(x+zI,y+rI)}{\partial(y+rI)^2}+8u_n(x+zI,y+rI)\}\}$

$u_{n+1}(x+zI,y+rI)=u_n(x+zI,y+rI)-M^{-1}\{\frac{1}{(v+uI)^2}M\{\frac{\partial^2 u_n(x,y+rI)}{\partial(y+rI)^2}+8u_n(x+zI,y+rI)\}\}$

*t*hus

$u_0+pu_1+p^2u_2+\ldots=u_n-pM^{-1}\{\frac{1}{(v+uI)^2}M\{(\frac{\partial^2 u_0}{\partial(y+rI)^2}+8u_0)+p(\frac{\partial^2 u_1}{\partial(y+rI)^2}+8u_1)+p^2(\frac{\partial^2 u_2}{\partial(y+rI)^2}+8u_2)+\ldots.\}$,

thus

$$p^0: u_0=u(0,y+rI)+x\ \ u_x(0,y+rI)=\sin(2[y+rI])$$
$$p^1: u_1=-M^{-1}\{\frac{1}{v^2}M\{(\frac{\partial^2 u_0}{\partial(y+rI)^2}+8u_0)\}\}$$

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$$= -M^{-1}\{\frac{1}{(v+uI)^2}M(-4sin(2[y+rI])+8\sin(2[y+rI]))\}$$
$$= -2(x+zI)^2 sin(2[y+rI])$$
$$p^2: u_2 = -M^{-1}\{\frac{1}{(v+uI)^2}M\{(\frac{\partial^2 u_1}{\partial(y+rI)^2}+8u_1)\}\} = \frac{2}{3}(x+zI)^4\sin(2[y+rI])$$

The solution is given by:

$u(x,y+rI) = u_0+u_1+u_2+\ldots = \sin(2[y+rI]) - 2(x+zI)^2\sin(2[y+rI]) + \frac{2}{3}(x+zI)^4\sin(2[y+rI])+\ldots = \sin(2[y+rI])(1-2(x+zI)^2+\frac{2}{3}(x+zI)^4+\ldots) = \sin(2[y+rI])\cos(2[x+zI])$.

Example 2: solve the following neutrosophic partial differential equation:

$u_t = u_{x+zIx+zI} - u.u_{x+zI}$ (13)

With the following condition

$$u(x+zI,0) = 1-\frac{2}{x+zI}$$

Apply Mohand transform:

$M[u_t - u_{x+zIx+zI} + u.u_{x+zI}] = 0$ (14)

hence

$\lambda\, M[\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial(x+zI)^2} + u.\frac{\partial u}{\partial(x+zI)}] = 0$ (15)

So that

$u_{n+1}(x+zI,v) = u_n(x+zI,v) + \lambda\, M[\frac{\partial u_n}{\partial t} - \frac{\partial^2 u_n}{\partial(x+zI)^2} + u_n.\frac{\partial u_n}{\partial x+zI}]$ (16)

$$\delta u_{n+1}(v) = 0$$
$$\delta u_{n+1} = \delta u_n + \delta\lambda\, M[\frac{\partial u_n}{\partial t} - \frac{\partial^2 u_n}{\partial(x+zI)^2} + u_n.\frac{\partial u_n}{\partial x+zI}]$$
$$\delta u_{n+1} = \delta u_n + \delta\lambda\,\{(vu_n(x+zI,v) - v^2u_n(x+zI,0))\} + \delta\lambda\, M[-\frac{\partial^2 \tilde{u}_n}{\partial(x+zI)^2} + \tilde{u}_n.\frac{\partial \tilde{u}_n}{\partial x+zI}\}$$
$$\delta u_{n+1} = \delta u_n + \lambda v\delta u_n$$
$$0 = 1+\lambda v$$
$$\lambda = \frac{-1}{v}$$

By using (16), we get

$u_{n+1}(x+zI,v) = u_n(x+zI,v) - \frac{1}{v}\, M[\frac{\partial u_n}{\partial t} - \frac{\partial^2 u_n}{\partial(x+zI)^2} + u_n.\frac{\partial u_n}{\partial(x+zI)}]$ (17)

