$Beta^{m}$-Closed Sets in Fuzzy Neutrosophic Topological Spaces

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Abstract

The current work offers a new concept of sets and called $f$uzzy $n$eutrosophic $Beta^{m}$-closed sets in fuzzy neutrosophic topology. In fact, the research is an extended form of a research conducted by F. M. Mohammed et.al. [1-7]. It explores a number of noteworthy examples to shed the light on the new characteristics and attributes of these recently formed conceptions, as well as some associated interactions between them.

Keywords: fuzzy neutrosophic closed sets; fuzzy neutrosophic Beta^{m}-closed sets; fuzzy neutrosophic topology.

1.Introduction

The fuzzy concept has invaded almost all branches of mathematics since the definition of the concept by Zadeh [8]. The applications of fuzzy sets has been presents in many fields such as the theory of fuzzy topological spaces which was studied and developed by Chang [9]. Since then various concepts in general topology have been generalized? (what do you mean? Rephrase) to Chang's fuzzy topological spaces. On the other hand, as a generalization of fuzzy topological spaces developed in many directions by the concept of neutrosophic sets where the term of neutrosophic set was defined with membership, non-membership and indeterminacy degrees by Smarandche [10] and topological spaces of neutrosophic sets by A. Salama et.al.[11]. After that a survey article of the developed areas of fuzzy neutrtrosophic topological spaces has been published by Arockiarani [12].

Additionally, the study has been conducted in other fruitful research areas (See for example, [1-7] and [13-15]) when they studied various notions in fuzzy neutrosophic topological space.

Hence, the aim of this paper is to introduce and study the concept of β^m-closed sets in the case of fuzzy neutrosophic. Several new characteristics and instances based on this specified idea have been addressed.

2. Basic definitions and terminologies

Definition 2. 1 [15]: Let X_N be a non-empty fixed set. The $f$uzzy $n$eutrosophic set (FNS), S_N is an object having the form $S_N= \{< x, \mu_{S_N}(x), \sigma_{S_N}(x), \upsilon_{S_N}(x) >: x \in X_N \}$ where the functions $\mu_{S_N}, \sigma_{S_N}, \upsilon_{S_N}: X_N \rightarrow [0, 1]$ denote the degree of membership function (namely $\mu_{S_N}(x)$), the degree of indeterminacy function (namely $\sigma_{S_N}(x)$) and the degree of non-membership function (namely $\upsilon_{S_N}(x)$) respectively of each element $x \in X_N$ to the set S_N and $0 \leq \mu_{S_N}(x)+\sigma_{S_N}(x)+\upsilon_{S_N}(x) \leq 3$, for each $x \in X_N$.

Remark 2. 2 [15]: FNS $S_N= \{< x, \mu_{S_N}(x), \sigma_{S_N}(x), \upsilon_{S_N}(x) >: x \in X_N \}$ can be identified

to an ordered triple $< x, \mu_{S_N}, \sigma_{S_N}, \upsilon_{S_N} >$ in [0, 1] on X_N.

Lemma 2. 3 [15]: Let X_N be a non-empty set and the FNSs λ_N and β_N be in the form:

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S_N= {< x, μ_{S_N}(x), σ_{S_N}(x), v_{S_N} (x) >: x ∈ X_N } and β_N = {< x, $\mu_{\beta N}$(x), $\sigma_{\beta N}$(x), $\nu_{\beta N}$(x) >: x ∈ X_N } on X_N .

Then,

i. $S_N \subseteq \beta_N$ iff $\mu_{S_N}(x) \le \mu_{\beta N}(x)$, $\sigma_{S_N}(x) \le \sigma_{\beta N}(x)$ and $v_{S_N}(x) \ge \nu_{\beta N}(x)$ for all x ∈ X_N ,

ii. $S_N = \beta_N$ iff $S_N \subseteq \beta_N$ and $\beta_N \subseteq S_N$,

iii. 1_N-S_N = {< x, v_{S_N}(x), 1-σ_{S_N}(x), μ_{S_N}(x)>: x∈ X_N },

iv. $S_N \cup \beta_N$ = {< x, Max(μ_{S_N}(x), $\mu_{\beta N}$(x)), Max(σ_{S_N}(x), $\sigma_{\beta N}$(x)), Min(v_{S_N}(x), $\nu_{\beta N}$(x)) >: x∈ X_N },

