Type-II q-rung neutrosophic interval valued soft sets

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Abstract

In this study, the theory of the Type-II q-rung neutrosophic interval valued soft set (Type-II q-rung NIVS) is introduced. We also define a few operations based on the Type-II q-rung NIVS set. Type-II q-rung NIVS sets are formed by extending neutrosophic interval valued soft (NIVS) sets and q-rung fuzzy soft sets. Type-II q-rung NIVS sets and their similarity measures. An illustrative example illustrates how they can be used to successfully address uncertainty-related problems.

Keywords: Type-II q-rung NIVS set, NIVS set, decision making problem.

1 Introduction

There has been an explosion of popularity among scientists across practically all disciplines due to Zadeh's demonstrations of FS (FS). When resolving ambiguous issues, decision makers must take into account membership value (MV).[1] According to Atanassov[2] an intuitionistic FS (IFS) is a relationship between a membership value and a non-membership value that does not exceed unity when the MV and NMV are added up. It is possible, however, to experience problems when the sum of MV and NMV for a given attribute exceeds unity. Pythagorean FS (PFS) as introduced by Yager.[3] The square sum of its MV and NMV was derived from IFSs, and the restriction that it does not exceed unity indicates that it has been extended. FSs and IFSs can be extended to picture FSs.[4] The picture FSs were created by Cuong in 2015. It may be appropriate to use picture FS models to model human opinions that contain multiple types of responses, such as yes, abstention, no, and refusal. Four types of human voters can be identified: supporters, abstainers, opposers, and refusers. As an example, voting can be helpful. A wider range of MV, NMV, and reluctance values can be assigned using these sets. As a result, Cuong et al.[5] employed three pointers in their FS logic: a positive MV, a neutral MV, and a negative MV, based on the sum of the three grades of these three points not exceeding 1, which was used to implement image FS logic. Lastly, it can offer more benefits than PFS and IFS in a few applications.

As Smarandache[6] points out, Neutosophy was developed in order to deal with ambiguous and inconsistent information. In recent years, NSs have become a popular theory. The main difference between FS and IFS is neutral cognition, which is the study of neutosophy. A development of NS reasoning is recognized in Smarandache.[6] Neutosophic sets (NS) are further generalizations of the FS and IFS. A classical set, FS, IVFS,

https://doi.org/10.54216/IJNS.230326

Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

etc., can be generalized by a NS from a philosophical perspective. The idea of Pythagorean NS has recently been discussed by Jansi et al.[7] There has been recent development of NS utilizing aggregation operators.[8–11]

An important contribution Molodtsov[12] made was the theory of soft sets. In real-world situations, soft sets describe decision-making more accurately than other uncertain theories. Soft sets need to be integrated with other mathematical models, which is another crucial topic for research. As proposed by Maji, the fuzzy soft set concept (FSS)[13] and intuitionistic fuzzy soft set concept (IFSS)[14] were developed. There are a variety of DM issues that can be addressed by using these two theories. A picture of a fuzzy soft set[15] was discussed with Yong Yang. The Pythagorean fuzzy soft set has recently been added to FSS by Peng.[16] MADM problems were resolved using this methodology when both MV and NMV were greater than one but the summation of the squares was equal to or less than one. Generalized FSS was discussed by Pinaki Majumdar.[17] Shawkat et al.[18] presented the concept of an interval-valued fuzzy soft set. The possibility fuzzy soft set was introduced by Alkhazaleh et al.[19] with practical applications. Using a decision-making approach, Faruk Karaaslan proposed a reason for neutral soft sets with the possibility.[20] The NS has been useful to many researchers for practical applications.[21–23] A number of ideal structures and their applications were discussed by Palanikumar et al. palanir3, palanir6, palani1. Type-II q-rung NIVS sets are examined in this article. Section **??** contains the introduction. There is also a PyNIVS set and a Generalized IVS set in Section 2. Section 3 conceptualizes Type-II q-rung NIVFS sets. The method for calculating the similarity measure for Type-II q-rung NIVFS sets can be found in Section 4. Providing numerical examples is beneficial when evaluating the Type-II q-rung NIVS set model.

2 Preliminaries

The purpose of this part is to introduce and review some ideas related to well-known literary concepts such as Pythagorean neutrosophics and generalized fuzzy soft sets.

Definition 2.1. [7] Let $\mathscr{X}$ be the universe, Pythagorean neutrosophic interval valued (PyNIV) set P in $\mathscr{X}$ is $\widehat{P} = \{c\widehat{\zeta}_P(\aleph), \widehat{\ell}_P(\aleph), \widehat{\hbar}_P(\aleph) | \aleph \in \mathscr{X}\}$, where $\widehat{\zeta}_P(\aleph) = [\zeta_P^-(\aleph), \zeta_P^+(\aleph)]$ and $\widehat{\ell}_P(\aleph) = [\ell_P^-(\aleph), \ell_P^+(\aleph)]$ and $\widehat{\hbar}_P(\aleph) = [\hbar_P^-(\aleph), \hbar_P^+(\aleph)]$ represents the degree of MV IMV and FV of P, respectively. The function $\widehat{\zeta}_P : \mathscr{X} \to D[0,1]$, $\widehat{\ell}_P : \mathscr{X} \to D[0,1]$, $\widehat{\hbar}_P : \mathscr{X} \to D[0,1]$ and $0 \leq (\widehat{\zeta}_P(\aleph))^2 + (\widehat{\ell}_P(\aleph))^2 + (\widehat{\hbar}_P(\aleph))^2 \leq 2$ means $0 \leq (\zeta_P^+(\aleph))^2 + (\ell_P^+(\aleph))^2 + (\hbar_P^+(\aleph))^2 \leq 2$. Here $\widehat{P} = \Big\langle [\zeta_P^-, \zeta_P^+], [\ell_P^-, \ell_P^+], [\hbar_P^-, \hbar_P^+] \Big\rangle$ is called a Pythagorean neutrosophic interval valued number(PyNIVN).

