Single-Valued Pentapartitioned Neutrosophic Bi-Topological Spaces

Suman Das[1], Prasanna Poojary[2], Vadiraja Bhatta G. R.[3,*], Sunny Kumar Sharma[4]

[1]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.
[2,4]Department of Mathematics, Manipal Institute of Technology Bengaluru, Manipal Academy of Higher Education, Manipal, India.
[3]Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, India.
Emails: sumandas18842@gmail.com; poojary.prasanna@manipal.edu; poojaryprasanna34@gmail.com; sunny.sharma@manipal.edu
*Correspondence: vadiraja.bhatta@manipal.edu

Abstract

In this article, we present the notion of Single-Valued Pentapartitioned Neutrosophic Bi-Topological Space (SVPNBTS) as a generalization of Single-Valued Pentapartitioned Neutrosophic Topological Space (SVPNTS) and Neutrosophic Bi-Topological Space (NBTS). Besides, we study the different types of open set and closed set namely single-valued pentapartitioned neutrosophic bi-open set (SVPNBOS), single-valued pentapartitioned neutrosophic bi-closed set (SVPNBCS), single-valued pentapartitioned neutrosophic bi-semi-open set (SVPNBSOS), single-valued pentapartitioned neutrosophic bi-semi-closed set (SVPNBSCS), single-valued pentapartitioned neutrosophic bi-pre-open set (SVPNBPOS), single-valued pentapartitioned neutrosophic bi-pre-closed set (SVPNBPCS), single-valued pentapartitioned neutrosophic bi-b-open set (SVPNBb-OS), single-valued pentapartitioned neutrosophic bi-b-closed set (SVPNBb-CS), etc. via SVPNBTSs. Besides, we introduce the notion of pairwise SVPNOS, pairwise SVPNCS, pairwise SVPNSOS, pairwise SVPNSCS, pairwise SVPNPOS, pairwise SVPNPCS, pairwise SVPNb-OS, pairwise SVPNb-CS, and furnish few illustrative examples on them. Further, we investigate several properties of these classes of sets and prove some interesting results in the form of propositions, theorems, etc. via SVPNBTSs.

Keywords: SVPNBTS; Bi-open set; Bi-closed set; Bi-semi-open set; Bi-pre-open set; NBTS.

1. Introduction

The notion of Neutrosophic Set (NS) was established by Smarandache [41] as an extension of Fuzzy Set and Intuitionistic FS. Wang et al. [27] then established the concept of the Single-Valued Neutrosophic Set (SVNS). Many mathematicians have used the concept of SVNS, Neutrosophy, and its extensions in theoretical work [2, 3, 10-14, 45, 46] as well as practical work [9, 39]. Salama and Alblowi [38] later introduced the concept of Neutrosophic Topological Space (NTS) in 2012. Iswaraya and Bageerathi [6] proposed the neutrosophic semi-closed and neutrosophic semi-open set through NTSs in 2016. Later, Arokiarani et al. [1] defined neutrosophic semi-open functions using NTSs and established some new notions on neutrosophic semi-open sets. Following that, Dhavaseelan and Jafari [22] used NTSs to establish the concept of generalized neutrosophic closed sets. Imran et al. [26] used NTSs to investigate the neutrosophic -open set. Rao and Srinivasa [36] established the concept of neutrosophic pre-open and neutrosophic pre-closed sets using NTSs in 2017. Ebenanjar et al. [24] introduced the concept of neutrosophic b-open set through NTSs in 2018. Following that, Sreeja and Sarankumar [42] proposed generalized alpha closed sets using NTSs, while Dhavaseelan et al. [23] investigated neutrosophic m-continuity with NTSs in 2019. Later, Mohammed Ali Jaffer and Ramesh [31] used NTSs to propose the neutrosophic generalized pre regular closed set. Following that, Pushpalatha and Nandhini [35] investigated the concept of generalized closed sets using NTS. Das and Pramanik [15] then established the notion of the generalized neutrosophic b-open set using NTSs. NTSs were used by Das and Pramanik [16] to introduce the concepts of neutrosophic -open sets and neutrosophic -continuous functions. Page and Imran [34] investigated the concept of neutrosophic generalized homomorphism. Das and Tripathy [20] developed the concept of

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

neutrosophic simply b-open set through NTSs in 2021. Das et al. [5] then established the notion of separation axioms through NTSs. Kelly [28] established the concept of bi-topological space for the first time in 1963. Till now, many researchers around the globe studied the notion of bitopological space and introduced different kinds of an open set. Tripathy and Sarma [44] investigated the concept of pairwise generalized b-R0 spaces using bi-topological spaces in 2017. Following that, Ozturk and Ozkan [33] investigated the notion of NBTS in 2019. Mwchahary and Basumatary [32] then explored NBTS further in 2020. Later, Das and Tripathy [19] used NBTS to introduce the notion of a pairwise neutrosophic b-open set. Tripathy and Das [43] have investigated the pairwise neutrosophic b-continuous functions using NBTS. Ganesan and Smarandache [25] introduced the concept of neutrosophic biminimal -open set in 2021. Mallick and Pramanik [30] have proposed SVPNS as an extension of NS and SVNS. Until now, numerous scholars throughout the world have used the notion of SVPNS in both theoretical [4, 8, 40] and practical work [7, 17-18, 29, 37]. Later, Das and Tripathy [21] investigated the notion of SVPNT and established the concept of SVPNTS. As a result, we were sufficiently motivated to do research on SVPNBTS to extend the concept of SVPNTS and NBTS.

In this study, we procure the notion of SVPNBTS as a generalization of the SVPNTS and NBTS. Besides, we introduce different types of open sets and closed sets namely, SVPNBOS, SVPNBCS, SVPNBSOS, SVPNBSCS, SVPNBPOS, SVPNBPCS, SVPNB*b*-OS, SVPNB*b*-CS, pairwise SVPNOS, pairwise SVPNCS, pairwise SVPNSOS, pairwise SVPNSCS, pairwise SVPNPOS, pairwise SVPNPCS, pairwise SVPN*b*-OS, pairwise SVPN*b*-CS, etc. via SVPNBTSs. Further, we investigate several properties of these kinds of sets via SVPNBTSs.

The remaining part of this article is divided into the following sections:

Preliminaries and definitions are covered in Section 2. This section contains definitions and theorems pertinent to the main results of this article. Section 3 introduces the concept of single-valued neutrosophic bi-topology (SVPNBT), SVPNBTS, and provides proofs of various SVPNBTS theorems. Finally, in section 4, we summarize the work done in this article.

For the purpose of clarity, we will use the following abbreviations throughout this article.

Short Terms	
Neutrosophic Set	NS
Neutrosophic Topological Space	NTS
Neutrosophic Open Set	NOS
Neutrosophic Closed Set	NCS
Single-Valued Neutrosophic Set	SVNS
Single-Valued Pentapartitioned Neutrosophic Set	SVPNS
Single-Valued Pentapartitioned Neutrosophic Topological Space	SVPNTS
Single-Valued Pentapartitioned Neutrosophic Open Set	SVPNOS
Single-Valued Pentapartitioned Neutrosophic Closed Set	SVPNCS
Single-Valued Pentapartitioned Neutrosophic Pre-Open Set	SVPNPOS
Single-Valued Pentapartitioned Neutrosophic Semi-Open Set	SVPNSOS
Single-Valued Pentapartitioned Neutrosophic Pre-Closed Set	SVPNPCS
Single-Valued Pentapartitioned Neutrosophic Semi-Closed Set	SVPNSCS
Single-Valued Pentapartitioned Neutrosophic Bi-Open Set	SVPNBOS
Single-Valued Pentapartitioned Neutrosophic Bi-Closed Set	SVPNBCS
Single-Valued Pentapartitioned Neutrosophic Bi-Pre-Open Set	SVPNBPOS
Single-Valued Pentapartitioned Neutrosophic Bi-Pre-Closed Set	SVPNBPCS
Single-Valued Pentapartitioned Neutrosophic Bi-Semi-Open Set	SVPNBSOS
Single-Valued Pentapartitioned Neutrosophic Bi-Semi-Closed Set	SVPNBSCS
Single-Valued Pentapartitioned Neutrosophic Bi-*b*-Open Set	SVPNB*b*-OS
Single-Valued Pentapartitioned Neutrosophic Bi-*b*-Closed Set	SVPNB*b*-CS

2. Some Relevant Definitions:

In this part, we give some important results that will be very useful in presenting the main results of this article.

