Prospects for Applied Mathematics and Data Analysis

♦ ISSN: 2836-4449 Vol: 03, No. 01, 2023 ♦ PP. 01–08

♦ Article DOI: https://doi.org/10.54216/PAMDA.030101

FULL LENGTH ARTICLE

Studying the isotherm of the complementary Schaefer Ignaczak thermodynamical process of the first plane state of elastic strains for the unbounded micropolar body—Fourier Schaefer-Ignaczak formulas

Kheder Manhal Al-Saleh[1] **Mountajab Al-Hasan**[2] **Monir Makhlouf**[3]

[1]PHD Student, Department of Mathematics, Albaath University, Homs, Syria

[2]Prof., Department of Mathematics, Albaath University, Homs, Syria

[3]Department of Mathematics, Albaath University, Homs, Syria

Emails: alsalehkheder@gmail.com; malhsan@albaath-univ.edu.sy; Monirmaklohf@albaath-univ.edu.sy

Received: May 2023 Revised: August 2023 Accepted: December 2023 ⋆ Corresponding author

ABSTRACT

This paper concerns the Ignaczak stress-temperature distribution [2] of the homogenous isotropic 2D micropolar thermodynamical in the first plane state of elastic strain, which discussed by Eringen [9] and Nowacki [8]. The analysis uses the Schaefer-Ignaczak method to separate the classical and complementary fields. In the paper, we do the following; We prove that the complementary Schaefer-Ignaczak process is an isothermal process for infinite 2D (E-N:5) [6,8], with no stresses and temperature at infinity, and then we find the related Fourier Schaefer-Ignaczak formulas for the classical and complementary behavior of a two-dimensional infinite body (E-N:5), which is a micropolar body.

Keywords: Isotherm ▪ Schaefer Ignaczak ▪ Thermodynamical process ▪ Elastic strains

1. INTRODUCTION

- In [10], the initial-boundary value problem of the elastic dynamical micropolar body was described by tow tensorial stress equations with the appropriate field relations and appropriate stress initial-boundary conditions.
- The present problem concerns the first plane state of elastic strain of a 2D (E-N:5) body, using the stress-temperature formulation of Ignaczak type [2] and the micropolar framework of Eringen [9] and Nowacki [8].
- The Schaefer-Ignaczak decomposition is used here to formulate separate classical and complementary problems, as detailed below.
- Attiah and Al-Hasan [1] derived generalized Beltrami–Michell stress-temperature equations for a classical homogeneous and isotropic Hooke body. This work provides related background for the classical part of the present decomposition, rather than a derivation of its micropolar complementary fields.

2. DISCUSSING

2.1 The aim and the importuncy of this paper

a) The aim of the paper is to prove the isotherm of the complementary Schaefer-Ignaczak process for the unbounded 2D (E-N:6) body free of stresses and temperature at infinity. Next using the integral transform theory, we drive the Fourier Schaefer-Ignaczak formulas for the Scheafer-

Ignaczak classical and complementary fields.

b) The importunacy of paper: the results of this paper have important applications in shells theory and material laboratories.

2.2 The methods of the paper

For the methods of this paper, we combine the integral transform theory [11] with the Schaefer-Ignaczak decomposition formulated below.

For the requirement of the results of the paper, need to introduce some following important results from the paper [5].

The scheafer-Ignaczak method for the 2D (E-N:6) elastic body

In [5] we write the micropolar thermodynamical process for the 2D (E-N:6) elastic body occupying the initial configuration Ω, in the following form in $\Omega\times\mathrm{T}$ ($\mathrm{T}=[0,\infty[$):

$$\begin{aligned} &\mathbf{u}=\mathbf{u}^0+\mathbf{u}', \quad \varphi=\varphi^0+\varphi',\\ &\theta=\theta^0+\theta', \quad \sigma=\sigma^0+\sigma',\\ &\mu=\mu^0+\mu', \quad \gamma=\varepsilon^0+\gamma',\\ &\kappa=\kappa^0+\kappa', \quad \mathbf{Y}=\mathbf{Y}^0+\mathbf{Y}'. \end{aligned} \tag{3.1}$$

Where $(\mathbf{u}^0,\varphi^0,\theta^0,\sigma^0,\mu^0,\varepsilon^0,\kappa^0)$, $\mathbf{Y}^0$ relates to the classical thermodynamical linear thermo elasticity (Thermodynamical Hooke Model) in the frame of first plane state of elastic strains, and $(\mathbf{u}',\varphi',\theta',\sigma',\mu',\gamma',\kappa')$, $\mathbf{Y}'$ are the micropolar complementary tensor fields. The above mentioned classical and complementary tensor fields can be written in $\Omega\times\mathrm{T}$, and in the inertial frame $\mathbf{e}_i$ $(i=1,2,3)$, in the following form:

$$\mathbf{u}^0\equiv(u_1^0,u_2^0,0), \quad \varphi^0\equiv(0,0,\varphi_3^0). \tag{3.2}$$

Where $\varphi_3^0=\frac{1}{2}\varepsilon_{\alpha\beta}u^0_{\beta,\alpha}$ and $\varepsilon_{\alpha\beta}$ is the Levi-Chevita semi-tensor [8] of second order, the coma index denotes the partial derivative with the position coordinates; $f_{,\alpha}=\partial f/\partial x_\alpha$;

$$\begin{aligned} &\varepsilon^0\equiv\begin{bmatrix}\varepsilon_{11}^0 & \varepsilon_{12}^0 & 0\\ \varepsilon_{21}^0 & \varepsilon_{22}^0 & 0\\ 0&0&0\end{bmatrix},\\ &\kappa^0\equiv\begin{bmatrix}0&0&\kappa_{13}^0\\ 0&0&\kappa_{23}^0\\ 0&0&0\end{bmatrix}. \end{aligned} \tag{3.3}$$

$$\begin{aligned} &\sigma^0\equiv\begin{bmatrix}\sigma_{11}^0 & \sigma_{12}^0 & 0\\ \sigma_{21}^0 & \sigma_{22}^0 & 0\\ 0&0&\sigma_{33}^0\end{bmatrix},\\ &\mu^0\equiv\begin{bmatrix}0&0&\mu_{13}^0\\ 0&0&\mu_{23}^0\\ \mu_{31}^0&\mu_{32}^0&0\end{bmatrix}. \end{aligned} \tag{3.4}$$

where:

$$\sigma_{33}^0=\nu\sigma_{\alpha\alpha}^0-\frac{\mu}{\mu+\lambda}\nu_T\theta^0, \quad \mu_{3\alpha}^0=\frac{\gamma-\varepsilon}{\gamma+\varepsilon}\mu_{\alpha3}^0.$$

And the matrixes representing ε^0, σ^0 are symmetric, $\nu=\lambda/[2(\mu+\lambda)]$ is the poisons ratio, $\nu_T=(3\lambda+2\mu)a_t$, where a_t is the linear thermal coefficient of the body, and $\mu,\lambda,\gamma,\varepsilon\in\mathbb{R}_+$ are the material coefficients of the body. Finally, the all components in relations (3.2)–(3.4) depend on the position $\mathbf{x}\equiv(x_1,x_2)\in\Omega$ and the time t; and the complementary Schaefer-Ignaczak tensor fields take the forms:

$$\mathbf{u}'\equiv(u_1',u_2',0), \quad \varphi'\equiv(0,0,\varphi_3'). \tag{3.5}$$

$$\begin{aligned} &\gamma'\equiv\begin{bmatrix}\gamma_{11}' & \gamma_{12}' & 0\\ \gamma_{21}' & \gamma_{22}' & 0\\ 0&0&0\end{bmatrix},\\ &\kappa'\equiv\begin{bmatrix}0&0&\kappa_{13}'\\ 0&0&\kappa_{23}'\\ 0&0&0\end{bmatrix}. \end{aligned} \tag{3.6}$$

$$\begin{aligned} &\sigma'\equiv\begin{bmatrix}\sigma_{11}' & \sigma_{12}' & 0\\ \sigma_{21}' & \sigma_{22}' & 0\\ 0&0&\sigma_{33}'\end{bmatrix},\\ &\mu'\equiv\begin{bmatrix}0&0&\mu_{13}'\\ 0&0&\mu_{23}'\\ \mu_{31}'&\mu_{32}'&0\end{bmatrix}. \end{aligned} \tag{3.7}$$

where

$$\sigma_{33}'=\nu\sigma_{\alpha\alpha}'-\frac{\mu}{\mu+\lambda}\nu_T\theta', \quad \mu_{3\alpha}'=\frac{\gamma-\varepsilon}{\gamma+\varepsilon}\mu_{\alpha3}'.$$

The matrixes representing $\gamma',\kappa',\sigma',\mu'$ are asymmetric.