Definition 2.2. [7] Given that $\widehat{\wp_1} = \langle \zeta_{\widehat{\wp_1}}, \ell_{\widehat{\wp_1}}, \hbar_{\widehat{\wp_1}} \rangle$, $\widehat{\wp_2} = \langle \zeta_{\widehat{\wp_2}}, \ell_{\widehat{\wp_2}}, \hbar_{\widehat{\wp_2}} \rangle$ and $\widehat{\wp_3} = \langle \zeta_{\widehat{\wp_3}}, \ell_{\widehat{\wp_3}}, \hbar_{\widehat{\wp_3}} \rangle$ are any three PyNIVNs over $(\mathscr{X}, E)$. Then
(i) $\widehat{\wp_1^c} = \langle \hbar_{\widehat{\wp_1}}, \ell_{\widehat{\wp_1}}, \zeta_{\widehat{\wp_1}} \rangle$
(ii) $\widehat{\wp_1} \sqcup \widehat{\wp_2} = \Big\langle \max(\zeta_{\widehat{\wp_1}}, \zeta_{\widehat{\wp_2}}), \min(\ell_{\widehat{\wp_1}}, \ell_{\widehat{\wp_2}}), \min(\hbar_{\widehat{\wp_1}}, \hbar_{\widehat{\wp_2}}) \Big\rangle$
(iii) $\widehat{\wp_1} \sqcap \widehat{\wp_2} = \Big\langle \min(\zeta_{\widehat{\wp_1}}, \zeta_{\widehat{\wp_2}}), \min(\ell_{\widehat{\wp_1}}, \ell_{\widehat{\wp_2}}), \max(\hbar_{\widehat{\wp_1}}, \hbar_{\widehat{\wp_2}}) \Big\rangle$
(iv) $\widehat{\wp_1} \leq \widehat{\wp_2}$ if and only if $\zeta_{\widehat{\wp_1}} \leq \zeta_{\widehat{\wp_2}}$ and $\ell_{\widehat{\wp_1}} \leq \ell_{\widehat{\wp_2}}$ and $\hbar_{\widehat{\wp_1}} \geq \hbar_{\widehat{\wp_2}}$
(v) $\widehat{\wp_1} = \widehat{\wp_2}$ if and only if $\zeta_{\widehat{\wp_1}} = \zeta_{\widehat{\wp_2}}$ and $\ell_{\widehat{\wp_1}} = \ell_{\widehat{\wp_2}}$ and $\hbar_{\widehat{\wp_1}} = \hbar_{\widehat{\wp_2}}$.

Definition 2.3. [18] Let $\mathscr{X} = \{\aleph_1, \aleph_2, ..., \aleph_n\}$ and $\mathscr{E} = \{\delta_1, \delta_2, ..., \delta_m\}$ be the universe and set of parameter, respectively. The notation $(\mathscr{X}, \mathscr{E})$ is a soft universe. The function $\widehat{\mathscr{U}} : \mathscr{E} \to D(I)^{\mathscr{X}}$ and $\widehat{\sigma}$ be an IVF subset of $\mathscr{E}$, ie. $\widehat{\sigma} : \mathscr{E} \to I = D[0,1]$. Let $\widehat{\mathscr{U}_\sigma} : \mathscr{E} \to D(I)^{\mathscr{X}} \times D(I)$ be the function defined as $\widehat{\mathscr{U}_\sigma}(\delta) = \Big(\widehat{\mathscr{U}}(\delta)(\aleph), \widehat{\sigma}(\delta)\Big), \forall \aleph \in \mathscr{X}$. Then $\widehat{\mathscr{U}_\sigma}$ is called a generalized interval valued fuzzy soft (GIVFS) set on $(\mathscr{X}, \mathscr{E})$. For each parameter δ_i, $\widehat{\mathscr{U}_\sigma}(\delta_i) = \Big(\widehat{\mathscr{U}}(\delta_i)(\aleph), \widehat{\sigma}(\delta_i)\Big), \forall \aleph \in \mathscr{X}$ not simply the degree to which the constituent parts of $\mathscr{X}$ in $\widehat{\mathscr{U}}(\delta_i)$ but moreover the interval valued likelihood of such belongingness, which is given by $\widehat{\sigma}(\delta_i)$. So we can write $\widehat{\mathscr{U}_\sigma}(\delta_i)$ as $\widehat{\mathscr{U}_\sigma}(\delta_i) = \Big(\Big\{\frac{\aleph_1}{\widehat{\mathscr{U}}(\delta_i)(\aleph_1)}, \frac{\aleph_2}{\widehat{\mathscr{U}}(\delta_i)(\aleph_2)}, ..., \frac{\aleph_n}{\widehat{\mathscr{U}}(\delta_i)(\aleph_n)}\Big\}, \widehat{\sigma}(\delta_i)\Big)$, where $\widehat{\mathscr{U}}(\delta_i)(\aleph_1)$, $\widehat{\mathscr{U}}(\delta_i)(\aleph_2)$,...,$\widehat{\mathscr{U}}(\delta_i)(\aleph_n)$ are the degrees of belongingness and the degree of such belongingness interval valued possibility is $\widehat{\sigma}(\delta_i)$.

Definition 2.4. [19] Let $\mathscr{X} = \{\aleph_1, \aleph_2, ..., \aleph_n\}$ and $\mathscr{E} = \{\delta_1, \delta_2, ..., \delta_m\}$ be the universe and set of parameters, respectively. The notation $(\mathscr{X}, \mathscr{E})$ is a soft universe. The function $\widehat{\mathscr{U}} : \mathscr{E} \to \widehat{\mathscr{U}}(\mathscr{X})$ and $\widehat{\sigma}$ be an IVF subset of $\mathscr{E}$, ie. $\widehat{\sigma} : \mathscr{E} \to \widehat{\mathscr{U}}(\mathscr{X})$. Let $\widehat{\mathscr{U}_\sigma} : \mathscr{E} \to \widehat{\mathscr{U}}(\mathscr{X}) \times \widehat{\mathscr{U}}(\mathscr{X})$ be a function defined as $\widehat{\mathscr{U}_\sigma}(\delta) =$

https://doi.org/10.54216/IJNS.230326
Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

$(\widehat{\mathscr{U}}(\delta)(\aleph), \widehat{\sigma}(\delta)(\aleph)), \forall \aleph \in \mathscr{X}$. Then $\widehat{\mathscr{U}_\sigma}$ is called a possibility interval valued fuzzy soft (PIVFS) set on $(\mathscr{X}, \mathscr{E})$. For each parameter δ_i, $\widehat{\mathscr{U}_\sigma}(\delta_i) = (\widehat{\mathscr{U}}(\delta_i)(\aleph), \widehat{\sigma}(\delta_i)(\aleph))$ not merely how much the components of $\mathscr{X}$ belong together in $\widehat{\mathscr{U}}(\delta_i)$, also the degree of IV possibilities of such belongingness which is $\widehat{\sigma}(\delta_i)$. Hence, $\widehat{\mathscr{U}_\sigma}(\delta_i) = \left\{ \left(\frac{\aleph_1}{\widehat{\mathscr{U}}(\delta_i)(\aleph_1)}, \widehat{\sigma}(\delta_i)(\aleph_1)\right), \left(\frac{\aleph_2}{\widehat{\mathscr{U}}(\delta_i)(\aleph_2)}, \widehat{\sigma}(\delta_i)(\aleph_2)\right), ..., \left(\frac{\aleph_n}{\widehat{\mathscr{U}}(\delta_i)(\aleph_n)}, \widehat{\sigma}(\delta_i)(\aleph_n)\right) \right\}$.

