Fermatean Shortest Route Problem with Interval Fermatean Neutrosophic Fuzzy Arc Length: Formulation and a Modified Dijkstra's Algorithm

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Abstract

Dijkstra's algorithm (DA) is a very popular approach for finding the shortest route (SR) in the shortest route problem (SRP). The SRP becomes a challenging and complex problem in real life scenarios. The Fermatean neutrosophic set is a mathematical model that combines Fermatean sets with neutrosophic sets. It can handle the unclear, ambiguous, inconsistent, confusing, and uncertain information that comes from real-world problems. Decision-makers face difficulty accurately determining the precise membership (MG) and non membership levels due to the lack of appropriate data available. The FNS can handle this problem. In this study, we consider the interval FNS to describe the arc weight of a neutrosophic graph (NG). This SRP is called an interval Fermatean neutrosophic shortest route problem (IFNSRP). A modified DA is presented to solve this IFNSRP in an uncertain environment. The effectiveness of the presented method is illustrated with a numerical instance of a neutrosophic network.

Keywords: DA; Fuzzy Set; Neutrosophic set; SRP

1 Introduction

The SRP is a basic and widely recognised network optimisation problem that arises as a sub-issue in several practical applications. The classical SRP problem[1] involves determining a route (between the starting node and the target node) with the smallest route length from a finite set of all possible routes. Several real-life problems, such as communications, shipping, routing information, and logistics management, may be considered specific variants of the SRP. During the 1950s and 1960s, Bellman, Dijkstra, Dreyfus, and Floyd introduced several efficient methods that elevated the SRP to a prominent place within network theory. Arc lengths are used in real-world situations to express factors like trip duration, distance, expense, or other relevant variables. Nevertheless, it is impossible to eliminate uncertainty completely, and typically, the exact values of arc lengths cannot be ascertained. On road networks, the lengths of arcs that represent the time it takes for vehicles to travel can vary due to factors such as congestion, crashes, or adverse weather, leading to ambiguity. Therefore, it could be difficult to discover the best route in such networks. These kinds of unclear situations can be handled by adding fuzzy logic/Neutrosophic logic[2] into a network or graph.

Prof. Zadeh[3] in 1965 described the principle of fuzzy Set (FS) as a means to address the challenges posed by uncertainties and ambiguities. Many real-life problems and practical applications are used to illustrate the

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FS theory.[4,5,7,8] The fuzzy set[9–11] possesses the capacity to effectively address a variety of problems, such as smart data analysis, data mining, decision-making processes, pattern identification, and optimisation. In 1986, Atanassov described the basic connection of the Intuitionistic FS (IFS) model. Within the framework of the IFS model, each and every item or element is described by its membership grade (μ) and non-membership grade (ν) (with a maximum limit of 1 for all totals) that fulfils $\mu + \nu \leq 1$. Using IFSs has several advantages over FSs, the main one being its ability to distinguish between positive ($+$) and negative ($-$), which supports an element's inclusion in the set. In certain scenarios, the decision maker may assign values of μ and ν that are exceedingly high, resulting in a sum that exceeds 1. To solve this problem, Yager has presented the concept of the Pythagorean FS (PytFS) as an upper version of the IFSs[12] to handle the complexities of inaccuracy and uncertainties associated with optimization problems seen in the actual world.

The PyFSs differ from other FS models by allowing the square root of the addition of the μ and ν to be less than or equal to one. Neutrosophic sets introduced by Smarandache[13] in 1995, offer solutions to problems arising from inadequate, unclear, vague, and imprecise data. Then, Senapati and Yager[14] propose the idea of Fermatean FS (FtFS), where the total of the cubes representing μ and ν must not exceed one. The FtFS is a valuable approach for accommodating ambiguities, uncertainties, and vagueness by augmenting the proportionate extent of μ and ν in fuzzy and PytFS. Akram et al.[15] introduced the concept of complex Fermatean fuzzy N-soft sets as a combination of FFS, N-soft set, and complex FS. They created a novel hybrid strategy based on the Fermatean fuzzy TOPSIS methods to address the formidable capability problem in the decision-making process.