3.a. The classical Schaefer-Ignaczak initial-boundary value problem

Consists of the following:

Classical Schaefer-Ignaczak equation of motion in $\Omega\times\mathrm{T}^+$:

$$\begin{aligned} &\hat{c}_2^2(R^0_{\alpha,\beta}+R^0_{\beta,\alpha})-\ddot{\sigma}^0_{\alpha\beta}\\ &+(\lambda\ddot{e}_1^0-\nu_T\ddot{\theta}^0)\delta_{\alpha\beta}=0, \end{aligned} \tag{3.8}$$

$$\theta^0_{,\alpha\alpha}-\frac{1}{\kappa}\dot{\theta}^0-\eta_0\dot{e}_1^0=-\frac{Q}{\kappa}, \tag{3.9}$$

where:

$$\begin{aligned} &R^0_\alpha=\hat{R}^0_\alpha+X_\alpha, && \hat{R}^0_\alpha=\sigma^0_{\beta\alpha,\beta},\\ &\dot{e}_1^0=\frac{\dot{\sigma}^0_{\varepsilon\varepsilon}+2\nu_T\dot{\theta}^0}{2(\mu+\lambda)}, && \ddot{e}_1^0=\frac{\ddot{\sigma}^0_{\varepsilon\varepsilon}+2\nu_T\ddot{\theta}^0}{2(\mu+\lambda)},\\ &\hat{c}_2^2=\frac{\mu}{\rho}. \end{aligned}$$

ρ,J are the density and micro polar inertia of the body, $\mathbf{X}\equiv(X_1,X_2,0)$ is the volume force vector of the body, $\mathbf{Y}\equiv(0,0,Y_3)$ is the volume moment vector of the body, Q is the heat sources, W is the heat quantity, in the volume unit and time unit, λ_0 is the heat conduction coefficient, c_ε the specific heat for through constant body deformation;

The boundary condition on $\partial\Omega\times\mathrm{T}$:

$$\sigma^0_{\beta\alpha}n_\beta=p_\alpha, \quad \theta^0=\vartheta, \tag{3.10}$$

where n_β are the components of the outer unit normal vector: $\mathbf{n}\equiv(n_1,n_2,0)$.

In the boundary condition (3.10) we refer that p_α and ϑ are the given components of the total traction vector and the given total thermal load on the surface of the body,

The initial condition in $\Omega\times\{0\}$:

$$\begin{aligned} &\sigma^0=\operatorname{sym}\sigma^{(0)}, \quad \theta^0=\ell,\\ &\dot{\sigma}^0=\operatorname{sym}\dot{\sigma}^{(0)}, \end{aligned} \tag{3.11}$$

where the symbol sym is the symmetric part of tensor field and $\sigma^{(0)}, \dot{\sigma}^{(0)}, \ell$ are the given initial total force stresses and total force stresses velocity and the given total initial temperature [2];

The inverse constitutive relations in $\Omega \times \mathrm{T}$:

$$2\mu\varepsilon^0_{\alpha\beta} = \sigma^0_{\alpha\beta} - (\lambda e^0_1 - \nu_T\theta^0)\delta_{\alpha\beta}, \tag{3.12}$$

where:

$$e^0_1 = \frac{1}{2(\mu+\lambda)}(\sigma^0_{\varepsilon\varepsilon} + 2\nu_T\theta^0).$$

The convolution relations in $\overline{\Omega} \times \mathrm{T}$:

$$u^0_\alpha = g_\alpha t + f_\alpha + \rho^{-1}(t * R^0_\alpha), \tag{3.13}$$

$$\varphi^0_3 = \frac{1}{2}\varepsilon_{\alpha\beta}[g_\beta t + f_\beta + \rho^{-1}(t * R^0_\beta)]_{,\alpha} \tag{3.14}$$

where f_α, g_α are the given initial total values of the total displacements u_α and its velocities.

3.b. The complementary Schaefer-Ignaczak initial-boundary value problem

The complementary Schaefer-Ignaczak initial-boundary value problem consists of the following:

The Schaefer-Ignaczak equations of motion in $\Omega \times \mathrm{T}^+$:

$$\begin{aligned} &\overset{*}{\square}_2\rho^{-1}R'_{\alpha,\beta} + J^{-1}\varepsilon_{\alpha\beta}\mathfrak{R}'_3 \\ &\quad - \overset{*}{\square}_2\Big\{\frac{1}{2\mu}\ddot{\sigma}'_{(\beta\alpha)} + \frac{1}{2\alpha}\ddot{\sigma}'_{[\beta\alpha]} \\ &\quad - \frac{1}{2\mu}(\lambda\ddot{e}'_1 - \nu_T\ddot{\theta}')\delta_{\alpha\beta}\Big\} = 0, \end{aligned} \tag{3.15}$$

$$c_4^2\mathfrak{R}'_{3,\alpha} - \overset{*}{\square}_2\mu'_{\alpha 3} = 0, \tag{3.16}$$

$$D\theta' - \eta_0\dot{e}'_1 = -\frac{\hat{Q}}{\kappa}, \tag{3.17}$$

where:

$$\begin{aligned} &R'_\alpha = \hat{R}'_\alpha + \hat{X}_\alpha, \qquad \hat{R}'_\alpha = \sigma'_{\beta\alpha,\beta}, \qquad \hat{X}_\alpha \equiv 0, \\ &\mathfrak{R}'_3 = \hat{\mathfrak{R}}'_3 + \hat{Y}_3, \qquad \hat{\mathfrak{R}}'_3 = \overset{*}{\square}_2\hat{R}'_3, \\ &\hat{R}'_3 = \varepsilon_{\alpha\beta}\sigma'_{\alpha\beta} + \mu'_{\beta 3,\beta}, \\ &\hat{Y}_3 = \overset{*}{\square}_2 Y_3 - \frac{1}{2}\overset{*}{\square}_4\varepsilon_{\alpha\beta}X_{\beta,\alpha}, \\ &\hat{Q} = 0, \\ &\dot{e}'_1 = \frac{\dot{\sigma}'_{\varepsilon\varepsilon} + 2\nu_T\dot{\theta}'}{2(\mu+\lambda)}, \qquad \ddot{e}'_1 = \frac{\ddot{\sigma}'_{\varepsilon\varepsilon} + 2\nu_T\ddot{\theta}'}{2(\mu+\lambda)}. \end{aligned}$$

and:

$$\begin{aligned} &\overset{*}{\square}_2 = \mu\Delta_1 - \rho\partial_t^2, \\ &\overset{*}{\square}_4 = (\gamma+\varepsilon)\Delta_1 - J\partial_t^2, \\ &D = \Delta_1 - \frac{1}{\kappa}\partial_t, \\ &\Delta_1 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \qquad \partial_t := \frac{\partial}{\partial t}. \end{aligned}$$

