q-rung square root interval-valued neutrosophic sets with respect to aggregated operators using multiple attribute decision making

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Abstract

This paper introduces the concept of multiple attribute decision making (MADM) using q-rung square root interval valued neutrosophic sets (q-rung SRIVNS). The interval valued neutrosophic set (IVNS) and the q-rung square root neutrosophic set (q-rung SRNS) deals with the q-rung SRIVNS. The purpose of this article is to provide an analysis of several aggregating operations. In this article, we discuss a novel idea for the q-rung square root interval valued neutrosophic weighted averaging (q-rung SRIVNWA), q-rung ortho square root interval valued neutrosophic weighted geometric (q-rung SRIVNWG), generalized q-rung SRIVN weighted averaging (q-rung GSRIVNWA) and generalized q-rung SRIVN weighted geometric (q-rung GSRIVNWG). Using Euclidean distances and Hamming distances is illustrated with examples. These sets will be subjected to various algebraic operations in this communication. By doing this, models will be more accurate and will be closed to an integer q. The four most important factors for courier services in India are reliability, turnaround time, payment options, and tracking capabilities. Expert judgments and criteria will determine the most appropriate options. Furthermore, several proposed and current models are compared to demonstrate their reliability and utility. A fascinating and intriguing conclusion can be drawn from the study.

Keywords: Aggregating operators; q-rung SRIVNWA; q-rung SRIVNWG; q-rung GSRIVNWA, q-rung GSRIVNWG.

1 Introduction

Due to the increasing complexity of real-world problems, decision-makers have difficulty identifying the optimal solution. Despite the difficulties of choosing between the alternatives, it is possible to select the best

https://doi.org/10.54216/IJNS.230314
Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

option. Creating opportunities, objectives, and viewpoint constraints is one of the biggest challenges facing firms. The result is that in decision-making (DM), individuals and groups must consider multiple objective simultaneously. It is a result of this that we have to learn how to deal with difficult situations in a more effective manner. In order to contribute to the development of this field of study, researchers have used a variety of methods in their research. Many uncertain theories have been proposed, including fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS)[3] and spherical FS (SFS)[4] and neutrosophic set (NSS) proposed by Smarandache.[5] An FS is a set of elements with levels of membership in a given set ranging from 0 to 1; these grades are referred to as an element membership value (MV) in the set. Later, Atanassov proposed the concept of an IFS that is divided into categories based on the non-membership value (NMV), which cannot exceed one.[2] When the combined grade value for MV and NMV is greater than one, we occasionally convey a single issue to the decision-making process. Yager[3] developed a new concept of PFS, a generalization of IFS, which is characterized by the square sum of its MV and NMV with a value of no more than one. These concepts are unable to capture a neutral state (neither favour nor disfavor). The concept of picture FS was developed by Cuong et al.[7] and it employs three pointers positive MV, neutral MV and negative MV with the sum of these three values not exceeding one. Finally, for a few applications, it offers more advantages than IFS and PFS. The generalized Pythagorean fuzzy aggregation operator (AO) and its applications were proposed by Liu et al.[8] The features of AOs[9] using Pythagorean interval values, on rare occasions convey a DM approach challenge, where the total of the values for the positive, neutral and negative MV is greater than 1. Ashraf et al.[4] presented the idea of SFSs, where the square sum of grade values for positive, neutral and negative memberships does not exceed 1. Fatmaa et al.[10] investigated the concept of SFSs by using the TOPSIS approach. Liu et al.[11] examined the topic of certain types of q-rung picture fuzzy AOs for DM in 2020.

Shamir et al.[12] applied the square root FS (SRFS) to the context of DM to study its weighted AOs. Smarandache[5] developed the NSS. FS and IFS differ in their neutrality, which is known as ”neutosophy.” This system includes truth degrees (TD), indeterminacy degrees (ID), and falsehood degrees (FD). A TD, ID, and FD consist of three components that lie between 0 and 1. Smarandache et al.[13] discussed Pythagorean NSIVs. In medical diagnostics as well as context analysis, a single NSS is applied.[14] The Hamming distance (HD), Euclidean distance (ED) and normalized Euclidean distance (NED) distances for IFSs were evaluated by Ejegwa[15] for MCDM and MADM problems. It was introduced by Palanikumar et al.[16] that Pythagorean NNIV with AOs could be solved by MADM. PNNIVSs are generalized PNSIVSs with the majority of distance functions. Several distance techniques have recently been applied to MADM challenges for ranking vague sets by Palanikumar.[17] Regression prediction for fuzzy time series is discussed in Xu et al.[18] A fuzzy c-number clustering procedure was introduced by Yang[19] for fuzzy data. Under MABAC and TOPSIS were proposed by Peng et al.[20] A generalization of PFS based on TOPSIS to MCDM was discussed by Zhang et al.[21] Hwang et al.[22] discuss a number of practical MADM applications. Many researchers[23]-[24] discussed the concept of Pythagorean with its extension based on DM.

1. based on q-rung SRIVS, we introduced the new type of ED and HD measures.
2. The new definition of q-rung SRIVFN with AO.
3. Using q-rung SRIVNWA, q-rung SRIVNWG, q-rung GSRIVNWA and q-rung GSRIVNWA, determine positive and negative ideal values.
4. An outcome is determined by the integer q.

An overview of the paper is provided in the following seven sections. An explanation of these concepts can be found in Section 2. In section 3, MADM using q-rung square root interval-valued neutrosophic number (q-rung SRIVNN) is discussed. According to the section 4, we use q-rung SRIVNN based on various distances between them. Section 5 of the reference list introduces a few AOs. Section 6 provides a numerical example, analysis, and algorithm for the q-rung SRIVNN, and also discusses the use of MADM in the implementation of q-rung SRIVNN. The conclusion can be found in the section 7.

2 Preliminaries

For our future studies, we will quickly review some fundamental terms in this section.

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

Definition 2.1. [3] Let Υ be the universe. The PFS $\Xi = \left\{\vartheta, \langle \xi_{\Xi}^{T}(\vartheta), \xi_{\Xi}^{F}(\vartheta)\rangle \middle| \vartheta \in \Upsilon\right\}$, where $\xi_{\Xi}^{T} : \Upsilon \overrightarrow{[} 0,1]$ and $\xi_{\Xi}^{F} : \Upsilon \overrightarrow{[} 0,1]$ are denotes the MD and NMD of $\vartheta \in \Upsilon$ to Ξ, respectively and $0 \leq (\xi_{\Xi}^{T}(\vartheta))^2 + (\xi_{\Xi}^{F}(\vartheta))^2 \leq 1$. For, $\Xi = \langle \xi_{\Xi}^{T}, \xi_{\Xi}^{F}\rangle$ is called a Pythagorean fuzzy number (PFN).

Definition 2.2. [12] The SRFS $\Xi = \left\{\vartheta, \langle \xi_{\Xi}^{T}(\vartheta), \xi_{\Xi}^{F}(\vartheta)\rangle \middle| \vartheta \in \Upsilon\right\}$, where $\xi_{\Xi}^{T} : \Upsilon \overrightarrow{[} 0,1]$ and $\xi_{\Xi}^{F} : \Upsilon \overrightarrow{[} 0,1]$ are denotes the MD and NMD of $\vartheta \in \Upsilon$ to Ξ, respectively and $0 \leq (\xi_{\Xi}^{T}(\vartheta))^2 + \sqrt{\xi_{\Xi}^{F}(\vartheta)} \leq 1$. For, $\Xi = \langle \xi_{\Xi}^{T}, \xi_{\Xi}^{F}\rangle$ is represent a square root fuzzy number (SRFN).