Definition 2.1.[41] Assume that W is a fixed set. Then P, an NS over W is defined as follows:

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

$$P=\{(q, T_P(q), I_P(q), F_P(q)): q\in W\},$$

where $T_P(q)$, $I_P(q)$, $F_P(q)$ ($\in[0, 1]$) are the truth membership, indeterminacy membership, and falsity membership values of $q\in W$. So, $0 \leq T_P(q) + I_P(q) + F_P(q) \leq 3$, for all $q\in W$.

Example 2.1. Suppose that $W=\{p, q\}$ is a non-empty set. Then, $P=\{(p, 0.8, 0.5, 0.6), (q, 0.9, 0.5, 0.3)\}$ is an NS over W, but $Q=\{(p, 0.9, -0.3, 0.2), (q, -0.5, 0.2, -0.3)\}$ is not an NS over W.

Definition 2.2.[41] The null NS (0_N) and whole NS (1_N) over a fixed set W are defined as follows:

$$0_N = \{(q, 0, 0, 1): q\in W\} \text{ and } 1_N = \{(q, 1, 0, 0): q\in W\}.$$

Clearly, $0_N \subseteq P \subseteq 1_N$, for any NS P over W.

Definition 2.3.[38] Suppose that W is a fixed set. Assume that τ be a collection of NSs over W satisfies the following properties:

(*i*) $0_N, 1_N\in\tau$;

(*ii*) $P_1, P_2\in\tau \Rightarrow P_1\cap P_2\in\tau$;

(*iii*) $\{P_i: i\in\Delta\} \subseteq \tau \Rightarrow \cup P_i\in\tau$.

Then, τ is called a neutrosophic topology on W, and the pair (W, τ) is called an NTS. If $P\in\tau$, then P is said to be a NOS and its complement i.e., P^c is said to be an NCS in (W, τ).

Example 2.2. Suppose that X, Y and Z is three NSs over a fixed set $W=\{p, q\}$ such that $X=\{(p, 0.7, 0.4, 0.6), (q, 0.5, 0.6, 0.4)\}$, $Y=\{(p, 0.6, 0.4, 0.7), (q, 0.5, 0.7, 0.6)\}$ and $Z=\{(p, 0.5, 0.5, 0.9), (q, 0.4, 0.9, 0.8)\}$. Then, the collection $\tau=\{0_N, 1_N, X, Y, Z\}$ forms a neutrosophic topology on W.

Example 2.3. Assume that X, Y and Z be three NSs over a fixed set $W=\{p, q\}$ such that $X=\{(p, 0.7, 0.6, 0.9), (q, 0.6, 0.8, 0.8)\}$, $Y=\{(p, 0.6, 0.4, 0.7), (q, 0.5, 0.7, 0.6)\}$ and $Z=\{(p, 0.5, 0.3, 0.8), (q, 0.4, 0.7, 0.6)\}$. Then, the collection $\tau=\{0_N, 1_N, X, Y, Z\}$ does not form a neutrosophic topology on W.

Definition 2.4.[33] Suppose that (W, τ_1) and (W, τ_2) are any two different NTSs. Then, the triplet (W, τ_1, τ_2) is called an NBTS.

Example 2.4. Suppose that X, Y, Z and Q are four NSs over a fixed set $W=\{p, q\}$ such that:

$X=\{(p, 0.7, 0.4, 0.6), (q, 0.5, 0.6, 0.4)\}$, $Y=\{(p, 0.6, 0.4, 0.7), (q, 0.5, 0.7, 0.6)\}$, $Z=\{(p, 0.5, 0.5, 0.9), (q, 0.4, 0.9, 0.8)\}$ and $Q=\{(p, 0.9, 0.1, 0.2), (q, 0.6, 0.8, 0.7)\}$. Then, the collection $\tau_1=\{0_N, 1_N, X, Y\}$ and $\tau_2=\{0_N, 1_N, Z, Q\}$ are two different neutrosophic topologies on W. So, the triplet (W, τ_1, τ_2) is an NBTS.

Definition 2.5.[33] Suppose that (W, τ_1, τ_2) is an NBTS. Then, an NS P over W is called a pairwise NOS in (W, τ_1, τ_2) if there exists a NOS P_1 in (W, τ_1) and a NOS P_2 in (W, τ_2) such that $P=P_1\cup P_2$.

Definition 2.6. [30] Let W be a universe of discourse. Then P, an SVPNS over W is defined by:

$$P=\{(q, T_P(q), C_P(q), G_P(q), U_P(q), F_P(q)): q\in W\},$$

where $T_P(q)$, $C_P(q)$, $G_P(q)$, $U_P(q)$ and $F_P(q)$ ($\in[0, 1]$) are the truth membership, contradiction membership, ignorance membership, unknown membership and falsity membership values of $q\in W$. So, $0 \leq T_P(q) + C_P(q) + G_P(q) + U_P(q) + F_P(q) \leq 5$, for all $q\in W$.

Definition 2.7. [30] The null SVPNS (0_{PN}) and the whole SVPNS (1_{PN}) over a fixed set W are defined as follows:

$$0_{PN} = \{(q, 0, 0, 1, 1, 1): q\in W\} \text{ and } 1_{PN} = \{(q, 1, 1, 0, 0, 0): q\in W\}.$$

The null SVPNS (0_{PN}) and whole SVPNS (1_{PN}) have other seven types of representations. They are given as follows:

$0_{PN} = \{(q, 0, 0, 1, 1, 0): q\in W\}$;

$0_{PN} = \{(q, 0, 0, 1, 0, 1): q\in W\}$;

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

0_{PN} = {(q, 0, 0, 0, 1, 1): $q\in W$};

0_{PN} = {(q, 0, 0, 1, 0, 0): $q\in W$};

0_{PN} = {(q, 0, 0, 0, 1, 0): $q\in W$};

0_{PN} = {(q, 0, 0, 0, 0, 1): $q\in W$};

0_{PN} = {(q, 0, 0, 0, 0, 0): $q\in W$};

1_{PN} = {(q, 1, 1, 0, 0, 1): $q\in W$};

1_{PN} = {(q, 1, 1, 0, 1, 0): $q\in W$};

1_{PN} = {(q, 1, 1, 1, 0, 0): $q\in W$};

1_{PN} = {(q, 1, 1, 0, 1, 1): $q\in W$};

1_{PN} = {(q, 1, 1, 1, 0, 1): $q\in W$};

1_{PN} = {(q, 1, 1, 1, 1, 0): $q\in W$};

1_{PN} = {(q, 1, 1, 1, 1, 1): $q\in W$};

Throughout this article we shall use 0_{PN} = {(q, 0, 0, 1, 1, 1): $q\in W$} and 1_{PN} = {(q, 1, 1, 0, 0, 0): $q\in W$}.