v. $S_N \cap \beta_N$ = {< x, Min(μ_{S_N}(x), $\mu_{\beta N}$(x)), Min(σ_{S_N}(x), $\sigma_{\beta N}$(x)), Max(v_{S_N}(x), $\nu_{\beta N}$(x)) >: x∈ X_N },

vi. 0_N = < x, 0, 0, 1 > and 1_N = < x, 1, 1, 0 >.

Definition 2.4 [15]: Fuzzy $n$eutrosophic $t$opology (FNT) on a non-empty set X_N is a family τ of fuzzy neutrosophic subsets in X_N satisfying the following axioms.

i. 0_N, $1_N \in \tau_N$,

ii. $S_{N_1} \cap S_{N_2} \in \tau_N$ for any λ_{N1}, $\lambda_{N2} \in \tau_N$,

iii. $\cup S_{N_j} \in \tau_N$, ∀ {S_{N_j}: j∈ J} ⊆ τ_N.

The pair (X_N , τ_N) is called $f$uzzy $n$eutrosophic $t$opological space ($FNTS$). Every elements of τ are called $f$uzzy $n$eutrosophic-open sets (FN-open set). The complement of FN-open set in the FNTS (X_N , τ_N) is called $f$uzzy $n$eutrosophic -closed set (FN-closed set).

Definition 2.5 [15]: Let (X_N , τ_N) is FNTS and S_N= {< x, μ_{S_N}(x), σ_{S_N}(x), v_{S_N} (x) >: x∈ X_N } is FNS in X_N . Then the $f$uzzy $n$eutrosophic -closure (FNcl) and the $f$uzzy $n$eutrosophic -interior (FNint) of λ_N are defined by:

FNcl(S_N) = ∩ {β_N: β_N is FN-closed set in X_N and $S_N \subseteq \beta_N$},

FNint (S_N) = ∪ {β_N: β_N is FN-open set in X_N and $\beta_N \subseteq S_N$}.

Now, the FNcl(S_N) is FN-closed set and FNint(S_N) is FN-open set in X_N .

Further,

i. S_N is FN-closed set in X_N iff FNcl(S_N) = S_N,

ii. S_N is FN-open set in X_N iff FNint(S_N) = S_N.

Proposition 2.6 [6]: Let (X_N , τ_N) is FNTS and S_N, β_N are FNSs in X_N . Then the following properties hold:

i. FNint(S_N) ⊆ S_N and S_N ⊆ FNcl(S_N),

ii. $S_N \subseteq \beta_N \Longrightarrow$ FNint(S_N) ⊆ FNint(β_N) and $S_N \subseteq \beta_N \Longrightarrow$ FNcl(S_N) ⊆ FNcl(β_N),

iii. FNint(FNint(S_N)) = FNint(S_N) and FNcl(FNcl(S_N)) = FNcl(S_N),

iv. FNint($S_N \cap \beta_N$) = FNint(S_N) ∩ FNint(β_N) and FNcl($S_N \cup \beta_N$) = FNcl(S_N) ∪ FNcl (β_N),

v. FNint(1_N) = 1_N and FNcl(0_N) = 0_N.

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Definition 2.7 [7]: FNS S_N in FNTS (X_N , τ_N) is called:

i. Fuzzy $n$eutrosophic semi-closed set (FNS-closed set) if FNint(FNcl(S_N)) $\subseteq S_N$.

ii. Fuzzy $n$eutrosophic pre-closed set (FNP-closed set) if FNcl(FNint(S_N)) $\subseteq S_N$.

iii. Fuzzy $n$eutrosophic α-closed set (FNα-closed set) if FNcl(FNint(FNcl(S_N)))$\subseteq S_N$.

v. Fuzzy $n$eutrosophic β-closed set (FN β -closed set) if FNint(FNcl (FNint (S_N)))$\subseteq S_N$.