Definition 2.3. [9] The PIVFS $\overrightarrow{\Xi} = \left\{\vartheta, \left\langle \overrightarrow{\xi_{\Xi}^{T}}(\vartheta), \overrightarrow{\xi_{\Xi}^{F}}(\vartheta)\right\rangle \middle| \vartheta \in \Upsilon\right\}$, where $\overrightarrow{\xi_{\Xi}^{T}} : \Upsilon \overrightarrow{I} nt([0,1])$ and $\overrightarrow{\xi_{\Xi}^{F}} : \Upsilon \overrightarrow{I} nt([0,1])$ denotes the MD and NMD of $\vartheta \in \Upsilon$ to Ξ, respectively, and $0 \leq (\xi_{\Xi}^{TU}(\vartheta))^2 + (\xi_{\Xi}^{FU}(\vartheta))^2 \leq 1$. For, $\overrightarrow{\Xi} = \left\langle \left[\xi_{\Xi}^{TL}, \xi_{\Xi}^{TU}\right], \left[\xi_{\Xi}^{FL}, \xi_{\Xi}^{FU}\right] \right\rangle$ is called a Pythagorean interval-valued fuzzy number (PIVFN).

Definition 2.4. [9] Let $\overrightarrow{\Xi} = \left\langle [\xi^{TL}, \xi^{TU}], [\xi^{FL}, \xi^{FU}] \right\rangle$, $\overrightarrow{\Xi}_1 = \left\langle [\xi_1^{TL}, \xi_1^{TU}], [\xi_1^{FL}, \xi_1^{FU}] \right\rangle$ and $\overrightarrow{\Xi}_2 = \left\langle [\xi_2^{TL}, \xi_2^{TU}], [\xi_2^{FL}, \xi_2^{FU}] \right\rangle$ be the PIVFNs, and $q > 0$. Then,

1. $\overrightarrow{\Xi}_1 \sqcup \overrightarrow{\Xi}_2 = \begin{bmatrix} \left[\sqrt{(\xi_1^{TL})^2 + (\xi_2^{TL})^2 - (\xi_1^{TL})^2 \cdot (\xi_2^{TL})^2}, \sqrt{(\xi_1^{TU})^2 + (\xi_2^{TU})^2 - (\xi_1^{TU})^2 \cdot (\xi_2^{TU})^2}\right], \\ \left[\xi_1^{FL} \cdot \xi_2^{FL}, \xi_1^{FU} \cdot \xi_2^{FU}\right] \end{bmatrix}$,
2. $\overrightarrow{\Xi}_1 \sqcap \overrightarrow{\Xi}_2 = \begin{bmatrix} \left[\xi_1^{TL} \cdot \xi_2^{TL}, \xi_1^{TU} \cdot \xi_2^{TU}\right], \\ \left[\sqrt{(\xi_1^{FL})^2 + (\xi_2^{FL})^2 - (\xi_1^{FL})^2 \cdot (\xi_2^{FL})^2}, \sqrt{(\xi_1^{FU})^2 + (\xi_2^{FU})^2 - (\xi_1^{FU})^2 \cdot (\xi_2^{FU})^2}\right] \end{bmatrix}$,
3. $q \cdot \overrightarrow{\Xi} = \left[\left[\sqrt{1 - \left(1 - (\xi^{TL})^2\right)^q}, \sqrt{1 - \left(1 - (\xi^{TU})^2\right)^q}\right], \left[(\xi^{FL})^q, (\xi^{FU})^q\right]\right]$,
4. $\overrightarrow{\Xi}^q = \left[\left[(\xi^{TL})^q, (\xi^{TU})^q\right], \left[\sqrt{1 - \left(1 - (\xi^{FL})^2\right)^q}, \sqrt{1 - \left(1 - (\xi^{FU})^2\right)^q}\right]\right]$.

Definition 2.5. [9] For any PIVFN $\overrightarrow{\Xi} = \left\langle [\xi^{TL}, \xi^{TU}], [\xi^{FL}, \xi^{FU}] \right\rangle$ and score function $S(\overrightarrow{\Xi})$ is defined as $S(\overrightarrow{\Xi}) = \frac{1}{2}\left((\xi^{TL})^2 + (\xi^{TU})^2 - (\xi^{FL})^2 - (\xi^{FU})^2\right)$, $S(\overrightarrow{\Xi}) \in [-1,1]$, and the accuracy function $H(\overrightarrow{\Xi})$ is defined as $H(\overrightarrow{\Xi}) = \frac{1}{2}\left((\xi^{TL})^2 + (\xi^{TU})^2 + (\xi^{FL})^2 + (\xi^{FU})^2\right)$, $H(\overrightarrow{\Xi}) \in [0,1]$.

Definition 2.6. For any SRIVFN $\overrightarrow{\Xi} = \left\langle [\xi^{TL}, \xi^{TU}], [\xi^{FL}, \xi^{FU}] \right\rangle$ and score function $S(\overrightarrow{\Xi})$ is defined as $S(\overrightarrow{\Xi}) = \frac{1}{2}\left((\xi^{TL})^2 + (\xi^{TU})^2 - \sqrt{\xi^{FL}} - \sqrt{\xi^{FU}}\right)$, $S(\overrightarrow{\Xi}) \in [-1,1]$, and the accuracy function $H(\overrightarrow{\Xi})$ is defined as $H(\overrightarrow{\Xi}) = \frac{1}{2}\left((\xi^{TL})^2 + (\xi^{TU})^2 + \sqrt{\xi^{FL}} + \sqrt{\xi^{FU}}\right)$, $H(\overrightarrow{\Xi}) \in [0,1]$.

3 Basic operations for q-rung SRIVNN

This article provides the q-rung SRIVNNs.

Definition 3.1. For any SRIVNN $\overrightarrow{\Xi} = \left\langle \left[\xi_{\Xi}^{TL}, \xi_{\Xi}^{TU}\right], \left[\xi_{\Xi}^{IL}, \xi_{\Xi}^{IU}\right], \left[\xi_{\Xi}^{FL}, \xi_{\Xi}^{FU}\right] \right\rangle$, the score function $S(\overrightarrow{\Xi}) = \frac{1}{2}\left(\frac{(\xi^{TL})^2 + (\xi^{TU})^2}{2} + \frac{(\xi^{IL})^2 + (\xi^{IU})^2}{2} + 1 - \frac{\sqrt{\xi^{FL}} + \sqrt{\xi^{FU}}}{2}\right)$, where $S(\overrightarrow{\Xi}) \in [-1,1]$.

Definition 3.2. Let $\overrightarrow{\Xi} = \left\langle [\xi^{TL}, \xi^{TU}], [\xi^{IL}, \xi^{IU}], [\xi^{FL}, \xi^{FU}] \right\rangle$ is a q-rung SRIVNN. The TD and ID are defined as and $\left[\xi^{TL}, \xi^{TU}\right]$, $\left[\xi^{IL}, \xi^{IU}\right]$, $\left[\xi^{FL}, \xi^{FU}\right] \in ([0,1])$ and $0 \leq \left(\xi^{TU}(\vartheta)\right)^q + \left(\xi^{IU}(\vartheta)\right)^q + \sqrt[q]{\xi^{FU}(\vartheta)} \leq 1$.