Remark 2.5. Using fundamental operations of arithmetic leads to the following:
(i) $[a, b] + [c, d] = [a + c, b + d]$
(ii) $[a, b] - [c, d] = [a - d, b - c]$
(iii) $[a, b] \cdot [c, d] = [ac, bd]$, whenever $a \geq 0$ and $b \geq 0$
(iv) $\frac{1}{[a,b]} = [\frac{1}{b}, \frac{1}{a}]$, whenever $0 \notin [a, b]$, $a, b, c, d \in \mathbb{R}$.

3 Type-II qNIVFS set

A Type-II q-rung NSIVS set is just in the beginning stages.

Definition 3.1. Let $\mathscr{X} = \{\aleph_1, \aleph_2, ..., \aleph_n\}$ and $\mathscr{E} = \{\delta_1, \delta_2, ..., \delta_m\}$ be the universal and set of parameters, respectively. The notation $(\mathscr{X}, \mathscr{E})$ represents a soft universe. Consider $\widehat{\mathscr{U}} : \mathscr{E} \to S\widehat{\mathscr{U}}(\mathscr{X})$ and $\widehat{u}$ is a PyNIV subset of $\mathscr{E}$. That is $\widehat{u} : \mathscr{E} \to D[0, 1]$, $S\widehat{\mathscr{U}}(\mathscr{X})$ denotes the collection of all PyNIV subsets of $\mathscr{X}$. If $\widehat{\mathscr{U}_u} : \mathscr{E} \to S\widehat{\mathscr{U}}(\mathscr{X}) \times D[0, 1]$ is a function defined as $\widehat{\mathscr{U}_u}(\delta) = \left(\widehat{\mathscr{U}}(\delta)(\aleph), \widehat{u}(\delta)\right)$, $\aleph \in \mathscr{X}$, then $\widehat{\mathscr{U}_u}$ is a Type-II q-rung-rung neutrosophic interval valued soft (Type-II q-rungNIVS) set on $(\mathscr{X}, \mathscr{E})$. For each parameter e, $\widehat{\mathscr{U}_u}(\delta_i) = \left(\left\{\frac{\aleph_1}{(\zeta_{\widehat{\mathscr{U}}(\delta)}(\aleph_1), \ell_{\widehat{\mathscr{U}}(\delta)}(\aleph_1), \hbar_{\widehat{\mathscr{U}}(\delta)}(\aleph_1))}, ..., \frac{\aleph_n}{(\zeta_{\widehat{\mathscr{U}}(\delta)}(\aleph_n), \ell_{\widehat{\mathscr{U}}(\delta)}(\aleph_n), \hbar_{\widehat{\mathscr{U}}(\delta)}(\aleph_n))}\right\}, (\widehat{u}_1(\delta_i), \widehat{u}_2(\delta_i), \widehat{u}_3(\delta_i))\right)$.

Definition 3.2. Let $\mathscr{X}$ and $\mathscr{E}$ be the universal and set of parameters, respectively. Let $\widehat{\mathscr{U}_u}$ and $\widehat{\mathscr{V}_v}$ be the Type-II q-rungNIVS sets on $(\mathscr{X}, \mathscr{E})$. Now $\widehat{\mathscr{U}_v}$ is a Type-II q-rung NIVS subset of $\widehat{\mathscr{V}_v}$ if and only if
(i) $\widehat{\mathscr{U}}(\delta)(\aleph) \sqsubseteq \widehat{\mathscr{V}}(\delta)(\aleph)$ if $\zeta_{\widehat{\mathscr{U}}(\delta)}(\aleph) \leq \zeta_{\widehat{\mathscr{V}}(\delta)}(\aleph)$, $\ell_{\widehat{\mathscr{U}}(\delta)}(\aleph) \leq \ell_{\widehat{\mathscr{V}}(\delta)}(\aleph)$, $\hbar_{\widehat{\mathscr{U}}(\delta)}(\aleph) \geq \hbar_{\widehat{\mathscr{V}}(\delta)}(\aleph)$,
(ii) $\widehat{u}(\delta) \leq \widehat{v}(\delta)$, $\forall \delta \in \mathscr{E}$ and $\forall \aleph \in \mathscr{X}$.

4 Method for similarity measure

Definition 4.1. Let $\mathscr{X} = \{\aleph_1, \aleph_2, ..., \aleph_m\}$ and $\mathscr{E} = \{\delta_1, \delta_2, ..., \delta_n\}$ be the universe and set of parameters, respectively. Let $\widehat{P}_u$ and $\widehat{Q}_v$ be the Type-II q-rungNIVS sets on $(\mathscr{X}, \mathscr{E})$. The similarity measure between two Type-II q-rungNIVS sets $\widehat{P}_u$ and $\widehat{Q}_v$ is defined as $Sim(\widehat{P}_u, \widehat{Q}_v) = \Phi(\widehat{P}, \widehat{Q}) \cdot \Psi(\widehat{u}, \widehat{v})$.
where $\Phi(\widehat{P}, \widehat{Q}) =$

$$\frac{1}{m}\sum_{j=1}^{m}\begin{bmatrix} \min\left\{T_1^-\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right), T_2^-\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right), S^-\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right)\right\}, \\ \max\left\{T_1^+\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right), T_2^+\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right), S^+\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right)\right\} \end{bmatrix}$$

and

$$\widehat{T_1}\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right) = \left[\frac{\sum_{i=1}^{n}\left(\zeta^-_{P(\delta_i)}(\aleph_j) \cdot \zeta^-_{Q(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n}\left(1 - \sqrt{\left(1 - \zeta^{q-}_{P(\delta_i)}(\aleph_j)\right) \cdot \left(1 - \zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right)}\right)}, \frac{\sum_{i=1}^{n}\left(\zeta^+_{P(\delta_i)}(\aleph_j) \cdot \zeta^+_{Q(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n}\left(1 - \sqrt{\left(1 - \zeta^{q+}_{P(\delta_i)}(\aleph_j)\right) \cdot \left(1 - \zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right)}\right)}\right]$$