The complement of Fuzzy $n$eutrosophic semi-closed set is Fuzzy $n$eutrosophic semi-open set, Fuzzy $n$eutrosophic pre-closed set is Fuzzy $n$eutrosophic pre-open set , Fuzzy $n$eutrosophic α-closed set is , Fuzzy $n$eutrosophic α-open set and Fuzzy $n$eutrosophic β-closed set is Fuzzy $n$eutrosophic β-open set respectively.

3. Fuzzy Neutrosophic Beta^^{m}-Closed Sets in Fuzzy Neutrosophic Topological Spaces.

Definition 3.1: Let (X_N , τ_N) be FNTS for each $S_N, M_N \in X_N$. A fuzzy set S_N is called fuzzy neutrosophic β^m-closed set (FN β^m-closed set). If FNcl (FNint (S_N)) $\subseteq M_N$ where $S_N \subseteq M_N$ andM_N is an $FN\beta$-open set in X_N . A S_N is called an fuzzy neutrosophic β^m open (FNβ^m-open set) if and only if 1_N- S_N is an FNβ^m-closed set.

Example 3.2: Let X_N = {x} define FNSs $\omega_N$and δ_N, in X_N as follows:

ω_N = < x, 0.2 ,0.5, 0.4> and δ_N= < x, 0.1 ,0.2, 0.4 >,

The family, τ_N = {0_N, 1_N, ω_N, δ_N } be FNT.

Now if, S_N = < x, 0.3 ,0.7, 0.6 > and M_N = < x, 0.5 ,0.9, 0.3> where,

M_N is FNβ-open set such that, $S_N \subseteq M_N$. Then, FNInt (S_N) = 0_N and FNCl(FNInt (S_N)) = 0_N.

That is, FNCl(FNInt (S_N)) $\subseteq M_N$. Since, < x, 0,0,1> $\subseteq$ < x, 0.5 ,0.9, 0.3>.

Therefore, S_N is FN$\beta^{\wedge m}$-closed set.

Theorem 3.3 : Let (X_N,τ_N) be FNTS and S_N is FNβ^m-open set in X_N if and only if S_N^C is FNβ^m-closed set.

Proof : Let S_N be an FNβ^m-open set such that $M_N \leq S_N$ where M_N is an FNβ-closed set .

That means, 1_N- $S_N \leq 1_N$- M_N where (1_N- M_N) is an FNβ-open set.

Now , if 1_N- S_N is an FNβ-closed set , so FNInt (FNCl(FNInt (1_N- S_N) $\leq$ (1_N- M_N)

That is $1_N-(1_N$- M_N) = $M_N \leq 1_N$- FNInt (FNCl (1_N- S_N) , FNCl(FNInt (S_N) $\subseteq$ M_N.

Therefor S_N is FNβ^m-closed set.

Conversely: Assume $M_N \leq$ FNCl (FNInt (S_N) , $M_N \leq S_N$ where M_N is an FNβ-closed set .

That mean 1_N- $S_N \leq 1_N$- M_N where (1_N- M_N) is an FNβ-open set.

And 1_N- $M_N \leq$ S_N so 1_N- $M_N \leq$ FNCl (FNInt (S_N)

1_N- FNCl (FNInt (S_N) $\leq$ M_N then FNInt (FNCl (1_N- S_N) $\leq$ M_N

1_N- S_N is an FNβ^m-closed set.Therefor S_N is FNβ^m-open set.

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Proposition 3. 4: For any FNS, the following statements satisfy:

i- Every FN-open set is FNβ-open set,
ii- Every FN-closed set is FNβ^m-closed set,
iii- Every FNS-closed set is FNβ^m-closed set,
iv- Every FNP-closed set is FNβ^m-closed set,
v- Every FNβ^m-closed set is FNP-closed set,
vi- Every FNα-closed set is FNβ^m-closed set,

Proof: i- Let S_N= {< x, μ_{S_N}(x), σ_{S_N}(x), ν_{S_N} (x) >: x∈ X_N} be FN-open set in FNTS (X_N, τ_N).Then, by **definition 2.5 (ii)**. We have, S_N = FNint(S_N)…… (1).

And, by **proposition 2.7 (i).** We get, S_N ⊆ FNcl(S_N).