Definition 3.3. Let $\overrightarrow{\Xi} = \left\langle [\xi^{TL}, \xi^{TU}], [\xi^{IL}, \xi^{IU}], [\xi^{FL}, \xi^{FU}] \right\rangle$, $\overrightarrow{\Xi}_1 = \left\langle [\xi_1^{TL}, \xi_1^{TU}], [\xi_1^{IL}, \xi_1^{IU}], [\xi_1^{FL}, \xi_1^{FU}] \right\rangle$ and $\overrightarrow{\Xi}_2 = \left\langle [\xi_2^{TL}, \xi_2^{TU}], [\xi_2^{IL}, \xi_2^{IU}], [\xi_2^{FL}, \xi_2^{FU}] \right\rangle$ be any three q-rung SRIVNNs, and $\Omega > 0$. Then,

https://doi.org/10.54216/IJNS.230314

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

1. $$\overrightarrow{\Xi}_1 \sqcup \overrightarrow{\Xi}_2 = \begin{bmatrix} \begin{bmatrix} \left(\sqrt[2q]{\xi_1^{TL}} + \sqrt[2q]{\xi_2^{TL}} - \sqrt[2q]{\xi_1^{TL}} \cdot \sqrt[2q]{\xi_2^{TL}}\right)^{2q}, \\ \left(\sqrt[2q]{\xi_1^{TU}} + \sqrt[2q]{\xi_2^{TU}} - \sqrt[2q]{\xi_1^{TU}} \cdot \sqrt[2q]{\xi_2^{TU}}\right)^{2q} \end{bmatrix}, \\ \begin{bmatrix} \left(\sqrt[q]{\xi_1^{IL}} + \sqrt[q]{\xi_2^{IL}} - \sqrt[q]{\xi_1^{IL}} \cdot \sqrt[q]{\xi_2^{IL}}\right)^{q}, \\ \left(\sqrt[q]{\xi_1^{IU}} + \sqrt[q]{\xi_2^{IU}} - \sqrt[q]{\xi_1^{IU}} \cdot \sqrt[q]{\xi_2^{IU}}\right)^{q} \end{bmatrix}, \\ \left[\xi_1^{FL} \cdot \xi_2^{FL}, \xi_1^{FU} \cdot \xi_2^{FU}\right] \end{bmatrix},$$

2. $$\overrightarrow{\Xi}_1 \sqcap \overrightarrow{\Xi}_2 = \begin{bmatrix} \left[\xi_1^{TL} \cdot \xi_2^{TL}, \xi_1^{TU} \cdot \xi_2^{TU}\right], \\ \begin{bmatrix} \left(\sqrt[q]{\xi_1^{IL}} + \sqrt[q]{\xi_2^{IL}} - \sqrt[q]{\xi_1^{IL}} \cdot \sqrt[q]{\xi_2^{IL}}\right)^{q}, \\ \left(\sqrt[q]{\xi_1^{IU}} + \sqrt[q]{\xi_2^{IU}} - \sqrt[q]{\xi_1^{IU}} \cdot \sqrt[q]{\xi_2^{IU}}\right)^{q} \end{bmatrix} \\ \begin{bmatrix} \left(\sqrt[2q]{\xi_1^{FL}} + \sqrt[2q]{\xi_2^{FL}} - \sqrt[2q]{\xi_1^{FL}} \cdot \sqrt[2q]{\xi_2^{FL}}\right)^{2q}, \\ \left(\sqrt[2q]{\xi_1^{FU}} + \sqrt[2q]{\xi_2^{FU}} - \sqrt[2q]{\xi_1^{FU}} \cdot \sqrt[2q]{\xi_2^{FU}}\right)^{2q} \end{bmatrix} \end{bmatrix},$$

3. $$\Omega \cdot \overrightarrow{\Xi} = \begin{bmatrix} \left[\left(1-\left(1-\sqrt[2q]{\xi^{TL}}\right)^{\Omega}\right)^{2q}, \left(1-\left(1-\sqrt[2q]{\xi^{TU}}\right)^{\Omega}\right)^{2q}\right], \\ \left[\left(1-\left(1-\sqrt[q]{\xi^{IL}}\right)^{\Omega}\right)^{q}, \left(1-\left(1-\sqrt[q]{\xi^{IU}}\right)^{\Omega}\right)^{q}\right], \\ \left[(\xi^{FL})^{\Omega}, (\xi^{FU})^{\Omega}\right] \end{bmatrix},$$

4. $$\overrightarrow{\Xi}^{\Omega} = \begin{bmatrix} \left[(\xi^{TL})^{\Omega}, (\xi^{TU})^{\Omega}\right], \\ \left[\left(1-\left(1-\sqrt[q]{\xi^{IL}}\right)^{\Omega}\right)^{q}, \left(1-\left(1-\sqrt[q]{\xi^{IU}}\right)^{\Omega}\right)^{q}\right] \\ \left[\left(1-\left(1-\sqrt[2q]{\xi^{FL}}\right)^{\Omega}\right)^{2q}, \left(1-\left(1-\sqrt[2q]{\xi^{FU}}\right)^{\Omega}\right)^{2q}\right] \end{bmatrix}.$$

4 Various distance for q-rung SRIVNN

ED and HD measures are presented along with some mathematical features of q-rung SRIVNNs and their relationship to the ED and HD measures.

Definition 4.1. For q-rung SRIVNNs $\overrightarrow{\Xi}_1 = \left\langle [\xi_1^{TL}, \xi_1^{TU}], [\xi_1^{IL}, \xi_1^{IU}], [\xi_1^{FL}, \xi_1^{FU}], \right\rangle$ and $\overrightarrow{\Xi}_2 = \left\langle [\xi_2^{TL}, \xi_2^{TU}], [\xi_2^{IL}, \xi_2^{IU}], [\xi_2^{FL}, \xi_2^{FU}] \right\rangle$. Then

$$\mathscr{D}_E\left(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2\right) = \frac{1}{2}\sqrt{\begin{matrix} \left[\begin{matrix} \frac{1+(\xi_1^{TL})^2+(\xi_1^{IL})^2-\sqrt{\xi_1^{FL}}+1+(\xi_1^{TU})^2+(\xi_1^{IU})^2-\sqrt{\xi_1^{FU}}}{4} \\ -\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4} \end{matrix}\right]^2 \\ +\frac{1}{2}\left[\begin{matrix} \frac{1+(\xi_1^{TL})^2+(\xi_1^{IL})^2-\sqrt{\xi_1^{FL}}+1+(\xi_1^{TU})^2+(\xi_1^{IU})^2-\sqrt{\xi_1^{FU}}}{4} \\ -\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4} \end{matrix}\right]^2 \end{matrix}}$$

where $\mathscr{D}_E\left(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2\right)$ is denote the ED between $\overrightarrow{\Xi}_1$ and $\overrightarrow{\Xi}_2$.

Also, the HD $\mathscr{D}_H\left(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2\right)$=

$$\frac{1}{2}\begin{bmatrix} \left|\begin{matrix} \frac{1+(\xi_1^{TL})^2+(\xi_1^{IL})^2-\sqrt{\xi_1^{FL}}+1+(\xi_1^{TU})^2+(\xi_1^{IU})^2-\sqrt{\xi_1^{FU}}}{4} \\ -\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4} \end{matrix}\right| \\ +\frac{1}{2}\left|\begin{matrix} \frac{1+(\xi_1^{TL})^2+(\xi_1^{IL})^2-\sqrt{\xi_1^{FL}}+1+(\xi_1^{TU})^2+(\xi_1^{IU})^2-\sqrt{\xi_1^{FU}}}{4} \\ -\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4} \end{matrix}\right| \end{bmatrix}$$

https://doi.org/10.54216/IJNS.230314
Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

where $\mathscr{D}_H\left(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2\right)$ is denote the HD between $\overrightarrow{\Xi}_1$ and $\overrightarrow{\Xi}_2$.