The boundary conditions on $\partial\Omega \times \mathrm{T}$:

$$\sigma'_{\beta\alpha}n_\beta = 0, \quad \mu'_{\alpha 3}n_\alpha = m_3 - m^0_3, \quad \theta' = 0 \tag{3.18}$$

where: $m^0_3 = \mu^0_{\alpha 3}n_\alpha$, in which $\mu^0_{\alpha 3}$ is related to the classical field in (3.14) through the relations stated below:

$$\kappa^0_{\alpha 3} = \frac{1}{2}\varepsilon_{\gamma\varepsilon}[g_\varepsilon t + f_\varepsilon + \rho^{-1}(t * R^0_\varepsilon)]_{,\alpha\gamma} \tag{3.19}$$

$$\kappa^0_{\alpha 3} = \frac{1}{\gamma+\varepsilon}\mu^0_{\alpha 3}, \tag{3.20}$$

The initial conditions in $\Omega \times \{0\}$ for (σ', μ', θ'):

$$\begin{aligned} &\sigma' = \operatorname{skew}\sigma^{(0)}, \quad \mu' = \mu^{(0)} - \mu^{0(0)}, \\ &\dot{\sigma}' = \operatorname{skew}\dot{\sigma}^{(0)}, \quad \dot{\mu}' = \dot{\mu}^{(0)} - \dot{\mu}^{0(0)}, \\ &\theta' = 0. \end{aligned} \tag{3.21}$$

where the symbol skew denotes the antisymmetric part of the tensor field, and $\sigma^{(0)}, \dot{\sigma}^{(0)}, \mu^{0(0)}, \dot{\mu}^{0(0)}$ are the given initial total force stress fields and couple stress fields and its velocities [2];

The inverse constitutes relations in $\Omega \times \mathrm{T}$:

$$\begin{aligned} \gamma'_{\alpha\beta} &= \frac{1}{2\mu}\sigma'_{(\alpha\beta)} + \frac{1}{2\alpha}\sigma'_{[\alpha\beta]} \\ &\quad - \frac{1}{2\mu}(\lambda e'_1 - \nu_T\theta')\delta_{\alpha\beta}, \\ \kappa'_{\alpha 3} &= \frac{1}{\gamma+\varepsilon}\mu'_{\alpha 3}, \\ e'_1 &= \frac{1}{2(\mu+\lambda)}(\sigma'_{\varepsilon\varepsilon} + 2\nu_T\theta'). \end{aligned} \tag{3.22}$$

The convolution relation in $\overline{\Omega} \times \mathrm{T}$:

$$\begin{aligned} u'_\alpha &= \rho^{-1}(t * R'_\alpha), \\ \varphi'_3 &= \left(g_3 - \tfrac{1}{2}\varepsilon_{\alpha\beta}g_{\beta,\alpha}\right) + \left(f_3 - \tfrac{1}{2}\varepsilon_{\alpha\beta}f_{\beta,\alpha}\right) \\ &\quad + J^{-1}[t * (\hat{R}'_3 + Y_3)] + J^{-1}(t * \hat{R}^0_3) \\ &\quad - \tfrac{1}{2}\varepsilon_{\alpha\beta}\rho^{-1}(t * R^0_{\beta,\alpha}). \end{aligned} \tag{3.23}$$

For the requirements of the following section, we use to rewrite the following integro-deferential equations relating to equations (3.8), (3.9), which are satisfied in $\Omega \times \mathrm{T}$ [2]:

$$\begin{aligned} &\hat{c}_2^2(t * R^0_{\alpha,\beta} + t * R^0_{\beta,\alpha}) - \sigma^0_{\alpha\beta} \\ &\quad + (\lambda e^0_1 - \nu_T\theta^0)\delta_{\alpha\beta} \\ &= -\mu[(g_{\alpha,\beta} + g_{\beta,\alpha})t + (f_{\alpha,\beta} + f_{\beta,\alpha})], \\ &\int_0^t \theta^0_{,\alpha\alpha}(\mathbf{x},\tau)\,d\tau - \frac{1}{\kappa}\theta^0(\mathbf{x},t) - \eta_0 e^0_1(\mathbf{x},t) \\ &= -\frac{1}{\kappa}\ell(\mathbf{x}) - \eta_0 f_{\varepsilon,\varepsilon}(\mathbf{x}) - \frac{1}{\kappa}\int_0^t Q(\mathbf{x},\tau)\,d\tau. \end{aligned} \tag{3.24}$$

where: $e^0_1 = [\sigma^0_{\varepsilon\varepsilon} + 2\nu_T\theta^0]/[2(\mu+\lambda)]$.

Using the constitutive relations (3.22) and the initial conditions (3.21), we consider the following complementary Schaefer-Ignaczak equations of motion in $\Omega \times \mathrm{T}^+$ and obtain their integro-differential form below:

$$\begin{aligned} &\tfrac{1}{2}\rho^{-1}(R'_{\alpha,\beta} + R'_{\beta,\alpha}) - \frac{1}{2\mu}\ddot{\sigma}'_{(\alpha\beta)} \\ &\quad + \frac{1}{2\mu}(\lambda\ddot{e}'_1 - \nu_T\ddot{\theta}')\delta_{\alpha\beta} + \frac{1}{2\alpha}\ddot{\sigma}'_{[\alpha\beta]} \\ &\quad + \tfrac{1}{2}\rho^{-1}(R^0_{\alpha,\beta} + R^0_{\beta,\alpha}) + J^{-1}\varepsilon_{\alpha\beta}(\hat{R}' + Y_3) = 0, \\ &D\theta' - \eta_0\dot{e}'_1 = -\frac{\hat{Q}}{\kappa}, \end{aligned} \tag{3.25}$$

$$c_4^2(\hat{R}'_3 + Y_3)_{,\alpha} - \ddot{\mu}'_{\alpha 3} = -c_4^2\hat{R}^0_{3,\alpha} + \ddot{\mu}^0_{\alpha 3} \tag{3.26}$$

where: $\hat{R}'_3 = \varepsilon_{\alpha\beta}\sigma'_{\alpha\beta} + \mu'_{\beta 3,\beta}$, $\hat{R}^0_3 = \mu^0_{\beta 3,\beta}$.

Appling the convolution operation to the above equation, and using the complementary initial conditions (3.21), we arrive to the following complementary Schaefer-Ignaczak system of integro-differential equations in $\Omega \times \mathrm{T}$:

$$
\begin{aligned}
&\tfrac{1}{2} t * \rho^{-1}(R'_{\alpha,\beta} + R'_{\beta,\alpha}) - \frac{1}{2\mu}\sigma'_{(\alpha\beta)} \\
&\quad + \frac{1}{2\alpha}\sigma'_{[\alpha\beta]} + \frac{1}{2\mu}(\lambda e'_1 - \nu_T\theta')\delta_{\alpha\beta} \\
&\quad + \tfrac{1}{2}[t(g_{\beta,\alpha} - g_{\alpha,\beta}) + (f_{\beta,\alpha} - f_{\alpha,\beta})] \\
&\quad + \tfrac{1}{2} t * \rho^{-1}(R^0_{\alpha,\beta} - R^0_{\beta,\alpha}) \\
&\quad + \tfrac{1}{2} t * \rho^{-1}(R'_{\alpha,\beta} - R'_{\beta,\alpha}) + J^{-1} t * \varepsilon_{\alpha\beta}(\hat{R}'_3 + Y_3) = 0, \qquad (3.27) \\
&\int_0^t \theta'_{,\alpha\alpha}(\mathbf{x},\tau)\,d\tau - \frac{1}{\kappa}\theta'(\mathbf{x},t) - \eta_0 e'_1(\mathbf{x},t) \\
&\quad = -\frac{1}{\kappa}\int_0^t \hat{Q}(\mathbf{x},\tau)\,d\tau, \\
&t * c_4^2(\hat{R}'_3 + Y_3)_{,\alpha} - \mu'_{\alpha 3} \\
&\quad = -t * c_4^2 \hat{R}^0_{3,\alpha} + \mu^0_{\alpha 3} - (\gamma + \varepsilon)(g_3 t + t_3)_{,\alpha}.
\end{aligned}
$$