But, S_N ⊆ FNInt(FNCl(S_N)). Then, FNCl(S_N) ⊆ FNCl(FNInt(FNCl(S_N))).

Therefore, by (1). We get, S_N⊆ FNCl (FNInt (FNCl(S_N))). Hence, S_N is FNβ-open set in (X_N, τ).

ii- Let S_N be FN-closed set in FNTS (X_N, τ). Then, by **definition 2.6 (i).** We have, S_N= FNcl(S_N)...... (1*).

And, by **Proposition 2.7 (i).** We get, S_N ⊆ FNCl(S_N) …… (2*).

But, FNCl(FNInt (S_N)) ⊆ FNcl(S_N).Then, by (1*). We get, FNCl(FNInt (S_N)) ⊆ S_N.

Now, let M_N be FN-open set such that, S_N ⊆ M_N.

By **Proposition 3.4 (i)** If, M_N is FN-open set. Then, M_N will be FNβ-open set in (X_N, τ_N).

Then, FNCl(FNInt (S_N))⊆ S_N⊆ M_N. Therefore, FNCl(FNInt (S_N))⊆ M_N. Hence, S_N is FNβ^m-closed set.

iii- Let S_N be FNS-closed set in FNTS (X_N, τ_N). Then, FNInt (FNCl(S_N)) ⊆ S_N.

Now, let M_N be FN-open set such that, S_N ⊆ M_N so by, **Proposition 3.4 (i).** If, M_N is FN-open set. Then,

M_N will be FNβ-open set.That is, FNCl(FNInt (S_N)) ⊆ S_N⊆ M_N.

Therefore, FNCl(FNInt (S_N)) ⊆ M_N. Hence, S_N is FNβ^m-closed set in (X_N, τ_N).

iv- Let S_N be FNP-closed set in FNTS (X_N, τ_N). Then, FNCl(FNInt (S_N)) ⊆ S_N.

Now, suppose M_N is FN-open set such that, S_N ⊆ M_N.So by, **Proposition 3.4 (i)**. If, M_N is FN-open set.

Then, is FNβ-open set. So, FNCl(FNInt (S_N)) ⊆ S_N⊆ M_N. Therefore, FNCl(FNInt (S_N)) ⊆ M_N.

Hence, S_N is FNβ^m-closed set in (X_N, τ_N).

v- Let S_N be FNβ-closed set in FNTS (X_N, τ_N). Then, FNCl(FNInt (S_N)) ⊆ M_N.Now, let M_N is FN-open set such that, S_N ⊆ M_N.So by, **Proposition 3.4 (i)**. If, M_N is FN-open set. Then, is FNβ-open set.

But, FNCl(FNInt (S_N)) ⊆ S_N⊆ M_N. Therefore, FNCl(FNInt (S_N)) ⊆ S_N. Hence, S_N is FNP-closed set.

vi- Let S_N be FNα-closed set in FNTS (X_N, τ_N). Then, FNcl(FNint(FNcl(S_N))) ⊆ S_N.

Now, let M_N be FN-open set such that, S_N ⊆ M_N.

So by **Proposition 3.4.(i)** If, M_N is FN-open set. Then, is FNβ-open set.

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Then, FNcl (FNint (S_N)) $\subseteq$ FNcl(FNint(FNcl(S_N))) $\subseteq S_N \subseteq M_N$.

Therefore, FNcl (FNint (S_N)) $\subseteq M_N$. Hence, S_N is FNβ^m-closed set in (X_N, τ_N).

Remark 3.5: The convers of Proposition 3.4 is not true in general as shown by the following example:

Example 3.6: i- Let X_N = {x} define the FNSs by ω_N= < x, 0.7, 0.6, 0.5 > in X_N.

And the family, τ_N = {0_N, $1_{N,}$ ω_N} is FNT. Now if, S_N = < x, 0.8, 0.6, 0.5 >.

Then, FNCl(S_N) = 1_N , FNInt (FNCl(S_N)) = 1_N and

FNCl (FNInt (FNCl(S_N))) =1_N. So, $S_N \subseteq$ FNCl (FNInt (FNCl(S_N))).