Definition 2.8. [30] Suppose that X={(q, $T_X(q)$, $C_X(q)$, $G_X(q)$, $U_X(q)$, $F_X(q)$): $q\in W$} and Y={(q, $T_Y(q)$, $C_Y(q)$, $G_Y(q)$, $U_Y(q)$, $F_Y(q)$): $q\in W$} are two SVPNSs over a non-empty set W. Then, $X\subseteq Y$ if and only if $T_X(q)\leq T_Y(q)$, $C_X(q)\leq C_Y(q)$, $G_X(q)\geq G_Y(q)$, $U_X(q)\geq U_Y(q)$, $F_X(q)\geq F_Y(q)$, for all $q\in W$.

Example 2.5. Assume that W={p, q} be a non-empty set. Let X={(p, 0.5, 0.7, 0.5, 0.6, 0.4), (q, 0.3, 0.4, 0.7, 0.8, 0.7)} and Y={(p, 0.6, 0.8, 0.4, 0.5, 0.4), (q, 0.5, 0.5, 0.3, 0.3, 0.3)} be two SVPNSs over W. Then, $X\subseteq Y$.

Definition 2.9. [30] Suppose that X={(q, $T_X(q)$, $C_X(q)$, $G_X(q)$, $U_X(q)$, $F_X(q)$): $q\in W$} and Y={(q, $T_Y(q)$, $C_Y(q)$, $G_Y(q)$, $U_Y(q)$, $F_Y(q)$): $q\in W$} are two SVPNSs over a fixed set W. Then, the intersection of X and Y is $X\cap Y$={(q, min{$T_X(q)$,$T_Y(q)$}, min{$C_X(q)$,$C_Y(q)$}, max{$G_X(q)$,$G_X(q)$}, max{$U_X(q)$,$U_X(q)$}, max{$F_X(q)$,$F_X(q)$}): $q\in W$}.

Example 2.6. Suppose that W={p, q} is a fixed set. Assume that X={(p, 0.4, 0.6, 0.5, 0.8, 0.5), (q, 0.7, 0.6, 0.5, 0.8, 0.7)} and Y={(p, 0.7, 0.7, 0.5, 0.6, 0.6), (q, 0.5, 0.8, 0.4, 0.4, 0.4)} be two SVPNSs over W. Then, $X\cap Y$={(p, 0.4, 0.6, 0.5, 0.8, 0.6), (q, 0.5, 0.6, 0.5, 0.8, 0.7)}.

Definition 2.10. [30] Suppose that X={(q, $T_X(q)$, $C_X(q)$, $G_X(q)$, $U_X(q)$, $F_X(q)$): $q\in W$} and Y={(q, $T_Y(q)$, $C_Y(q)$, $G_Y(q)$, $U_Y(q)$, $F_Y(q)$): $q\in W$} are two SVPNSs over a fixed set W. Then, the union of X and Y is $X\cup Y$={(q, max{$T_X(q)$,$T_Y(q)$}, max{$C_X(q)$,$C_Y(q)$}, min{$G_X(q)$,$G_X(q)$}, min{$U_X(q)$,$U_X(q)$}, min{$F_X(q)$,$F_X(q)$}): $q\in W$}.

Example 2.7. Suppose that X and Y are two SVPNSs over a fixed set W as shown in Example 2.6. Then, $X\cup Y$={(p, 0.7, 0.7, 0.5, 0.6, 0.5), (q, 0.7, 0.8, 0.4, 0.4, 0.4)}.

Definition 2.11.[21] Assume that X={(q, $T_X(q)$, $C_X(q)$, $G_X(q)$, $U_X(q)$, $F_X(q)$): $q\in W$} be an SVPNS over a fixed set W. Then, the complement of X i.e., X^c is defined as follows:

$$X^c=\{(q,\ 1-T_X(q),\ 1-C_X(q),\ 1-G_X(q),\ 1-U_X(q),\ 1-F_X(q)): q\in W\}.$$

Example 2.8. Assume that X be an SVPNS over W as shown in Example 2.6. Then, the complement of X i.e., X^c={(p, 0.6, 0.4, 0.5, 0.2, 0.5), (q, 0.3, 0.4, 0.5, 0.2, 0.3)}.

Definition 2.12.[21] Suppose that W is a non-empty set. Then, a family τ of SVPNSs over W is said to be an SVPNT on W, if the following axioms hold:

(i) 0_{PN}, 1_{PN} $\in\tau$;

(ii) P_1, $P_2\in\tau\Rightarrow P_1\cap P_2\in\tau$;

(iii) {P_i: $i\in\Delta$} $\in\tau\Rightarrow\cup P_i\in\tau$.

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

Then, the pair (W, τ) is said to be an SVPNTS. Each member of τ is said to be an SVPNOS. If $P\in\tau$, then P^c is said to be an SVPNCS.

Example 2.9. Assume that X, Y and Z be three SVPNSs over a fixed set W={p, q} such that:

X={(p, 0.4, 0.5, 0.6, 0.7, 0.8), (q, 0.8, 0.6, 0.5, 0.5, 0.4)};

Y={(p, 0.6, 0.6, 0.5, 0.4, 0.4), (q, 0.9, 0.8, 0.4, 0.2, 0.3)};

and Z={(p, 0.8, 0.7, 0.4, 0.3, 0.2), (q, 0.9, 0.9, 0.2, 0.2, 0.2)}.

Then, the family $\tau=\{0_{PN}, 1_{PN}, X, Y, Z\}$ forms an SVPNT on W. Therefore, (W, τ) is an SVPNTS.

3. Single-Valued Pentapartitioned Neutrosophic Bi-Topological Space:

In this section, we define SVPNBTS as a generalization of SVPNTS and NBTS. Furthermore, we investigate the various kinds of an open and closed set, such as SVPNBOS, SVPNBCS, SVPNBSOS, SVPNBSCS, SVPNBPOS, SVPNBPCS, etc, using SVPNBTSs. Furthermore, we establish several intriguing findings using SVPNBTSs in the form of propositions, and theorems.

Definition 3.1. Suppose that (W, τ_1) and (W, τ_2) are any two different SVPNTSs. Then, the structure (W, τ_1, τ_2) is called an SVPNBTS.

Example 3.1. Assume that W={u, v, w} be a fixed set. Suppose that X_1, X_2, Y_1, Y_2, and Y_3 be five SVPNSs over W such that:

X_1={(u, 0.8, 0.5, 0.3, 0.4, 0.7), (v, 0.5, 0.6, 0.6, 0.7, 0.6), (w, 0.2, 0.5, 0.4, 0.5, 0.3)};

X_2={(u, 0.7, 0.3, 0.6, 0.4, 0.8), (v, 0.4, 0.4, 0.8, 0.8, 0.9), (w, 0.1, 0.4, 0.5, 0.6, 0.4)};

Y_1={(u, 0.8, 0.7, 0.3, 0.8, 0.6), (v, 0.5, 0.7, 0.6, 0.6, 0.8), (w, 0.3, 0.4, 0.6, 0.6, 0.2)};

Y_2={(u, 1.0, 0.8, 0.2, 0.5, 0.5), (v, 0.8, 0.8, 0.5, 0.6, 0.8), (w, 0.6, 0.5, 0.2, 0.5, 0.1)};

and Y_3={(u, 1.0, 1.0, 0.5, 0.4, 0.3), (v, 1.0, 0.9, 0.3, 0.6, 0.8), (w, 0.9, 0.7, 0.1, 0.5, 0.0)}.

Then, clearly $\tau_1=\{0_{PN}, 1_{PN}, X_1, X_2\}$ and $\tau_2=\{0_{PN}, 1_{PN}, Y_1, Y_2, Y_3\}$ are two different SVPNTs on W. Therefore, the structure (W, τ_1, τ_2) is an SVPNBTS.

Definition 3.2. Suppose that (W, τ_1, τ_2) is an SVPNBTS. Then R, an SVPNS over W is called an SVPNBOS if $R\in\tau_1\cup\tau_2$. An SVPNS N is called an SVPNBCS if and only if N^c is an SVPNBOS in the SVPNBTS (W, τ_1, τ_2).

