New type neutrosophic set applied to power aggregating operators

K.Raja[1], P. Maragatha Meenakshi[2], N. Rajesh[3], M. Palanikumar[4,*], Faisal Al-Sharqi[5], Ashraf Al-Quran[6,7], Ali Mohammad Alorsan Bany Awad[8]

[1,2]Department of Mathematics, Thanthai Periyar Government Arts and Science College (affiliated to Bharathidasan University), Tiruchirappalli 624024, Tamilnadu, India

[1]raju729467@gmail.com, [2]maragathameenakship@gmail.com

[3]Department of Mathematics, Rajah Serfoji Government College, Thanjavur 613005, Tamilnadu, India

[3]nrajesh_topology@yahoo.co.in

[4]Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai-602105, India

[4]palanimaths86@gmail.com

[5]Department of Mathematics, Faculty of Education for Pure Sciences, University Of Anbar, Ramadi, Anbar, Iraq

[5]College of Pharmacy, National University of Science and Technology, Dhi Qar, Iraq

[6]Department of Basic Sciences, Preparatory Year, King Faisal University, Al-Ahsa 31982, Saudi Arabia

[7]Department of mathematics and statistics, College of Science, King Faisal University, Al-Ahsa 31982, Saudi Arabia

[8]Deanship of Development and Quality Assurance, King Faisal University, Al-Ahsa 31982, Saudi Arabia

* Corresponding author: M.Palanikumar

Abstract

We introduce the new type neutrosophic set (NS) problems relevant to multiple attribute decision making (MADM). Pythagorean fuzzy set (PFS) and neutrosophic set (NS) can be extended into new type neutrosophic set. We discusses new type neutrosophic weighted averaging (New type NWA), new type neutrosophic weighted geometric (New type NWG), generalized new type neutrosophic weighted averaging (new type GNWA) and generalized new type neutrosophic weighted geometric (new type GNWG). A number of algebraic properties of new type NSs have been established such as associativity, distributivity and idempotency.

Keywords: MADM; new type NWA; new type NWG; new type GNWA; new type GNWG.

1 Introduction

Many authors have contributed to this field of study by using different techniques. As a result of the uncertainties, fuzzy set (FS),[1] intuitionistic FS (IFS),[2] interval valued FS (IVFS),[3] vague set (VS),[4] Pythagorean FS (PFS),[5] IVPFS,[6] spherical FS (SFS)[7] and neutrosophic set (NS).[8] The membership degree (MD) in a set consists of degrees of belongingness that lie between 0 and 1. Atanassov introduced the concept of an IFS, which is classified by the total of its MD with a non-membership degree (NMD) less than one. The DM approach sometimes generates a single problem when the total of the MDs and NMDs exceeds one. The concept of PFS

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was developed by[5] to generalize IFS by ensuring that the square total of its MD and NMD does not exceed one. The neutral state cannot be demonstrated (neither favor nor disfavor).[9] proposed the notion of a picture FS, and used three pointers; positive, neutral, and negative, with a total of not more than one value. Liu et al.[10] discussed the idea of generalized PFS with an aggregation operator (AO) was introduced and its applications were described. DM approach challenge where the sum of the truth membership value (TMD), the indeterminacy membership value (IMD), and the false membership value (FMD) is greater than one. Thus,[7] propose a SFS with a square total of TMD, IMD and FMD less than one. The SFS[11] can be conceptualized using the TOPSIS approach.

Smarandache et al.[12] introduced the concept of neutrosophic sets. Neutrosophy refers to the knowledge of neutral thought, and that neutrality is the main difference between FS and IFS. The concept of NSS was introduced by.[12] A value of TMD, an IMD, and FD is assigned to each proposition in this logic. NS in which every element of the universe has a value between 0 and 1. NS generalizes a classical set, FS and IVFS etc. A method based on AOs for MCDM under interval NS was presented by Ye.[13] Palanikumar et al. discussed many algebraic structures and aggregation operators with applications,[14].[16] Many researchers,[17][18] discussed the concept of Pythagorean with its extension based on DM. An introduction is found in section 1. The PNS and FS information is described in section 2. The definition and some operations of new type NNs are provided in section 3. AOs for new type neutrosophic set in section 4. The conclusion is provided in Section 6. Accordingly, the paper has the following outcomes: 1) A number of algebraic properties of new type NSs have been established such as associativity, distributivity and idempotency. 2) We discussed the new type NWA, new type NWG, new type GNWA and new type GNWG.

2 Basic concepts

In this section, we will review the concepts of PFS and PIVFS.

Definition 2.1. [5] Let $\mathbb{U}$ be the universal set. The PFS $\Xi = \Big\{\varepsilon, \big\langle \varsigma_{\Xi}^{T}(\varepsilon), \varsigma_{\Xi}^{F}(\varepsilon)\big\rangle \big| \varepsilon \in \mathbb{U}\Big\}$, $\varsigma_{\Xi}^{T} : \mathbb{U} \to [0,1]$ and $\varsigma_{\Xi}^{F} : \mathbb{U} \to [0,1]$ denotes MD and NMD of $\varepsilon \in \mathbb{U}$ to Ξ, respectively and $0 \preceq (\varsigma_{\Xi}^{T}(\varepsilon))^2 + (\varsigma_{\Xi}^{F}(\varepsilon))^2 \preceq 1$. For convenience, $\Xi = \big\langle \varsigma_{\Xi}^{T}, \varsigma_{\Xi}^{F}\big\rangle$ is called the Pythagorean fuzzy number(PFN).

Definition 2.2. Let $\mathbb{U}$ be the universal set. The NS $\Xi = \Big\{\varepsilon, \big\langle \varsigma_{\Xi}^{T}(\varepsilon), \varsigma_{\Xi}^{F}(\varepsilon)\big\rangle \big| \varepsilon \in \mathbb{U}\Big\}$, $\varsigma_{\Xi}^{T} : \mathbb{U} \to [0,1]$ and $\varsigma_{\Xi}^{F} :$ $\mathbb{U} \to [0,1]$ denotes MD, IMD and NMD of $\varepsilon \in \mathbb{U}$ to Ξ, respectively and $0 \preceq (\varsigma_{\Xi}^{T}(\varepsilon)) + (\varsigma_{\Xi}^{I}(\varepsilon)) + (\varsigma_{\Xi}^{F}(\varepsilon)) \preceq 2$. For convenience, $\Xi = \big\langle \varsigma_{\Xi}^{T}, \varsigma_{\Xi}^{I}, \varsigma_{\Xi}^{F}\big\rangle$ is called the neutrosophic number(NN).

Definition 2.3. [6] The Pythagorean IVFS (PIVFS) $\Xi = \Big\{\varepsilon, \Big\langle \widetilde{\varsigma_{\Xi}^{T}}(\varepsilon), \widetilde{\varsigma_{\Xi}^{F}}(\varepsilon)\Big\rangle \Big| \varepsilon \in \mathbb{U}\Big\}$, where $\widetilde{\varsigma_{\Xi}^{T}} : \mathbb{U} \to Int([0,1])$ and $\widetilde{\varsigma_{\Xi}^{F}} : \mathbb{U} \to Int([0,1])$ denotes MD and NMD of $\varepsilon \in \mathbb{U}$ to Ξ, respectively, and $0 \preceq (\varsigma_{\Xi}^{T+}(\varepsilon))^2 + (\varsigma_{\Xi}^{F+}(\varepsilon))^2 \preceq 1$. For convenience, $\Xi = \Big\langle \Big[\varsigma_{\Xi}^{T-}, \varsigma_{\Xi}^{T+}\Big], \Big[\varsigma_{\Xi}^{F-}, \varsigma_{\Xi}^{F+}\Big]\Big\rangle$ is called the PIVFN.

Definition 2.4. The Pythagorean neutrosophic set $\Xi = \Big\{\varepsilon, \big\langle \varsigma_{\Xi}^{T}(\varepsilon), \varsigma_{\Xi}^{I}(\varepsilon), \varsigma_{\Xi}^{F}(\varepsilon)\big\rangle \big| \varepsilon \in \mathbb{U}\Big\}$, where $\varsigma_{\Xi}^{T} : \mathbb{U} \to [0,1]$, $\varsigma_{\Xi}^{I} : \mathbb{U} \to [0,1]$ and $\varsigma_{\Xi}^{F} : \mathbb{U} \to [0,1]$ denotes TMD, IMD and FMD of $\varepsilon \in \mathbb{U}$ to Ξ, respectively and $0 \preceq (\varsigma_{\Xi}^{T}(\varepsilon))^2 + (\varsigma_{\Xi}^{I}(\varepsilon))^2 + (\varsigma_{\Xi}^{F}(\varepsilon))^2 \preceq 2$. For convenience, $\Xi = \big\langle \varsigma_{\Xi}^{T}, \varsigma_{\Xi}^{I}, \varsigma_{\Xi}^{F}\big\rangle$ is called the Pythagorean neutrosophic number.