Hence, S_N is FNβ-open set. But, not FN-open set.

ii-Take, **Example 3.2.** Then, S_N is FNβ^m-closed set but, not FN-closed set.

iii- Let X_N = {a, b, c} define FNSs λ_N and $\delta_N$in X as follows:

λ_N= < x, a(0.05, 0.25 , 0.75), b(0.25, 0.3, 0.2) , c(0.05, 0.07, 0.25)>, and

δ_N = < x, a(0.25, 0.3 , 0.3), b(0.4, 0.4, 0.1) , c(0.1, 0.1, 0.2)>.

And the family, τ_N = {0_N, 1_N, λ_N, δ_N} is FNT.

Such that, 1_N-τ_N = {1_N, 0_N, < x, a(0.75, 0.75, 0.05) b(0.2, 0.7, 0.25) , c(0.25, 0.93, 0.05) > , < x, a(0.3, 0.7 ,0.25) ,b(0.1, 0.6, 0.4) , c(0.2, 0.9, 0.1)> }.

Now if, S_N = < x, a(0.05, 0.35 , 0.70), b(0.2, 0.4, 0.15) , c(0.2, 0.1, 0)> and

M_N = < x, a(0. 5, 0.4 , 0.3), b(0.3, 0.5, 0.1) , c(0.3, 0.4, 0) > where, M_N is FNβ-open set such that,

$S_N \subseteq M_N$. Then, FNInt (S_N) = 0_N and FNCl(FNInt (0_N)) = 0_N

Since, FNCl(FNInt (S_N)) $\subseteq M_N$. Hence, S_N is FNβ^m-closed set.

But,FNInt (S_N) =1_N, FNInt (FNCl (1_N)) = 1_N

Therefore, FNInt (FNCl (S_N)) $\nsubseteq S_N$. Hence, S_N is not FNS-closed set.

iv- Let X_N = $\{x\}$ define FNSs λ_N and $\delta_N$in X_N as follows:

λ_N= < x, 0.05 , 0.5, 0.8>, and δ_N = < x, 0.4, 0.5, 0.2>.

And the family, τ_N = {0_N, 1_N, λ_N, δ_N} is FNT.

Now if, S_N = < x, 0.2, 0.5 ,0.3> and M_N = < x, 0.3, 0.5, 0.1 > where, M_N is FNβ-open set such that, $S_N \subseteq M_N$.Then, FNInt (S_N) = 0_N and FNCl(FNInt (0_N)) = 0_N

Therefore, FNCl(FNInt (S_N)) $\subseteq M_N$.

Hence, S_N is FNβ^m-closed set. But, FNInt (S_N) =1_N, FNInt (FNCl (1_N)) = 1_N .

Therefore, FNInt (FNCl (S_N)) $\nsubseteq S_N$. Hence, S_N is not FNα-closed set.

Proposition 3.7: If S_N is FNβ^m-closed set and $S_N \subseteq \eta_N \subseteq$ FNCl(FNInt (S_N)). Then η_N is FNβ^m-closed set.

Proof: Let S_N be FNβ^m-closed set such that, $S_N \subseteq \eta_N \subseteq$ FNCl(FNInt (S_N)).

Now, let M_N be FNβ-open set such that, $\eta_N \subseteq M_N$. Since, S_N is FNβ^m-closed set.

Then we have, FNCl(FNInt (S_N))$\subseteq M_N$, where $S_N \subseteq M_N$.

Since, $\lambda_N \subseteq \eta_N$ and $\eta_N \subseteq$ FNCl(FNInt (S_N))we get,

FNCl(FNInt (η_N)) $\subseteq$ (FNcl(FNint(FNcl(λ_N))) $\subseteq$ FNCl(FNInt (S_N))$\subseteq M_N$.

Therefore, FNCl(FNInt (η_N))$\subseteq M_N$. Hence, η_N is FNβ^m-closed set in (X_N, τ_N).

Proposition 3.8: The intersection of two FNβ^m-closed sets is also FNβ^m-closed set.

Proof: Let S_N and β_N be FNS-closed sets on FNTS (X_N, τ_N).