$$\widehat{T_2}\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right) = \left[\frac{\sum_{i=1}^{n}\left(\ell^{q-}_{P(\delta_i)}(\aleph_j) \cdot \ell^{q-}_{Q(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n}\left(1 - \sqrt[q]{\left(1 - \ell^{2q-}_{P(\delta_i)}(\aleph_j)\right) \cdot \left(1 - \ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)}\right)}, \frac{\sum_{i=1}^{n}\left(\ell^{q+}_{P(\delta_i)}(\aleph_j) \cdot \ell^{q+}_{Q(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n}\left(1 - \sqrt[q]{\left(1 - \ell^{2q+}_{P(\delta_i)}(\aleph_j)\right) \cdot \left(1 - \ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)}\right)}\right]$$

$$\widehat{S}\left(\widehat{P}(\delta)(\aleph_j), \widehat{Q}(\delta)(\aleph_j)\right) = \left[1 - \sqrt[q]{\left|\left[\frac{\sum_{i=1}^{n}\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j) - \hbar^{q+}_{Q(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1 + \left(\hbar^{q+}_{P(\delta_i)}(\aleph_j) \cdot \hbar^{q+}_{Q(\delta_i)}(\aleph_j)\right)}, \frac{\sum_{i=1}^{n}\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j) - \hbar^{q-}_{Q(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1 + \left(\hbar^{q-}_{P(\delta_i)}(\aleph_j) \cdot \hbar^{q-}_{Q(\delta_i)}(\aleph_j)\right)}\right]\right|}\right],$$

Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

for $j = 1, 2, ..., m$.
and $\Psi(\widehat{u}, \widehat{v}) = 1 - \left[\frac{\sum_{i=1}^{n} \min\left\{\left[u^-(\delta_i), v^-(\delta_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^+(\delta_i)+v^+(\delta_i)\right)}, \frac{\sum_{i=1}^{n} \max\left\{\left[u^+(\delta_i), v^+(\delta_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^-(\delta_i)+v^-(\delta_i)\right)} \right]$

Theorem 4.2. *Let $\widehat{P}_u,\ \widehat{Q}_v$ and $\widehat{R}_w$ be the any three Type-II q-rungNIVS sets over $(\mathscr{X}, \mathscr{E})$. Prove that $\widehat{P}_u \sqsubseteq \widehat{Q}_v \sqsubseteq \widehat{R}_w \implies Sim(\widehat{P}_u, \widehat{R}_w) \leq Sim(\widehat{Q}_v, \widehat{R}_w)$.*

Proof. For $j = 1, 2, ..., m$

$$\left\{\begin{array}{l} \widehat{P}_u \sqsubseteq \widehat{Q}_v \implies \left\{\begin{array}{l} \left[\zeta^-_{P(\delta_i)}(\aleph_j), \zeta^+_{P(\delta_i)}(\aleph_j)\right] \leq \left[\zeta^-_{Q(\delta_i)}(\aleph_j), \zeta^+_{Q(\delta_i)}(\aleph_j)\right] \\ \left[\ell^-_{P(\delta_i)}(\aleph_j), \ell^+_{P(\delta_i)}(\aleph_j)\right] \leq \left[\ell^-_{Q(\delta_i)}(\aleph_j), \ell^+_{Q(\delta_i)}(\aleph_j)\right] \\ \left[\hbar^-_{P(\delta_i)}(\aleph_j), \hbar^+_{P(\delta_i)}(\aleph_j)\right] \geq \left[\hbar^-_{Q(\delta_i)}(\aleph_j), \hbar^+_{Q(\delta_i)}(\aleph_j)\right] \\ \left[u^-(\delta_i), u^+(\delta_i)\right] \leq \left[v^-(\delta_i), v^+(\delta_i)\right] \end{array}\right\} \\ \widehat{P}_u \sqsubseteq \widehat{R}_w \implies \left\{\begin{array}{l} \left[\zeta^-_{P(\delta_i)}(\aleph_j), \zeta^+_{P(\delta_i)}(\aleph_j)\right] \leq \left[\zeta^-_{R(\delta_i)}(\aleph_j), \zeta^+_{R(\delta_i)}(\aleph_j)\right] \\ \left[\ell^-_{P(\delta_i)}(\aleph_j), \ell^+_{P(\delta_i)}(\aleph_j)\right] \leq \left[\ell^-_{R(\delta_i)}(\aleph_j), \ell^+_{R(\delta_i)}(\aleph_j)\right] \\ \left[\hbar^-_{P(\delta_i)}(\aleph_j), \hbar^+_{P(\delta_i)}(\aleph_j)\right] \geq \left[\hbar^-_{R(\delta_i)}(\aleph_j), \hbar^+_{R(\delta_i)}(\aleph_j)\right] \\ \left[u^-(\delta_i), u^+(\delta_i)\right] \leq \left[w^-(\delta_i), w^+(\delta_i)\right] \end{array}\right\} \\ \widehat{Q}_v \sqsubseteq \widehat{R}_w \implies \left\{\begin{array}{l} \left[\zeta^-_{Q(\delta_i)}(\aleph_j), \zeta^+_{Q(\delta_i)}(\aleph_j)\right] \leq \left[\zeta^-_{R(\delta_i)}(\aleph_j), \zeta^+_{R(\delta_i)}(\aleph_j)\right] \\ \left[\ell^-_{Q(\delta_i)}(\aleph_j), \ell^+_{Q(\delta_i)}(\aleph_j)\right] \leq \left[\ell^-_{R(\delta_i)}(\aleph_j), \ell^+_{R(\delta_i)}(\aleph_j)\right] \\ \left[\hbar^-_{Q(\delta_i)}(\aleph_j), \hbar^+_{Q(\delta_i)}(\aleph_j)\right] \geq \left[\hbar^-_{R(\delta_i)}(\aleph_j), \hbar^+_{R(\delta_i)}(\aleph_j)\right] \\ \left[v^-(\delta_i), v^+(\delta_i)\right] \leq \left[w^-(\delta_i), w^+(\delta_i)\right] \end{array}\right\} \end{array}\right\}$$