3 New type neutrosophic set and its operations

Several intriguing fundamental operations are described for the new type neutrosophic set. In this section onwards L denotes logarithmic operator.

Definition 3.1. The new type NS $\Xi = \Big\{\varepsilon, \Big\langle LT_{\Xi}(\varepsilon), LI_{\Xi}(\varepsilon), LF_{\Xi}(\varepsilon)\Big\rangle \Big| \varepsilon \in \mathbb{U}\Big\}$, $\varsigma_{\Xi}^{T} : \mathbb{U} \to Int([0,1])$, $\varsigma_{\Xi}^{I} : \mathbb{U} \to Int([0,1])$ and $\varsigma_{\Xi}^{F} : \mathbb{U} \to Int([0,1])$ denotes TMD, IMD and FMD of $\varepsilon \in \mathbb{U}$ to Ξ, respectively and $0 \preceq (L_{\Delta_i} T_{\Xi}(\varepsilon))^{l_1} + (L_{\Delta_i} I_{\Xi}(\varepsilon))^{l_2} + (L_{\Delta_i} F_{\Xi}(\varepsilon))^{l_3} \preceq 2$, where l_1, l_2, l_3 are positive integers and $\Delta = \bigcirc T_{\Xi}, I_{\Xi}, F_{\Xi}$. For convenience, $\Xi = \Big\langle LT_{\Xi}, LI_{\Xi}, LF_{\Xi}\Big\rangle$ is called the new type NN.

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Definition 3.2. Let $\Xi = \left\langle LT_\Xi, LI_\Xi, LF_\Xi \right\rangle$ be the new type neutrosophic number, the score function of Ξ is defined as $-1 \preceq \mathbb{S}(\Xi) \preceq 1$. where

$$\mathbb{S}(\Xi) = \mathcal{P} + 1 - \mathcal{Q} + 1 - \mathcal{R}$$

The accuracy function of Ξ is $\mathbb{A}(\Xi)$, where $0 \preceq \mathbb{A}(\Xi) \preceq 1$.

$$\mathbb{A}_1(\Xi) = \mathcal{P} + 1 + \mathcal{Q} + 1 + \mathcal{R}$$

where $\mathcal{P} = (L_{\Delta_i} T_\Xi)^2, \mathcal{Q} = (L_{\Delta_i} I_\Xi)^2, \mathcal{R} = (L_{\Delta_i} F_\Xi)^2$ and

Definition 3.3. Let $\Xi = \left\langle LT_\Xi, LI_\Xi, LF_\Xi \right\rangle$, $\Xi_1 = \left\langle LT_{\Xi 1}, LI_{\Xi 1}, LF_{\Xi 1} \right\rangle$ and $\Xi_2 = \left\langle LT_{\Xi 2}, LI_{\Xi 2}, LF_{\Xi 2} \right\rangle$ be any three new type NNs and $\Delta = \bigcirc[T_{\Xi_i}, I_{\Xi_i}, F_{\Xi_i}]$. Their following operations are defined as follows:

1. $\Xi_1 \oplus \Xi_2 = \begin{bmatrix} \sqrt[l_1]{(L_{\Delta_i} T_{\Xi 1})^{l_1} + (L_{\Delta_i} T_{\Xi 2})^{l_1} - (L_{\Delta_i} T_{\Xi 1})^{l_1} \cdot (L_{\Delta_i} T_{\Xi 2})^{l_1}}, \\ \sqrt[l_2]{(L_{\Delta_i} I_{\Xi 1})^{l_2} + (L_{\Delta_i} I_{\Xi 2})^{l_2} - (L_{\Delta_i} I_{\Xi 1})^{l_2} \cdot (L_{\Delta_i} I_{\Xi 2})^{l_2}}, \\ L_{\Delta_i}(F_{\Xi 1})^{l_3} \cdot L_{\Delta_i}(F_{\Xi 2})^{l_3} \end{bmatrix}$,

2. $\Xi_1 \odot \Xi_2 = \begin{bmatrix} L_{\Delta_i}(T_{\Xi 1})^{l_1} \cdot L_{\Delta_i}(T_{\Xi 2})^{l_1}, \\ \sqrt[l_2]{(L_{\Delta_i} I_{\Xi 1})^{l_2} + (L_{\Delta_i} I_{\Xi 2})^{l_2} - (L_{\Delta_i} I_{\Xi 1})^{l_2} \cdot (L_{\Delta_i} I_{\Xi 2})^{l_2}}, \\ \sqrt[l_3]{(L_{\Delta_i} F_{\Xi 1})^{l_3} + (L_{\Delta_i} F_{\Xi 2})^{l_3} - (L_{\Delta_i} F_{\Xi 1})^{l_3} \cdot (L_{\Delta_i} F_{\Xi 2})^{l_3}} \end{bmatrix}$,

3. $\Lambda \cdot \Xi = \left[\sqrt[l_1]{1 - \left(1 - (L_{\Delta_i} T_\Xi)^{l_1}\right)^\Lambda}, \sqrt[l_2]{1 - \left(1 - (L_{\Delta_i} I_\Xi)^{l_2}\right)^\Lambda}, (L_{\Delta_i} F_\Xi)^{l_3 \Lambda} \right]$,

4. $\Xi^\Lambda = \left[(L_{\Delta_i} T_\Xi)^{l_1 \Lambda}, \sqrt[l_2]{1 - \left(1 - (L_{\Delta_i} I_\Xi)^{l_2}\right)^\Lambda}, \sqrt[l_3]{1 - \left(1 - (L_{\Delta_i} F_\Xi)^{l_3}\right)^\Lambda} \right]$.

4 AOs based on new type NS approach

Based on the operational rules of new type neutrosophic numbers, the weighed averaging operators for new type neutrosophic numbers are presented.

4.1 New type neutrosophic weighted averaging(new type NWA) operator

Definition 4.1. Let $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi i}, LF_{\Xi i} \right\rangle$ be the family of new type NNs, $\Gamma = (\Gamma_1, \Gamma_2, ..., \Gamma_n)$ be the weight of Ξ_i, $\Gamma_i \succeq 0$ and $\odot_{i=1}^{n} \Gamma_i = 1$ and $\Delta = \bigcirc[T_{\Xi_i}, I_{\Xi_i}, F_{\Xi_i}$. Then new type NWA operator is new type NWA $(\Xi_1, \Xi_2, ..., \Xi_n) = \odot_{i=1}^{n} \Gamma_i \Xi_i$ for $i = 1, 2, ..., n$.

Theorem 4.2. *Let $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi}^{-+}{}_i, LF_{\Xi i} \right\rangle$ be the family of new type NNs. Then*

new type NWA $(\Xi_1, \Xi_2, ..., \Xi_n) = \begin{bmatrix} \sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} T_{\Xi i})^{l_1}\right)^{\Gamma_i}}, \\ \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi i})^{l_2}\right)^{\Gamma_i}}, \\ \bigcirc_{i=1}^{n} (L_{\Delta_i} F_{\Xi i})^{l_3 \Gamma_i} \end{bmatrix}$. *(associativity property).*

Proof. If $n = 2$, then new type NWA $(\Xi_1, \Xi_2) = \Gamma_1 \Xi_1 \oplus \Gamma_2 \Xi_2$, where

$$\Gamma_1 \Xi_1 = \left[\sqrt[l_1]{1 - \left(1 - (L_{\Delta_i} T_{\Xi 1})^{l_1}\right)^{\Gamma_1}}, \sqrt[l_2]{1 - \left(1 - (L_{\Delta_i} I_{\Xi 1})^{l_2}\right)^{\Gamma_1}}, (L_{\Delta_i} F_{\Xi 1})^{l_3 \Gamma_1} \right]$$

and

$$\Gamma_2 \Xi_2 = \left[\sqrt[l_1]{1 - \left(1 - (L_{\Delta_i} T_{\Xi 2})^{l_1}\right)^{\Gamma_2}}, \sqrt[l_2]{1 - \left(1 - (L_{\Delta_i} I_{\Xi 2})^{l_2}\right)^{\Gamma_2}}, (L_{\Delta_i} F_{\Xi 2})^{l_3 \Gamma_2} \right].$$