Example 3.2. Let us consider an SVPNBTS (W, τ_1, τ_2) as shown in Example 3.1. Then, 0_{PN}, 1_{PN}, X_1, X_2, Y_1, Y_2 and Y_3 are SVPNBOSs in (W, τ_1, τ_2). Similarly, $(0_{PN})^c = 1_{PN}$, $(1_{PN})^c=0_{PN}$, X_1^c={(u,0.8, 0.5, 0.7, 0.6, 0.3), (v, 0.5, 0.4, 0.4, 0.3, 0.4), (w, 0.8, 0.5, 0.6, 0.5, 0.7)} , X_2^c={(u, 0.3, 0.7, 0.4, 0.6, 0.2), (v, 0.6, 0.6, 0.2, 0.2, 0.1), (w, 0.9, 0.6, 0.5, 0.4, 0.6)}, Y_1^c={(u, 0.2, 0.3, 0.7, 0.2, 0.4), (v, 0.5, 0.3, 0.4, 0.4, 0.2), (w, 0.7, 0.6, 0.4, 0.4, 0.8)}, Y_2^c={(u, 0.0, 0.2, 0.8, 0.5, 0.5), (v, 0.2, 0.2, 0.5, 0.4, 0.2), (w, 0.4, 0.5, 0.8, 0.5, 0.9)} and Y_3^c={(u, 0.0, 0.0, 0.5, 0.6, 0.7), (v, 0.0, 0.1, 0.7, 0.4, 0.2), (w, 0.1, 0.3, 0.9, 0.5, 1.0)} are SVPNBCSs in (W, τ_1, τ_2).

Remark 3.1. The family of all SVPNBOSs and SVPNBCSs in (W, τ_1, τ_2) are denoted by SVPNBO(W) and SVPNBC(W) respectively.

Proposition 3.1. Every SVPNOS in (W, τ_i), i = 1, 2 is an SVPNBOS in (W, τ_1, τ_2).

Proof. Suppose that R is an SVPNOS in (W, τ_i), i = 1, 2. Therefore, $R\in\tau_i$, i = 1, 2. This implies, $R\in \bigcup_{i\in\{1,2\}}\tau_i$. Hence, R is an SVPNBOS in (W, τ_1, τ_2). Therefore, every SVPNOS in (W, τ_i), i = 1, 2 is an SVPNBOS in (W, τ_1, τ_2).

Proposition 3.2. Every SVPNCS in (W, τ_i), i = 1, 2 is an SVPNBCS in (W, τ_1, τ_2).

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

Proof. Let R be an SVPNCS in (W, τ_i), i = 1, 2. So R^c is an SVPNOS in (W, τ_i) , i = 1, 2. Therefore, $R^c \in \tau_i$, i = 1, 2. This implies, $R^c \in \bigcup_{i\in\{1,2\}} \tau_i$. This implies, R^c is an SVPNBOS in (W, τ_1, τ_2). Therefore, R is an SVPNBCS in (W, τ_1, τ_2). Hence, every SVPNCS in (W, τ_i), i = 1, 2 is an SVPNBCS in (W, τ_1, τ_2).

Remark 3.2. In an SVPNBTS (W, τ_1, τ_2), the union of two SBPNBOSs may not be an SVPNBOS. This follows from the following example.

Example 3.3. Consider the SVPNBTS (W, τ_1, τ_2) which has been shown in Example 3.1. Then, clearly X_2 and Y_1 are two SVPNBOSs in (W, τ_1, τ_2). But their union $X_2 \cup Y_1$={(u, 0.8, 0.7, 0.3, 0.4, 0.6), (v, 0.5, 0.7, 0.6, 0.6, 0.8), (w, 0.3, 0.4, 0.5, 0.6, 0.2)} is not an SVPNBOS in (W, τ_1, τ_2), because $X_2 \cup Y_1 \notin \bigcup_{i\in\{1,2\}} \tau_i$. Hence, the union of two SVPNBOSs may not be an SVPNBOS.

Remark 3.3. In an SVPNBTS (W, τ_1, τ_2), the intersection of two SVPNBOSs may not be an SVPNBOS. This follows from the following example.

Example 3.4. Consider the SVPNBTS (W, τ_1, τ_2) which has been shown in Example 3.1. Then, clearly X_1 and Y_2 are two SVPNBOSs in (W, τ_1, τ_2). But their intersection $X_1 \cap Y_2$={(u, 0.8, 0.5, 0.3, 0.5, 0.7), (v, 0.5, 0.6, 0.6, 0.7, 0.8), (w, 0.2, 0.5, 0.4, 0.5, 0.3)} is not an SVPNBOS, because $X_1 \cap Y_2 \notin \bigcup_{i\in\{1,2\}} \tau_i$. Hence, the intersection of two SVPNBOSs may not be an SVPNBOS.

Definition 3.3. In an SVPNBTS (W, τ_1, τ_2), an SVPNS G over W is called an SVPNBSOS if G is an SVPNSOS in at least one of two SVPNTSs (W, τ_1) and (W, τ_2).

Example 3.5. Consider the SVPNBTS (W, τ_1, τ_2) which is shown in Example 3.1. Then, M={(u, 0.9, 0.6, 0.2, 0.4, 0.5), (v, 0.6, 0.6, 0.4, 0.5, 0.4), (w, 0.4, 0.5, 0.4, 0.3, 0.2)} is an SVPNBSOS in (W, τ_1, τ_2), because M is an SVPNSOS in (W, τ_1).

Definition 3.4. In an SVPNBTS (W, τ_1, τ_2), an SVPNS G over W is called an SVPNBPOS if G is an SVPNPOS in at least one of two SVPNTSs (W, τ_1) and (W, τ_2).

Example 3.6. Consider the SVPNBTS (W, τ_1, τ_2) which is shown in Example 3.1. Then, Q={(u, 1.0, 0.7, 0.1, 0.3, 0.4), (v, 0.7, 0.7 0.3, 0.4, 0.3), (w, 0.5, 0.6, 0.3, 0.2, 0.1)} is an SVPNBPOS (W, τ_1, τ_2), because Q is an SVPNPOS in (W, τ_1).

Definition 3.5. In an SVPNBTS (W, τ_1, τ_2), an SVPNS G over W is called an SVPNB*b*-OS if G is an SVPN*b*-OS in at least one of two SVPNTSs (W, τ_1) and (W, τ_2).

Example 3.7. Consider the SVPNBTS (W, τ_1, τ_2) which is shown in Example 3.1. Then, P={(u, 1.0, 0.8, 0.0, 0.2, 0.3), (v, 0.8, 0.8, 0.2, 0.3 0.2), (w, 0.6, 0.7, 0.2, 0.1, 0.0)} is an SVPNB*b*-OS in (W, τ_1, τ_2), because P is an SVPN*b*-OS in(W, τ_1).

Remark 3.4. Suppose that (W, τ_1, τ_2) is an SVPNBTS. Assume that $\tau_{1,2} = \tau_1 \cup \tau_2$. Then, $\tau_{1,2}$ may not be an SVPNT on W in general. This follows from the following example.

Example 3.8. Consider the SVPNBTS (W, τ_1, τ_2) which is shown in Example 3.1. Clearly, X_2, $Y_1 \in \tau_{1,2}$, but their intersection $X_2 \cap Y_1 \notin \tau_{1,2}$. Hence, $\tau_{1,2}$ does not form a SVPNT on W.

Definition 3.6. Assume that (W, τ_1, τ_2) be an SVPNBTS. Then R, an SVPNS over W is called an pairwise SVPNOS in (W, τ_1, τ_2) if and only if there exist SVPNOSs R_1 in τ_1 and R_2 in τ_2 such that $R = R_1 \cup R_2$.