Theorem 4.2. *If any three q-rung SRIVNNs* $\overrightarrow{\Xi}_1 = \left\langle [\xi_1^{TL}, \xi_1^{TU}], [\xi_1^{IL}, \xi_1^{IU}], [\xi_1^{FL}, \xi_1^{FU}] \right\rangle$, $\overrightarrow{\Xi}_2 = \left\langle [\xi_2^{TL}, \xi_2^{TU}], [\xi_2^{IL}, \xi_2^{IU}], [\xi_2^{FL}, \xi_2^{FU}] \right\rangle$, $\overrightarrow{\Xi_3} = \left\langle [\xi_3^{TL}, \xi_3^{TU}], [\xi_3^{IL}, \xi_3^{IU}], [\xi_3^{FL}, \xi_3^{FU}] \right\rangle$, *then* $\mathscr{D}_E(\Xi_1, \Xi_2)$. *Then*

1. $\mathscr{D}_E(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2) = 0$ *iff* $\overrightarrow{\Xi}_1 = \overrightarrow{\Xi}_2$.
2. $\mathscr{D}_E(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2) = \mathscr{D}_E(\overrightarrow{\Xi}_2, \overrightarrow{\Xi}_1)$.
3. $\mathscr{D}_E(\overrightarrow{\Xi}_1, \overrightarrow{\Xi_3}) \leq \mathscr{D}_E(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2) + \mathscr{D}_E(\overrightarrow{\Xi}_2, \overrightarrow{\Xi_3})$.

Proof. Now, $\left(\mathscr{D}_E(\Xi_1, \Xi_2 + \mathscr{D}_E(\Xi_2, \Xi_3)\right)^2 =$

$$\left[\begin{array}{l} \frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\xi_1^{TL})^2+(\xi_1^{IL})^2-\sqrt{\xi_1^{FL}}+1+(\xi_1^{TU})^2+(\xi_1^{IU})^2-\sqrt{\xi_1^{FU}}}{4}\\ -\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4}\end{array}\right]^2\\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\xi_1^{TL})^2+(\xi_1^{IL})^2-\sqrt{\xi_1^{FL}}+1+(\xi_1^{TU})^2+(\xi_1^{IU})^2-\sqrt{\xi_1^{FU}}}{4}\\ -\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4}\end{array}\right]^2\end{array}} \\ +\frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4}\\ -\frac{1+(\xi_3^{TL})^2+(\xi_3^{IL})^2-\sqrt{\xi_3^{FL}}+1+(\xi_3^{TU})^2+(\xi_3^{IU})^2-\sqrt{\xi_3^{FU}}}{4}\end{array}\right]^2\\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4}\\ -\frac{1+(\xi_3^{TL})^2+(\xi_3^{IL})^2-\sqrt{\xi_3^{FL}}+1+(\xi_3^{TU})^2+(\xi_3^{IU})^2-\sqrt{\xi_3^{FU}}}{4}\end{array}\right]^2\end{array}} \end{array}\right]^2$$

implies

$$\begin{aligned}&\frac{1}{4}\Big((\Gamma_a-\Gamma_b)^2+\frac{1}{2}(\Gamma_a-\Gamma_b)^2\Big)+\frac{1}{4}\Big((\Gamma_b-\Gamma_c)^2+\frac{1}{2}(\Gamma_b-\Gamma_c)^2\Big)\\ &+\frac{1}{2}\left(\sqrt{(\Gamma_a-\Gamma_b)^2+\frac{1}{2}(\Gamma_a-\Gamma_b)^2}\times\sqrt{(\Gamma_b-\Gamma_c)^2+\frac{1}{2}(\Gamma_b-\Gamma_c)^2}\right),\end{aligned}$$

Since,

$$\Gamma_a = \frac{1+(\xi_1^{TL})^2+(\xi_1^{IL})^2-\sqrt{\xi_1^{FL}}+1+(\xi_1^{TU})^2+(\xi_1^{IU})^2-\sqrt{\xi_1^{FU}}}{4},$$

$$\Gamma_b = \frac{1+(\xi_2^{TL})^2+(\xi_2^{IL})^2-\sqrt{\xi_2^{FL}}+1+(\xi_2^{TU})^2+(\xi_2^{IU})^2-\sqrt{\xi_2^{FU}}}{4},$$

a

$$\Gamma_c = \frac{1+(\xi_3^{TL})^2+(\xi_3^{IL})^2-\sqrt{\xi_3^{FL}}+1+(\xi_3^{TU})^2+(\xi_3^{IU})^2-\sqrt{\xi_3^{FU}}}{4}.$$

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

Hence, $\Big(\mathscr{D}_E(\Xi_1,\Xi_2+\mathscr{D}_E(\Xi_2,\Xi_3)\Big)^2$

$$\begin{aligned}
&\geq \frac{1}{4}\Big((\Gamma_a-\Gamma_b)^2+\frac{1}{2}(\Gamma_a-\Gamma_b)^2\Big)+\frac{1}{4}\Big((\Gamma_b-\Gamma_c)^2+\frac{1}{2}(\Gamma_b-\Gamma_c)^2\Big)\\
&\quad+\frac{1}{2}\Big((\Gamma_a-\Gamma_b)\times(\Gamma_b-\Gamma_c)+\frac{1}{2}(\Gamma_a-\Gamma_b)\times(\Gamma_b-\Gamma_c)\Big)\\
&= \frac{1}{4}\Big((\Gamma_a-\Gamma_b)^2+(\Gamma_b-\Gamma_c)^2+2(\Gamma_a-\Gamma_b)\times(\Gamma_b-\Gamma_c)\Big)\\
&\quad+\frac{1}{4}\Big(\frac{1}{2}(\Gamma_a-\Gamma_b)^2+\frac{1}{2}(\Gamma_b-\Gamma_c)^2+(\Gamma_a-\Gamma_b)\times(\Gamma_b-\Gamma_c)\Big)\\
&= \frac{1}{4}(\Gamma_a-\Gamma_b+\Gamma_b-\Gamma_c)^2+\frac{1}{8}(\Gamma_a-\Gamma_b+\Gamma_b-\Gamma_c)^2\\
&= \frac{1}{4}(\Gamma_a-\Gamma_c)^2+\frac{1}{8}(\Gamma_a-\Gamma_c)^2\\
&= \frac{1}{4}\Big[(\Gamma_a-\Gamma_c)^2+\frac{1}{2}(\Gamma_a-\Gamma_c)^2\Big]\\
&= \mathscr{D}_E(\Xi_1,\Xi_3)^2.
\end{aligned}$$

Corollary 4.3. *If any three q-rung SRIVNNs* $\overrightarrow{\Xi}_1=\Big\langle[\xi_1^{TL},\xi_1^{TU}],[\xi_1^{IL},\xi_1^{IU}],[\xi_1^{FL},\xi_1^{FU}]\Big\rangle$, $\overrightarrow{\Xi}_2=\Big\langle[\xi_2^{TL},\xi_2^{TU}],[\xi_2^{IL},\xi_2^{IU}],[\xi_2^{FL},\xi_2^{FU}]\Big\rangle$, $\overrightarrow{\Xi}_3=\Big\langle[\xi_3^{TL},\xi_3^{TU}],[\xi_3^{IL},\xi_3^{IU}],[\xi_3^{FL},\xi_3^{FU}]\Big\rangle$. *Then*

1. $\mathscr{D}_H(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2)=0$ *if and only if* $\overrightarrow{\Xi}_1=\overrightarrow{\Xi}_2$.
2. $\mathscr{D}_H(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2)$ *and* $\mathscr{D}_H(\overrightarrow{\Xi}_2,\overrightarrow{\Xi}_1)$ *are co-occur.*
3. $\mathscr{D}_H(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_3)\leq\mathscr{D}_H(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2)+\mathscr{D}_H(\overrightarrow{\Xi}_2,\overrightarrow{\Xi}_3)$.