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Hence,

$$\Gamma_1\Xi_1 \oplus \Gamma_2\Xi_2 = \begin{bmatrix} \sqrt[l_1]{\begin{matrix}\left(1-\left(1-(L_{\Delta_i}T_{\Xi 1})^{l_1}\right)^{\Gamma_1}\right)+\left(1-\left(1-(L_{\Delta_i}T_{\Xi 2})^{l_1}\right)^{\Gamma_2}\right) \\ -\left(1-\left(1-(L_{\Delta_i}T_{\Xi 1})^{l_1}\right)^{\Gamma_1}\right)\cdot\left(1-\left(1-(L_{\Delta_i}T_{\Xi 2})^{l_1}\right)^{\Gamma_2}\right),\end{matrix}}, \\ \sqrt[l_2]{\begin{matrix}\left(1-\left(1-(L_{\Delta_i}I_{\Xi 1})^{l_2}\right)^{\Gamma_1}\right)+\left(1-\left(1-(L_{\Delta_i}I_{\Xi 2})^{l_2}\right)^{\Gamma_2}\right) \\ -\left(1-\left(1-(L_{\Delta_i}I_{\Xi 1})^{l_2}\right)^{\Gamma_1}\right)\cdot\left(1-\left(1-(L_{\Delta_i}I_{\Xi 2})^{l_2}\right)^{\Gamma_2}\right),\end{matrix}} \\ (L_{\Delta_i}F_{\Xi 1})^{l_3\Gamma_1}\cdot(L_{\Delta_i}F_{\Xi 2})^{l_3\Gamma_2} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[l_1]{1-\left(1-(L_{\Delta_i}T_{\Xi 1})^{l_1}\right)^{\Gamma_1}\cdot\left(1-(L_{\Delta_i}T_{\Xi 2})^{l_1}\right)^{\Gamma_2}}, \\ \sqrt[l_2]{1-\left(1-(L_{\Delta_i}I_{\Xi 1})^{l_2}\right)^{\Gamma_1}\cdot\left(1-(L_{\Delta_i}I_{\Xi 2})^{l_2}\right)^{\Gamma_2}}, \\ (L_{\Delta_i}F_{\Xi 1})^{l_3\Gamma_1}\cdot(L_{\Delta_i}F_{\Xi 2})^{l_3\Gamma_2} \end{bmatrix}.$$

Thus, new type NWA $(\Xi_1,\Xi_2) = \begin{bmatrix} \sqrt[l_1]{1-\bigcirc_{i=1}^{l_1}\left(1-(L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{\Gamma_i}}, \\ \sqrt[l_2]{1-\bigcirc_{i=1}^{l_2}\left(1-(L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{\Gamma_i}}, \bigcirc_{i=1}^{l_2}(L_{\Delta_i}F_{\Xi i})^{l_3\Gamma_i} \end{bmatrix}$.

It is valid for $n = l$ and $l \succeq 3$. Hence,

new type NWA $(\Xi_1,\Xi_2,...,\Xi_l) = \begin{bmatrix} \sqrt[l_1]{1-\bigcirc_{i=1}^{l}\left(1-(L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{\Gamma_i}}, \\ \sqrt[l_2]{1-\bigcirc_{i=1}^{l}\left(1-(L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{\Gamma_i}}, \\ \bigcirc_{i=1}^{l}(L_{\Delta_i}F_{\Xi i})^{l_3\Gamma_i} \end{bmatrix}$.

If $n = l+1$ and we apply, new type NWA $(\Xi_1,\Xi_2,...,\Xi_l,\Xi_{l+1})$

$$= \begin{bmatrix} \sqrt[l_1]{\begin{matrix}\bigcirc_{i=1}^{l}\left(1-\left(1-(L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{\Gamma_i}\right)+\left(1-\left(1-(L_{\Delta_i}T_{\Xi l+1})^{l_1}\right)^{\Gamma_{l+1}}\right) \\ -\bigcirc_{i=1}^{l}\left(1-\left(1-(L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{\Gamma_i}\right)\cdot\left(1-\left(1-(L_{\Delta_i}T_{\Xi l+1})^{l_1}\right)^{\Gamma_{l+1}}\right),\end{matrix}} \\ \sqrt[l_2]{\begin{matrix}\bigcirc_{i=1}^{l}\left(1-\left(1-(L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{\Gamma_i}\right)+\left(1-\left(1-(L_{\Delta_i}I_{\Xi l+1})^{l_2}\right)^{\Gamma_{l+1}}\right) \\ -\bigcirc_{i=1}^{l}\left(1-\left(1-(L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{\Gamma_i}\right)\cdot\left(1-\left(1-(L_{\Delta_i}I_{\Xi l+1})^{l_2}\right)^{\Gamma_{l+1}}\right),\end{matrix}} \\ \bigcirc_{i=1}^{l}(L_{\Delta_i}F_{\Xi i})^{l_3\Gamma_i}\cdot(L_{\Delta_i}F_{\Xi l+1})^{l_3\Gamma_{l+1}} \end{bmatrix}$$

$$= \left[\sqrt[l_1]{1-\bigcirc_{i=1}^{l+1}\left(1-(L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{\Gamma_i}}, \sqrt[l_2]{1-\bigcirc_{i=1}^{l+1}\left(1-(L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{\Gamma_i}}, \bigcirc_{i=1}^{l+1}(L_{\Delta_i}F_{\Xi i})^{l_3\Gamma_i}\right].$$

Theorem 4.3. *(idempotency property) If all $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi i}, LF_{\Xi i}\right\rangle (i = 1,2,...,n)$ are equal and $\Xi_i = \Xi$. Then new type NWA $(\Xi_1,\Xi_2,...,\Xi_n) = \Xi$.*

Proof. Given that $LT_{\Xi_i} = LT_{\Xi}$, $LI_{\Xi_i} = LI_{\Xi}$ and $LF_{\Xi_i} = LF_{\Xi}$, for $i = 1,2,...,n$ and $\odot_{i=1}^{n}\Gamma_i = 1$. Now, new type NWA $(\Xi_1,\Xi_2,...,\Xi_n)$

$$\begin{aligned} &= \left[\sqrt[l_1]{1-\bigcirc_{i=1}^{n}\left(1-(L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{\Gamma_i}}, \sqrt[l_2]{1-\bigcirc_{i=1}^{n}\left(1-(L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{\Gamma_i}}, \bigcirc_{i=1}^{n}(L_{\Delta_i}F_{\Xi i})^{l_3\Gamma_i}\right] \\ &= \left[\sqrt[l_1]{1-\left(1-(L_{\Delta_i}T_{\Xi})^{l_1}\right)^{\odot_{i=1}^{n}\Gamma_i}}, \sqrt[l_2]{1-\left(1-(L_{\Delta_i}I_{\Xi})^{l_2}\right)^{\odot_{i=1}^{n}\Gamma_i}}, (L_{\Delta_i}F_{\Xi})^{\odot_{i=1}^{n}l_3\Gamma_i}\right] \\ &= \left[\sqrt[l_1]{1-\left(1-(L_{\Delta_i}T_{\Xi})^{l_1}\right)}, \sqrt[l_2]{1-\left(1-(L_{\Delta_i}I_{\Xi})^{l_2}\right)}, (L_{\Delta_i}F_{\Xi})^{l_3}\right] \\ &= \Xi. \end{aligned}$$

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Theorem 4.4. *(boundedness property) Let* $\Xi_i = \left\langle LT_{\Xi ij}, LI_{\Xi ij}, LF_{\Xi ij} \right\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the collection of new type NWA, where* $\overleftarrow{L_{\Delta_i} T_{\Xi}} = \min L_{\Delta_i} T_{\Xi ij}$, $\overrightarrow{L_{\Delta_i} T_{\Xi}} = \max L_{\Delta_i} T_{\Xi ij}$, $\overleftarrow{L_{\Delta_i} I_{\Xi}} = \min L_{\Delta_i} I_{\Xi ij}$, $\overleftarrow{L_{\Delta_i} I_{\Xi}} = \min L_{\Delta_i} I_{\Xi ij}$, $\overleftarrow{L_{\Delta_i} F_{\Xi}} = \min L_{\Delta_i} F_{\Xi ij}$, $\overrightarrow{L_{\Delta_i} F_{\Xi}} = \max L_{\Delta_i} F_{\Xi ij}$.
Then, $\left\langle \overleftarrow{L_{\Delta_i} T_{\Xi}}, \overleftarrow{L_{\Delta_i} I_{\Xi}}, \overrightarrow{L_{\Delta_i} F_{\Xi}} \right\rangle$

$$\begin{aligned} &\preceq \text{ new type } NWA(\Xi_1, \Xi_2, ..., \Xi_n) \\ &\preceq \left\langle \overrightarrow{L_{\Delta_i} T_{\Xi}}, \overrightarrow{L_{\Delta_i} I_{\Xi}}, \overleftarrow{L_{\Delta_i} (F_{\Xi})} \right\rangle. \end{aligned}$$

where $1 \preceq i \preceq n, \ j = 1, 2, ..., i_j$.