Example 3.9. Consider the SVPNBTS (W, τ_1, τ_2) as shown in Example 3.1. Then, R={(u, 0.8, 0.7, 0.3, 0.4, 0.6), (v, 0.5, 0.7, 0.6, 0.6, 0.6), (w, 0.3, 0.5, 0.4, 0.5, 0.2)} is a pairwise SVPNOS, since there exist SVPNOSs X_1 in τ_1 and Y_1 in τ_2 such that $R = X_1 \cup Y_1$.

Remark 3.5. In an SVPNBTS (W, τ_1, τ_2), an SVPNS G is called a pairwise SVPNCS if and only if G^c is a pairwise SVPNOS in (W, τ_1, τ_2).

Theorem 3.1. Let (W, τ_1, τ_2) be an SVPNBTS.

(i) The null SVPNS (0_{PN}) and the whole SVPNS (1_{PN}) are always a pairwise SVPNOS in (W, τ_1, τ_2);

(ii) Every SVPNOS in (W, τ_1) and (W, τ_2) are pairwise SVPNOS in (W, τ_1, τ_2);

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

(*iii*) Every SVPNCS in (W, τ_1) and (W, τ_2) are pairwise SVPNCS in (W, τ_1, τ_2).

Proof. (*i*) We can write the null SVPNS (0_{PN}) as $0_{PN}=M\cup N$, where $M=0_{PN}$, $N=0_{PN}$ are SVPNOSs in (W, τ_1) and (W, τ_2) respectively. Hence, 0_{PN} is a pairwise SVPNOS in (W, τ_1, τ_2).

Similarly, we can write the whole SVPNS (1_{PN}) as $1_{PN}=M\cup N$, where $M=1_{PN}$, $N=1_{PN}$ are SVPNOSs in (W, τ_1) and (W, τ_2) respectively. Hence, 1_{PN} is a pairwise SVPNOS in (W, τ_1, τ_2).

(*ii*) Let Q be an SVPNOS in (W, τ_1). Now, we can write $Q=Q\cup 0_{PN}$. Therefore, there exist SVPNOSs Q in (W, τ_1) and 0_{PN} in (W, τ_2) such that $Q=Q\cup 0_{PN}$. Hence, Q is a pairwise SVPNOS in (W, τ_1, τ_2).

Let Q be an SVPNOS in (W, τ_2). Now, we can write $Q=0_{PN}\cup Q$. Therefore, there exist SVPNOSs 0_{PN} in (W, τ_1) and Q in (W, τ_2) respectively such that $Q=0_{PN}\cup Q$. Hence, Q is a pairwise SVPNOS in (W, τ_1, τ_2).

(*iii*) Suppose that Q be an SVPNCS in (W, τ_1). So Q^c is an SVPNOS in (W, τ_1). By the second part of this theorem, Q^c is a pairwise SVPNOS in (W, τ_1, τ_2). Hence, Q is a pairwise SVPNCS in (W, τ_1, τ_2).

Suppose that Q be an SVPNCS in (W, τ_2). So Q^c is an SVPNOS in (W, τ_2). By the second part of this theorem, Q^c is a pairwise SVPNOS in (W, τ_1, τ_2). Hence, Q is a pairwise SVPNCS in (W, τ_1, τ_2).

Theorem 3.2. In an SVPNBTS (W, τ_1, τ_2), the union of two pairwise SVPNOSs is also a pairwise SVPNOS.

Proof. Suppose that X and Y are two pairwise SVPNOSs in an SVPNBTS (W, τ_1, τ_2). So there exist SVPNOSs X_1, Y_1 in (W, τ_1), and X_2, Y_2 in (W, τ_2), such that $X=X_1\cup X_2$ and $Y=Y_1\cup Y_2$. Now, $X\cup Y=(X_1\cup X_2)\cup(Y_1\cup Y_2)=(X_1\cup Y_1)\cup(X_2\cup Y_2)$. Since X_1 and Y_1 are SVPNOSs in (W, τ_1), so $X_1\cup Y_1$ is an SVPNOS in (W, τ_1). Further, since X_2 and Y_2 are SVPNOSs in (W, τ_2), so $X_2\cup Y_2$ is an SVPNOS in (W, τ_2). Therefore, $X\cup Y$ is a pairwise SVPNOS in (W, τ_1, τ_2).

Remark 3.6. In an SVPNBTS (W, τ_1, τ_2), the intersection of any two pairwise SVPNOSs may not be a pairwise SVPNOS. This follows from the following example.

Example 3.10. Consider the SVPNBTS (W, τ_1, τ_2) as shown in Example 3.1. Then, P={(u, 1.0, 0.8, 0.2, 0.4, 0.5), (v, 0.8, 0.8, 0.5, 0.6, 0.8), (w, 0.6, 0.5, 0.2, 0.5, 0.1)} and R={(u, 0.8, 0.7, 0.3, 0.4, 0.6), (v, 0.5, 0.7, 0.6, 0.6, 0.6), (w, 0.3, 0.5, 0.4, 0.5, 0.2)} are two pairwise SVPNOSs in (W, τ_1, τ_2), but their intersection $P\cap R$={(u, 0.8, 0.7, 0.3, 0.4, 0.6), (v, 0.5, 0.7, 0.6, 0.6, 0.8), (w, 0.3, 0.5, 0.4, 0.5, 0.2)} is not a pairwise SVPNOS in (W, τ_1, τ_2).

Definition 3.7. Assume that (W, τ_1, τ_2) be an SVPNBTS. Then Q, an SVPNS over W is called a pairwise SVPNSOS in (W, τ_1, τ_2) if and only if there exists SVPNSOSs Q_1 in (W, τ_1) and Q_2 in (W, τ_2) such that $Q=Q_1\cup Q_2$.

Example 3.11. Let us consider the SVPNBTS (W, τ_1, τ_2) as shown in Example 3.1. Then, M = {(u, 0.9, 0.6, 0.2, 0.4, 0.5), (v, 0.6, 0.6, 0.4, 0.5, 0.4), (w, 0.4, 0.5, 0.4, 0.3, 0.2)} is an SVPNSOS in (W, τ_1) and N = {(u, 1.0, 1.0, 0.5, 0.3, 0.2), (v, 1.0, 1.0, 0.3, 0.6, 0.3), (w, 1.0, 1.0, 0.1, 0.5, 0.0)} is an SVPNSOS in (W, τ_2). Therefore, Z = $M\cup N$ = {(u, 1.0, 1.0, 0.2, 0.3, 0.2), (v, 1.0, 1.0, 0.3, 0.5, 0.4), (w, 1.0, 1.0, 0.1, 0.3, 0.0)} is a pairwise SVPNSOS in (W, τ_1, τ_2).

Theorem 3.3. In an SVPNBTS (W, τ_1, τ_2), every SVPNBSOS is also a pairwise SVPNSOS.

Proof. Suppose that X is an SVPNBSOS in an SVPNBTS (W, τ_1, τ_2). So, X must be an SVPNSOS in at least one of the SVPNTS (W, τ_1), (W, τ_2). So, there will be three cases.

Case-1: X is an SVPNSOS in (W, τ_1);

Case-2: X is an SVPNSOS in (W, τ_2);

Case-4: X is an SVPNSOS in both (W, τ_1) and (W, τ_2).

In case-1, we can express, $X=X\cup 0_{PN}$, that is X is the union of SVPNSOSs X (in (W, τ_1)) and 0_{PN} (in (W, τ_2)). Therefore, X is a pairwise SVPNSOS in (W, τ_1, τ_2).

In case-2, we can express, $X = 0_N\cup X$, that is X is the union of SVPNSOSs 0_{PN} (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a pairwise SVPNSOS in (W, τ_1, τ_2).

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

In case-3, we can express, $X=X\cup X$, that is X is the union of SVPNSOSs X (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a pairwise SVPNSOS in (W, τ_1, τ_2).