3.c. Separating the classical Schaefer-Ignaczak equations of motion in $\Omega \times \mathrm{T}^+$

For this purpose, we need to derive the independent equation for the quantities: $e^0, \theta^0, \rho^{-1}(t * R^0_\alpha) + g_\alpha t + f_\alpha$, proceeding as following. Taking the contract of the equations $(3.24)_1$, for $\alpha = \beta$ we found:

$$\rho^{-1}(t * R^0_{\beta,\beta}) + g_{\beta,\beta} t + f_{\beta,\beta} = e^0_1 \qquad (3.28)$$

Taking the partial derivative of $(3.24)_1$ with respect to x_β, we find that:

$$
\begin{aligned}
&\hat{c}_2^2(t * R^0_{\alpha,\beta\beta} + t * R^0_{\beta,\alpha\beta}) - \sigma^0_{\alpha\beta,\beta} \\
&\quad + (\lambda e^0_1 - \nu_T\theta^0)_{,\beta} \\
&\quad = -\mu[(g_{\alpha,\beta\beta} + g_{\beta,\alpha\beta})t + (f_{\alpha,\beta\beta} + f_{\beta,\alpha\beta})],
\end{aligned}
$$

or

$$
\begin{aligned}
&\mu\{\rho^{-1}(t * R^0_{\alpha,\beta\beta}) + g_{\alpha,\beta\beta} t + f_{\alpha,\beta\beta} \\
&\quad + \rho^{-1}(t * R^0_{\beta,\beta\alpha}) + g_{\beta,\alpha\beta} t + f_{\beta,\beta\alpha}\} \qquad (3.29) \\
&\quad - R^0_\alpha + X_\alpha = 0,
\end{aligned}
$$

where we use the relations:

$$-R^0_\alpha = -\rho\frac{\partial^2}{\partial t^2}\left[t * (\rho^{-1}R^0_\alpha) + g_\alpha t + f_\alpha\right],$$

Equations (3.29) take the following operator form:

$$
\begin{aligned}
&\overset{*}{\Box}_2\left[\rho^{-1} t * R^0_\alpha + g_\alpha t + f_\alpha\right] \\
&\quad + \left\{\mu(\rho^{-1} t * R^0_{\beta\beta} + g_{\beta\beta} t + f_{\beta\beta}) \right. \qquad (3.30) \\
&\qquad \left. + \lambda e^0_1 - \nu_T\theta^0\right\}_{,\alpha} + X_\alpha = 0.
\end{aligned}
$$

Where: $\overset{*}{\Box}_2 f = \mu\Delta_1 f - \rho\partial^2 f/\partial t^2$.

Using (3.28), (3.29) equation (3.30) takes the form:

$$
\begin{aligned}
&\overset{*}{\Box}_2\left[\rho^{-1} t * R^0_\alpha + g_\alpha t + f_\alpha\right] \qquad (3.31) \\
&\quad + \left\{(\mu + \lambda)e^0_1 - \nu_T\theta^0\right\}_{,\alpha} + X_\alpha = 0.
\end{aligned}
$$

Or the following form in $\Omega \times \mathrm{T}$:

$$
\begin{aligned}
&\overset{*}{\Box}_2\left[\rho^{-1} t * R^0_\alpha + g_\alpha t + f_\alpha\right] \qquad (3.32) \\
&\quad + (\mu + \lambda)e^0_{1,\alpha} - \nu_T\theta^0_{,\alpha} + X_\alpha = 0.
\end{aligned}
$$

Now, taking the partial derivative of (3.32), with respect to x_α, we find:

$$
\begin{aligned}
&\overset{*}{\Box}_2\left[\rho^{-1} t * R^0_{\alpha,\alpha} + g_{\alpha,\alpha} t + f_{\alpha,\alpha}\right] \qquad (3.33) \\
&\quad + (\mu + \lambda)e^0_{1,\alpha\alpha} - \nu_T\theta^0_{,\alpha\alpha} + X_{\alpha,\alpha} = 0,
\end{aligned}
$$

or:

$$\overset{*}{\Box}_2 e^0_1 = (\mu + \lambda)\Delta_1 e^0_1 - \nu_T\Delta_1\theta^0 + X_{\alpha,\alpha} = 0$$

or:

$$\Box_1 e^0_1 - \nu_T\Delta_1\theta^0 + X_{\alpha,\alpha} = 0, \qquad (3.34)$$

where: $\Box_1 e^0_1 = (\lambda + 2\mu)\Delta_1 e^0_1 - \rho\partial^2 e^0_1/\partial t^2$.

Taking the partial derivative of $(3.24)_2$ with respect to the time "t" we find the following equation in $\Omega \times \mathrm{T}^+$:

$$\Delta_1\theta^0 - \frac{1}{\kappa}\frac{\partial}{\partial t}\theta^0 - \eta_0\dot{e}^0_1 = -\frac{1}{\kappa}Q, \qquad (3.35)$$

or:

$$D\theta^0 - \eta_0\dot{e}^0_1 = -\frac{1}{\kappa}Q, \qquad (3.36)$$

where: $D\theta^0 = \Delta_1 - \kappa^{-1}\partial\theta^0/\partial t$.

Now applying operator D to the equation (3.34) and using (3.36), we find the following equation in $\Omega \times \mathrm{T}^+$:

$$De^0_1 = -\left(DX_{\alpha,\alpha} + \frac{\nu_T}{\kappa}\Delta_1 Q\right), \qquad (3.37)$$

Where: $D_2 = D\Box_1 - \nu_T\eta_0\Delta_1\partial_t$.

On other hand applying the operator $\Box_1$ on the equation (3.34) and using (3.36), we arrive to the following equation in $\Omega \times \mathrm{T}^+$:

$$D_2\theta^0 = -\left(\eta_0\partial_t X_{\alpha,\alpha} + \frac{1}{\kappa}\Box_1 Q\right), \qquad (3.38)$$

Finally, we find the last separating equations for the quantity: $\rho^{-1} t * R^0_\alpha + g_\alpha t + f_\alpha$, applying operator D_2 on equation (3.32), when we find the following separated equation in $\Omega \times \mathrm{T}$:

$$
\begin{aligned}
&D_2\overset{*}{\Box}_2(\rho^{-1} t * R^0_\alpha + g_\alpha t + f_\alpha) \\
&\quad = -D_2 X_\alpha + [(\mu + \lambda)D - \nu_T\eta_0\partial_t]X_{\beta,\beta\alpha} \qquad (3.39) \\
&\quad + \frac{\nu_T}{\kappa}[(\mu + \lambda)\Delta_1 - \Box_1]Q_{,\alpha}
\end{aligned}
$$

or:

$$
\begin{aligned}
&D_2\overset{*}{\Box}_2(\rho^{-1} t * R^0_\alpha + g_\alpha t + f_\alpha) \\
&\quad = -D_2 X_\alpha + D_1 X_{\gamma,\gamma\alpha} - \frac{\nu_T}{\kappa}\overset{*}{\Box}_2 Q_{,\alpha},
\end{aligned}
$$

where: $D_1 = (\mu + \lambda)D - \nu_T\eta_0\partial_t$.