Proof. Since, $\overleftarrow{L_{\Delta_i} T_{\Xi}} = \min L_{\Delta_i} T_{\Xi ij}$, $\overrightarrow{L_{\Delta_i} T_{\Xi}} = \max L_{\Delta_i} T_{\Xi ij}$
and $\overleftarrow{L_{\Delta_i} T_{\Xi}} \preceq L_{\Delta_i} T_{\Xi ij} \preceq \overrightarrow{L_{\Delta_i} T_{\Xi}}$ Now, $\overleftarrow{L_{\Delta_i} T_{\Xi}}$

$$\begin{aligned} &= \sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overleftarrow{L_{\Delta_i} T_{\Xi}})^{l_1}\right)^{\Gamma_i}} \\ &\preceq \sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} T_{\Xi ij})^{l_1}\right)^{\Gamma_i}} \\ &\preceq \sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overrightarrow{L_{\Delta_i} T_{\Xi}})^{l_1}\right)^{\Gamma_i}} \\ &= \overrightarrow{L_{\Delta_i} T_{\Xi}}. \end{aligned}$$

Since, $\overleftarrow{L_{\Delta_i} I_{\Xi}} = \min L_{\Delta_i} I_{\Xi ij}$, $\overleftarrow{L_{\Delta_i} I_{\Xi}} = \min L_{\Delta_i} I_{\Xi ij}$, $\overleftarrow{L_{\Delta_i} I_{\Xi}} \preceq L_{\Delta_i} I_{\Xi ij} \preceq \overrightarrow{L_{\Delta_i} I_{\Xi}}$. Now,

$$\begin{aligned} \overleftarrow{L_{\Delta_i} I_{\Xi}} &= \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overleftarrow{L_{\Delta_i} I_{\Xi}})^{l_2}\right)^{\Gamma_i}} \\ &\preceq \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi ij})^{l_2}\right)^{\Gamma_i}} \\ &\preceq \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overrightarrow{L_{\Delta_i} I_{\Xi}})^{l_2}\right)^{\Gamma_i}} \\ &= \overrightarrow{L_{\Delta_i} I_{\Xi}}. \end{aligned}$$

Since, $\overleftarrow{L_{\Delta_i} F_{\Xi}} = \min L_{\Delta_i} F_{\Xi ij}$, $\overrightarrow{L_{\Delta_i} F_{\Xi}} = \max L_{\Delta_i} F_{\Xi ij}$ and $\overleftarrow{L_{\Delta_i} F_{\Xi}} \preceq L_{\Delta_i} F_{\Xi ij} \preceq \overrightarrow{L_{\Delta_i} F_{\Xi}}$. Now,

$$\begin{aligned} \overleftarrow{L_{\Delta_i} F_{\Xi}} &= \bigcirc_{i=1}^{n} (\overleftarrow{L_{\Delta_i} F_{\Xi}})^{l_3 \Gamma_i} \\ &\preceq \bigcirc_{i=1}^{n} (L_{\Delta_i} F_{\Xi ij})^{l_3 \Gamma_i} \\ &\preceq \bigcirc_{i=1}^{n} (\overrightarrow{L_{\Delta_i} F_{\Xi}})^{l_3 \Gamma_i} \\ &= \overrightarrow{L_{\Delta_i} F_{\Xi}}. \end{aligned}$$

Therefore,

$$\begin{aligned} &\left[\begin{array}{c} \left(\sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overleftarrow{L_{\Delta_i} T_{\Xi}})^{l_1}\right)^{\Gamma_i}} \right)^2 + 1 - \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overleftarrow{L_{\Delta_i} I_{\Xi}})^{l_2}\right)^{\Gamma_i}} \right)^2 \\ +1 - \left(\bigcirc_{i=1}^{n} (\overrightarrow{L_{\Delta_i} F_{\Xi}})^{l_3 \Gamma_i} \right)^2 \end{array} \right] \\ = &\left[\begin{array}{c} \left(\sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} T_{\Xi ij})^{l_1}\right)^{\Gamma_i}} \right)^2 + 1 - \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi ij})^{l_2}\right)^{\Gamma_i}} \right)^2 \\ +1 - \left(\bigcirc_{i=1}^{n} (L_{\Delta_i} F_{\Xi ij})^{l_3 \Gamma_i} \right)^2 \end{array} \right] \\ = &\left[\begin{array}{c} \left(\sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overrightarrow{L_{\Delta_i} T_{\Xi}})^{l_1}\right)^{\Gamma_i}} \right)^2 + 1 - \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (\overrightarrow{L_{\Delta_i} I_{\Xi}})^{l_2}\right)^{\Gamma_i}} \right)^2 \\ +1 - \left(\bigcirc_{i=1}^{n} (\overleftarrow{L_{\Delta_i} F_{\Xi}})^{l_3 \Gamma_i} \right)^2 \end{array} \right]. \end{aligned}$$

Hence, $\left\langle \overleftarrow{LT_{\Xi}}, \overleftarrow{LI_{\Xi}}, \overrightarrow{LF_{\Xi}} \right\rangle \preceq \text{new type } NWA(\Xi_1, \Xi_2, ..., \Xi_n) \preceq \left\langle \overrightarrow{LT_{\Xi}}, \overrightarrow{LI_{\Xi}}, \overleftarrow{LF_{\Xi}} \right\rangle$.

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Theorem 4.5. *(monotonicity property) Let* $\Xi_i = \left\langle LT_{\Xi_{t_{ij}}}, LI_{\Xi_{t_{ij}}}, LF_{\Xi_{t_{ij}}} \right\rangle$ *and* $\Gamma_i = \left\langle LT_{\Xi_{h_{ij}}}, LI_{\Xi_{h_{ij}}}, L(F_{\Xi_{h_{ij}}}) \right\rangle (i = 1,2,...,n); (j = 1,2,...,i_j)$ *be the families of new type NWAs. For any* i*, if there is* $\left(L_{\Delta_i} T_{\Xi_{t_{ij}}}\right)^2 \preceq \left(L_{\Delta_i} T_{\Xi_{h_{ij}}}\right)^2$ *and* $\left(L_{\Delta_i} I_{\Xi_{t_{ij}}}\right)^2 \succeq \left(L_{\Delta_i} I_{\Xi_{h_{ij}}}\right)^2$ *and* $\left(L_{\Delta_i} F_{\Xi_{t_{ij}}}\right)^2 \succeq \left(L_{\Delta_i} F_{\Xi_{h_{ij}}}\right)^2$ *or* $\Xi_i \preceq \mathscr{W}_i$*. Then new type NWA* $(\Xi_1, \Xi_2, ..., \Xi_n) \preceq$ *new type NWA* $(\mathscr{W}_1, \mathscr{W}_2, ..., \mathscr{W}_n)$*.*