In 2020, Aydemer et al.[16] presented the Fermatean fuzzy TOPSIS technique, which utilises Dombi aggregation operators. There are several works in SRP using fuzzy set.[17] Based on the Fermatean Neutrosophic relationship, Broumi et al.[18] introduced the concepts and practical uses of FNGs in 2022. In 2022, Ganie[19] introduced a decision-making technique with multiple criteria that uses the knowledge and distance evaluations of FtFSs. In,[20] Broumi investigates NGs and their corresponding features. Sundareswaran et al. investigated and characterised the sensitive aspects of the neutrosophic environment in.[21] In 2019, the authors presented a correlation metric for Pythagorean Neutrosophic Sets. Keshavarz-Ghorabaee et al.[22] introduced an algorithm for evaluating green construction suppliers in the year 2020 that was based on fermatean FS and WASPAS. Mishra presented the idea of the Fermatean fuzzy WASPAS technique for selecting medical waste recycling locations based on multiple parameters. Broumi et al.[23] conducted a study on interval neutrosophic sets, focusing on the operations performed on interval-valued Fermatean neutrosophic sets and their application in multicriteria optimization problems. Broumi et al.[24] introduced a smart technique for evaluating trapezoidal interval-valued neutrosophic networks. In 2016, Dey et al.[25] investigated the shortest route problem (SRP) fuzzy variant. Ebrahimnejad et al.[26] proposed an optimisation technique in 2020 to solve SRPs using interval-valued triangular fuzzy arc weight. In 2020, Singh[27] introduced a fuzzy single-point perspective (SRP) focusing on the viewpoint of a startup's founder. The concept of SRP was initially introduced by Jan A et al.[28] in 2022, employing Pythagorean fuzzy components characterised by interval values. In 2021, Antony and Jansi[29] proposed the notion of Fermatean neutrosophic set by combining the ideas of Neutrosophic sets[25] and Fermatean fuzzy sets. Asim Bash et al.[30] propose a method for solving the SRP in a neutrosophic Pythagorean environment. Then, the idea of Fermatean Neutrosophic Dombi Fuzzy Graphs is interpreted by Sasikala.[31] In a fermatean fuzzy environment, Mary et al.[32] presented an algorithm to solve the minimum spanning tree problem. An algorithmic approach by Vidhya[15] is proposed for searching the SR in a Pythagorean fuzzy environment with interval numbers. Broumi et al.[33] examined the notion of interval-valued FNGs. Raut et al.[34] investigated the issue of finding the SR on Fermatean Neutrosophic Networks.

The membership function or degree of a neutrosophic or fuzzy set is generally assessed through human perception. Uncertainty also exists in the degree of membership, making it challenging to identify the precise membership functions. For instance, we lack understanding about the mathematical representation of how traffic frequency changes over time, or the parameter values obtained from experts through questionnaires may contain ambiguous terms. This type of uncertainty can be handled by using interval Fermatean Neutrosophic Numbers (IFNN).

The main objective of the paper is to discover a straightforward and proper algorithm for SRP that can be applied effectively in real-life circumstances. Let G be an interval Fermatean NG (IFNG) in which the sets of arcs are represented by E and the nodes by V. IFNN is a representation of the arc weights. An alternating series of nodes and arcs that begins and ends with nodes is referred to as a path between two nodes. By adding up the total weight of all the arcs along the way, one can calculate the time, distance, or cost of a route.

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Nevertheless, due to the existence of multiple possible routes connecting any pair of nodes, it is meaningful to address the problem of determining the route with the lowest cost between two specified nodes. We propose a linear programming model for solving the SRP in a FNG. We also present a novel technique that uses DA to determine the SR in a FNG. In this algorithm, we employ a de-neutrosophication technique to select the SR that corresponds to the minimum value.

The organisation of this manuscript is outlined as follows: Section 2 provides the background and relevant applications that served as inspiration for the proposed study. A mathematical formulation is presented in Section 3. The proposed SRP algorithm, which incorporates the IFNNs uncertainty into Dijkstra's algorithm, is explained in Section 4. To demonstrate the algorithm's effectiveness, we provide numerical examples in Section 5. In the end, we present a conclusion in Section 6.

2 Preliminaries

The subsequent section provides an overview of fundamental principles and explanations for PytFS, FtFS, interval Fermatean neutrosophic sets (IFeNS), and interval-valued Fermatean NG (IFeNG) as documented in the available literature.