Applying inverse Mohand transform on (17) gives:

$u_{n+1}(x+zI,t) = u_n(x+zI,t) - M^{-1}[\frac{1}{v}\, M[-\frac{\partial^2 u_n}{\partial(x+zI)^2} + u_n.\frac{\partial u_n}{\partial(x+zI)}]$ (18)

Thus

$u_0+pu_1+p^2u_2+\ldots = u_n - pM^{-1}\{\frac{1}{v}M\{(-\frac{\partial^2 u_0}{\partial(x+zI)^2} + u_0\frac{\partial u_0}{\partial(x+zI)}) + p(-\frac{\partial^2 u_1}{\partial(x+zI)^2} + u_1\frac{\partial u_0}{\partial x+zI} + u_0\frac{\partial u_1}{\partial x+zI}) + p^2(-\frac{\partial^2 u_2}{\partial x^2} + u_2\frac{\partial u_0}{\partial x+zI} + u_1\frac{\partial u_1}{\partial x+zI} + u_0\frac{\partial u_2}{\partial x+zI})+\ldots\}$

$$p^0: u_0 = 1-\frac{2}{x+zI}$$
$$p^1: u_1 = -M^{-1}\{\frac{1}{v}M\{(-\frac{\partial^2 u_0}{\partial(x+zI)^2} + u_0\frac{\partial u_0}{\partial x+zI})\}\} = -\frac{2}{(x+zI)^2}t$$
$$p^2: u_2 = -M^{-1}\{\frac{1}{v}M\{(-\frac{\partial^2 u_1}{\partial(x+zI)^2} + u_1\frac{\partial u_0}{\partial x+zI} + u_0\frac{\partial u_1}{\partial x+zI})\}\} = -\frac{2}{(x+zI)^3}t^2$$

Thus, the solution is given by:

$u(x+zI,t) = u_0+u_1+u_2+\ldots = 1-\frac{2}{x+zI} - \frac{2}{(x+zI)^2}t - \frac{2}{(x+zI)^3}t^2+\ldots. = 1-\frac{2}{x+zI}(1+\frac{t}{x+zI}+\ldots) = 1-\frac{2}{x+zI-t};\ \left|\frac{t}{x+zI}\right| < 1 \Longrightarrow |t| < |x+zI|$.

Example 3:

Consider the following partial neutrosophic differential equation:

$$u_t = u_{x+zIx+zI} + u.u_{x+zI}$$

With conditions:

$$u(x+zI,0) = 1-x-zI, u_{x+zI}(0,t) = \frac{1}{1+t}, u(1,t) = 0$$

By applying Mohand transform, we get:

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$M[u_t - u_{x+zIx+zI} - u.u_{x+zI}] = 0$ (20)

hence

$\lambda\, M[\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial (x+zI)^2} - u.\frac{\partial u}{\partial x+zI}] = 0$ (21)

This implies

$u_{n+1}(x+zI,v) = u_n(x+zI,v) + \lambda\, M[\frac{\partial u_n}{\partial t} - \frac{\partial^2 u_n}{\partial x^2} - u_n.\frac{\partial u_n}{\partial x+zI}]$ (22)

$$\delta u_{n+1}(v) = 0$$

$$\delta u_{n+1} = \delta u_n + \delta\lambda\, M[\frac{\partial u_n}{\partial t} - \frac{\partial^2 u_n}{\partial (x+zI)^2} - u_n.\frac{\partial u_n}{\partial x+zI}]$$

$\delta u_{n+1} = \delta u_n + \delta\lambda\,\{(v\,u_n(x+zI,v) - v^2 u_n(x+zI,0))\} + \lambda\,\delta\,\mathrm{M}\{-\frac{\partial^2 \tilde{u}_n}{\partial (x+zI)^2} + \tilde{u}_n.\frac{\partial \tilde{u}_n}{\partial x+zI}\}$

$$\delta u_{n+1} = \delta u_n + \lambda\, v\, \delta u_n$$

$$0 = 1 + \lambda v$$

$$\lambda = \frac{-1}{v}$$

By substituting in (22), we get:

$u_{n+1}(x+zI,v) = u_n(x+zI,v) - \frac{1}{v}\, M[\frac{\partial u_n}{\partial t} - \frac{\partial^2 u_n}{\partial (x+zI)^2} - u_n.\frac{\partial u_n}{\partial x+zI}]$ (23)

We apply the inverse transform to get:

$u_{n+1}(x+zI,t) = u_n(x+zI,t) - M^{-1}[\frac{1}{v}\, M[-\frac{\partial^2 u_n}{\partial (x+zI)^2} - u_n.\frac{\partial u_n}{\partial x+zI}]$ **(24)**

$u_0 + pu_1 + p^2u_2 + \ldots = u_n - pM^{-1}\{\frac{1}{v}M\{(-\frac{\partial^2 u_0}{\partial (x+zI)} - u_0\frac{\partial u_0}{\partial x+zI}) + p(-\frac{\partial^2 u_1}{\partial (x+zI)^2} - u_1\frac{\partial u_0}{\partial x+zI} - u_0\frac{\partial u_1}{\partial x+zI}) + p^2(-\frac{\partial^2 u_2}{\partial x^2} - u_2\frac{\partial u_0}{\partial x+zI} - u_1\frac{\partial u_1}{\partial x+zI} - u_0\frac{\partial u_2}{\partial x+zI}) + \ldots\}$,

thus

$$p^0 : u_0 = 1 - (x+zI)$$

$$p^1 : u_1 = -M^{-1}\{\frac{1}{v}M\{(-\frac{\partial^2 u_0}{\partial (x+zI)^2} - u_0\frac{\partial u_0}{\partial x+zI})\}\} = -(1-x-zI)t$$

$$p^2 : u_2 = -M^{-1}\{\frac{1}{v}M\{(-\frac{\partial^2 u_1}{\partial (x+zI)^2} + u_1\frac{\partial u_0}{\partial (x+zI)} + u_0\frac{\partial u_1}{\partial x+zI})\}\} = -(1-(x+zI))t^2$$

So that, the solution is given by

$u(x+zI,t) = u_0 + u_1 + u_2 + \ldots = 1-(x+zI) - (1-(x+zI))t + (1-(x+zI))t^2 + \ldots = \frac{1-(x+zI)}{1+t}$

Example 4:

Consider the following neutrosophic partial differential equation:

$u_{x_1+x_2Ix_1+x_2I} = u_{t_1+t_2It_1+t_2I} + u_{t_1+t_2I} - u$ (25)

$u(x_1+x_2I,0) = e^{x_1+x_2I}$ $u_t(x_1+x_2I,0) = -2e^{x_1+x_2I}$

By applying Mohand transform, we get:

$M\{u_{t_1+t_2It_1+t_2I} + u_{t_1+t_2I} - u - u_{x_1+x_2Ix_1+x_2I}\} = 0$ (26)

$\lambda\, M\{u_{t_1+t_2It_1+t_2I} + u_{t_1+t_2I} - u - u_{x_1+x_2Ix_1+x_2I}\} = 0$ (27)

$u_{n+1}(x_1+x_2I,v) = u_n(x_1+x_2I,v) + \lambda\, M[\frac{\partial^2 u_n}{\partial (t_1+t_2I)^2} - \frac{\partial^2 u_n}{\partial (x_1+x_2I)^2} - u_n + \frac{\partial u_n}{\partial t_1+t_2I}]$ (28)

$$\delta u_{n+1}(v) = 0$$

$$\delta u_{n+1} = \delta u_n + \delta\,\lambda\, M[\frac{\partial^2 u_n}{\partial (t_1+t_2I)^2} - \frac{\partial^2 u_n}{\partial (x_1+x_2I)^2} - u_n + \frac{\partial u_n}{\partial t_1+t_2I}]$$

$\delta u_{n+1} = \delta u_n + \delta\,\lambda\{(v^2u_n(x_1+x_2I,v) - v^3\widetilde{u'}_n(x,0)' - v^2\tilde{u}_n(x_1+x_2I,0))\} + \lambda\,\delta\,M[-\frac{\partial^2 \tilde{u}_n}{\partial (x_1+x_2I)^2} - \tilde{u}_n + \frac{\partial \tilde{u}_n}{\partial t_1+t_2I}\}$

$$\delta u_{n+1} = \delta u_n + \lambda\, v^2\, \delta u_n$$

$$0 = 1 + \lambda v^2$$

$$\lambda = \frac{-1}{v^2}$$

By using (28), we get:

$u_{n+1}(x_1+x_2I,v) = u_n(x_1+x_2I,v) - \frac{1}{v^2}\, M[\frac{\partial^2 u_n}{\partial (t_1+t_2I)^2} - \frac{\partial^2 u_n}{\partial (x_1+x_2I)^2} - u_n + \frac{\partial u_n}{\partial t_1+t_2I}]$ (29)

By using the inverse transform, we get:

$u_{n+1}(x_1+x_2I,t) = u_n(x_1+x_2I,t_1+t_2I) - M^{-1}[\frac{1}{v^2}M[-\frac{\partial^2 u_n}{\partial (x_1+x_2I)^2} - u_n + \frac{\partial u_n}{\partial t_1+t_2I}]$ (30)

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$u_0 + pu_1 + p^2u_2 + \ldots = u_n - pM^{-1}\{\frac{1}{v^2}M\{(-\frac{\partial^2 u_0}{\partial(x_1+x_2I)^2} - u_0 + \frac{\partial u_0}{\partial t_1+t_2I}) + p(-\frac{\partial^2 u_1}{\partial(x_1+x_2I)^2} - u_1 + \frac{\partial u_1}{\partial t_1+t_2I}) + p^2(-\frac{\partial^2 u_2}{\partial(x_1+x_2I)^2} - u_2 + \frac{\partial u_2}{\partial t_1+t_2I}) + \ldots\}$

thus

$$p^0: u_0 = (1 - 2[t_1 + t_2I])e^{x_1+x_2I}$$

$$p^1: u_1 = -M^{-1}\{\frac{1}{v^2}\mathrm{M}\{(-\frac{\partial^2 u_0}{\partial(x_1+x_2I)^2} - u_0 + \frac{\partial u_0}{\partial x_1 + x_2I})\}\} = (2[t_1 + t_2I] - \frac{2}{3}[t_1 + t_2I]^3)e^{x_1+x_2I}$$

$p^2: u_2 = -M^{-1}\{\frac{1}{v^2}\mathrm{M}\{(-\frac{\partial^2 u_1}{\partial(x_1+x_2I)^2} - u_1 + \frac{\partial u_1}{\partial t_1+t_2I})\}\} = (\frac{-2}{3}[t_1 + t_2I]^3 + \frac{1}{2}[t_1 + t_2I]^4 - \frac{1}{15}[t_1 + t_2I]^5)e^{x_1+x_2I}$

So that, the solution is given by:

$u(x_1 + x_2I, t_1 + t_2I) = u_0 + u_1 + u_2 + \ldots = (1 - \frac{4}{3}[t_1 + t_2I]^3 + \frac{1}{2}[t_1 + t_2I]^4 - \frac{1}{15}[t_1 + t_2I]^5 + \ldots)e^{x_1+x_2I}$.

Example 5:

Consider the equation:

$\frac{d^2u}{dt^2} + u + \frac{3}{2}u^3 + 3\sin u = 0$ (31)

$u(0) = \frac{\pi}{3} \quad \frac{du}{dt}(0) = 0$

$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + u + \frac{3}{2}u^3 + 3(\mathrm{u} - \frac{u^3}{6}) = 0$

$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + 4u + u^3 = 0$ (32)

By Mohand transform, we get:

$M\{\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + 4u + u^3\} = 0$ (33)

$\lambda\, M\{\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + 4u + u^3\} = 0$ (34)