Proof. For any i, $\left(L_{\Delta_i} T_{\Xi_{t_{ij}}}\right)^{l_1} \preceq \left(L_{\Delta_i} T_{\Xi_{h_{ij}}}\right)^{l_1}$.
Therefore, $1 - (L_{\Delta_i} T_{\Xi_{t_i}})^{l_1} \succeq 1 - (L_{\Delta_i} T_{\Xi_{h_i}})^{l_1}$.
Hence, $\bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} T_{\Xi_{t_i}})^{l_1}\right)^{\Gamma_i} \succeq \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} T_{\Xi_{h_i}})^{l_1}\right)^{\Gamma_i}$
and $\sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} T_{\Xi_{t_i}})^{l_1}\right)^{\Gamma_i}} \preceq \sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} T_{\Xi_{h_i}})^{l_1}\right)^{\Gamma_i}}$.
For any i, $\left(L_{\Delta_i} I_{\Xi_{t_{ij}}}\right)^{l_2} \succeq \left(L_{\Delta_i} I_{\Xi_{h_{ij}}}\right)^{l_2}$.
Therefore, $1 - (L_{\Delta_i} I_{\Xi_{t_i}})^{l_2} \preceq 1 - (L_{\Delta_i} I_{\Xi_{h_i}})^{l_2}$.
Hence, $\bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi_{t_i}})^{l_2}\right)^{\Gamma_i} \preceq \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi_{h_i}})^{l_2}\right)^{\Gamma_i}$
implies that $\sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi_{t_i}})^{l_2}\right)^{\Gamma_i}} \succeq \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi_{h_i}})^{l_2}\right)^{\Gamma_i}}$.
Hence, $1 - \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi_{t_i}})^{l_2}\right)^{\Gamma_i}} \preceq 1 - \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - (L_{\Delta_i} I_{\Xi_{h_i}})^{l_2}\right)^{\Gamma_i}}$.

For any i, $\left(L_{\Delta_i} F_{\Xi_{t_{ij}}}\right)^{l_3} \succeq \left(L_{\Delta_i} F_{\Xi_{h_{ij}}}\right)^{l_3}$.
Therefore, $1 - \left(\bigcirc_{i=1}^{n} L_{\Delta_i} F_{\Xi_{t_{ij}}}\right)^{l_3} \preceq 1 - \left(\bigcirc_{i=1}^{n} L_{\Delta_i} F_{\Xi_{h_{ij}}}\right)^{l_3}$.

$$= \left[\begin{array}{c} \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n}\left(1 - (L_{\Delta_i} T_{\Xi ti})^{l_2}\right)^{\Gamma_i}}\right)^2 + 1 - \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n}\left(1 - (L_{\Delta_i} I_{\Xi ti})^{l_2}\right)^{\Gamma_i}}\right)^2 \\ +1 - \left(\bigcirc_{i=1}^{n} (L_{\Delta_i} F_{\Xi tij})\right)^2 \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n}\left(1 - (L_{\Delta_i} T_{\Xi hi})^{l_2}\right)^{\Gamma_i}}\right)^2 + 1 - \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n}\left(1 - (L_{\Delta_i} I_{\Xi hi})^{l_2}\right)^{\Gamma_i}}\right)^2 \\ +1 - \left(\bigcirc_{i=1}^{n} (L_{\Delta_i} F_{\Xi hij})\right)^2 \end{array}\right].$$

Hence, new type NWA $(\Xi_1, \Xi_2, ..., \Xi_n) \preceq$ new type NWA $(\mathscr{W}_1, \mathscr{W}_2, ..., W_n)$.

4.2 new type neutrosophic weighted geometric(new type NWG) operator

Definition 4.6. Let $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi i}, LF_{\Xi i}] \right\rangle$ be the family of new type NNs. Then new type NWG operator is new type NWG $(\Xi_1, \Xi_2, ..., \Xi_n) = \bigcirc_{i=1}^{n} \Xi_i^{\Gamma_i}$ $(i = 1,2,...,n)$.

Theorem 4.7. *Let* $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi i}, LF_{\Xi i} \right\rangle$ *be the family of new type NNs. Then*

$$\textit{new type NWG } (\Xi_1, \Xi_2, ..., \Xi_n) = \left[\begin{array}{c} \bigcirc_{i=1}^{n} (L_{\Delta_i} T_{\Xi i})^{l_1 \Gamma_i},\ \sqrt[l_2]{1 - \bigcirc_{i=1}^{n}\left(1 - (L_{\Delta_i} I_{\Xi i})^{l_2}\right)^{\Gamma_i}}, \\ \sqrt[l_3]{1 - \bigcirc_{i=1}^{n}\left(1 - (L_{\Delta_i} F_{\Xi i})^{l_3}\right)^{\Gamma_i}} \end{array}\right].$$

Proof. This proof based on Theorem 4.2.

Theorem 4.8. *If all* $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi i}, LF_{\Xi i} \right\rangle$ *are equal and* $\Xi_i = \Xi$*, for* $i = 1,2,...,n$*. Then new type NWG* $(\Xi_1, \Xi_2, ..., \Xi_n) = \Xi$*.*

Proof. This proof based on Theorem 4.3.

Corollary 4.9. *The new type NWG operator is used to satisfy the boundedness and monotonicity properties.*

Proof. This proof based on Theorem 4.4 and Theorem 4.5.

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4.3 Generalized new type NWA (new type GNWA) operator

Definition 4.10. Let $\Xi_i = \left\langle LT_{\Xi_i}, LI_{\Xi_i}, LF_{\Xi_i} \right\rangle$ be the family of new type NN. Then new type GNWA $(\Xi_1, \Xi_2, ..., \Xi_n) = \left(\odot_{i=1}^{n} \Gamma_i \Xi_i^{\Lambda} \right)^{1/\Lambda}$ is called the new type GNWA operator.

Theorem 4.11. *Let $\Xi_i = \left\langle LT_{\Xi_i}, LI_{\Xi_i}, LF_{\Xi_i} \right\rangle$ be the family of new type NNs. Then new type GNWA*

$$(\Xi_1, \Xi_2, ..., \Xi_n) = \begin{bmatrix} \left(\sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - \left((L_{\Delta_i} T_{\Xi_i})^{l_1} \right)^{l_1} \right)^{\Gamma_i}} \right)^{1/\Lambda}, \\ \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - \left((L_{\Delta_i} I_{\Xi_i})^{l_2} \right)^{l_2} \right)^{\Gamma_i}} \right)^{1/\Lambda}, \\ \sqrt[l_3]{1 - \left(1 - \left(\bigcirc_{i=1}^{n} \left(\sqrt[l_3]{1 - \left(1 - (L_{\Delta_i} F_{\Xi_i})^{l_3} \right)^{l_3}} \right)^{\Gamma_i} \right) \right)^{l_3}}^{\,1/\Lambda} \end{bmatrix}.$$

Proof. We have,

$$\odot_{i=1}^{n} \Gamma_i \Xi_i^{\Lambda} = \begin{bmatrix} \sqrt[l_1]{1 - \bigcirc_{i=1}^{n} \left(1 - \left((L_{\Delta_i} T_{\Xi_i})^{l_1} \right)^{l_1} \right)^{\Gamma_i}}, \sqrt[l_2]{1 - \bigcirc_{i=1}^{n} \left(1 - \left((L_{\Delta_i} I_{\Xi_i})^{l_2} \right)^{l_2} \right)^{\Gamma_i}}, \\ \bigcirc_{i=1}^{n} \left(\sqrt[l_3]{1 - \left(1 - (L_{\Delta_i} F_{\Xi_i})^{l_3} \right)^{l_3}} \right)^{\Gamma_i} \end{bmatrix}.$$