Then, FNCl(FNInt(S_N)) $\subseteq S_N$……. (1).

And, FNCl(FNInt(β_N)) $\subseteq \beta_N$……. (2).

Consider $S_N \cap \beta_N \supseteq$ FNCl(FNInt(S_N)) $\cap$ FNCl(FNInt(β_N))

= FNCl (FNInt (S_N) $\cap$ FNInt(β_N)) $\supseteq$ FNCl(FNInt ($S_N \cap \beta_N$)).

Therefore, FNCl(FNInt ($S_N \cap \beta_N$)) $\subseteq S_N \cap \beta_N$.

Now, let M_N be FN-open set such that, $S_N \cap \beta_N \subseteq M_N$. So by Proposition 2.1.4. If, M_N is FN-open set.

Then, M_N is FNβ-open set. Then, FNCl(FNInt ($S_N \cap \beta_N$)) $\subseteq S_N \cap \beta_N \subseteq M_N$.

So, FNCl(FNInt ($S_N \cap \beta_N$)) $\subseteq M_N$. Hence, $S_N \cap \beta_N$ is FNβ^m-closed set in (X_N, τ_N).

Remark 3.9: The union of two FNβ^m-closed sets not necessary to be FNβ^m- closed set as shown by the following example:

Example 3.10: Let $X_N = \{x\}$ define FNSs λ_N , δ_N ,ϑ_N and ∂_N in X as follows:

λ_N= < x, 0.05 , 0.5, 0.8> and δ_N = < x, 0.4, 0.5, 0.2> and the family, $\tau_{N_1} = \{0_N, 1_N, \lambda_N, \delta_N\}$ is FNT.

Now if, S_{N_1}= < x, 0.2, 0.5 ,0.3> and M_{N_1} = < x, 0.3, 0.5, 0.1 > where, M_N is FNβ-open set such that,

$S_{N_1} \subseteq M_{N_1}$.Then, FNInt ($S_{N_1}$) = 0_N and FNCl(FNInt (0_N)) = 0_N

Therefore, FNCl(FNInt (S_{N_1})) $\subseteq M_{N_1}$. That is, $S_{N_1}$is FN$\beta^{\wedge m}$-closed set.

ϑ_N= < x, 0.3 , 0.1, 0.6> and ∂_N = < x, 0.1, 0, 0.8>.

The family, $\tau_{N_2} = \{0_N, 1_N, \vartheta_N, \partial_N\}$ is FNT.

Such that, 1_N-τ_{N_2} = {1_N, 0_N, < x, 0.6, 0.9, 0.3>, < x, 0.8, 1, 0.1> }.

Now if, S_{N_2}= < x, 0.4, 0.5 ,0.7> and M_{N_2}= < x, 0.9, 0.5, 0.5 > where, $M_{N_2}$is FNβ-open set such that,

$S_{N_2} \subseteq M_{N_2}$.Then, FNInt ($S_{N_2}$) = 0_N and FNCl(FNInt (0_N)) = 0_N

Therefore, FNCl(FNInt (S_{N_2})) $\subseteq M_{N_2}$. Hence, $S_{N_2}$is FNβ^m-closed set. But $S_{N_1} \cup S_{N_2}$is not FNβ^m-closed set .

Definition 3.11: Let (X_N, τ_N) be FNTS and $S_N$be FNS in X_N. Then, the fuzzy neutrosophic β^m-closure

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(FNβ^mcl) and the fuzzy neutrosophic β^m-interior (FNβ^mint) of S_N are defined by:

i. FNβ^mcl(S_N) = ∩ {β_N: β_N is FNβ^m-closed set in X_N and $S_N \subseteq \beta_N$},

ii. FNβ^mint(S_N) = ∪ {β_N: β_N is FNβ^m-open set in X_N and $\beta_N \subseteq S_N$ }.