Clearly,

$$\begin{aligned} & \left[\left(\zeta^-_{P(\delta_i)}(\aleph_j) \cdot \zeta^-_{R(\delta_i)}(\aleph_j)\right), \left(\zeta^+_{P(\delta_i)}(\aleph_j) \cdot \zeta^+_{R(\delta_i)}(\aleph_j)\right)\right] \\ \leq\ & \left[\left(\zeta^-_{Q(\delta_i)}(\aleph_j) \cdot \zeta^-_{R(\delta_i)}(\aleph_j)\right), \left(\zeta^+_{Q(\delta_i)}(\aleph_j) \cdot \zeta^+_{R(\delta_i)}(\aleph_j)\right)\right] \end{aligned}$$

implies that

$$\begin{aligned} & \left[\sum_{i=1}^{n}\left(\zeta^-_{P(\delta_i)}(\aleph_j) \cdot \zeta^-_{R(\delta_i)}(\aleph_j)\right), \sum_{i=1}^{n}\left(\zeta^+_{P(\delta_i)}(\aleph_j) \cdot \zeta^+_{R(\delta_i)}(\aleph_j)\right)\right] \\ \leq\ & \left[\sum_{i=1}^{n}\left(\zeta^-_{Q(\delta_i)}(\aleph_j) \cdot \zeta^-_{R(\delta_i)}(\aleph_j)\right), \sum_{i=1}^{n}\left(\zeta^+_{Q(\delta_i)}(\aleph_j) \cdot \zeta^+_{R(\delta_i)}(\aleph_j)\right)\right] \end{aligned} \tag{1}$$

for $j = 1, 2, ..., m$
Clearly,

$$\left[\zeta^{q-}_{P(\delta_i)}(\aleph_j),\ \zeta^{q+}_{P(\delta_i)}(\aleph_j)\right] \leq \left[\zeta^{q-}_{Q(\delta_i)}(\aleph_j),\ \zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right] \leq \left[\zeta^{q-}_{R(\delta_i)}(\aleph_j),\ \zeta^{q+}_{R(\delta_i)}(\aleph_j)\right]$$

implies that

$$\left[-\zeta^{q+}_{P(\delta_i)}(\aleph_j),\ -\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right] \geq \left[-\zeta^{q+}_{Q(\delta_i)}(\aleph_j),\ -\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right] \geq \left[-\zeta^{q+}_{R(\delta_i)}(\aleph_j),\ -\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right]$$

and

$$\left[1-\zeta^{q-}_{P(\delta_i)}(\aleph_j),\ 1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right] \geq \left[1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j),\ 1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right] \geq \left[1-\zeta^{q-}_{R(\delta_i)}(\aleph_j),\ 1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right]$$

and

$$\begin{aligned} & \left[\left(\left(1-\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right) \cdot \left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right),\ \left(\left(1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right) \cdot \left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)\right] \\ \geq\ & \left[\left(\left(1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right) \cdot \left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right),\ \left(\left(1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right) \cdot \left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)\right] \end{aligned}$$

https://doi.org/10.54216/IJNS.230326
Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

and

$$\begin{aligned}&\left[\sqrt[q]{\left(\left(1-\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right]\\ \geq\ &\left[\sqrt[q]{\left(\left(1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right]\end{aligned}$$

and

$$\begin{aligned}&1-\left[\sqrt[q]{\left(\left(1-\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right]\\ \leq\ &1-\left[\sqrt[q]{\left(\left(1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right]\end{aligned}$$

and

$$\begin{aligned}&\left[1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)},\ 1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right]\\ \leq\ &\left[1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)},\ 1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right]\end{aligned}$$

and

$$\begin{aligned}&\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]\\ \leq\ &\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]\end{aligned} \quad (2)$$

Equations 1 and 2,

$$\begin{aligned}&\frac{\left[\sum_{i=1}^{n}\left(\zeta^{-}_{P(\delta_i)}(\aleph_j)\cdot\zeta^{-}_{R(\delta_i)}(\aleph_j)\right),\ \sum_{i=1}^{n}\left(\zeta^{+}_{P(\delta_i)}(\aleph_j)\cdot\zeta^{+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]}\\ \leq\ &\frac{\left[\sum_{i=1}^{n}\left(\zeta^{-}_{Q(\delta_i)}(\aleph_j)\cdot\zeta^{-}_{R(\delta_i)}(\aleph_j)\right),\ \sum_{i=1}^{n}\left(\zeta^{+}_{Q(\delta_i)}(\aleph_j)\cdot\zeta^{+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]}\end{aligned}$$

implies that

$$\begin{aligned}&\left[\frac{\left[\sum_{i=1}^{n}\left(\zeta^{-}_{P(\delta_i)}(\aleph_j)\cdot\zeta^{-}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\zeta^{+}_{P(\delta_i)}(\aleph_j)\cdot\zeta^{+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]}\right]\\ \leq\ &\left[\frac{\left[\sum_{i=1}^{n}\left(\zeta^{-}_{Q(\delta_i)}(\aleph_j)\cdot\zeta^{-}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\zeta^{+}_{Q(\delta_i)}(\aleph_j)\cdot\zeta^{+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\zeta^{q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\zeta^{q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]}\right]\end{aligned}$$

Therefore

$$\widehat{T_1}\left(\widehat{P}(\delta)(\aleph_j),\widehat{R}(\delta)(\aleph_j)\right)\leq\widehat{T_1}\left(\widehat{Q}(\delta)(\aleph_j),\widehat{R}(\delta)(\aleph_j)\right) \quad (3)$$

Clearly,

$$\begin{aligned}&\left[\left(\ell^{q-}_{P(\delta_i)}(\aleph_j)\cdot\ell^{q-}_{R(\delta_i)}(\aleph_j)\right),\left(\ell^{q+}_{P(\delta_i)}(\aleph_j)\cdot\ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\\ \leq\ &\left[\left(\ell^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\ell^{q-}_{R(\delta_i)}(\aleph_j)\right),\left(\ell^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\end{aligned}$$

https://doi.org/10.54216/IJNS.230326

Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

implies that

$$\begin{aligned} &\left[\sum_{i=1}^{n}\left(\ell^{q-}_{P(\delta_i)}(\aleph_j)\cdot \ell^{q-}_{R(\delta_i)}(\aleph_j)\right), \sum_{i=1}^{n}\left(\ell^{q+}_{P(\delta_i)}(\aleph_j)\cdot \ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right] \\ \leq\ &\left[\sum_{i=1}^{n}\left(\ell^{q-}_{Q(\delta_i)}(\aleph_j)\cdot \ell^{q-}_{R(\delta_i)}(\aleph_j)\right), \sum_{i=1}^{n}\left(\ell^{q+}_{Q(\delta_i)}(\aleph_j)\cdot \ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right] \end{aligned} \tag{4}$$