Hence, every SVPNBSOS is a pairwise SVPNSOS in (W, τ_1, τ_2).

Definition 3.8. Assume that (W, τ_1, τ_2) be an SVPNBTS. Then Q, an SVPNS over W is called a pairwise SVPNPOS in (W, τ_1, τ_2) if and only if there exists SVPNPOSs Q_1 in (W, τ_1) and Q_2 in (W, τ_2) such that $Q=Q_1\cup Q_2$.

Example 3.12. Suppose that (W, τ_1, τ_2) is an SVPNBTS as shown in Example 3.1. Then, L = {(u, 1.0, 0.7, 0.1, 0.3, 0.4), (v, 0.7, 0.7, 0.3, 0.4, 0.3), (w, 0.5, 0.6, 0.3, 0.2, 0.1)} is an SVPNPOS in (W, τ_1) and K = {(u, 0.8, 0.7, 0.5, 0.2, 0.5), (v, 0.7, 0.8, 0.2, 0.5, 0.6), (w, 0.8, 0.8, 0.2, 0.1, 0.1)} is an SVPNPOS in (W, τ_2). Therefore, $S = L\cup K$ = {(u, 1.0, 0.7, 0.1, 0.2, 0.4), (v, 0.7, 0.8, 0.2, 0.4, 0.3), (w, 0.8, 0.8, 0.2, 0.1, 0.1)} is a pairwise SVPNPOS in (W, τ_1, τ_2).

Theorem 3.4. In an SVPNBTS (W, τ_1, τ_2), every SVPNBPOS is also a pairwise SVPNPOS.

Proof. Suppose that X be an SVPNBPOS in an SVPNBTS (W, τ_1, τ_2). So, X must be an SVPNPOS in at least one of the SVPNTS (W, τ_1), (W, τ_2). So, there will be three cases.

Case-1: X is an SVPNPOS in (W, τ_1);

Case-2: X is an SVPNPOS in (W, τ_2);

Case-4: X is an SVPNPOS in both (W, τ_1) and (W, τ_2).

In case-1, we can express, $X=X\cup 0_{PN}$, that is X is the union of SVPNPOSs X (in (W, τ_1)) and 0_{PN} (in (W, τ_2)). Therefore, X is a pairwise SVPNPOS in (W, τ_1, τ_2).

In case-2, we can express, $X = 0_{PN}\cup X$, that is X is the union of SVPNPOSs 0_{PN} (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a pairwise SVPNPOS in (W, τ_1, τ_2).

In case-3, we can express, $X=X\cup X$, that is X is the union of SVPNPOSs X (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a pairwise SVPNPOS in (W, τ_1, τ_2).

Hence, every SVPNBPOS is a pairwise SVPNPOS in (W, τ_1, τ_2).

Definition 3.9. Let (W, τ_1, τ_2) be an SVPNBTS. Then Q, an SVPNS over W is called a pairwise SVPNb-OS in (W, τ_1, τ_2) if and only if there exists SVPNb-OSs Q_1 in (W, τ_1) and Q_2 in (W, τ_2) such that $Q=Q_1\cup Q_2$.

Example 3.13. Suppose that (W, τ_1, τ_2) is an SVPNBTS as shown in Example 3.1. Then, M = {(u, 0.9, 0.6, 0.2, 0.4, 0.5), (v, 0.6, 0.6, 0.4, 0.5, 0.4), (w, 0.4, 0.5, 0.4, 0.3, 0.2)} is an SVPNb-OS in (W, τ_1) and

K = {(u, 0.8, 0.7, 0.5, 0.2, 0.5), (v, 0.7, 0.8, 0.2, 0.5, 0.6), (w, 0.8, 0.8, 0.2, 0.1, 0.1)} is an SVPNb-OS in (W, τ_2). Therefore, $Y = M\cup K$ = {(u, 0.9, 0.7, 0.2, 0.2, 0.5), (v, 0.7, 0.8, 0.2, 0.5, 0.4), (w, 0.8, 0.8, 0.2, 0.1, 0.1)} is a pairwise SVPNb-OS in (W, τ_1, τ_2).

Theorem 3.5. In an SVPNBTS (W, τ_1, τ_2), every SVPNBb-OS is also a pairwise SVPNb-OS.

Proof. Suppose that X be an SVPNBb-OS in an SVPNBTS (W, τ_1, τ_2). So, X must be an SVPNb-OS in at least one of the SVPNTS (W, τ_1), (W, τ_2). So, there will be three cases.

Case-1: X is an SVPNBb-OS in (W, τ_1);

Case-2: X is an SVPNBb-OS in (W, τ_2);

Case-4: X is an SVPNBb-OS in both (W, τ_1) and (W, τ_2);

In case-1, we can express, $X=X\cup 0_{PN}$, that is X is the union of SVPNBb-OSs X (in (W, τ_1)) and 0_{PN} (in (W, τ_2)). Therefore, X is a pairwise SVPNb-OS in (W, τ_1, τ_2).

In case-2, we can express, $X = 0_{PN}\cup X$, that is X is the union of SVPNBb-OSs 0_{PN} (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a pairwise SVPNb-OS in (W, τ_1, τ_2).

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

In case-3, we can express, $X=X\cup X$, that is X is the union of SVPNBb-OSs X (in (W, τ_1)) and X (in (W, τ_2)). Therefore, X is a pairwise SVPNb-OS in (W, τ_1, τ_2).

Hence, every SVPNBb-OS is a pairwise SVPNb-OS.

Definition 3.10. Suppose that (W, τ_1, τ_2) is an SVPNBTS. Assume that X be an SVPNS over W. Then, the pairwise single-valued pentapartitioned neutrosophic interior ($P\text{-}P_{int}$) and pairwise single-valued pentapartitioned neutrosophic closure ($P\text{-}P_{cl}$) of X is defined as follows:

$$P\text{-}P_{int}(X) = \cup\{Y: Y \text{ is a pairwise SVPNOS and } Y\subseteq X\},$$

$$\text{and } P\text{-}P_{cl}(X) = \cap\{Y: Y \text{ is a pairwise SVPNCS and } X\subseteq Y\}.$$

It is clearly observed that $P\text{-}P_{int}(X)$ is the largest pairwise SVPNOS which is contained in X and $P\text{-}P_{cl}(X)$ is the smallest pairwise SVPNCS which contains X.

Theorem 3.6. Suppose that (W, τ_1, τ_2) is an SVPNBTS. Assume that X and Y be two SVPNSs over W. Then, the following holds:

(*i*) $P\text{-}P_{int}(X) \subseteq X$;

(*ii*) $X\subseteq Y \Rightarrow P\text{-}P_{int}(X)\subseteq P\text{-}P_{int}(Y)$;

(*iii*) If X is a pairwise SVPNOS, then $P\text{-}P_{int}(X)=X$;

(*iv*) $P\text{-}P_{int}(0_N)=0_N$ and $P\text{-}P_{int}(1_N)=1_N$.

Proof. (i) From the Definition 3.10., we have $P\text{-}P_{int}(X) = \cup\{B: B$ is a pairwise SVPNOS and $B\subseteq X\}$. Since $B\subseteq X$, so $\cup\{B: B$ is a pairwise SVPNOS and $B\subseteq X\} \subseteq X$. Therefore, $P\text{-}P_{int}(X)\subseteq X$.

(ii) Assume that X and Y be two SVPNSs over W such that $X\subseteq Y$. Then,

$P\text{-}P_{int}(X)$

$= \cup\{B: B$ is a pairwise SVPNOS and $B\subseteq X\}$

$\subseteq \cup\{B: B$ is a pairwise SVPNOS and $B\subseteq Y\}$ [since $X\subseteq Y$]

$= P\text{-}P_{int}(Y)$

$\Rightarrow P\text{-}P_{int}(X) \subseteq P\text{-}P_{int}(Y)$.