$u_{n+1} = u_n + \lambda\, M\{\frac{d^2u_n}{dt^2} + 4u_n + u_n^3\}$ (35)

$$\delta u_{n+1}(v) = 0$$

$\delta u_{n+1} = \delta u_n + \lambda\, \delta\{(v^2u_n(t,v) - v^3u_n(t,0) - v^2u_n(t,0)')\} + \lambda\, \delta\, M\{4u_n + u_n^3\}$

$$\delta u_{n+1} = \delta u_n + \lambda\, v^2 \delta\, u_n$$

$$0 = 1 + \lambda\, v^2$$

$$\lambda = \frac{\mathbf{-1}}{v^2}$$

$u_{n+1} = u_n - \frac{\mathbf{1}}{v^2}\boldsymbol{M}\{\frac{d^2u_n}{\mathrm{d}t^2} + 4u_n + u_n^3\}$ (36)

The inverse transform implies:

$$u_{n+1} = u_n - \mathrm{M}^{-1}\{\frac{\mathbf{1}}{v^2}\boldsymbol{M}\{\frac{\mathrm{d}^2u_n}{\mathrm{d}t^2} + 4u_n + u_n^3\}\}$$

hence

$u_0 + pu_1 + p^2u_2 + \ldots = u_n - pM^{-1}\{\frac{1}{v^2}\mathrm{M}\{\frac{d^2u_n}{dt^2} + 4u_n + (u_{0\,1} + pu_{1\,1} + \ldots)^3\} = u_n - p\mathrm{M}^{-1}\{\frac{1}{v^2}\mathrm{M}\{(4u_0 + u_{0\,1}^3) + p(4u_1 + 3u_{0\,1}^2u_{1\,1}) + \ldots.\}$

$u_0 = \frac{\pi}{3}$

$u_1 = -\frac{1}{54}\pi(36 + \pi^2)t^2$

$$u_2 = \frac{1}{1944}\pi(12 + \pi^2)(36 + \pi^2)t^4$$

$$u_3 = -\frac{1}{58320}\pi(36 + \pi^2)(\pi^4 + 32\pi^2 + 48)t^6$$

So that, the approximate solution is:

$$u(t) = u_0 + u_1 + u_2 + u_3 + \cdots.$$

$$= \frac{\pi}{3} - \frac{2}{3}\pi t^2 - \frac{1}{54}\pi^3t^2 + \frac{2}{9}\pi t^4 + \frac{2}{81}\pi^3t^4 + \frac{1}{1944}\pi^5t^4 - \frac{4}{135}\pi t^6 - \frac{5}{243}\pi^3t^6 - \frac{17}{14580}\pi^5t^6 - \frac{1}{58320}\pi^7t^6 + \ldots$$

The solution generated by applying [VIM] method:

$u_3 = \frac{\pi}{3} - \frac{2\pi}{3}t^2 - \frac{\pi^3}{54}t^2 + \frac{\pi(36+\pi^2)}{162}t^4 + \frac{\pi^3(36+\pi^2)}{1944}t^4 - \frac{\pi^3(36+\pi^2)^2}{87480}t^6 + \frac{\pi^3(36+\pi^2)^3}{8817984}t^8 + \frac{\pi(36+\pi^2)(12+\pi^2)^2}{174960}t^6 + \frac{\pi^3(12+\pi^2)(36+\pi^2)^2}{249440}t^8 + \ldots.$

We compare the errors:

Table (1)

T	*He-Mohand*	*VIM*

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0	0	0
0.1	5.387961487×10^{-5}	265.5047847×10^{-5}
0.2	334.4578056×10^{-5}	5663.795292×10^{-5}
0.3	3597.560179×10^{-5}	40867.65427×10^{-5}
0.4	18796.48632×10^{-5}	184513.2469×10^{-5}
0.5	65677.64747×10^{-5}	629110.0189×10^{-5}
0.6	177675.8258×10^{-5}	1771259.748×10^{-5}
0.7	403499.1531×10^{-5}	4341043.094×10^{-5}
0.8	809870.9658×10^{-5}	$96246377.763\times10^{-5}$
0.9	1492035.749×10^{-5}	20662080.64×10^{-5}
1	2611474.237×10^{-5}	55157440.02×10^{-5}

4. Conclusion:

In most of the applications studied, we obtained accurate and approximate solutions with a lower number of iterations compared to other methods, and that the error of the He-Mohand method used in this research was less and better than the methods used in previous research, we recommend as a future work hybridizing the he-Mohand method with some integral transformations such as the Elzaki, Sumdu and Fox transformations..

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