Definition 2.1. A PytFS[12] D is a structure denoted by W and has the following form:

$$D_{\text{PYTFS}} = \{\langle z, t_D(w), f_D(w)\rangle \mid w \in W\} \tag{1}$$

The function $t_D(w)$ maps elements from the set W to the interval $[0,1]$.The symbol specifies the grade of membership, whereas $f_D(w) : W \to [0,1]$ represents a function that maps elements from the set W to values between 0 and 1. The symbol signifies the degree of nonmembership of each element $w \in W$ to the set D, respectively, subject to the given constraints:

$$0 \leq (t_D(w))^2 + (f_D(w))^2 \leq 1 \tag{2}$$

Senapati et al.[14] proposed the concept of FtFS to account for a wider range of uncertainties. The following are explicitly stated:

Definition 2.2. A FtFS[14] D on a universe of discourse W is a structure defined as,

$$D_{FtFS} = \{\langle z, t_D(w), f_D(w)\rangle \mid w \in W\} \tag{3}$$

The function $t_D(w)$ maps elements from the set W to the interval $[0,1]$. The symbol specifies the grade of membership, whereas $f_D(w) : W \to [0,1]$ represents a function that maps elements from the set W to values between 0 and 1. The symbol signifies the degree of nonmembership of each element $w \in W$ to the set D, accordingly, subject to the given constraints:

$$0 \leq (t_D(w))^3 + (f_D(w))^3 \leq 1 \tag{4}$$

Definition 2.3. A Fermatean neutrosophic number (FNN)[23] D is represented as an interval value.

$$\left\langle \left[t_D^L, t_D^U\right], \left[i_D^L, i_D^U\right], \left[f_D^L, f_D^U\right]\right\rangle \tag{5}$$

A FNN is zero-valued if

$$\left\langle \left[t_D^L = 0, t_D^U = 0\right], \left[i_D^L = 1, i_D^U = 1\right], \left[f_D^L = 1, f_D^U = 1\right]\right\rangle \tag{6}$$

Definition 2.4. [23] The universe of discourse is associated with an interval-valued Fermatean neutrosophic set (IVeFNS) denoted as D. The variable W can be precisely characterized as:

$$D_{\text{IVeNSS}} = \left\{\left\langle D, \tilde{t}_D(\omega), \tilde{i}_D(\omega), \tilde{f}_D(\omega)\right\rangle |\omega \in W\rangle\right\} \tag{7}$$

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where, $\tilde{t}_D(\omega) = \left(t_D^L(\omega), t_D^U(\omega)\right), \tilde{i}_D(\omega) = \left(i_D^L(\omega), i_D^U(\omega)\right)$ and $\tilde{f}_D(\omega) = \left(f_D^L(\omega), f_D^U(\omega)\right)$ describes the truth, indeterminacy and falsity membership degrees, respectively. Consider the mapping $\tilde{t}_D(\omega) : Z \rightarrow D[0,1], \tilde{i}_D(\omega) : W \rightarrow D[0,1], \tilde{f}_D(\omega) : W \rightarrow D[0,1]$ and $0 \leq (t_D(\omega))^3 + (f_D(\omega))^3 \leq 1$ and $0 \leq \left(i_A^U(\omega)\right)^3 \leq 1$,

$$0 \leq \left(t_D^U(\omega)\right)^3 + \left(i_D^U(\omega)\right)^3 + \left(f_D^U(\omega)\right)^3 \leq 2, \forall \omega \in W \tag{8}$$

Definition 2.5. An Interval-Valued Fermatean NG (IVFeNG) is defined as a pair $T = (R, S)$, where $(R = \left\langle \left[t_(R^L, t_(R^U\right], \left[i_(R^L, i_(R^U\right], \left[f_(R^L, f_(R^U\right] \right\rangle$ is an IVeFNS on nodes (β) and
$S = \left\langle \left[t_{\underline{S}}^L, t_{\underline{S}}^U\right], \left[i_{\underline{S}}^L, i_{\underline{S}}^U\right], \left[f_{\underline{S}}^L, f_{\underline{S}}^U\right] \right\rangle$ is an IVeFNS relationship on $\operatorname{arcs}(A)$ meeting a specific condition: i. $\beta = \{\eta_1, \eta_2, \ldots, \eta_\eta\}$, such that $t_(R^L : \beta \rightarrow [0,1], t_(R^U : \beta \rightarrow [0,1], i_(R^L : \beta \rightarrow [0,1], i_(R^U : \beta \rightarrow [0,1]$ and $f_(R^L : \beta \rightarrow [0,1], f_(R^U : \beta \rightarrow [0,1]$ Accordingly, indicate the level of truth-membership, indeterminacy membership, and falsity-membership of the element $y \in V$, and