for $j = 1, 2, ..., m$
Clearly,

$$\left[\ell^{2q-}_{P(\delta_i)}(\aleph_j),\ \ell^{2q+}_{P(\delta_i)}(\aleph_j)\right] \leq \left[\ell^{2q-}_{Q(\delta_i)}(\aleph_j),\ \ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right] \leq \left[\ell^{2q-}_{R(\delta_i)}(\aleph_j),\ \ell^{2q+}_{R(\delta_i)}(\aleph_j)\right]$$

implies that

$$\left[-\ell^{2q+}_{P(\delta_i)}(\aleph_j),\ -\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right] \geq \left[-\ell^{2q+}_{Q(\delta_i)}(\aleph_j),\ -\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right] \geq \left[-\ell^{2q+}_{R(\delta_i)}(\aleph_j),\ -\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right]$$

and

$$\left[1-\ell^{2q-}_{P(\delta_i)}(\aleph_j),\ 1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right] \geq \left[1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j),\ 1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right] \geq \left[1-\ell^{2q-}_{R(\delta_i)}(\aleph_j),\ 1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right]$$

and

$$\begin{aligned} &\left[\left(\left(1-\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right),\ \left(\left(1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)\right] \\ \geq\ &\left[\left(\left(1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right),\ \left(\left(1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)\right] \end{aligned}$$

and

$$\begin{aligned} &\left[\sqrt[q]{\left(\left(1-\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right] \\ \geq\ &\left[\sqrt[q]{\left(\left(1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right] \end{aligned}$$

and

$$\begin{aligned} &1-\left[\sqrt[q]{\left(\left(1-\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right] \\ \leq\ &1-\left[\sqrt[q]{\left(\left(1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)},\ \sqrt[q]{\left(\left(1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right] \end{aligned}$$

and

$$\begin{aligned} &\left[1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)},\ 1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right] \\ \leq\ &\left[1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)},\ 1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right] \end{aligned}$$

and

$$\begin{aligned} &\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right] \\ \leq\ &\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right] \end{aligned} \tag{5}$$

Equations 4 and 5,

$$\begin{aligned} &\frac{\left[\sum_{i=1}^{n}\left(\ell^{q-}_{P(\delta_i)}(\aleph_j)\cdot \ell^{q-}_{R(\delta_i)}(\aleph_j)\right),\ \sum_{i=1}^{n}\left(\ell^{q+}_{P(\delta_i)}(\aleph_j)\cdot \ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]} \\ \leq\ &\frac{\left[\sum_{i=1}^{n}\left(\ell^{q-}_{Q(\delta_i)}(\aleph_j)\cdot \ell^{q-}_{R(\delta_i)}(\aleph_j)\right),\ \sum_{i=1}^{n}\left(\ell^{q+}_{Q(\delta_i)}(\aleph_j)\cdot \ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]} \end{aligned}$$

https://doi.org/10.54216/IJNS.230326

Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

implies that

$$\left[\frac{\left[\sum_{i=1}^{n}\left(\ell^{q-}_{P(\delta_i)}(\aleph_j)\cdot\ell^{q-}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]},\frac{\left[\sum_{i=1}^{n}\left(\ell^{q+}_{P(\delta_i)}(\aleph_j)\cdot\ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{P(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]}\right]$$
$$\leq\left[\frac{\left[\sum_{i=1}^{n}\left(\ell^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\ell^{q-}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q-}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q-}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]},\frac{\left[\sum_{i=1}^{n}\left(\ell^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\ell^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt[q]{\left(\left(1-\ell^{2q+}_{Q(\delta_i)}(\aleph_j)\right)\cdot\left(1-\ell^{2q+}_{R(\delta_i)}(\aleph_j)\right)\right)}\right)\right]}\right]$$

Therefore

$$\widehat{T_2}\left(\widehat{P}(\delta)(\aleph_j),\widehat{R}(\delta)(\aleph_j)\right)\leq\widehat{T_2}\left(\widehat{Q}(\delta)(\aleph_j),\widehat{R}(\delta)(\aleph_j)\right) \tag{6}$$

Clearly,

$$\left[\hbar^{q-}_{P(\delta_i)}(\aleph_j),\hbar^{q+}_{P(\delta_i)}(\aleph_j)\right]\geq\left[\hbar^{q-}_{Q(\delta_i)}(\aleph_j),\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\right]\geq\left[\hbar^{q-}_{R(\delta_i)}(\aleph_j),\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right]$$

and

$$\left[\hbar^{q-}_{P(\delta_i)}(\aleph_j),\hbar^{q+}_{P(\delta_i)}(\aleph_j)\right]-\left[\hbar^{q-}_{R(\delta_i)}(\aleph_j),\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right]\geq\left[\hbar^{q-}_{Q(\delta_i)}(\aleph_j),\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\right]-\left[\hbar^{q-}_{R(\delta_i)}(\aleph_j),\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right]$$

implies

$$\left|\left[\hbar^{q-}_{P(\delta_i)}(\aleph_j),\hbar^{q+}_{P(\delta_i)}(\aleph_j)\right]-\left[\hbar^{q-}_{R(\delta_i)}(\aleph_j),\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right]\right|\geq\left|\left[\hbar^{q-}_{Q(\delta_i)}(\aleph_j),\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\right]-\left[\hbar^{q-}_{R(\delta_i)}(\aleph_j),\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right]\right|$$

and

$$\left|\left[\hbar^{q-}_{P(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\ ,\ \hbar^{q+}_{P(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right]\right|\geq\left|\left[\hbar^{q-}_{Q(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\ ,\ \hbar^{q+}_{Q(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right]\right|$$

Hence

$$\left|\left[\sum_{i=1}^{n}\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\ ,\ \sum_{i=1}^{n}\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)\right]\right|$$
$$\geq\left|\left[\sum_{i=1}^{n}\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\ ,\ \sum_{i=1}^{n}\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)\right]\right| \tag{7}$$