5 Several different aggregation operators are available for q-rung SRIVNN

As a result of the work we have done, we are introducing the new operators for the q-rung SRIVNWA, q-rung SRIVNWG, q-rung GSRIVNWA, and q-rung GSRIVNWG.

5.1 q-rung SRIV weighted averaging(q-rung SRIVNWA) operator

Definition 5.1. Let $\overrightarrow{\Xi}_i=\Big\langle[\xi_i^{TL},\xi_i^{TU}],[\xi_i^{IL},\xi_i^{IU}],[\xi_i^{FL},\xi_i^{FU}]\Big\rangle$ be the q-rung SRIVNNs, $W=(\vartheta_1,\vartheta_2,...,\vartheta_n)$ be the weight of $\overrightarrow{\Xi}_i$, $\vartheta_i\geq 0$ and $\sqcup_{i=1}^{n}\vartheta_i=1$. Then q-rung SRIVNWA $(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_n)=\sqcup_{i=1}^{n}\vartheta_i\overrightarrow{\Xi}_i$.

Theorem 5.2. *Let* $\overrightarrow{\Xi}_i=\Big\langle[\xi_i^{TL},\xi_i^{TU}],[\xi_i^{IL},\xi_i^{IU}],[\xi_i^{FL},\xi_i^{FU}]\Big\rangle$ *be the q-rung SRIVNNs. Then* $q-rungSRIVNWA(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_n)$=

$$\left[\begin{array}{c}\left[\Big(1-\oslash_{i=1}^{n}\Big(1-\sqrt[2q]{(\xi_i^{TL})}\Big)^{\vartheta_i}\Big)^{2q},\Big(1-\oslash_{i=1}^{n}\Big(1-\sqrt[2q]{(\xi_i^{TU})}\Big)^{\vartheta_i}\Big)^{2q}\right],\\ \left[\Big(1-\oslash_{i=1}^{n}\Big(1-\sqrt[q]{(\xi_i^{IL})}\Big)^{\vartheta_i}\Big)^{q},\Big(1-\oslash_{i=1}^{n}\Big(1-\sqrt[q]{(\xi_i^{IU})}\Big)^{\vartheta_i}\Big)^{q}\right],\\ \left[\oslash_{i=1}^{n}(\xi_i^{FL})^{\vartheta_i},\oslash_{i=1}^{n}(\xi_i^{FU})^{\vartheta_i}\right]\end{array}\right].$$

Proof. It was based on mathematical induction that we were able to prove the hypothesis.
Put $n=2$, q-rung SRIVNWA$(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2)=\vartheta_1\overrightarrow{\Xi}_1\sqcup\vartheta_2\overrightarrow{\Xi}_2$, where

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

$$\vartheta_1\overrightarrow{\Xi}_1=\begin{bmatrix}\left[\left(1-\left(1-\sqrt[2q]{(\xi_1^{TL})}\right)^{\vartheta_1}\right)^{2q},\left(1-\left(1-\sqrt[2q]{(\xi_1^{TU})}\right)^{\vartheta_1}\right)^{2q}\right],\\ \left[\left(1-\left(1-\sqrt[q]{(\xi_1^{IL})}\right)^{\vartheta_1}\right)^{q},\left(1-\left(1-\sqrt[q]{(\xi_1^{IU})}\right)^{\vartheta_1}\right)^{q}\right],\\ \left[(\xi_1^{FL})^{\vartheta_1},(\xi_1^{FU})^{\vartheta_1}\right]\end{bmatrix}$$

$$\vartheta_2\overrightarrow{\Xi}_2=\begin{bmatrix}\left[\left(1-\left(1-\sqrt[2q]{(\xi_2^{TL})}\right)^{\vartheta_2}\right)^{2q},\left(1-\left(1-\sqrt[2q]{(\xi_2^{TU})}\right)^{\vartheta_2}\right)^{2q}\right],\\ \left[\left(1-\left(1-\sqrt[q]{(\xi_2^{IL})}\right)^{\vartheta_2}\right)^{q},\left(1-\left(1-\sqrt[q]{(\xi_2^{IU})}\right)^{\vartheta_2}\right)^{q}\right],\\ \left[(\xi_2^{FL})^{\vartheta_2},(\xi_2^{FU})^{\vartheta_2}\right]\end{bmatrix}.$$

Now,

$$\vartheta_1\overrightarrow{\Xi}_1\sqcup\vartheta_2\overrightarrow{\Xi}_2=\begin{bmatrix}\begin{bmatrix}\begin{pmatrix}\left(1-\left(1-\sqrt[2q]{(\xi_1^{TL})}\right)^{\vartheta_1}\right)+\left(1-\left(1-\sqrt[2q]{(\xi_2^{TL})}\right)^{\vartheta_2}\right)\\-\left(1-\left(1-\sqrt[2q]{(\xi_1^{TL})}\right)^{\vartheta_1}\right)\cdot\left(1-\left(1-\sqrt[2q]{(\xi_2^{TL})}\right)^{\vartheta_2}\right)\end{pmatrix}^{2q},\\ \begin{pmatrix}\left(1-\left(1-\sqrt[2q]{(\xi_1^{TU})}\right)^{\vartheta_1}\right)+\left(1-\left(1-\sqrt[2q]{(\xi_2^{TU})}\right)^{\vartheta_2}\right)\\-\left(1-\left(1-\sqrt[2q]{(\xi_1^{TU})}\right)^{\vartheta_1}\right)\cdot\left(1-\left(1-\sqrt[2q]{(\xi_2^{TU})}\right)^{\vartheta_2}\right)\end{pmatrix}^{2q}\end{bmatrix},\\ \begin{bmatrix}\begin{pmatrix}\left(1-\left(1-\sqrt[q]{(\xi_1^{IL})}\right)^{\vartheta_1}\right)+\left(1-\left(1-\sqrt[q]{(\xi_2^{IL})}\right)^{\vartheta_2}\right)\\-\left(1-\left(1-\sqrt[q]{(\xi_1^{IL})}\right)^{\vartheta_1}\right)\cdot\left(1-\left(1-\sqrt[q]{(\xi_2^{IL})}\right)^{\vartheta_2}\right)\end{pmatrix}^{q},\\ \begin{pmatrix}\left(1-\left(1-\sqrt[q]{(\xi_1^{IU})}\right)^{\vartheta_1}\right)+\left(1-\left(1-\sqrt[q]{(\xi_2^{IU})}\right)^{\vartheta_2}\right)\\-\left(1-\left(1-\sqrt[q]{(\xi_1^{IU})}\right)^{\vartheta_1}\right)\cdot\left(1-\left(1-\sqrt[q]{(\xi_2^{IU})}\right)^{\vartheta_2}\right)\end{pmatrix}^{q}\end{bmatrix},\\ \left[(\xi_1^{FL})^{\vartheta_1}(\xi_2^{FL})^{\vartheta_2},(\xi_1^{FU})^{\vartheta_1}(\xi_2^{FU})^{\vartheta_2}\right]\end{bmatrix}$$