$$0 \leq \left(t_D^U(\eta_1)\right)^3 + \left(i_D^U(\eta_1)\right)^3 + \left(f_D^U(\eta_1)\right)^3 \leq 2, \forall \eta_1 \in \beta \tag{9}$$

ii. The membership degree $t_S^L : \beta \times \beta \rightarrow [0,1], t_S^U : \beta \times \beta \rightarrow [0,1], i_S^L : \beta \times \beta \rightarrow [0,1], i_S^U : \beta \times \beta \rightarrow [0,1]$ and $f_S^L : \beta \times \beta \rightarrow [0,1], f_S^U : \beta \times \beta \rightarrow [0,1]$ can be defined as follows

Here, $t_S^L, t_S^U, i_S^L, i_S^U, f_S^L$ and f_S^U represent the lower and upper bounds of truth, indeterminacy, and falsity membership degrees for the edge $(n_1, n_2) \in E$, where $0 \leq (t_S(n_i, n_\eta))^3 + (\boldsymbol{i}_S(n_i, n_\eta))^3 + (\boldsymbol{f}_S(n_i, n_\eta))^3 \leq 2$ for all $\{n_i, n_\eta\} \in E(i, \eta = 1, 2, \ldots, n)$ means $0 \leq (t_S(n_i, n_\eta))^3 + (\boldsymbol{i}_S(n_i, n_\eta))^3 + (\boldsymbol{f}_S(n_i, n_\eta))^3 \leq 2, \forall x \in \mathbf{X}$.

Definition 2.6. Broumi et al.,[2335] established the average potential degree of membership of item W to interval-valued Fermateanneutrosophic set $P = \left\langle \left[t_P^L, t_P^U\right], \left[i_P^L, i_P^U\right], \left[f_P^L, f_P^U\right] \right\rangle$ as follows:

$$S = \frac{\left(t_P^L(w)\right)^3 + \left(t_P^U(w)\right)^3 + \left(i_P^L(w)\right)^3 + \left(i_P^U(w)\right)^3 + \left(f_P^U(w)\right)^3 + \left(f_P^U(w)\right)^3}{2} \tag{10}$$

Definition 2.7. [23] Let

$$\begin{aligned} P &= \left[\left(t^L, t^U\right), \left(i^L, i^U\right), \left(f^L, f^U\right)\right] \\ P_1 &= \left[\left({t_1}^L, {t_1}^U\right), \left({i_1}^L, {i_1}^U\right), \left({f_1}^L, f_1^U\right)\right] \\ P_2 &= \left[\left({t_2}^L, {t_2}^U\right), \left({i_2}^L, {i_2}^U\right), \left({f_2}^L, {f_2}^U\right)\right] \end{aligned} \tag{11}$$

be three IVeFNS numbers and $\alpha > 0$. The operation of IVeFNS are defined as follows

$$\begin{aligned} &P_1 \oplus P_2 = \left(\left[\sqrt[3]{t_1^{L^3} + t_2^{L^3} - t_1^{L^3} t_2^{L^3}}, \sqrt[3]{t_1^{U^3} + t_2^{U^3} - t_1^{U^3} t_2^{U^3}}\right], \left[i_1^L i_2^L, i_1^U i_2^U\right], \left[f_1^L f_2^L, f_1^U f_2^U\right]\right) \\ &P_1 \otimes P_2 = \\ &\left(\left[t_1^L t_2^L, t_1^U t_2^U\right], \left[\sqrt[3]{i_1^{L^3} + i_2^{L^3} - i_1^{L^3} i_2^{L^3}}, \sqrt[3]{i_1^{U^3} + i_2^{U^3} - i_1^{U^3} i_2^{U^3}}\right], \left[\sqrt[3]{f_1^{L^3} + f_2^{L^3} - f_1^{L^3} f_2^{L^3}}, \sqrt[3]{f_1^{U^3} + f_2^{U^3} - f_1^{U^3} f_2^{U^3}}\right]\right) \\ &\alpha P = \left(\left[\sqrt[3]{1 - \left(1 - t^{L^3}\right)^\alpha}, \sqrt[3]{1 - \left(1 - t^{U^3}\right)^\alpha}\right], \left[i^{L^\alpha}, i^{U^\alpha}\right], \left[f^{L^\alpha}, f^{U^\alpha}\right]\right) \\ &P^\alpha = \left(\left[t^{L^\alpha}, t^{U^\alpha}\right], \left[\sqrt[3]{1 - \left(1 - i^{L^3}\right)^\alpha}, \sqrt[3]{1 - \left(1 - i^{U^3}\right)^\alpha}\right], \left[\sqrt[3]{1 - \left(1 - f^{L^3}\right)^\alpha}, \sqrt[3]{1 - \left(1 - f^{U^3)^\alpha}\right.}\right]\right) \end{aligned} \tag{12}$$