Theorem 3.12: Let (X_N, τ_N) be FNTS and S_N, β_N are FNSs in X_N. Then, the following properties hold:

i. FNβ^mcl(0_N) = 0_N and FNβ^m-cl (1_N) =1_N,

ii. $S_N \subseteq$ FNβ^m-cl(S_N),

iii. If, $S_N \subseteq \beta_N$. Then, FNβ^mcl(S_N) ⊆ FNβ^mcl(β_N),

iv. S_N is FNβ^m-closed set in X_N iff FNβ^mcl (S_N) = S_N,

v. FNβ^mcl(S_N) = FNβ^mcl(FNβ^mcl(S_N)).

Proof: i. By, **definition 3.11 (i)**. We have,

FNβ^mcl(0_N) = ∩ {β_N: β_N is FNβ^m-closed set in X_N and $0_N \subseteq \beta_N$} = 0_N.

And, FNβ^mcl(1_N) = ∩ {β_N: β_N is FNβ^m-closed set in X_N and $1_N \subseteq \beta_N$} =1_N.

ii. $\lambda_N \subseteq$ ∩ {β_N: β_N is FNβ^m-closed set in X_N and $\lambda_N \subseteq \beta_N$ } = FNβ^mcl (λ_N).

iii. Suppose that $S_N \subseteq \beta_N$. Then,

∩ {β_N: β_N is FNβ^m-closed set in X_N and $S_N \subseteq \beta_N$} ⊆ ∩ {η_N: η_N is FNβ^m-closed set in X_N and $\beta_N \subseteq \eta_N$}.

Therefore, FNβ^mcl(S_N) ⊆ FNα^mcl(β_N).

iv. If, S_N is FNβ^m-closed set. Then,

FNβ^mcl (S_N) = ∩ {β_N: β_N is FNβ^m-closed set in X_N and $S_N \subseteq \beta_N$}.

And, by (ii). We get, $S_N \subseteq$ FNβ^mcl (S_N) but, S_N is necessarily to be the smallest set.

Thus, S_N = ∩ {β_N: β_N is FNβ^m-closed set in $X_N$and $S_N \subseteq \beta_N$}.Therefore, S_N= FNβ^mcl (S_N).

Conversely; assume that S_N= FNβ^mcl (S_N) by using **definition 3.11 (i).** We get, S_N is FNβ^m-closed set.

v. By, (iv). We get, S_N = FNβ^mcl (S_N). Then, FNβ^mcl (S_N) = FNβ^mcl(FNβ^mcl(S_N)).

Theorem 3.13: Let (X_N, τ_N) is FNTS and S_N, β_N are FNSs in X_N. Then, the following properties hold:

i. FNβ^mint(0_N) = 0_N and FNβ^mint (1_N) =1_N,

ii. FNβ^mint(S_N) ⊆ S_N,

iii. If, $S_N \subseteq \beta_N$. Then, FNβ^mint(S_N) ⊆ FNβ^mint(β_N),

iv. S_N is FNβ^m-open in X_N iff S_N = FNβ^mint (S_N),

v. FNβ^mint(S_N) = FNβ^mint(FNβ^mint(S_N)).

Proof: i. By, **Definition 3.11**

(ii). We have, FNβ^mint(0_N) = ∪ {β_N: β_N is FNβ^m-open set in X_N and $\beta_N \subseteq 0_N$} = 0_N.

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And, FNβ^mint(1_N) = ∪ {β_N: β_N is FNβ^m-open set in X_N and $\beta_N \subseteq 1_N$} =1_N.

ii. Follows from **definition 3.11 (ii).**

iii. FNβ^mint(S_N) = ∪ {β_N: β_N is FNβ^m-open set in X_N and $\beta_N \subseteq S_N$}.

Since, $S_N \subseteq \beta_N$. Then, ∪ {β_N: β_N is FNβ^m-open set in X_N and $\beta_N \subseteq S_N$} ⊆ ∪ {η_N: η_N is FNβ^m-open set in X_N and $\eta_N \subseteq \beta_N$}.

Therefore, FNβ^mint(S_N) ⊆ FNβ^mint(β_N).

iv. Suppose that S_N is FNβ^m-open set in X_N.

Then, $S_N \subseteq$ FNβ^mint(S_N) …… (1).

By using (ii). We get, FNβ^mint(S_N) ⊆ S_N…… (2).