If $n = 2$, then $\Gamma_1 \Xi_1 \oplus \Gamma_2 \Xi_2$

$$= \begin{bmatrix} \sqrt[l_1]{\begin{matrix} \left(\sqrt[l_1]{1 - \left(1 - \left((L_{\Delta_i} T_{\Xi_1})^{l_1} \right)^{l_1} \right)^{\Gamma_1}} \right)^{l_1} + \left(\sqrt[l_1]{1 - \left(1 - \left((L_{\Delta_i} T_{\Xi_2})^{l_1} \right)^{l_1} \right)^{\Gamma_1}} \right)^{l_1}, \\ - \left(\sqrt[l_1]{1 - \left(1 - \left((L_{\Delta_i} T_{\Xi_1})^{l_1} \right)^{l_1} \right)^{\Gamma_1}} \right)^{l_1} \cdot \left(\sqrt[l_1]{1 - \left(1 - \left((L_{\Delta_i} T_{\Xi_2})^{l_1} \right)^{l_1} \right)^{\Gamma_1}} \right)^{l_1} \end{matrix}} \\ \sqrt[l_2]{\begin{matrix} \left(\sqrt[l_2]{1 - \left(1 - \left((L_{\Delta_i} I_{\Xi_1})^{l_2} \right)^{l_2} \right)^{\Gamma_1}} \right)^{l_2} + \left(\sqrt[l_2]{1 - \left(1 - \left((L_{\Delta_i} I_{\Xi_2})^{l_2} \right)^{l_2} \right)^{\Gamma_1}} \right)^{l_2}, \\ - \left(\sqrt[l_2]{1 - \left(1 - \left((L_{\Delta_i} I_{\Xi_1})^{l_2} \right)^{l_2} \right)^{\Gamma_1}} \right)^{l_2} \cdot \left(\sqrt[l_2]{1 - \left(1 - \left((L_{\Delta_i} I_{\Xi_2})^{l_2} \right)^{l_2} \right)^{\Gamma_1}} \right)^{l_2} \end{matrix}} \\ \left(\sqrt[l_3]{1 - \left(1 - (L_{\Delta_i} F_{\Xi_1})^{l_3} \right)^{l_3}} \right)^{\Gamma_1} \cdot \left(\sqrt[l_3]{1 - \left(1 - (L_{\Delta_i} F_{\Xi_2})^{l_3} \right)^{l_3}} \right)^{\Gamma_1}, \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[l_1]{1 - \bigcirc_{i=1}^{l_1} \left(1 - \left((L_{\Delta_i} T_{\Xi_1})^{l_1} \right)^{l_1} \right)^{\Gamma_i}}, \sqrt[l_2]{1 - \bigcirc_{i=1}^{l_2} \left(1 - \left((L_{\Delta_i} I_{\Xi_1})^{l_2} \right)^{l_2} \right)^{\Gamma_i}}, \\ \bigcirc_{i=1}^{l_3} \left(\sqrt[l_3]{1 - \left(1 - (L_{\Delta_i} F_{\Xi_i})^{l_3} \right)^{l_3}} \right)^{\Gamma_i} \end{bmatrix}.$$

It is valid for $n = l$ and $l \succeq 3$.

Hence, $$\odot_{i=1}^{l} \Gamma_i \Xi_i^{\Lambda} = \begin{bmatrix} \sqrt[l_1]{1 - \bigcirc_{i=1}^{l} \left(1 - \left((L_{\Delta_i} T_{\Xi_1})^{l_1} \right)^{l_1} \right)^{\Gamma_i}}, \sqrt[l_2]{1 - \bigcirc_{i=1}^{l} \left(1 - \left((L_{\Delta_i} I_{\Xi_1})^{l_2} \right)^{l_2} \right)^{\Gamma_i}}, \\ \bigcirc_{i=1}^{l} \left(\sqrt[l_3]{1 - \left(1 - (L_{\Delta_i} F_{\Xi_i})^{l_3} \right)^{l_3}} \right)^{\Gamma_i}, \end{bmatrix}.$$

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If $n = l + 1$ and we apply, then $\odot_{i=1}^{l}\Gamma_i\Xi_i^{\Lambda} + \Gamma_{l+1}\Xi_{l+1}^{\Lambda} = \odot_{i=1}^{l+1}\Gamma_i\Xi_i^{\Lambda}$.
Now,$\odot_{i=1}^{l}\Gamma_i\Xi_i^{\Lambda} + \Gamma_{l+1}\Xi_{l+1}^{\Lambda} = \Gamma_1\Xi_1^{\Lambda} \oplus \Gamma_2\Xi_2^{\Lambda} \oplus ... \oplus \Gamma_l\Xi_l^{\Lambda} \oplus \Gamma_{l+1}\Xi_{l+1}^{\Lambda}$

$$= \left[\begin{array}{c} \sqrt[l_1]{\begin{array}{c}\left(\sqrt[l_1]{1-\bigcirc_{i=1}^{l}\left(1-\left((L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{l_1}\right)^{\Gamma_i}}\right)^{l_1} + \left(\sqrt[l_1]{1-\left(1-\left((L_{\Delta_i}T_{\Xi l+1})^{l_1}\right)^{l_1}\right)^{\Gamma_1}}\right)^{l_1}, \\ -\left(\sqrt[l_1]{1-\bigcirc_{i=1}^{l}\left(1-\left((L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{l_1}\right)^{\Gamma_i}}\right)^{l_1}\cdot\left(\sqrt[l_1]{1-\left(1-\left((L_{\Delta_i}T_{\Xi l+1})^{l_1}\right)^{l_1}\right)^{\Gamma_1}}\right)^{l_1}\end{array}} \\ \sqrt[l_2]{\begin{array}{c}\left(\sqrt[l_2]{1-\bigcirc_{i=1}^{l}\left(1-\left((L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{l_2}\right)^{\Gamma_i}}\right)^{l_2} + \left(\sqrt[l_2]{1-\left(1-\left((L_{\Delta_i}I_{\Xi l+1})^{l_2}\right)^{l_2}\right)^{\Gamma_1}}\right)^{l_2}, \\ -\left(\sqrt[l_2]{1-\bigcirc_{i=1}^{l}\left(1-\left((L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{l_2}\right)^{\Gamma_i}}\right)^{l_2}\cdot\left(\sqrt[l_2]{1-\left(1-\left((L_{\Delta_i}I_{\Xi l+1})^{l_2}\right)^{l_2}\right)^{\Gamma_1}}\right)^{l_2}\end{array}} \\ \bigcirc_{i=1}^{l}\left(\sqrt[l_3]{1-\left(1-(L_{\Delta_i}F_{\Xi i})^{l_3}\right)^{l_3}}\right)^{\Gamma_i}\cdot\left(\sqrt[l_3]{1-\left(1-(L_{\Delta_i}F_{\Xi l+1})^{l_3}\right)^{l_3}}\right)^{\Gamma_1} \end{array}\right].$$

Thus,

$$\odot_{i=1}^{l+1}\Gamma_i\Xi_i^{\Lambda} = \left[\begin{array}{c}\sqrt[l_1]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((L_{\Delta_i}T_{\Xi 1})^{l_1}\right)^{l_1}\right)^{\Gamma_i}},\ \sqrt[l_2]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((L_{\Delta_i}I_{\Xi 1})^{l_2}\right)^{l_2}\right)^{\Gamma_i}}, \\ \bigcirc_{i=1}^{l+1}\left(\sqrt[l_3]{1-\left(1-(L_{\Delta_i}F_{\Xi i})^{l_3}\right)^{l_3}}\right)^{\Gamma_i}\end{array}\right].$$

Hence,

$$\left(\odot_{i=1}^{l+1}\Gamma_i\Xi_i^{\Lambda}\right)^{1/\Lambda} = \left[\begin{array}{c}\left(\sqrt[l_1]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((L_{\Delta_i}T_{\Xi i})^{l_1}\right)^{l_1}\right)^{\Gamma_i}}\right)^{1/\Lambda}, \\ \left(\sqrt[l_2]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((L_{\Delta_i}I_{\Xi i})^{l_2}\right)^{l_2}\right)^{\Gamma_i}}\right)^{1/\Lambda}, \\ \sqrt[l_3]{1-\left(1-\left(\bigcirc_{i=1}^{l+1}\left(\sqrt[l_3]{1-\left(1-(L_{\Delta_i}F_{\Xi i})^{l_3}\right)^{l_3}}\right)^{\Gamma_i}\right)^{l_3}\right)^{1/\Lambda}}\end{array}\right].$$

It is valid for $l \succeq 1$.

Remark 4.12. If $\Gamma_i = 1$, then new type GNWA operator is modified to the new type NWA operator.

Theorem 4.13. *If all* $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi i}, LF_{\Xi i}\right\rangle (i = 1, 2, ..., n)$ *are equal and* $\Xi_i = \Xi$*. Then new type GNWA* $(\Xi_1, \Xi_2, ..., \Xi_n) = \Xi$*.*

Proof. This proof based on Theorem 4.3.

Remark 4.14. To satisfy the boundedness and monotonicity conditions, we use the new type GNWA operator.

Proof. This proof based on Theorem 4.4 and Theorem 4.5.