Therefore, $X\subseteq Y \Rightarrow P\text{-}P_{int}(X)\subseteq P\text{-}P_{int}(Y)$.

(iii) Assume that X be a pairwise SVPNOS in an SVPNBTS (W, τ_1, τ_2). Now, $P\text{-}P_{int}(X) = \cup\{B: B$ is a pairwise SVPNOS and $B\subseteq X\}$. Since X is a pairwise SVPNOS in (W, τ_1, τ_2), so X is the largest pairwise SVPNOS in (W, τ_1, τ_2), which is contained in X. Hence $\cup\{B: B$ is a pairwise SVPNOS and $B\subseteq X\} = X$. Therefore, $P\text{-}P_{int}(X) = X$.

(iv) It is known that, both 0_{PN} and 1_{PN} are pairwise SVPNOSs in (W, τ_1, τ_2), so by the third part of this theorem, we have $P\text{-}P_{int}(0_{PN}) = 0_{PN}$, $P\text{-}P_{int}(1_{PN}) = 1_{PN}$.

Theorem 3.7. Suppose that (W, τ_1, τ_2) is an SVPNBTS. Let X and Y be two SVPNSs over W. Then,

(*i*) $X \subseteq P\text{-}P_{cl}(X)$;

(*ii*) $X \subseteq Y \Rightarrow P\text{-}P_{cl}(X) \subseteq P\text{-}P_{cl}(Y)$;

(*iii*) X is a pairwise SVPNCS iff $P\text{-}P_{cl}(X) = X$;

(*iv*) $P\text{-}P_{cl}(0_{PN}) = 0_{PN}$ and $P\text{-}P_{cl}(1_{PN}) = 1_{PN}$.

Proof. (i) From the Definition 3.10, we see that $P\text{-}P_{cl}(X) = \cap\{B: B$ is a pairwise SVPNCS and $X\subseteq B\}$. Since each $X\subseteq B$, so $X \subseteq \cap\{B: B$ is a pairwise SVPNCS and $X\subseteq B\}$. Therefore, $X \subseteq P\text{-}P_{cl}(X)$.

(ii) Suppose that X and Y be two SVPNSs over W such that $X\subseteq Y$. Then,

$P\text{-}P_{cl}(X)$

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

$= \cap\{B$: B is a pairwise SVPNCS and $X\subseteq B\}$

$\subseteq \cap\{B$: B is a pairwise SVPNCS and $Y\subseteq B\}$ [since $X\subseteq Y$]

$= P\text{-}P_{cl}(Y)$.

Therefore, $X\subseteq Y \Rightarrow P\text{-}P_{cl}(X)\subseteq P\text{-}P_{cl}(Y)$.

(iii) Suppose that X be a pairwise SVPNCS in (W, τ_1, τ_2). Now, $P\text{-}P_{cl}(X) = \cap\{B$: B is a pairwise SVPNCS and $X\subseteq B\}$. Since X is a pairwise SVPNCS in (W, τ_1, τ_2), so X is the smallest pairwise SVPNCS in (W, τ_1, τ_2), which contains X. Therefore, $\cap\{B$: B is a pairwise SVPNCS and $X\subseteq B\} = X$. Therefore, $P\text{-}P_{cl}(X) = X$.

(iv) It is known that, both the SVPNSs 0_{PN} and 1_{PN} are pairwise SVPNCSs in (W, τ_1, τ_2). So, by the third part of this theorem, we have $P\text{-}P_{cl}(0_{PN}) = 0_{PN}$, $P\text{-}P_{cl}(1_{PN}) = 1_{PN}$.

Theorem 3.8. Suppose that (W, τ_1, τ_2) is an SVPNBTS. Then, for any SVPNS X, $\tau_i\text{-}P_{int}(X) = P\text{-}P_{int}(X)$.

Proof. Suppose that (W, τ_1, τ_2) is an SVPNBTS, and X be an SVPNS over W. Now, $\tau_i\text{-}P_{int}(X) = \cup\{Y$: Y is an SVPNOS in (W, τ_i) and $Y\subseteq X\}$, i=1, 2. Since Y is an SVPNOS in (W, τ_i), so by second part of Theorem 3.1., Y is a pairwise SVPNOS in (W, τ_1, τ_2).

Therefore, $\tau_i\text{-}P_{int}(X)$

$= \cup\{Y$: Y is an SVPNOS in (W, τ_i) and $Y\subseteq X\}$

$= \cup\{Y$: Y is a pairwise SVPNOS in (W, τ_1, τ_2), and $Y\subseteq X\}$

$= P\text{-}P_{int}(X)$.

Hence, in an SVPNBTS (W, τ_1, τ_2), $\tau_i\text{-}P_{int}(X) = P\text{-}P_{int}(X)$ for any SVPNS X over W.

Theorem 3.9. Suppose that (W, τ_1, τ_2) is an SVPNBTS. Then, for any SVPNS X, $\tau_i\text{-}P_{cl}(X) \subseteq P\text{-}P_{cl}(X)$.

Proof. Assume that X be an SVPNS over W, and (W, τ_1, τ_2) be an SVPNBTS. Now, $\tau_i\text{-}P_{cl}(X) = \cap\{Y$: Y is an SVPNCS in (W, τ_i) and $X\subseteq Y\}$. Since Y is an SVPNCS in (W, τ_i), so by third part of Theorem 3.1., Y is a pairwise SVPNCS in (W, τ_1, τ_2).

Therefore, $\tau_i\text{-}P_{cl}(X)$

$= \cap\{Y$: Y is an SVPNCS in (W, τ_i) and $X\subseteq Y\}$

$= \cap\{Y$: Y is a pairwise SVPNCS in (W, τ_1, τ_2) and $X\subseteq Y\}$

$= P\text{-}P_{cl}(X)$.

Hence, $\tau_i\text{-}P_{cl}(X) = P\text{-}P_{cl}(X)$, for any SVPNS X in (W, τ_1, τ_2).

4. Conclusions

In this study, we establish the notion SVPNBTS as a generalization of SVPNTS and NBTS. Also, we present some of their basic properties. By defining SVPNBT and SVPNBTS, we present some well-described examples and establish some interesting results in the form of the theorem, remark, etc. via SVPNBTSs.

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, *On some new notations and functions in neutrosophic topological spaces*, Neutrosophic Sets and Systems, 16, 16-19 (2017).

[2] S. Das, *Neutrosophic Supra Simply Open Set and Neutrosophic Supra Simply Compact Space*. Neutrosophic Sets and Systems, 43, 105-113 (2021). DOI: 10.5281/zenodo.4914821

[3] S. Das, R. Das and C. Granados, *Topology on Quadripartitioned Neutrosophic Sets*. Neutrosophic Sets and Systems, 45, 54-61 (2021).

[4] S. Das, R. Das, C. Granados and A. Mukherjee, *Pentapartitioned Neutrosophic Q-Ideals of Q-Algebra*. Neutrosophic Sets and Systems, 41, 52-63 (2021).