From (1) and (2) we have, S_N = FNβ^mint (S_N). Conversely; assume that S_N = FNβ^mint (S_N) by using

definition 3.11 (ii). We get, S_N is FNβ^m-open set in X_N.

v. By, (iv). We get, S_N = FNβ^mint(S_N). Then, FNβ^mint(S_N) = FNβ^mint(FNβ^mint(S_N)).

Theorem 3.14: Let (X_N, τ_N) be FNTS. Then, for any fuzzy neutrosophic subsets $S_N$of X_N, so we get:

i. 1_N- (FNβ^mint(S_N)) = FNβ^mcl(1_N-S_N),

ii. 1_N- (FNβ^mcl(S_N)) = FNβ^mint(1_N-S_N).

Proof: i. FNβ^mint(S_N) = ∪ {β_N: β_N is FNβ^m-open set in X_N and $\beta_N \subseteq S_N$}.

By the complement we have, 1_N- (FNβ^mint(S_N)) = 1_N- (∪ {β_N: β_N is FNβ^m-open set in X_N and $\beta_N \subseteq S_N$}).

So, 1_N- (FNβ^mint(S_N)) = ∩ {(1_N-β_N): (1_N-β_N) is FNβ^m-closed set in X_N

and (1_N-S_N) ⊆ (1_N-β_N)}. Now, replacing (1_N-β_N) by η_N we have,

1_N- (FNβ^mint(S_N)) = ∩ {η_N: η_N is FNβ^m-closed set in X_N and (1_N-S_N) ⊆ η_N} = FNβ^mcl(1_N-S_N)

ii. FNβ^mcl(S_N) = ∩ {β_N: β_N is FN β^m-closed set in X_N and $S_N \subseteq \beta_N$ }.By the complement we have,

1_N- (FNβ^mcl(S_N)) = 1_N- (∩ {β_N: β_N is FNβ^m-closed set in X_N and $S_N \subseteq \beta_N$}).

So, 1_N- (FNβ^mcl(S_N)) = ∪ {(1_N-β_N): (1_N-β_N) is FNβ^m-open set in X_N and (1_N-β_N) ⊆ (1_N- S_N)}.

Again replacing (1_N-β_N) by η_N we have,

1_N- (FNβ^mcl(S_N)) = ∪ {η_N: η_N is FNβ^m-open set in X_N and η_N ⊆ (1_N- S_N) } = FNβ^mint(1_N-S_N).

Remark 3.15: The relationship between different sets in FNTS (X_N, τ_N) can be shown in the next diagram. But, the convers is not true in general.

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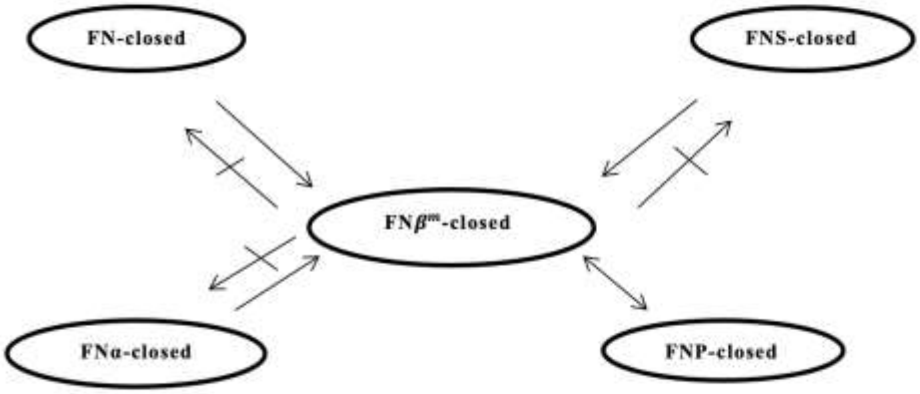

Figure 1: FNBeta^m-closed sets

4. Conclusion

In the course of study, a new terminology with respect to the theory of fuzzy neutrosophic sets has been defined, which is called called fuzzy neutrosophic Betam- closed. The study has proposed some essential characteristics and attributes of this recently established concept. Some relations between the defined model with other sets based of fuzzy neutrosophic topological spaces has been clarified.

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