4.4 Generalized new type NWG (new type GNWG) operator

Definition 4.15. Let $\Xi_i = \left\langle LT_{\Xi i}, LI_{\Xi i}, LF_{\Xi i}\right\rangle$ be the family of new type NNs. Then new type GNWG $(\Xi_1, \Xi_2, ..., \Xi_n) = \frac{1}{\Lambda}\left(\bigcirc_{i=1}^{n}(\Lambda\Xi_i)^{\Gamma_i}\right)$ $(i = 1, 2, ..., n)$ is called the new type GNWG operator.

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Theorem 4.16. *Let* $\Xi_i = \left\langle LT_{\Xi_i}, LI_{\Xi_i}, LF_{\Xi_i} \right\rangle$ *be the family of new type NNs. Then new type GNWG*$(\Xi_1, \Xi_2, ..., \Xi_n)$

$$= \left[\begin{array}{c} \sqrt[l_1]{1 - \left(1 - \left(\bigcirc_{i=1}^{n}\left(\sqrt[l_1]{1 - \left(1 - (L_{\Delta_i} T_{\Xi_i})^{l_1}\right)^{l_1}}\right)^{\Gamma_i}\right)^{l_1}\right)^{1/\Lambda}}, \\ \left(\sqrt[l_2]{1 - \bigcirc_{i=1}^{n}\left(1 - \left((L_{\Delta_i} I_{\Xi_i})^{l_2}\right)^{l_2}\right)^{\Gamma_i}}\right)^{1/\Lambda}, \left(\sqrt[l_3]{1 - \bigcirc_{i=1}^{n}\left(1 - \left((L_{\Delta_i} F_{\Xi_i})^{l_3}\right)^{l_3}\right)^{\Gamma_i}}\right)^{1/\Lambda} \end{array} \right].$$

Proof. This proof based on the Theorem 4.11.

Remark 4.17. If $\Gamma_i = 1$, then new type GNWG operator is converted to the new type NWG operator.

Remark 4.18. Boundedness and monotonicity properties can be met using the new type GNWG operator.

Proof. This proof based on Theorem 4.4 and Theorem 4.5.

Corollary 4.19. *If all* $\Xi_i = \left\langle LT_{\Xi_i}, LI_{\Xi_i}, LF_{\Xi_i} \right\rangle$ *are equal and* $\Xi_i = \Xi$*, for* $i = 1, 2, ..., n$*. Then new type GNWG*$(\Xi_1, \Xi_2, ..., \Xi_n) = \Xi$.

Proof. This proof based on Theorem 4.3.

5 MADM using new type NS data

Let $\overrightarrow{\Xi} = \{\overrightarrow{\Xi}_a, \overrightarrow{\Xi}_b, ..., \overrightarrow{\Xi}_n\}$ be the alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the attributes, $w = \{\vartheta_1, \vartheta_2, ..., \vartheta_m\}$ be the weights of attributes, $\Xi_{ij} = \left\langle LT_{\Xi_{ij}}, LI_{\Xi_{ij}}, LF_{\Xi_{ij}} \right\rangle$ is denote new type NS of $\overrightarrow{\Xi}_i$ in C_j. Here, $\mathbb{U}\Big\}$, $\varsigma_{\Xi}^{T} : \mathbb{U} \to Int([0, 1])$, $\varsigma_{\Xi}^{I} : \mathbb{U} \to Int([0, 1])$ and $\varsigma_{\Xi}^{F} : \mathbb{U} \to Int([0, 1])$ denotes TMD, IMD and FMD of $\varepsilon \in \mathbb{U}$ to Ξ, respectively and $0 \preceq (L_{\Delta_i} T_{\Xi}(\varepsilon))^{l_1} + (L_{\Delta_i} I_{\Xi}(\varepsilon))^{l_2} + (L_{\Delta_i} F_{\Xi}(\varepsilon))^{l_3} \preceq 2$, where l_1, l_2, l_3 are positive integers and $\Delta = \bigcirc(T_{\Xi}, I_{\Xi}, F_{\Xi})$, where $\bigcirc$ denote usual product.

5.1 Algorithm

Step-1: As the new type NS is one of the most important values, it should have a choice value.
Step-2: A choice is made regarding which values to use for the weighted averaging (geometric) process. Let $\mathscr{D} = (\overrightarrow{\Xi}_{ij})_{n \times m}$ is into $\overrightarrow{\mathscr{D}} = (\overrightarrow{\Xi}_{ij})_{n \times m}$.
Step-3: Calculate the positive and negative ideal values: $\overrightarrow{\Xi}^{P} = \left\langle 1, 1, 0 \right\rangle$, and $\overrightarrow{\Xi}^{N} = \left\langle 0, 0, 1 \right\rangle$.
Step-4: It is important to find the difference between the ideal values of each option in order to find the ED:

$$\mathscr{D}_i^P = \mathscr{D}_E\Big(\overrightarrow{\Xi}_i, \overrightarrow{\Xi}^P\Big);\ \mathscr{D}_i^N = \mathscr{D}_E\Big(\overrightarrow{\Xi}_i, \overrightarrow{\Xi}^N\Big).$$

Step-5: Using the following formula, one can calculate how close two points are to each other:

$$\mathscr{D}_i^{\star} = \frac{\mathscr{D}_i^N}{\mathscr{D}_i^P + \mathscr{D}_i^N}.$$

Step-6: If you are looking for the best output in this case, you will need to use $\sup \mathscr{D}_i^{\star}$. A decision is the process of choosing the right option in order to solve a given problem. The Figure-1 shows that algorithm for the proposed methods.

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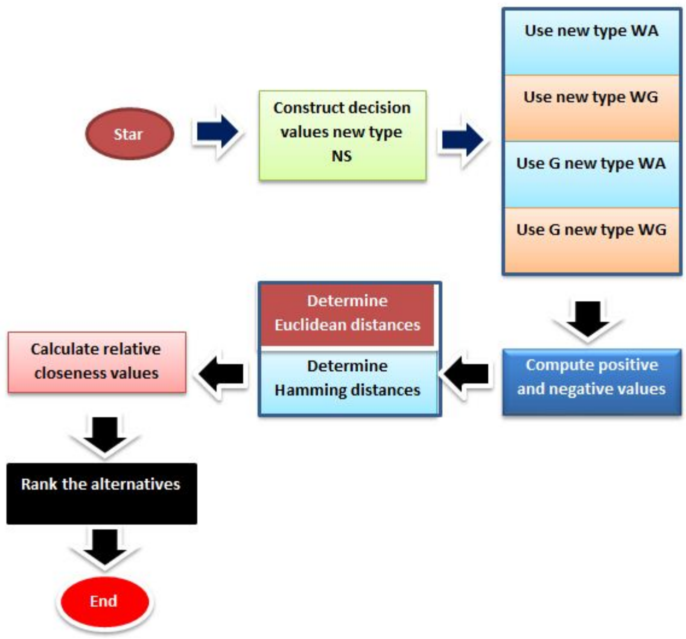

Figure-1. algorithm for proposed models.

5.2 Selection of Courier Service in India

Deliveries are a necessity for every business today. Now that the e-commerce industries require road transportation, small and medium enterprises have joined this sector. The courier services in India are also fiercely competitive, with each company claiming to be the best. As a result, you should take great care in selecting a courier partner. A client may have a difficult time choosing a reliable courier company when there are so many options available on the market? For you to avoid wasting time and money, here are the most important factors to consider when selecting the best courier service in India. Five courier companies have been selected at random, namely company-1, company-2, company-3, company-4, and company-5. When it comes to courier companies, what factors are important?

1. Reliability C_1: Choosing a courier service should be based on reliability. A different supplier may be a better choice if your shipments arrive late or damaged. The prices advertised by courier services are often very low, but they cost a lot more than what they advertise. It is likely that they will deliver within a particular time frame if they promise it. Providing reliable service at an affordable price is the hallmark of the best courier service. You will save money on shipping costs in the long run if you invest in a reliable courier first.

2. Turnaround Time C_2: A courier service's turnaround time is probably something you consider when choosing a service. There are a number of factors that determine the speed of delivery, including the delivery method (ground or air), the origin and destination locations, and even the season (that last one is obvious). Prioritizing your business is the key to success. If it takes two days to ship your product, does it matter if it arrives three days after it expires? It's a good idea to research the estimated speed of a courier company before selecting one for a specific shipment if speed is important to you.