Also,

$$\left[\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right),\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]$$
$$\geq\left[\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right),\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]$$

implies that

$$\left|\left[\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right),\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\right|$$
$$\geq\left|\left[\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right),\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\right|$$

$$\left|\left[\sum_{i=1}^{n}1+\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right),\sum_{i=1}^{n}1+\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\right|$$
$$\geq\left|\left[\sum_{i=1}^{n}1+\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right),\sum_{i=1}^{n}1+\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\right| \tag{8}$$

https://doi.org/10.54216/IJNS.230326

Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

for $j = 1, 2, ..., m$.
Equations 7 and 8, we get

$$\frac{\left|\left[\sum_{i=1}^{n}\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j) - \hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\ ,\ \sum_{i=1}^{n}\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j) - \hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)\right]\right|}{\left|\left[\sum_{i=1}^{n} 1 + \left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)\cdot \hbar^{q-}_{R(\delta_i)}(\aleph_j)\right), \sum_{i=1}^{n} 1 + \left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)\cdot \hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\right|}$$
$$\geq \frac{\left|\left[\sum_{i=1}^{n}\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j) - \hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\ ,\ \sum_{i=1}^{n}\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j) - \hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)\right]\right|}{\left|\left[\sum_{i=1}^{n} 1 + \left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)\cdot \hbar^{q-}_{R(\delta_i)}(\aleph_j)\right), \sum_{i=1}^{n} 1 + \left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\cdot \hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)\right]\right|}$$

implies that

$$\left|\left[\frac{\sum_{i=1}^{n}\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}\right]\right|$$
$$\geq \left|\left[\frac{\sum_{i=1}^{n}\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}\right]\right|$$

and

$$\sqrt[q]{\left|\left[\frac{\sum_{i=1}^{n}\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}\right]\right|}$$
$$\geq \sqrt[q]{\left|\left[\frac{\sum_{i=1}^{n}\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}\right]\right|}$$

and

$$\left[1-\sqrt[q]{\left|\left[\frac{\sum_{i=1}^{n}\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\hbar^{q+}_{P(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q-}_{P(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}\right]\right|}\right]$$
$$\leq \left[1-\sqrt[q]{\left|\left[\frac{\sum_{i=1}^{n}\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)-\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q+}_{R(\delta_i)}(\aleph_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\hbar^{q+}_{Q(\delta_i)}(\aleph_j)-\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}{\sum_{i=1}^{n} 1+\left(\hbar^{q-}_{Q(\delta_i)}(\aleph_j)\cdot\hbar^{q-}_{R(\delta_i)}(\aleph_j)\right)}\right]\right|}\right]$$

Therefore

$$\widehat{S}\left(\widehat{P}(\delta)(\aleph_j), \widehat{R}(\delta)(\aleph_j)\right) \leq \widehat{S}\left(\widehat{Q}(\delta)(\aleph_j), \widehat{R}(\delta)(\aleph_j)\right) \tag{9}$$

Equations 3, 6 and 9, for each $j = 1, 2, ..., m$.

$$\Phi(\widehat{P}, \widehat{R}) \leq \Phi(\widehat{Q}, \widehat{R}) \tag{10}$$

Clearly

$$\left[u^-(\delta_i), u^+(\delta_i)\right] \leq \left[v^-(\delta_i), v^+(\delta_i)\right] \leq \left[w^-(\delta_i), w^+(\delta_i)\right]$$

and

$$\left[\min\left\{[u^-(\delta_i), w^-(\delta_i)]\right\}, \max\left\{[u^+(\delta_i), w^+(\delta_i)]\right\}\right] \leq \left[\min\left\{[v^-(\delta_i), w^-(\delta_i)]\right\}, \max\left\{[v^+(\delta_i), w^+(\delta_i)]\right\}\right]$$

Hence

$$\Big[\sum_{i=1}^{n}\min\left\{[u^-(\delta_i), w^-(\delta_i)]\right\}, \sum_{i=1}^{n}\max\left\{[u^+(\delta_i), w^+(\delta_i)]\right\}\Big]$$
$$\leq \Big[\sum_{i=1}^{n}\min\left\{[v^-(\delta_i), w^-(\delta_i)]\right\}, \sum_{i=1}^{n}\max\left\{[v^+(\delta_i), w^+(\delta_i)]\right\}\Big] \tag{11}$$

https://doi.org/10.54216/IJNS.230326

Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

Plso,

$$\Big[\big(u^-(\delta_i)+w^-(\delta_i)\big),\big(u^+(\delta_i)+w^+(\delta_i)\big)\Big] \leq \Big[\big(v^-(\delta_i)+w^-(\delta_i)\big),\big(v^+(\delta_i)+w^+(\delta_i)\big)\Big]$$

$$\begin{aligned} &\Big[\sum_{i=1}^{n}\big(u^-(\delta_i)+w^-(\delta_i)\big),\sum_{i=1}^{n}\big(u^+(\delta_i)+w^+(\delta_i)\big)\Big] \\ \leq\quad &\Big[\sum_{i=1}^{n}\big(v^-(\delta_i)+w^-(\delta_i)\big),\sum_{i=1}^{n}\big(v^+(\delta_i)+w^+(\delta_i)\big)\Big] \end{aligned} \tag{12}$$

Equations 11 and 12, we get

$$\begin{aligned} &-\frac{\Big[\sum_{i=1}^{n}\min\Big\{\big[u^-(\delta_i),w^-(\delta_i)\big]\Big\},\sum_{i=1}^{n}\max\Big\{\big[u^+(\delta_i),w^+(\delta_i)\big]\Big\}\Big]}{\Big[\sum_{i=1}^{n}\left(u^-(\delta_i)+w^-(\delta_i)\right),\sum_{i=1}^{n}\left(u^+(\delta_i)+w^+(\delta_i)\right)\Big]} \\ \geq\quad &-\frac{\Big[\sum_{i=1}^{n}\min\Big\{\big[v^-(\delta_i),w^-(\delta_i)\big]\Big\},\sum_{i=1}^{n}\max\Big\{\big[v^+(\delta_i),w^+(\delta_i)\big]\Big\}\Big]}{\Big[\sum_{i=1}^{n}\left(v^-(\delta_i)+w^-(\delta_i)\right),\sum_{i=1}^{n}\left(v^+(\delta_i)+w^+(\delta_i)\right)\Big]} \end{aligned}$$

implies that

$$\begin{aligned} &-\left[\frac{\sum_{i=1}^{n}\min\Big\{\big[u^-(\delta_i),w^-(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(u^+(\delta_i)+w^+(\delta_i)\big)},\ \frac{\sum_{i=1}^{n}\max\Big\{\big[u^+(\delta_i),w^+(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(u^-(\delta_i)+w^-(\delta_i)\big)}\right] \\ \geq\quad &-\left[\frac{\sum_{i=1}^{n}\min\Big\{\big[v^-(\delta_i),w^-(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(v^+(\delta_i)+w^+(\delta_i)\big)},\ \frac{\sum_{i=1}^{n}\max\Big\{\big[v^+(\delta_i),w^+(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(v^-(\delta_i)+w^-(\delta_i)\big)}\right] \end{aligned}$$