Finally let us rewrite the first of the equation (3.24) in the form:

$$
\begin{aligned}
&\mu\{[\rho^{-1}(t * R^0_\alpha) + g_\alpha t + f_\alpha]_{,\beta} \\
&\quad + [\rho^{-1}(t * R^0_\beta) + g_\beta t + f_\beta]_{,\alpha}\} \qquad (3.40) \\
&\quad - \sigma^0_{\alpha\beta} + (\lambda e^0_1 - \nu_T\theta^0)\delta_{\alpha\beta} = 0.
\end{aligned}
$$

Now applying the double operator $D_2\overset{*}{\Box}_2$ on equation (3.40), and using equations (3.37)–(3.38), we arrive to the following separated equation in $\Omega \times \mathrm{T}^{+}$:

$$\begin{aligned}D_2\overset{*}{\Box}_2\sigma^0_{\alpha\beta} &= -\mu D_2(X_{\alpha,\beta}+X_{\beta,\alpha})\\&+2\mu D_1 X_{\gamma,\gamma\alpha\beta} - 2\mu\frac{\nu_T}{\kappa}\overset{*}{\Box}_2 Q_{,\alpha\beta}\\&+\overset{*}{\Box}_2\Big\{(\mu D - D_1)X_{\gamma,\gamma}\\&+\frac{\nu_T}{\kappa}(\Box_1-\lambda\Delta_1)Q\Big\}\delta_{\alpha\beta}.\end{aligned}\tag{3.41}$$

3.d. Separating the complementary Schaefer-Ignaczak equations of motion in $\Omega \times \mathrm{T}^{+}$

Using similar way as in the previous one, from complementary integro-differential equations (3.27), we arrive to the following independent equations for the quantities e'_1, θ' in $\Omega \times \mathrm{T}^{+}$:

$$D_2 e'_1 = -\Big(D\hat{X}_{\alpha,\alpha} + \frac{\nu_T}{\kappa}\Delta_1\hat{Q}\Big) \tag{3.42}$$

$$D\theta' = -\Big(\frac{1}{\kappa}\Box_1\hat{Q} + \eta_0\partial_1\hat{X}_{\alpha,\alpha}\Big) \tag{3.43}$$

By the view of: $\hat{X}_\alpha = 0$, $\hat{Q}=0$, the above equations take the form in $\Omega \times \mathrm{T}^{+}$:

$$D_2 e'_1 = 0, \quad D\theta' = 0 \tag{3.44}$$

Theorem: The complementary Schaefer-Ignaczak proses for the unbounded (E-N:5) occupying $\mathbb{R}^2$, of free complementary stresses and free complementary temperature at ∞, is isothermal proses.

Proof: Applying the double Fourier integral transform of third order of the position (x_1,x_2) and time t [11], and using the properties of this transform, we arrive to the following transformed equations:

$$-(\lambda+2\mu)W_4(\xi,\tau)\overline{\theta}'(\xi,\tau) = 0, \tag{3.45}$$

$$-(\lambda+2\mu)W_4(\xi,\tau)\overline{e}'(\xi,\tau) = 0, \tag{3.46}$$

where the symbol:

$$\begin{aligned}\mathscr{F}_3[f(\mathbf{x},t)] &= \overline{f}(\xi,\tau)\\&:= \frac{1}{2\pi\sqrt{2\pi}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\\&\quad f(\mathbf{x},t)e^{i(\mathbf{x}\cdot\xi+t\tau)}\,d\mathbf{x}\,dt\end{aligned}$$

where: $\mathbf{x}=(x_1,x_2)$, $\xi=(\xi_1,\xi_2)$, $d\mathbf{x}=dx_1\,dx_2$, $\mathbf{x}\cdot\xi = x_\alpha\xi_\alpha$, $i=\sqrt{-1}$ and:

$$\begin{aligned}W_4(\xi,\tau) &= \xi^4 - [\sigma_1^2(\tau)+(1+\varepsilon)q(\tau)]\xi^2\\&+q(\tau)\sigma_1^2(\tau).\end{aligned}$$

The equation (3.44), (3.45) take the form:

$$\overline{\theta}'(\xi,\tau) = 0, \quad \overline{e}'(\xi,\tau) = 0.$$

Now applying the invers double Fourier integral transform of third order for the parameters (ξ_1,ξ_2), τ [11], we arrive to the following tow identities:

$$\theta'(\mathbf{x},t) = 0, \quad e'(\mathbf{x},t) = 0 \tag{3.47}$$

From the above we conclude that the complementary temperature and dilatation vanish, so the complementary Schaefer-Ignaczak proses for the unbounded considerable 2D (E-N:5) body is isothermal.

The independent complementary Schaefer-Ignaczak equations for the unbounded considerable 2D (E-N:5) body:

Proceeding in similar way as in the previous subsection 3.a, from the complementary Schaefer-Ignaczak integro-differential equations and using (3.47), we arrive to the following complementary Schaefer-Ignaczak independent equations for the isothermal complementary fields $\sigma'_{\alpha\beta}, \mu'_{\alpha 3}$ in $\Omega \times \mathrm{T}$ $(\Omega = \mathbb{R}^2)$:

$$\begin{aligned}\overset{*}{\Box}_2 L\sigma'_{\alpha\beta} &= 2\alpha\big[(\mu+\alpha)\varepsilon_{\alpha\beta}\hat{Y}_{3,\gamma\alpha}\\&+(\mu-\alpha)\varepsilon_{\alpha\gamma}\hat{Y}_{3,\gamma\alpha}\big]\\&+2\alpha\varepsilon_{\alpha\beta}\Box_2\hat{Y}_3\end{aligned}\tag{3.48}$$

$$\overset{*}{\Box}_2 L\mu'_{\alpha 3} = -(\gamma+\varepsilon)\Box_2\hat{Y}_{3,\alpha} \tag{3.49}$$

3.e. Fourier classical and complementary formulas for the Schaefer-Ignaczak fields

In order to obtain the classical and complementary Fourier formulas for the Schaefer-Ignaczak fields, we apply the double Fourier integral transform of third order of the suppurating classical and complementary equations for $\sigma^0_{\alpha\beta}, \theta^0, \sigma'_{\alpha\beta}, \mu'_{\alpha 3}$, taking into account that the variables are x_1, x_2, t, and using the transform relations:

$$\begin{aligned}&\mathscr{F}_3(f_{,\beta}) = (-i\xi_\beta)\overline{f},\\&\mathscr{F}_3(\partial_t f) = (-i\tau)\overline{f},\\&\mathscr{F}_3(\Delta_1 f) = -\xi^2\overline{f},\\&\mathscr{F}_3(\overset{*}{\Box}_2 f) = -\mu[\xi^2-\hat{\sigma}_2^2(\tau)]\overline{f},\\&\mathscr{F}_3(\Box_1 f) = -(\lambda+2\mu)[\xi^2-\sigma_1^2(\tau)]\overline{f},\\&\mathscr{F}_3(Df) = -[\xi^2-q(\tau)]\overline{f},\\&\mathscr{F}_3(D_1 f) = -\big\{(\lambda+\mu)[\xi^2-q(\tau)]\\&\qquad -(\lambda+2\mu)\varepsilon q(\tau)\big\}\overline{f}\\&\qquad = -\big\{(\lambda+2\mu)[\xi^2-(1+\varepsilon)q(\tau)]\\&\qquad -\mu[\xi^2-q(\tau)]\big\}\overline{f},\\&\mathscr{F}_3(D_2 f) = (\lambda+2\mu)\big\{[\xi^2-q(\tau)][\xi^2-\sigma_1^2(\tau)]\\&\qquad -\varepsilon q(\tau)\xi^2\big\}\overline{f}\\&\qquad = (\lambda+2\mu)W_4(\xi,\tau)\overline{f},\end{aligned}$$