$$=\begin{bmatrix}\begin{bmatrix}\left(1-\left(1-\sqrt[2q]{(\xi_1^{TL})}\right)^{\vartheta_1}\cdot\left(1-\sqrt[2q]{(\xi_2^{TL})}\right)^{\vartheta_2}\right)^{2q},\\ \left(1-\left(1-\sqrt[2q]{(\xi_1^{TU})}\right)^{\vartheta_1}\cdot\left(1-\sqrt[2q]{(\xi_2^{TU})}\right)^{\vartheta_2}\right)^{2q}\end{bmatrix},\\ \begin{bmatrix}\left(1-\left(1-\sqrt[q]{(\xi_1^{IL})}\right)^{\vartheta_1}\cdot\left(1-\sqrt[q]{(\xi_2^{IL})}\right)^{\vartheta_2}\right)^{q},\\ \left(1-\left(1-\sqrt[q]{(\xi_1^{IU})}\right)^{\vartheta_1}\cdot\left(1-\sqrt[q]{(\xi_2^{IU})}\right)^{\vartheta_2}\right)^{q}\end{bmatrix},\\ \left[(\xi_1^{FL})^{\vartheta_1}\cdot(\xi_2^{FL})^{\vartheta_2},(\xi_1^{FU})^{\vartheta_1}\cdot(\xi_2^{FU})^{\vartheta_2}\right]\end{bmatrix}$$

$$q{-}rungSRIVNWA(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2)=\begin{bmatrix}\left[\left(1-\oslash_{i=1}^{2}\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\vartheta_i}\right)^{2q},\left(1-\oslash_{i=1}^{2}\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\vartheta_i}\right)^{2q}\right],\\ \left[\left(1-\oslash_{i=1}^{2}\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\vartheta_i}\right)^{q},\left(1-\oslash_{i=1}^{2}\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\vartheta_i}\right)^{q}\right],\\ \left[\oslash_{i=1}^{2}(\xi_i^{FL})^{\vartheta_i},\oslash_{i=1}^{2}(\xi_i^{FU})^{\vartheta_i}\right]\end{bmatrix}.$$

It valid for $n\geq 3$, hence $q-rungSRIVNWA(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_l)=$

$$\begin{bmatrix}\left[\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\vartheta_i}\right)^{2q},\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\vartheta_i}\right)^{2q}\right],\\ \left[\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\vartheta_i}\right)^{q},\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\vartheta_i}\right)^{q}\right],\\ \left[\oslash_{i=1}^{l}(\xi_i^{FL})^{\vartheta_i},\oslash_{i=1}^{l}(\xi_i^{FU})^{\vartheta_i}\right]\end{bmatrix}.$$

https://doi.org/10.54216/IJNS.230314

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

If $n = l + 1$, then q-rung SRIVNWA $(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_l, \overrightarrow{\Xi}_{l+1})$

$$= \begin{bmatrix} \begin{bmatrix} \begin{pmatrix} \sqcup_{i=1}^{l}\left(1-\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\vartheta_i}\right)+\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TL})}\right)^{\vartheta_{l+1}}\right) \\ -\oslash_{i=1}^{l}\left(1-\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\vartheta_i}\right)\cdot\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TL})}\right)^{\vartheta_{l+1}}\right) \end{pmatrix}^{2q}, \\ \begin{pmatrix} \sqcup_{i=1}^{l}\left(1-\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\vartheta_i}\right)+\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TU})}\right)^{\vartheta_{l+1}}\right) \\ -\oslash_{i=1}^{l}\left(1-\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\vartheta_i}\right)\cdot\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TU})}\right)^{\vartheta_{l+1}}\right) \end{pmatrix}^{2q} \end{bmatrix}, \\ \begin{bmatrix} \begin{pmatrix} \sqcup_{i=1}^{l}\left(1-\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\vartheta_i}\right)+\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IL})}\right)^{\vartheta_{l+1}}\right) \\ -\oslash_{i=1}^{l}\left(1-\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\vartheta_i}\right)\cdot\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IL})}\right)^{\vartheta_{l+1}}\right) \end{pmatrix}^{q}, \\ \begin{pmatrix} \sqcup_{i=1}^{l}\left(1-\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\vartheta_i}\right)+\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IU})}\right)^{\vartheta_{l+1}}\right) \\ -\oslash_{i=1}^{l}\left(1-\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\vartheta_i}\right)\cdot\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IU})}\right)^{\vartheta_{l+1}}\right) \end{pmatrix}^{q} \end{bmatrix}, \\ \left[\oslash_{i=1}^{l}(\xi_i^{FL})^{\vartheta_i}\cdot(\xi_{l+1}^{FL})^{\vartheta_{l+1}}, \oslash_{i=1}^{l}(\xi_i^{FU})^{\vartheta_i}\cdot(\xi_{l+1}^{FU})^{\vartheta_{l+1}}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left[\left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\vartheta_i}\right)^{2q}, \left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\vartheta_i}\right)^{2q}\right], \\ \left[\left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\vartheta_i}\right)^{q}, \left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\vartheta_i}\right)^{q}\right], \\ \left[\oslash_{i=1}^{l+1}(\xi_i^{FL})^{\vartheta_i}, \oslash_{i=1}^{l+1}(\xi_i^{FU})^{\vartheta_i}\right] \end{bmatrix}.$$

Theorem 5.3. *If all* $\overrightarrow{\Xi}_i = \left\langle[\xi_i^{TL}, \xi_i^{TU}], [\xi_i^{IL}, \xi_i^{IU}], [\xi_i^{FL}, \xi_i^{FU}]\right\rangle$ *are equal, then q-rung SRIVNWA*$(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n) = \overrightarrow{\Xi}$ *(idempotency property).*

Proof. Given that $[\xi_i^{TL}, \xi_i^{TU}] = [\xi^{TL}, \xi^{TU}]$, $[\xi_i^{IL}, \xi_i^{IU}] = [\xi^{IL}, \xi^{IU}]$ and $[\xi_i^{FL}, \xi_i^{FU}] = [\xi^{FL}, \xi^{FU}]$ and $\sqcup_{i=1}^{n}\vartheta_i = 1$. Now, $q-rungSRIVNWA(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n)$

$$= \begin{bmatrix} \left[\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\vartheta_i}\right)^{2q}, \left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\vartheta_i}\right)^{2q}\right], \\ \left[\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\vartheta_i}\right)^{q}, \left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\vartheta_i}\right)^{q}\right], \\ \left[\oslash_{i=1}^{n}(\xi_i^{FL})^{\vartheta_i}, \oslash_{i=1}^{n}(\xi_i^{FU})^{\vartheta_i}\right] \end{bmatrix},$$

$$= \begin{bmatrix} \left[\left(1-\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\sqcup_{i=1}^{n}\vartheta_i}\right)^{2q}, \left(1-\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\sqcup_{i=1}^{n}\vartheta_i}\right)^{2q}\right], \\ \left[\left(1-\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\sqcup_{i=1}^{n}\vartheta_i}\right)^{q}, \left(1-\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\sqcup_{i=1}^{n}\vartheta_i}\right)^{q}\right], \\ \left[(\xi_i^{FL})^{\sqcup_{i=1}^{n}\vartheta_i}, (\xi_i^{FU})^{\sqcup_{i=1}^{n}\vartheta_i}\right] \end{bmatrix},$$

$$= \begin{bmatrix} \left[\left(1-\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)\right)^{2q}, \left(1-\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)\right)^{2q}\right], \\ \left[\left(1-\left(1-\sqrt[q]{(\xi_i^{IL})}\right)\right)^{q}, \left(1-\left(1-\sqrt[q]{(\xi_i^{IU})}\right)\right)^{q}\right], \\ \left[(\xi_i^{FL}), (\xi_i^{FU})\right] \end{bmatrix},$$