3. Payment Options C_3: It is well known that every business transaction involves money. Third parties are usually involved in the process of processing and facilitating transactions. This role is typically filled by banks, but other parties can also fulfill it. A courier service plays a similar role when it comes to payments between individuals or businesses. What are the differences between different courier services? A variety of payment options are available. In addition to accepting in-person payments and credit cards (like PayPal), many services also accept electronic payments. Depending on your shipping volume and your customers' payment preferences, an all-in-one option may not be feasible.

4. Tracking Capability C_4: As soon as the courier service picks up your package in India, you must be notified. You can monitor the progress of your shipment at any time while it is in transit. Tracking

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systems also provide you with information about delays in shipments. In the absence of it, tracking your package is impossible, causing unnecessary anxiety. Late deliveries can also cost a business if an order is missed. Decide carefully which courier you want to use.

Table-1 shows that decision making information's are

Table 1: Decision making information

	C_1	C_2	C_3	C_4
$\vec{\Xi}_a$	$\langle 0.75, 0.7, 0.65\rangle$	$\langle 0.75, 0.75, 0.65\rangle$	$\langle 0.55, 0.55, 0.5\rangle$	$\langle 0.75, 0.6, 0.55\rangle$
$\vec{\Xi}_b$	$\langle 0.85, 0.65, 0.75\rangle$	$\langle 0.65, 0.65, 0.7\rangle$	$\langle 0.65, 0.65, 0.75\rangle$	$\langle 0.65, 0.65, 0.35\rangle$
$\vec{\Xi}_c$	$\langle 0.85, 0.75, 0.8\rangle$	$\langle 0.8, 0.85, 0.65\rangle$	$\langle 0.75, 0.45, 0.4\rangle$	$\langle 0.5, 0.7, 0.65\rangle$
$\vec{\Xi}_d$	$\langle 0.8, 0.8, 0.65\rangle$	$\langle 0.65, 0.55, 0.6\rangle$	$\langle 0.85, 0.45, 0.5\rangle$	$\langle 0.7, 0.85, 0.65\rangle$
$\vec{\Xi}_e$	$\langle 0.75, 0.75, 0.65\rangle$	$\langle 0.85, 0.65, 0.75\rangle$	$\langle 0.8, 0.65, 0.7\rangle$	$\langle 0.85, 0.8, 0.5\rangle$

The following aggregate data can be obtained using the new type NWA operator. In table-2 it is shown that new type NWA operates.

Table 2: new type NWA operator

	$newtypeNWA\ operator\ (l_1, l_2, l_3 = (1,1,1)$
$\vec{\Xi}_a$	$\langle 0.2448, 0.231, 0.2274\rangle$
$\vec{\Xi}_b$	$\langle 0.2274, 0.25, 0.1762\rangle$
$\vec{\Xi}_c$	$\langle 0.1992, 0.2395, 0.1924\rangle$
$\vec{\Xi}_d$	$\langle 0.2501, 0.2626, 0.2414\rangle$
$\vec{\Xi}_e$	$\langle 0.2719, 0.2593, 0.218\rangle$

Analyze the positive and negative values of the following alternatives and determine the optimum value $\vec{\Xi}^P = \langle 1, 1, 0\rangle$ and $\vec{\Xi}^N = \langle 0, 0, 1\rangle$.
ED between each alternatives with both ideal values are $\mathscr{D}_1^P$=0.0381, $\mathscr{D}_2^P$=0.0396, $\mathscr{D}_3^P$=0.03, $\mathscr{D}_4^P$=0.0336, $\mathscr{D}_5^P$=0.043 and $\mathscr{D}_1^N$=0.2681, $\mathscr{D}_2^N$=0.2666, $\mathscr{D}_3^N$=0.2762, $\mathscr{D}_4^N$=0.2725, $\mathscr{D}_5^N$=0.2632.
The following formula is used to calculate relative closeness: $\mathscr{D}_1^\star$= 0.8755, $\mathscr{D}_2^\star$= 0.8706, $\mathscr{D}_3^\star$= 0.902, $\mathscr{D}_4^\star$= 0.8901, $\mathscr{D}_5^\star$= 0.8597.
The following are the alternatives ranked in order of preference: $\vec{\Xi}_c \succ \vec{\Xi}_d \succ \vec{\Xi}_a \succ \vec{\Xi}_b \succ \vec{\Xi}_e$. As a result, company-3 is the best option.

5.3 Sensitive analysis

If you change the values of $l_1, l_2, l_3 = (1, 1, 2)$ ED between each alternatives with both ideal values are $\mathscr{D}_1^P$=0.0804, $\mathscr{D}_2^P$=0.0898, $\mathscr{D}_3^P$=0.0785, $\mathscr{D}_4^P$=0.0785, $\mathscr{D}_5^P$=0.0887 and $\mathscr{D}_1^N$=0.2258, $\mathscr{D}_2^N$=0.2164, $\mathscr{D}_3^N$=0.2277, $\mathscr{D}_4^N$=0.2276, $\mathscr{D}_5^N$=0.2175.
The following formula is used to calculate relative closeness: $\mathscr{D}_1^\star$= 0.7374, $\mathscr{D}_2^\star$= 0.7068, $\mathscr{D}_3^\star$= 0.7435, $\mathscr{D}_4^\star$= 0.7435, $\mathscr{D}_5^\star$= 0.7103.
The following are the alternatives ranked in order of preference: $\vec{\Xi}_c \succ \vec{\Xi}_d \succ \vec{\Xi}_a \succ \vec{\Xi}_e \succ \vec{\Xi}_b$. As a result, company-3 is the best option.

If you change the values of $l_1, l_2, l_3 = (1, 2, 2)$, ED between each alternatives with both ideal values are $\mathscr{D}_1^P$=0.0877, $\mathscr{D}_2^P$=0.09, $\mathscr{D}_3^P$=0.0815, $\mathscr{D}_4^P$=0.0837, $\mathscr{D}_5^P$=0.0914 and $\mathscr{D}_1^N$=0.2184, $\mathscr{D}_2^N$=0.2162, $\mathscr{D}_3^N$=0.2247, $\mathscr{D}_4^N$=0.2225, $\mathscr{D}_5^N$=0.2148.
The following formula is used to calculate relative closeness: $\mathscr{D}_1^\star$= 0.7134, $\mathscr{D}_2^\star$= 0.706, $\mathscr{D}_3^\star$= 0.7339, $\mathscr{D}_4^\star$= 0.7266, $\mathscr{D}_5^\star$= 0.7015.
The following are the alternatives ranked in order of preference: $\vec{\Xi}_c \succ \vec{\Xi}_d \succ \vec{\Xi}_a \succ \vec{\Xi}_b \succ \vec{\Xi}_e$. As a result, company-3 is the best option.

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If you change the values of $l_1, l_2, l_3 = (2, 2, 2)$, ED between each alternatives with both ideal values are $\mathscr{D}_1^P$=0.0897, $\mathscr{D}_2^P$=0.0923, $\mathscr{D}_3^P$=0.0866, $\mathscr{D}_4^P$=0.086, $\mathscr{D}_5^P$=0.0924 and $\mathscr{D}_1^N$=0.2164, $\mathscr{D}_2^N$=0.2138, $\mathscr{D}_3^N$=0.2196, $\mathscr{D}_4^N$=0.2202, $\mathscr{D}_5^N$=0.2138.
The following formula is used to calculate relative closeness: $\mathscr{D}_1^\star$= 0.7069, $\mathscr{D}_2^\star$= 0.6984, $\mathscr{D}_3^\star$= 0.7172, $\mathscr{D}_4^\star$= 0.7192, $\mathscr{D}_5^\star$= 0.6983.
The following are the alternatives ranked in order of preference: $\overrightarrow{\Xi}_d \succ \overrightarrow{\Xi}_c \succ \overrightarrow{\Xi}_a \succ \overrightarrow{\Xi}_b \succ \overrightarrow{\Xi}_e$. As a result, company-4 is the best option. Therefore, if the change of values (l_1, l_2, l_3) is more efficient.

6 Conclusion

A number of methods have been developed for dealing with uncertain information such as FSs, IFSs, PFSs and NSs. AO rules have been proposed for new type NWA, new type NWG, new type GNWA, and new type GNWG. We discussed the score function based on new type of NS. Future discussions will cover the following topics:

1. An investigation of the new type NS of soft sets and expert sets.
2. Investigating Pythagorean cubic FSs and neutrosophic cubic FSs.
3. The problem of MADM can be solved using other decision-making methodologies based on square root cubic fuzzy sets.

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