Where:

$$\xi = (\xi_\alpha\xi_\alpha)^{1/2}, \qquad \sigma_1(\tau) = \frac{\tau}{c_1}, \qquad \hat{\sigma}_2(\tau) = \frac{\tau}{\hat{c}_2},$$

$$c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}, \qquad \hat{c}_2 = \sqrt{\frac{\mu}{\rho}}, \qquad q(\tau) = \frac{i\tau}{\kappa},$$

$$m = \frac{\nu_T}{\lambda+2\mu}, \qquad \varepsilon = m\kappa\eta_0,$$

$$\begin{aligned}W_4(\xi,\tau) &= \xi^4 - [\sigma_1^2(\tau)+(1+\varepsilon)q(\tau)]\xi^2\\&+q(\tau)\sigma_1^2(\tau).\end{aligned}$$

We arrive to:

Classical transformed stresses and temperature:

$$
\begin{aligned}
\overline{\sigma}^0_{\alpha\beta} &= \frac{(-i\xi_\beta)\overline{X}_\alpha + (-i\xi_\alpha)\overline{X}_\beta}{\xi^2 - \hat{\sigma}_2^2(\tau)} \\
&\quad + \frac{2}{(\lambda+2\mu)[\xi^2 - \hat{\sigma}_2^2(\tau)]W_4(\xi,\tau)} \\
&\qquad \times \{(\lambda+2\mu)[\xi^2 - (1+\varepsilon)q(\tau)] \\
&\qquad - \mu[\xi^2 - q(\tau)]\} \\
&\qquad \times (-i\xi_\gamma)(-i\xi_\alpha)(-i\xi_\beta)\overline{X}_\gamma \\
&\quad - \frac{2\mu\nu_T}{\kappa}\frac{[\xi^2 - \sigma_1^2(\tau)](-i\xi_\alpha)(-i\xi_\beta)\overline{Q}}{(\lambda+2\mu)[\xi^2 - \hat{\sigma}_2^2(\tau)]W_4(\xi,\tau)} \\
&\quad + \frac{1}{(\lambda+2\mu)W_4(\xi,\tau)} \\
&\qquad \times \Big\{\big[-\mu(\xi^2 - q(\tau)) \\
&\qquad + (\lambda+2\mu)[\xi^2 - (1+\varepsilon)q(\tau)] \\
&\qquad - \mu(\xi^2 - q(\tau))\big](-i\xi_\gamma)\overline{X}_\gamma \\
&\qquad + \frac{\nu_T}{\kappa}[-(\lambda+2\mu)[\xi^2 - q(\tau)] \\
&\qquad + \lambda\xi^2]\overline{Q}\Big\}\delta_{\alpha\beta}.
\end{aligned} \tag{3.50}
$$

$$
\begin{aligned}
\overline{\theta}^0 &= \frac{\kappa\eta_0}{(\lambda+2\mu)W_4(\xi,\tau)}q(\tau)(-i\xi_\alpha)\overline{X}_\alpha \\
&\quad + \frac{1}{\kappa W_4(\xi,\tau)}[\xi^2 - \sigma_1^2(\tau)]\overline{Q}.
\end{aligned} \tag{3.51}
$$

On the other hand, applying the double Fourier transform of the third order to the complementary Scheaefer-Ignaczak equations (3.48), (3.49), and using the transform relations:

$$
\begin{aligned}
\mathscr{F}_3(\Box_2 f) &= -(\mu+\alpha)[\xi^2 - \sigma_2^2(\tau)]\overline{f}, \\
\mathscr{F}_3(\Box_4 f) &= -[(\gamma+\varepsilon)\xi^2 + 4\alpha - J\tau^2]\overline{f} \\
&= -(\gamma+\varepsilon)[\xi^2 + \nu_0^2 - \sigma_4^2(\tau)]\overline{f}, \\
\mathscr{F}_3(Lf) &= \mathscr{F}_3[(\Box_2\Box_4 + 4\alpha^2\Delta_1)f] \\
&= (\mu+\alpha)(\gamma+\varepsilon)[\xi^2 - \sigma_2^2(\tau)] \\
&\qquad \times [\xi^2 + \nu_0^2 - \sigma_4^2(\tau)]\overline{f} - 4\alpha^2\xi^2\overline{f} \\
&= (\mu+\alpha)(\gamma+\varepsilon)\{[\xi^2 - \sigma_2^2(\tau)] \\
&\qquad \times [\xi^2 + \nu_0^2 - \sigma_4^2(\tau)] - ps\}\overline{f} \\
&= (\mu+\alpha)(\gamma+\varepsilon)\Delta_4(\xi;\tau),
\end{aligned}
$$

where:

$$
\begin{aligned}
\Delta_4(\xi;\tau) &= \xi^4 - [\sigma_2^2(\tau) + \sigma_4^2(\tau) + \eta_0^2 - \nu_0^2]\xi^2 \\
&\quad + \sigma_2^2(\tau)[\sigma_4^2(\tau) - \nu_0^2], \\
c_4 &= \sqrt{\frac{\gamma+\varepsilon}{J}}, \quad c_2 = \sqrt{\frac{\mu+\alpha}{\rho}}, \\
\sigma_4(\tau) &= \frac{\tau}{c_4}, \quad \sigma_2(\tau) = \frac{\tau}{c_2}, \\
\nu_0^2 &= 2p = \frac{4\alpha}{\gamma+\varepsilon}, \quad \eta_0^2 = ps, \quad s = \frac{2\alpha}{\mu+\alpha}.
\end{aligned}
$$

We arrive to the following complementary Fourier transforms for the stresses: $\sigma'_{\alpha\beta}, \mu'_{\alpha 3}$:

$$
\begin{aligned}
&-\mu[\xi^2 - \hat{\sigma}_2^2(\tau)](\mu+\alpha)(\gamma+\varepsilon)\Delta_4(\xi;\tau)\overline{\sigma}'_{\alpha\beta} \\
&= 2\alpha\big[(\mu+\alpha)(-i\xi_\alpha)(-i\xi_\gamma)\varepsilon_{\beta\gamma}\overline{\hat{Y}}_3 \\
&\qquad + (\mu-\alpha)(-i\xi_\beta)(-i\xi_\gamma)\varepsilon_{\alpha\gamma}\overline{\hat{Y}}_3\big] \\
&\quad - 2\alpha(\mu+\alpha)\varepsilon_{\alpha\beta}[\xi^2 - \sigma_2^2(\tau)]\overline{\hat{Y}}_3, \\
&-\mu(\mu+\alpha)(\gamma+\varepsilon)[\xi^2 - \hat{\sigma}_2^2(\tau)]\Delta_4(\xi;\tau)\overline{\mu}'_{\alpha 3} \\
&= (\gamma+\varepsilon)(\mu+\alpha)[\xi^2 - \sigma_2^2(\tau)](-i\xi_\alpha)\overline{\hat{Y}}_3,
\end{aligned}
$$

or:

$$
\begin{aligned}
\overline{\sigma}'_{\alpha\beta} &= \frac{-2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)\Delta_4(\xi;\tau)} \\
&\qquad \times \big[(\mu+\alpha)(-i\xi_\alpha)(-i\xi_\gamma)\varepsilon_{\beta\gamma}\overline{\hat{Y}}_3 \\
&\qquad + (\mu-\alpha)(-i\xi_\beta)(-i\xi_\gamma)\varepsilon_{\alpha\gamma}\overline{\hat{Y}}\big] \\
&\quad + \frac{2\alpha\varepsilon_{\alpha\beta}[\xi^2 - \sigma_2^2(\tau)]\overline{\hat{Y}}_3}{\mu(\gamma+\varepsilon)[\xi^2 - \hat{\sigma}_2^2(\tau)]\Delta_4(\xi;\tau)},
\end{aligned} \tag{3.52}
$$

$$
\overline{\mu}'_{\alpha 3} = \frac{-[\xi^2 - \sigma_2^2(\tau)](-i\xi_\alpha)\overline{\hat{Y}}}{\mu[\xi^2 - \hat{\sigma}_2^2(\tau)]\Delta_4(\xi;\tau)}. \tag{3.53}
$$