$$= \overrightarrow{\Xi}.$$

Theorem 5.4. *Let* $\overrightarrow{\Xi}_i = \left\langle[\xi_{ij}^{TL}, \xi_{ij}^{TU}], [\xi_{ij}^{IL}, \xi_{ij}^{IU}], [\xi_{ij}^{FL}, \xi_{ij}^{FU}]\right\rangle$ *be the q-rung SRIVNWA, where*
$\overrightarrow{\xi^{TL}} = \inf \xi_{ij}^{TL}$, $\underline{\xi^{TL}} = \sup \xi_{ij}^{TL}$, $\overrightarrow{\xi^{TU}} = \inf \xi_{ij}^{TU}$, $\underline{\xi^{TU}} = \sup \xi_{ij}^{TU}$,

https://doi.org/10.54216/IJNS.230314

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

$\overrightarrow{\xi^{IL}} = \inf \xi_{ij}^{IL}, \underline{\xi^{IL}} = \sup \xi_{ij}^{IL}, \overrightarrow{\xi^{IU}} = \inf \xi_{ij}^{IU}, \underline{\xi^{IU}} = \sup \xi_{ij}^{IU},$
$\overrightarrow{\xi^{FL}} = \inf \xi_{ij}^{FL}, \underline{\xi^{FL}} = \sup \xi_{ij}^{FL}, \overrightarrow{\xi^{FU}} = \inf \xi_{ij}^{FU}, \underline{\xi^{FU}} = \sup \xi_{ij}^{FU}.$
Then, $\left\langle [\overrightarrow{\xi^{TL}}, \overrightarrow{\xi^{TU}}], [\overrightarrow{\xi^{IL}}, \overrightarrow{\xi^{IU}}], [\underline{\xi^{FL}}, \underline{\xi^{FU}}] \right\rangle \leq q-rungSRIVNWA(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n)$
$\leq \left\langle [\underline{\xi^{TL}}, \underline{\xi^{TU}}], [\underline{\xi^{IL}}, \underline{\xi^{IU}}], [\overrightarrow{\xi^{FL}}, \overrightarrow{\xi^{FU}}] \right\rangle$*, where* $1 \leq i \leq n,\ j = 1, 2, ..., i_j$ *(boundedness property).*

Proof. Since, $\overrightarrow{\xi^{TL}} = \inf \xi_{ij}^{TL}, \underline{\xi^{TL}} = \sup \xi_{ij}^{TL}$ $\overrightarrow{\xi^{TU}} = \inf \xi_{ij}^{TU}, \underline{\xi^{TU}} = \sup \xi_{ij}^{TU}$ and $\overrightarrow{\xi^{TL}} \leq \xi_{ij}^{TL} \leq \underline{\xi^{TL}}$ and $\overrightarrow{\xi^{TU}} \leq \xi_{ij}^{TU} \leq \underline{\xi^{TU}}$. Now,

$$\begin{aligned}
\overrightarrow{\xi^{TL}} + \overrightarrow{\xi^{TU}} &= \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[2q]{(\overrightarrow{\xi^{TL}})}\right)^{\vartheta_i}\right)^{2q} + \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[2q]{(\overrightarrow{\xi^{TU}})}\right)^{\vartheta_i}\right)^{2q} \\
&\leq \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[2q]{(\xi_{ij}^{TL})}\right)^{\vartheta_i}\right)^{2q} + \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[2q]{(\xi_{ij}^{TU})}\right)^{\vartheta_i}\right)^{2q} \\
&\leq \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[2q]{\underline{(\xi^{TL})}}\right)^{\vartheta_i}\right)^{2q} + \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[2q]{\underline{(\xi^{TU})}}\right)^{\vartheta_i}\right)^{2q} \\
&= \underline{\xi^{TL}} + \underline{\xi^{TU}}.
\end{aligned}$$

Since, $\overrightarrow{\xi^{IL}} = \inf \xi_{ij}^{IL}, \underline{\xi^{IL}} = \sup \xi_{ij}^{IL}$ $\overrightarrow{\xi^{IU}} = \inf \xi_{ij}^{IU}, \underline{\xi^{IU}} = \sup \xi_{ij}^{IU}$ and $\overrightarrow{\xi^{IL}} \leq \xi_{ij}^{IL} \leq \underline{\xi^{IL}}$ and $\overrightarrow{\xi^{IU}} \leq \xi_{ij}^{IU} \leq \underline{\xi^{IU}}$. Now,

$$\begin{aligned}
\overrightarrow{\xi^{IL}} + \overrightarrow{\xi^{IU}} &= \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[q]{(\overrightarrow{\xi^{IL}})}\right)^{\vartheta_i}\right)^{q} + \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[q]{(\overrightarrow{\xi^{IU}})}\right)^{\vartheta_i}\right)^{q} \\
&\leq \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[q]{(\xi_{ij}^{IL})}\right)^{\vartheta_i}\right)^{q} + \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[q]{(\xi_{ij}^{IU})}\right)^{\vartheta_i}\right)^{q} \\
&\leq \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[q]{\underline{(\xi^{IL})}}\right)^{\vartheta_i}\right)^{q} + \left(1 - \oslash_{i=1}^{n}\left(1 - \sqrt[q]{\underline{(\xi^{IU})}}\right)^{\vartheta_i}\right)^{q} \\
&= \underline{\xi^{IL}} + \underline{\xi^{IU}}.
\end{aligned}$$

Since, $\overrightarrow{\xi^{FL}} = \inf \xi_{ij}^{FL}, \underline{\xi^{FL}} = \sup \xi_{ij}^{FL}$ $\overrightarrow{\xi^{FU}} = \inf \xi_{ij}^{FU}, \underline{\xi^{FU}} = \sup \xi_{ij}^{FU}$ and $\overrightarrow{\xi^{FL}} \leq \xi_{ij}^{FL} \leq \underline{\xi^{FL}}$ and $\overrightarrow{\xi^{FU}} \leq \xi_{ij}^{FU} \leq \underline{\xi^{FU}}$. Now,

$$\begin{aligned}
\overrightarrow{\xi^{FL}} + \overrightarrow{\xi^{FU}} &= \oslash_{i=1}^{n}(\overrightarrow{\xi^{FL}})^{\vartheta_i} + \oslash_{i=1}^{n}(\overrightarrow{\xi^{FU}})^{\vartheta_i} \\
&\leq \oslash_{i=1}^{n}(\xi_{ij}^{FL})^{\vartheta_i} + \oslash_{i=1}^{n}(\xi_{ij}^{FU})^{\vartheta_i} \\
&\leq \oslash_{i=1}^{n}(\underline{\xi^{FL}})^{\vartheta_i} + \oslash_{i=1}^{n}(\underline{\xi^{FU}})^{\vartheta_i} \\
&= \underline{\xi^{FL}} + \underline{\xi^{FU}}.
\end{aligned}$$

https://doi.org/10.54216/IJNS.230314

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

Hence,

$$\frac{1}{2}\times\left[\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\overrightarrow{(\xi^{TL})}}\right)^{\vartheta_i}\right)^{2q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\overrightarrow{(\xi^{TU})}}\right)^{\vartheta_i}\right)^{2q}}{\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{\overrightarrow{(\xi^{IL})}}\right)^{\vartheta_i}\right)^{q}{}^{2}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{\overrightarrow{(\xi^{IU})}}\right)^{\vartheta_i}\right)^{q}}{2}+1-\frac{\sqrt{\left(\oslash_{i=1}^{n}(\underline{\xi^{FL}})^{\vartheta_i}\right)}+\sqrt{\left(\oslash_{i=1}^{n}(\underline{\xi^{FU}})^{\vartheta_i}\right)}}{2}}\right]$$