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

[5] S. Das, R. Das, S. Pramanik and F. Smarandache, *Neutrosophic-Separation-Axioms.* Neutrosophic Sets and Systems, 49(1), 103-110, (2022).
[6] S. Das, R. Das and S. Pramanik, *Topology on Ultra Neutrosophic Set*, Neutrosophic Sets and Systems, 47, 93-104 (2021). DOI: 10.5281/zenodo.5775098
[7] S. Das, R. Das and S. Pramanik, *Single Valued Bipolar Pentapartitioned Neutrosophic Set and Its Application in MADM Strategy*, Neutrosophic Sets and Systems, 49, 145-163, (2022).
[8] S. Das, R. Das and S. Pramanik, Single Valued Pentapartitioned Neutrosophic Graphs. Neutrosophic Sets and Systems, 50, 225-238, (2022).
[9] S. Das, R. Das and B.C. Tripathy, *Multi-criteria group decision making model using single-valued neutrosophic set*. LogForum, 16(3), 421-429, (2020).
[10] R. Das, S. Das and B.C. Tripathy, *On Multiset Minimal Structure Topological Space*. Proceedings of International Mathematical Sciences, III(2), 88-97 (2021).
[11] S. Das, R. Das and B.C. Tripathy, *Neutrosophic Pre-I-open Set in Neutrosophic Ideal Bitopological Space*, Soft Computing, 26(12), 5457-5464, (2022).
[12] S. Das and A.K. Hassan, *Neutrosophic d-Ideal of Neutrosophic d-Algebra*. Neutrosophic Sets and Systems, 46, 246-253 (2021).
[13] S. Das and S. Pramanik, *Neutrosophic simply soft open set in neutrosophic soft topological space*. Neutrosophic Sets and Systems, 38, 235-243 (2020).
[14] S. Das and S. Pramanik, *Neutrosophic Tri-Topological Space*. Neutrosophic Sets and Systems, 45, 366-377 (2021).
[15] S. Das and S. Pramanik, *Generalized neutrosophic b-open sets in neutrosophic topological space*, Neutrosophic Sets and Systems, 35, 522-530 (2020).
[16] S. Das and S. Pramanik, *Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions*, Neutrosophic Sets and Systems, 38, 355-367 (2020).
[17] S. Das, B. Shil and S. Pramanik, *SVPNS-MADM strategy based on GRA in SVPNS Environment*, Neutrosophic Sets and Systems, 47, 50-65 (2021).
[18] S. Das, B. Shil and B.C. Tripathy, *Tangent Similarity Measure Based MADM-Strategy under SVPNS-Environment*, Neutrosophic Sets and Systems, 43, 93-104 (2021).
[19] S. Das and B.C. Tripathy, *Pairwise neutrosophic-b-open set in neutrosophic bitopological spaces*, Neutrosophic Sets and Systems, 38, 135-144 (2020).
[20] S. Das and B.C. Tripathy, *Neutrosophic simply b-open set in neutrosophic topological spaces*, Iraqi Journal of Science, 62(12), 4830-4838 (2021).
[21] S. Das and B.C. Tripathy, *Pentapartitioned Neutrosophic Topological Space*. Neutrosophic Sets and Systems, 45, 121-132 (2021).
[22] R. Dhavaseelan and S. Jafari, *Generalized neutrosophic closed sets*, New trends in neutrosophic theory and applications, 2, 261-273 (2018).
[23] R. Dhavaseelan, R. Narmada Devi, S. Jafari and Q.H. Imran, *Neutrosophic α^m-continuity*, Neutrosophic Sets and Systems, 27, 171-179 (2019).
[24] E. Ebenanjar, J. Immaculate and C.B. Wilfred, *On neutrosophic b-open sets in neutrosophic topological space*, Journal of Physics Conference Series, 1139(1), ID. 012062 (2018).
[25] S. Ganesan and F. Smarandache, *Neutrosophic biminimal α-open sets*, Bulletin Of The International Mathematical Virtual Institute, 11(3), 545-553 (2021).
[26] Q.H. Imran, F. Smarandache, R.K. Al-Hamido and R. Dhavaseelan, *On neutrosophic semi-α-open sets*, Neutrosophic Sets and Systems, 18, 37-42 (2017).
[27] H. Wang, F. Smarandache, Y.Q. Zhang and R. Sunderraman, *Single valued neutrosophic sets*. Multispace and Multistructure, 4, 410-413 (2010).
[28] J. C. Kelly, *Bitopological Spaces*, Proceedings of the London Mathematical Society, 3(1), 71-89 (1963).
[29] P. Majumder, S. Das, R. Das and B. C. Tripathy, *Identification of the Most Significant Risk Factor of COVID-19 in Economy Using Cosine Similarity Measure under SVPNS-Environment*, Neutrosophic Sets and Systems, 46, 112-127 (2021).
[30] R. Mallick and S. Pramanik, *Pentapartitioned neutrosophic set and its properties*. Neutrosophic Sets and Systems, 36, 184-192 (2020).
[31] I. Mohammed Ali Jaffer and K. Ramesh, *Neutrosophic generalized pre regular closed sets*. Neutrosophic Sets and Systems, 30, 171-181 (2019).
[32] D.D. Mwchahary and B. Basumatary, *A Note on Neutrosophic Bitopological Spaces*, Neutrosophic Sets and Systems, 33, 134-144 (2020).
[33] T.Y. Ozturk and A. Ozkan, *Neutrosophic Bitopological Spaces*, Neutrosophic Sets and Systems, 30, 88-97 (2019).
[34] M.H. Page and Q.H. Imran, *Neutrosophic generalized homeomorphism*, Neutrosophic Sets and Systems, 35, 340-346 (2020).

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024

[35] A. Pushpalatha and T. Nandhini, *Generalized closed sets via neutrosophic topological spaces*, Malaya Journal of Matematik, 7(1), 50-54 (2019).
[36] V.V. Rao and R. Srinivasa, *Neutrosophic pre-open sets and pre-closed sets in neutrosophic topology*, International Journal of Chem Tech Research, 10(10), 449-458 (2017).
[37] P. Saha, P. Majumder, S. Das, P.K. Das and B.C. Tripathy, *Single-Valued Pentapartitioned Neutrosophic Dice Similarity Measure and Its Application in the Selection of Suitable Metal Oxide Nano-Additive for Biodiesel Blend on Environmental Aspect*. Neutrosophic Sets and Systems, 48, 154-171 (2022).
[38] A.A. Salama and S.A. Alblowi, *Neutrosophic set and neutrosophic topological space*, ISOR Journal of Mathematics, 3(4), 31-35 (2012).
[39] B. Shil, R. Das, C. Granados, S. Das and B.D. Chowdhury, *Hyperbolic Cosine Similarity Measure Based MADM-Strategy under the SVNS Environment*. Neutrosophic Sets and Systems, 50, 409-419, (2022).
[40] B. Shil, S. Das, R. Das and S. Pramanik, *Single Valued Pentapartitioned Neutrosophic Soft Set*. Neutrosophic Sets and Systems, 50, 225-238, (2022).
[41] F. Smarandache, *A unifying field in logics, neutrosophy: neutrosophic probability, set and logic*, Rehoboth: American Research Press, (1998).
[42] D. Sreeja and T. Sarankumar, *Generalized Alpha Closed sets in Neutrosophic topological spaces*, Journal of Applied Science and Computations, 5(11), 1816-1823 (2018).
[43] B.C. Tripathy and S. Das, *Pairwise Neutrosophic b-Continuous Function in Neutrosophic Bitopological Spaces*, Neutrosophic Sets and Systems, 43, 82-92 (2021).
[44] B.C. Tripathy and D.J. Sarma, *Pairwise generalized b-R_0 spaces in bitopological spaces*, Proyecciones Journal of Mathematics, 36(4), 589-600 (2017).
[45] G.R. Vadiraja Bhatta, K.J. Manasa, B. Gautham Shenoy, P. Prasanna and B.J. Chaithra, *Introduction to NeutroNearring*. Neutrosophic Sets and Systems, 46, 445-455, (2021).
[46] G.R. Vadiraja Bhatta, P. Shashi Kant, M. Prasanna and P. Prasanna, *ClassicalBalanced, AntiBalanced and NeutroBalanced functions*. Neutrosophic Sets and Systems, 48, 386-398, (2022).

Doi: https://doi.org/10.54216/IJNS.230415
Received: June 24, 2023 Revised: January 19, 2024 Accepted: March 08, 2024