Now, suppose that:

$$
\begin{aligned}
\hat{F}_2(\mathbf{x},t) &:= \mathscr{F}_3^{-1}\Big[\frac{1}{\xi^2 - \hat{\sigma}_2^2(\tau)}\Big], \\
F_1(\mathbf{x},t) &:= \mathscr{F}_3^{-1}\Big[\frac{1}{\xi^2 - \sigma_2^2(\tau)}\Big], \\
G_2(\mathbf{x},t) &:= \mathscr{F}_3^{-1}\Big[\frac{1}{W_4(\xi,\tau)}\Big], \\
G_1(\mathbf{x},t) &:= \mathscr{F}_3^{-1}\Big[\frac{1}{\Delta_4(\xi,\tau)}\Big],
\end{aligned}
$$

applying the inverse double Fourier transform with respect to $(\xi;\tau)$, then using the Fourier convolution theorem [11], we arrive to the following:

Classical Schaefer-Ignaczak Fourier formulas for $\sigma^0_{\alpha\beta}, \theta^0$:

$$
\begin{aligned}
\sigma^0_{\alpha\beta} &= \partial_\beta(\hat{F}_2 * X_\alpha) + \partial_\alpha(\hat{F}_2 * X_\beta) \\
&\quad - \frac{2}{\lambda+2\mu}\partial_\gamma\partial_\beta\partial_\alpha D_1(\hat{F}_2 * G_2 * X_\gamma) \\
&\quad - \frac{2\mu\nu_T}{\kappa}\partial_\alpha\partial_\beta(G_2 * Q) \\
&\quad + \frac{1}{\lambda+2\mu}\Big\{[\mu D - D_1]\partial_\alpha(G_2 * X_\gamma) \\
&\qquad + \frac{\nu_T}{\kappa}[(\lambda+2\mu)D - \lambda\Delta_1](G_2 * Q)\Big\}\delta_{\alpha\beta},
\end{aligned} \tag{3.54}
$$

$$
\begin{aligned}
\theta^0 &= -\frac{\kappa\eta_0}{(\lambda+2\mu)\kappa}\partial_t\partial_\alpha(G_2 * X_\alpha) \\
&\quad - \frac{1}{\kappa(\lambda+2\mu)}\Box(G_2 * Q).
\end{aligned} \tag{3.55}
$$

Complementary Schaefer-Ignaczak Fourier formulas for $\sigma'_{\alpha\beta}, \mu'_{\alpha 3}$:

$$
\begin{aligned}
\sigma'_{\alpha\beta} &= -\frac{2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)} \\
&\qquad \times [(\mu+\alpha)\varepsilon_{\beta\gamma}\partial_\alpha + (\mu-\alpha)\varepsilon_{\alpha\gamma}\partial_\beta] \\
&\qquad \times \partial_\gamma(\hat{F}_2 * G_1 * \hat{Y}_3) \\
&\quad - \frac{2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)}\varepsilon_{\alpha\beta}\Box_2(\hat{F}_2 * G_1 * \hat{Y}_3),
\end{aligned} \tag{3.56}
$$

$$
\mu'_{\alpha 3} = \frac{1}{\mu(\mu+\alpha)}\Box_2(\hat{F}_2 * G_1 * \hat{Y}_3). \tag{3.57}
$$

Remark 1: We can arrive to the corresponding classical Schaefer-Ignaczak fields: $(u^0_\alpha, \varphi^0_3, \varepsilon^0_{\alpha\beta}, \kappa^0_{\alpha 3}, \mu^0_{\alpha 3})$ using the Scheafer-Ignaczak relation (3.2), (3.11)–(3.14), (3.20), (3.21), and we arrive to the following results:

$$u_\alpha^0 = \frac{1}{\mu}(\hat{F}_2 * X_\alpha) - \frac{1}{\mu(\lambda+2\mu)} D_1 \partial_\alpha \partial_\beta (\hat{F}_2 * G_2 * X_\beta) - \frac{m}{\kappa}\partial_\alpha (G_2 * Q), \tag{3.58}$$

$$\varphi_3^0 = \frac{1}{2\mu}\varepsilon_{\alpha\beta}\partial_\alpha(\hat{F}_2 * X_\beta), \tag{3.59}$$

$$\varepsilon_{\alpha\beta}^0 = \frac{1}{2\mu}[(\hat{F}_2 * X_\alpha)_{,\beta} + (\hat{F}_2 * X_\beta)_{,\alpha}] - \frac{1}{\mu(\lambda+2\mu)} D_1 \partial^3_{\alpha\beta\gamma}(\hat{F}_2 * G_2 * X_\gamma) - \frac{m}{\kappa}\partial^2_{\alpha\beta}(G_2 * Q), \tag{3.60}$$

$$\mu_{\alpha 3}^0 = \frac{\gamma+\varepsilon}{2\mu}\varepsilon_{\gamma\beta}\partial^2_{\alpha\gamma}(\hat{F}_2 * X_\beta), \tag{3.61}$$

$$\kappa_{\alpha 3}^0 = \frac{1}{2\mu}\varepsilon_{\gamma\beta}\partial^2_{\alpha\gamma}(\hat{F}_2 * X_\beta). \tag{3.62}$$

Remark 2: we also can arrive to the complementary Schaefer-Ignaczak fields $(u'_\alpha, \varphi'_3, \gamma'_{\alpha\beta}, \kappa'_{\alpha 3}, \mu'_{\alpha 3})$, using Schaefer-Ignaczak relations (3.22)–(3.24), when we arrive to the following results:

$$u'_\alpha = -\frac{2\alpha}{\mu(\gamma+\varepsilon)(\mu+\alpha)}\varepsilon_{\alpha\gamma}\partial_\gamma(\hat{F}_2 * G_1 * \hat{Y}_3) \tag{3.63}$$

$$\varphi'_3 = \frac{1}{\mu(\mu+\alpha)(\gamma+\varepsilon)}\Box_2(\hat{F}_2 * G_1 * \hat{Y}_3) \tag{3.64}$$

$$\gamma'_{\alpha\beta} = \frac{1}{\mu(\mu+\alpha)(\gamma+\varepsilon)} \times [-2\alpha\varepsilon_{\beta\gamma}\partial^2_{\alpha\gamma} + \varepsilon_{\beta\alpha}\Box_2](\hat{F}_2 * G_1 * \hat{Y}_3) \tag{3.65}$$

$$\kappa'_{\alpha 3} = \frac{1}{\mu(\mu+\alpha)(\gamma+\varepsilon)}\partial_\alpha \Box_2(\hat{F}_2 * G_1 * \hat{Y}_3) \tag{3.66}$$