Thus,

$$\begin{aligned} &1-\left[\frac{\sum_{i=1}^{n}\min\Big\{\big[u^-(\delta_i),w^-(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(u^+(\delta_i)+w^+(\delta_i)\big)},\ \frac{\sum_{i=1}^{n}\max\Big\{\big[u^+(\delta_i),w^+(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(u^-(\delta_i)+w^-(\delta_i)\big)}\right] \\ \geq\quad &1-\left[\frac{\sum_{i=1}^{n}\min\Big\{\big[v^-(\delta_i),w^-(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(v^+(\delta_i)+w^+(\delta_i)\big)},\ \frac{\sum_{i=1}^{n}\max\Big\{\big[v^+(\delta_i),w^+(\delta_i)\big]\Big\}}{\sum_{i=1}^{n}\big(v^-(\delta_i)+w^-(\delta_i)\big)}\right] \end{aligned}$$

Hence

$$\Psi(\widehat{u},\widehat{w}) \geq \Psi(\widehat{v},\widehat{w}). \tag{13}$$

Equations 10 and 13,

$$\Phi(\widehat{P},\widehat{R})\cdot\Psi(\widehat{u},\widehat{w}) \leq \Phi(\widehat{Q},\widehat{R})\cdot\Psi(\widehat{v},\widehat{w}).$$

Hence $Sim(\widehat{P}_u,\widehat{R}_w) \leq Sim(\widehat{Q}_v,\widehat{R}_w)$.

Example 4.3. Determine the similarity between the two Type-II q-rungNIVS sets. We choose $\mathscr{X} = \{\aleph_1,\aleph_2,\aleph_3\}$ and parameter $\mathscr{E} = \{\delta_1,\delta_2,\delta_3\}$ can be defined below:

$P_u(\delta)$	δ_1	δ_2	δ_3
$P(\delta)(\aleph_1)$	$[0.6,0.7],[0.4,0.5],[0.7,0.75]$	$[0.8,0.85],[0.3,0.45],[0.6,0.65]$	$[0.7,0.75],[0.5,0.65],[0.65,0.7]$
$P(\delta)(\aleph_2)$	$[0.5,0.6],[0.35,0.45],[0.75,0.8]$	$[0.7,0.8],[0.25,0.45],[0.65,0.7]$	$[0.4,0.55],[0.45,0.65],[0.8,0.85]$
$P(\delta)(\aleph_3)$	$[0.4,0.6],[0.65,0.7],[0.45,0.65]$	$[0.5,0.6],[0.55,0.6],[0.55,0.7]$	$[0.3,0.45],[0.75,0.8],[0.65,0.75]$
$p(\delta)$	$[0.6,0.7],[0.6,0.8],[0.5,0.6]$	$[0.5,0.65],[0.7,0.75],[0.4,0.45]$	$[0.7,0.75],[0.55,0.6],[0.3,0.45]$

$Q_v(\delta)$	δ_1	δ_2	δ_3
$Q(\delta)(\aleph_1)$	$[0.5,0.6],[0.6,0.7],[0.6,0.65]$	$[0.6,0.65],[0.4,0.6],[0.5,0.55]$	$[0.5,0.55],[0.7,0.75],[0.45,0.55]$
$Q(\delta)(\aleph_2)$	$[0.45,0.55],[0.55,0.65],[0.55,0.65]$	$[0.55,0.65],[0.75,0.85],[0.45,0.55]$	$[0.45,0.55],[0.65,0.75],[0.65,0.75]$
$Q(\delta)(\aleph_3)$	$[0.75,0.8],[0.75,0.8],[0.3,0.35]$	$[0.65,0.75],[0.55,0.65],[0.45,0.5]$	$[0.55,0.65],[0.45,0.75],[0.45,0.55]$
$q(\delta)$	$[0.55,0.65],[0.65,0.75],[0.55,0.65]$	$[0.65,0.7],[0.25,0.35],[0.75,0.9]$	$[0.75,0.8],[0.35,0.45],[0.65,0.8]$

https://doi.org/10.54216/IJNS.230326

Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024

Using Definition 4.1 and assume that $q = 2$, we get
$T_1\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right) = \Big[0.934878, 0.938375\Big]$,
$T_2\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right) = \Big[0.772088, 0.878488\Big]$ and
$S\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right) = \Big[0.069727, 0.197646\Big]$.
Now,

$$\begin{bmatrix} \min\left\{T_1^-\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right), T_2^-\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right), S^-\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right)\right\}, \\ \max\left\{T_1^+\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right), T_2^+\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right), S^+\left(P(\delta)(\aleph_1), Q(\delta)(\aleph_1)\right)\right\} \end{bmatrix} = \Big[0.555426, 0.938375\Big].$$

Similarly, $T_1\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right) = \Big[0.97325, 0.978015\Big]$,
$T_2\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right) = \Big[0.462308, 0.68444\Big]$ and
$S\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right) = \Big[0.088235, 0.262329\Big]$.
Now,

$$\begin{bmatrix} \min\left\{T_1^-\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right), T_2^-\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right), S^-\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right)\right\}, \\ \max\left\{T_1^+\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right), T_2^+\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right), S^+\left(P(\delta)(\aleph_2), Q(\delta)(\aleph_2)\right)\right\} \end{bmatrix} = \Big[0.462308, 0.978015\Big].$$

Similarly, $T_1\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right) = \Big[0.841353, 0.931637\Big]$,
$T_2\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right) = \Big[0.832117, 0.972205\Big]$ and
$S\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right) = \Big[0.075499, 0.309633\Big]$.
Now,

$$\begin{bmatrix} \min\left\{T_1^-\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right), T_2^-\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right), S^-\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right)\right\}, \\ \max\left\{T_1^+\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right), T_2^+\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right), S^+\left(P(\delta)(\aleph_3), Q(\delta)(\aleph_3)\right)\right\} \end{bmatrix} = \Big[0.443553, 0.972205\Big].$$

Thus, $\Phi(\widehat{P}, \widehat{Q}) = \left[\frac{0.555426+0.462308+0.443553}{3}, \frac{0.938375+0.978015+0.972205}{3}\right] = \Big[0.487096, 0.962865\Big]$.
Also, $\Psi(\widehat{u}, \widehat{v}) = \Big[0.351695, 0.67\Big]$.
Hence, $Sim(\widehat{P}_u, \widehat{Q}_v) = \Big[0.1607416, 0.62423\Big]$.

Acknowledgments: This research was supported by University of Phayao and Thailand Science Research and Innovation Fund (Fundamental Fund 2024).

Conflicts of Interest: The authors declare no conflict of interest.

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Received: July 13, 2023 Revised: November 29, 2023 Accepted: February 25, 2024