3 Linear Programming Model for IFNSRP:

We formulate a linear programming approach to solve the IFNSRP. This IFNSRP aims to determine a route with a minimum interval FNN while ensuring that the route begins at node i, terminates at node d, and does not stop at any other node. We do this by determining whether each edge is in the route.

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$$\text{Minimize} \sum_{\eta \in \hat{E}} D(\eta) \cdot Y_\eta \tag{13}$$

Subject to constant

$$\begin{aligned}
&\sum_{i,\mu \in \hat{E}} Y_{i,\mu} = 1 \\
&-\sum_{\mu,d \in E} Y_{i,\mu} = -1 \\
&\sum_{\mu,\nu \in \tilde{E}} Y_{\mu,\nu} - \sum_{w,\mu \in \tilde{E}} Y_{w,\nu} = 0 \quad \forall \mu \neq i, d \\
&Y_\eta \geq 0 \quad \forall \eta \in \tilde{E}
\end{aligned} \tag{14}$$

We consider that there are no edges leading to the vertex i and no edges originating from the vertex d. In the presence of certain conditions, it is possible for two cycles to occur instead of a single route, with one cycle revolving around i and the other around d.

4 Proposed DA for IFNSRP:

The proposed approach is a modification of DA for the IFNSRP. Using IVeFNS as an arc length, we have integrated the idea of uncertainty into DA. The pseudocode for the proposed DA for IFNSRP is included in Algorithm 1. This algorithm determines the SR between a starting node and a target node of an IVFeNG. The starting and target vertices are distinct and indicated by the symbols i and t, respectively. The set P holds all the unvisited vertices of the IVFeNG $\tilde{G}$. Consider the variable r as a vertex in the IVFeNG network $\tilde{G}$. In this context, dis$[r]$ represents the current shortest distance between the starting node and node r. Additionally, edge weight (w, y) denotes the arc length that connects two adjacency vertices between w and v. The values of dis [r] and edge weight (w, v) are represented using IVeFNS, whereas the symbols pre[w] record the vertex that comes before the current vertex w on the SR from the starting vertex i. The ranking method is applied to calculate the score value of an IVeFNS based on its mean value, as described in Definition 2.6. The de-neutrosophication of the SR from the starting node to w is stored in the variable score $[w]$. The node v is now the lowest score node in the set P.

Our proposed algorithm keeps track of the distances in the form of IVeFNS to each and every node of IVFeNG $\tilde{G}$, starting with the initial/starting node i at null IVeFNS and all other nodes at infinite IVeFNS. During each iteration, the algorithm chooses the vertex with the lowest score value, examines its neighbouring vertices, and modifies their tentative distances if a shorter route is discovered. This process persists until the target node t is reached.

[H] [1] The graph comprises IFNN edges and includes a solitary initial node and a target/destination node. Determining the SR between the staring node to the destination/target node. **Begin** score[i] $\leftarrow$ 0 dis[i] $\leftarrow$ null The associated IVeFNS is empty. add i to P each node j (excluding the i) in the IVFeNG score[i] $\leftarrow \infty$ add node j to P Assign all the nodes of the IVFeNG to the variable P. $v \leftarrow start$ (i is not same the target node t) SR from i to t P=$P \setminus v$ every node w adjacent node to v tem_dis[w] $\leftarrow$ dist[v] $\oplus$ edge_weight(w, v) tem_score[w] $\leftarrow$ score (tem_dis[w]) (tem_score[w] $<$ score[w]) dis[w] $\leftarrow$ temp_dis[w] score[w] $\leftarrow$ temp_score[w] pre[w]$\leftarrow$ v v $\leftarrow$ node in P with the smallest score value

End

5 Numerical Example

Suppose the starting vertex is 1 and the target vertex is 6. Table 1 displays the IVeFNSs matching the graph's arcs. We have obtained the IVeFNS values from [30] and randomly allocated them to the edges of the IVFeNG.