Remark 3: Finally substituting the results for $(\sigma^0_{\alpha\beta}, \theta^0, u^0_\alpha, \varphi^0_3, \varepsilon^0_{\alpha\beta}, \kappa^0_{\alpha 3}, \mu^0_{\alpha 3})$ and for $(\sigma'_{\alpha\beta}, u'_\alpha, \varphi'_3, \gamma'_{\alpha\beta}, \kappa'_{\alpha 3}, \mu'_{\alpha 3})$ in the relation (3.1), we arrive the final solution $(\sigma_{\alpha\beta}, \theta, u_\alpha, \varphi_3, \kappa_{\alpha 3}, \gamma_{\alpha\beta}, \mu_{\alpha 3})$, in which:

$$\begin{aligned}\sigma_{\alpha\beta} &= \partial_\beta(\hat{F}_2 * X_\alpha) + \partial_\alpha(\hat{F}_2 * X_\beta) \\ &- \frac{2}{\lambda+2\mu}\partial_\gamma\partial_\beta\partial_\alpha D_1(\hat{F}_2 * G_2 * X_\gamma) \\ &- \frac{2\mu v_T}{\kappa}\partial_\alpha\partial_\beta(G_2 * Q) \\ &+ \frac{1}{\lambda+2\mu}\Big\{[\mu D - D_1]\partial_\alpha(G_2 * X_\gamma) \\ &+ \frac{v_T}{\kappa}[(\lambda+2\mu)D - \lambda\Delta_1](G_2 * Q)\Big\}\delta_{\alpha\beta} \\ &- \frac{2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)} \times [(\mu+\alpha)\varepsilon_{\beta\gamma}\partial_\alpha + (\mu-\alpha)\varepsilon_{\alpha\gamma}\partial_\beta] \times \partial_\gamma(\hat{F}_2 * G_1 * \hat{Y}_3) \\ &- \frac{2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)}\varepsilon_{\alpha\beta}\Box_2(\hat{F}_2 * G_1 * \hat{Y}_3),\end{aligned} \tag{3.67}$$

$$\theta = -\frac{\eta_0}{\lambda+2\mu}\partial_t\partial_\alpha(G_2 * X_\alpha) - \frac{1}{\kappa(\lambda+2\mu)}\Box(G_2 * Q), \tag{3.68}$$

$$\mu_{\alpha 3} = \frac{\gamma+\varepsilon}{2\mu}\varepsilon_{\gamma\beta}\partial^2_{\alpha\gamma}(\hat{F}_2 * X_\beta) + \frac{1}{\mu(\mu+\alpha)}\Box_2\partial_\alpha(\hat{F}_2 * G_1 * \hat{Y}_3), \tag{3.69}$$

$$\begin{aligned}\gamma_{\alpha\beta} &= \frac{1}{2\mu}[(\hat{F}_2 * X_\alpha)_{,\beta} + (\hat{F}_2 * X_\beta)_{,\alpha}] \\ &- \frac{1}{\mu(\lambda+2\mu)}D_1\partial^3_{\alpha\beta\gamma}(\hat{F}_2 * G_2 * X_\gamma) \\ &- \frac{m}{\kappa}\partial^2_{\alpha\beta}(G_2 * Q) \\ &+ \frac{1}{\mu(\mu+\alpha)(\gamma+\varepsilon)} \times [-2\alpha\varepsilon_{\beta\gamma}\partial^2_{\alpha\gamma} + \varepsilon_{\beta\alpha}\Box_2](\hat{F}_2 * G_1 * \hat{Y}_3),\end{aligned} \tag{3.70}$$

$$\kappa_{\alpha 3} = \frac{1}{2\mu}\varepsilon_{\gamma\beta}\partial^2_{\alpha\gamma}(\hat{F}_2 * X_\beta) + \frac{1}{\mu(\mu+\alpha)(\gamma+\varepsilon)}\partial_\alpha\Box_2(\hat{F}_2 * G_1 * \hat{Y}_3), \tag{3.71}$$

$$\begin{aligned}u_\alpha &= \frac{1}{\mu}(\hat{F}_2 * X_\alpha) \\ &- \frac{1}{\mu(\lambda+2\mu)}D_1\partial_\alpha\partial_\beta(\hat{F}_2 * G_2 * X_\beta) \\ &- \frac{m}{\kappa}\partial_\alpha(G_2 * Q) \\ &- \frac{2\alpha}{\mu(\gamma+\varepsilon)(\mu+\alpha)}\varepsilon_{\alpha\gamma}\partial_\gamma(\hat{F}_2 * G_1 * \hat{Y}_3),\end{aligned} \tag{3.72}$$

$$\varphi_3 = \frac{1}{2\mu}\varepsilon_{\alpha\beta}\partial_\alpha(\hat{F}_2 * X_\beta) + \frac{1}{\mu(\mu+\alpha)(\gamma+\varepsilon)}\Box_2(\hat{F}_2 * G_1 * \hat{Y}_3). \tag{3.73}$$

3. CONCLUSIONS AND SUGGESTIONS

In paper, first we prove the isotherm of the complementary Scheafer-Ignaczak of the 2D(E-N:5) plane micropolar and unbounded body. Next for the same unbounded 2D(E-N:5) plane thermodynamical body, we derive the Fourier classical and complementary tensorial fields, using the combination of the transform theory and the method of Scheafer-Ignaczak. This result may have impotency in material resistance and composites theory.

Suggestions: we can suggest the following problems for discussing:

1. discuss the green functions (Fundamental Solutions) for the above Scheafer-Ignaczak initial-boundary problems.

2. Reputing the above problem for the second state of plane elastic strains.

REFERENCES

[1] W. S. Attiah and M. Al-Hasan, "The generalized thermodynamical Beltrami–Michell tensorial equations for the general thermodynamical state of the Hooke body", *Journal of Nature, Life and Applied Sciences*, vol. 4, no. 1, pp. 60–71, 2020.

[2] M. Al-Hasan and J. Dyszlewicz, "Coupled, dynamic micropolar problems of thermoelasticity: Stress–temperature equations of motion of Ignaczak type", in *Encyclopedia of Thermal Stresses*, R. B. Hetnarski, Ed., Dordrecht: Springer, 2014.

[3] R. B. Hetnarski, J. Ignaczak, M. R. Eslami, N. Noda, N. Sumi, and Y. Tanigawa, *Theory of Elasticity and Thermal Stresses*, Dordrecht: Springer Science+Business Media, 2013.

[4] R. B. Hetnarski and J. Ignaczak, *The Mathematical Theory of Elasticity*, 2nd ed., 6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL 33487-2742: CRC Press, Taylor & Francis Group, 2011.

[5] J. Ignaczak and M. O. Starzewski, *Thermoelasticity with Finite Wave Speeds*, New York: Oxford University Press Inc., 2010.

[6] J. Dyszlewicz, *Micropolar Theory of Elasticity*, ser. Lecture Notes in Applied and Computational Mechanics, vol. 15, Springer, 356 p., 2004.

[7] J. Dyszlewicz, “Selected problems of linear asymmetrical thermoelasticity”, *Journal of Thermal Stresses*, vol. 19, 1996.

[8] W. Nowacki, *Theory of Asymmetric Elasticity*, Warsaw: PWN, 1986.

[9] A. C. Eringen, “Linear theory of micropolar elasticity”, *J. Math*, 1966.

[10] J. Ignaczak, “Tensorial equations of motion for elastic materials with microstructure”, in *Trends of Elasticity and Thermoelasticity*, Groningen: Wolters-Noordhoff, Witold Nowacki Ann. Volume, 1971.

[11] L. Debnath and D. Bhatta, *Integral Transforms and their Applications*, 2nd ed., Boca Raton, Florida: CRC Press, 2007.