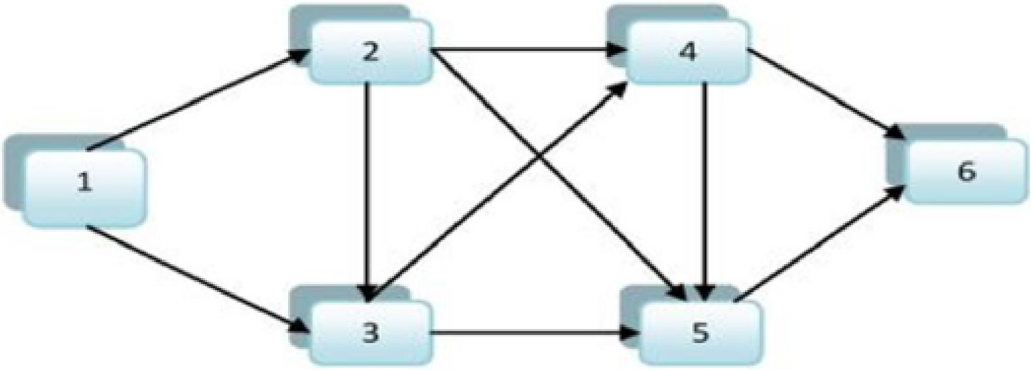

Figure 1: A FNG with IVeFNS

Table 1: The edges of the graph, shown in Fig. 1

Edge	Distance
(1,2)	{(0.5, 0.7), (0.2, 0.4), (0.3, 0.3)}
(1,3)	{(0.2, 0.7), (0.1, 0.5), (0.1, 0.3)}
(2,3)	{(0.1, 0.7), (0.2, 0.4), (0.3, 0.5)}
(2,4)	{(0.4, 0.5), (0.7, 0.8), (0.1, 0.2)}
(2,5)	{(0.5, 0.6), (0.5, 0.7), (0.3, 0.4)}
(3,4)	{(0.6, 0.7), (0.4, 0.6), (0.3, 0.5)}
(3,5)	{(0.6, 0.7), (0.3, 0.6), (0.2, 0.5)}
(4,5)	{(0.4, 0.7), (0.3, 0.8), (0.1, 0.2)}
(5,6)	{(0.3, 0.8), (0.3, 0.6), (0.2, 0.4)}

Table 2: Comparison between Disktra's Algorithm and LPP method

Solution using LINGO	Solution using Disktra's Algorithm
Min Z= $[\langle[0.009, 0.042], [0.0095, 0.1715],$ $[0.0003, 0.0027]\rangle, 4]$ $\mathbf{Y}_{1.2} = \mathbf{1}, \mathbf{Y}_{2.4} = \mathbf{1}$ and $\mathbf{Y}_{4.6} = \mathbf{1},$	The SR cost: $[\langle[0.009, 0.042], [0.0095, 0.1715],$ $[0.0003, 0.0027]\rangle, 4]$ The SR: $\mathbf{1} \rightarrow \mathbf{2} \rightarrow \mathbf{4} \rightarrow \mathbf{6}$

We must determine the most efficient route, i.e., SR, between nodes 1 and 6. We employed a Linear Programming Problem (LPP) framework to solve this problem. The solution is obtained using LINGO software. Table 2 presents the outcome derived from the computational analysis performed using LINGO. The decision variable $Y_{u,x}$ equals 1 if the edge (u, v) is included in the SR. The outcome of DA is additionally displayed in Table 2. The results obtained from LINGO and our proposed approach are identical.

6 Conclusion

This study investigates the SRP, where the costs of arcs are represented using IFNS. This study describes the importance of utilizing IFNS in SRP. We improve the traditional Dijkstra approach by integrating uncertainty through an SRP between a single starting and a single destination. The proposed algorithm's usefulness is demonstrated through the use of numerical examples. The main benefit of this research is introducing an algorithmic strategy for solving the SRP in an uncertain environment. The suggested approach is both straightforward and efficient for practical applications.

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