$$\leq\ \frac{1}{2}\times\left[\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_{ij}^{TL})}\right)^{\vartheta_i}\right)^{2q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_{ij}^{TU})}\right)^{\vartheta_i}\right)^{2q}}{\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{ij}^{IL})}\right)^{\vartheta_i}\right)^{q}{}^{2}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{ij}^{IU})}\right)^{\vartheta_i}\right)^{q}}{2}+1-\frac{\sqrt{\left(\oslash_{i=1}^{n}(\xi_{ij}^{FL})^{\vartheta_i}\right)}+\sqrt{\left(\oslash_{i=1}^{n}(\xi_{ij}^{FU})^{\vartheta_i}\right)}}{2}}\right]$$

$$\leq\ \frac{1}{2}\times\left[\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\underline{(\xi_{ij}^{TL})}}\right)^{\vartheta_i}\right)^{2q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\underline{(\xi_{ij}^{TU})}}\right)^{\vartheta_i}\right)^{2q}}{\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{\underline{(\xi_{ij}^{IL})}}\right)^{\vartheta_i}\right)^{q}{}^{2}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{\underline{(\xi_{ij}^{IU})}}\right)^{\vartheta_i}\right)^{q}}{2}+1-\frac{\sqrt{\left(\oslash_{i=1}^{n}(\xi_{ij}^{FL})^{\vartheta_i}\right)}+\sqrt{\left(\oslash_{i=1}^{n}(\xi_{ij}^{FU})^{\vartheta_i}\right)}}{2}}\right].$$

Therefore, $\left\langle[\overrightarrow{\xi^{TL}},\overrightarrow{\xi^{TU}}],[\overrightarrow{\xi^{IL}},\overrightarrow{\xi^{IU}}],[\underline{\xi^{FL}},\underline{\xi^{FU}}]\right\rangle\leq q-rungSRIVNWA(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_n)$
$\leq\left\langle[\underline{\xi^{TL}},\underline{\xi^{TU}}],[\underline{\xi^{IL}},\underline{\xi^{IU}}],\overrightarrow{\xi^{FL}},\overrightarrow{\xi^{FU}}]\right\rangle$.

Theorem 5.5. *Let* $\overrightarrow{\Xi}_i=\left\langle[\xi_{t_{ij}}^{TL},\xi_{t_{ij}}^{TU}],[\xi_{t_{ij}}^{IL},\xi_{t_{ij}}^{IU}],[\xi_{t_{ij}}^{FL},\xi_{t_{ij}}^{FU}]\right\rangle$ *and* $\overrightarrow{W}_i=\left\langle[\xi_{h_{ij}}^{TL},\xi_{h_{ij}}^{TU}],[\xi_{h_{ij}}^{IL},\xi_{h_{ij}}^{IU}],[\xi_{h_{ij}}^{FL},\xi_{h_{ij}}^{FU}]\right\rangle$ *be the two families of q-rung SRIVNWAs. For any* i, *if there is* $\sqrt{\left(\xi_{t_{ij}}^{TL}\right)}+\sqrt{\left(\xi_{t_{ij}}^{TU}\right)}\leq\sqrt{\left(\xi_{h_{ij}}^{TL}\right)}+\sqrt{\left(\xi_{h_{ij}}^{TU}\right)}$ *and* $\sqrt{\left(\xi_{t_{ij}}^{IL}\right)}+\sqrt{\left(\xi_{t_{ij}}^{IU}\right)}\leq\sqrt{\left(\xi_{h_{ij}}^{IL}\right)}+\sqrt{\left(\xi_{h_{ij}}^{IU}\right)}$ *and* $\left(\xi_{t_{ij}}^{FL}\right)+\left(\xi_{t_{ij}}^{FU}\right)\geq\left(\xi_{h_{ij}}^{FL}\right)+\left(\xi_{h_{ij}}^{FU}\right)$ *or* $\overrightarrow{\Xi}_i\leq\overrightarrow{W}_i$, *then* $q-rungSRIVNWA\left(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_n\right)\leq q-rungSRIVNWA\left(\overrightarrow{W}_1,\overrightarrow{W}_2,...,\overrightarrow{W}_n\right)$, *where* $(i=1\ to\ n);(j=1\ to\ i_j)$ *(monotonicity property).*

Proof. For any i, $\sqrt{\left(\xi_{t_{ij}}^{TL}\right)}+\sqrt{\left(\xi_{t_{ij}}^{TU}\right)}\leq\sqrt{\left(\xi_{h_{ij}}^{TL}\right)}+\sqrt{\left(\xi_{h_{ij}}^{TU}\right)}$.
Therefore, $1-\sqrt[2q]{\left(\xi_{t_i}^{TL}\right)}+1-\sqrt[2q]{\left(\xi_{t_i}^{TU}\right)}\geq1-\sqrt[2q]{\left(\xi_{h_i}^{TL}\right)}+1-\sqrt[2q]{\left(\xi_{h_i}^{TU}\right)}$.
Hence,

$$\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{t_i}^{TL}\right)}\right)^{\vartheta_i}+\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{t_i}^{TU}\right)}\right)^{\vartheta_i}\geq$$
$$\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{h_i}^{TL}\right)}\right)^{\vartheta_i}+\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{h_i}^{TU}\right)}\right)^{\vartheta_i}$$

and $\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{t_i}^{TL}\right)}\right)^{\vartheta_i}\right)^{2q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{t_i}^{TU}\right)}\right)^{\vartheta_i}\right)^{2q}\leq$

$$\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{h_i}^{TL}\right)}\right)^{\vartheta_i}\right)^{2q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{\left(\xi_{h_i}^{TU}\right)}\right)^{\vartheta_i}\right)^{2q}.$$

For any i, $\sqrt{\left(\xi_{t_{ij}}^{IL}\right)}+\sqrt{\left(\xi_{t_{ij}}^{IU}\right)}\leq\sqrt{\left(\xi_{h_{ij}}^{IL}\right)}+\sqrt{\left(\xi_{h_{ij}}^{IU}\right)}$.
Iherefore, $1-\sqrt[q]{\left(\xi_{t_i}^{IL}\right)}+1-\sqrt[q]{\left(\xi_{t_i}^{IU}\right)}\geq1-\sqrt[q]{\left(\xi_{h_i}^{IL}\right)}+1-\sqrt[q]{\left(\xi_{h_i}^{IU}\right)}$.

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

Hence,

$$\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{t_i}^{IL})}\right)^{\vartheta_i}+\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{t_i}^{IU})}\right)^{\vartheta_i}\geq \oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{h_i}^{IL})}\right)^{\vartheta_i}+\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{h_i}^{IU})}\right)^{\vartheta_i}$$

and $$\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{t_i}^{IL})}\right)^{\vartheta_i}\right)^{q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{t_i}^{IU})}\right)^{\vartheta_i}\right)^{q}\leq \left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{h_i}^{IL})}\right)^{\vartheta_i}\right)^{q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{h_i}^{IU})}\right)^{\vartheta_i}\right)^{q}.$$

For any i, $\left(\xi_{t_{ij}}^{FL}\right)+\left(\xi_{t_{ij}}^{FU}\right)\geq\left(\xi_{h_{ij}}^{FL}\right)+\left(\xi_{h_{ij}}^{FU}\right)$.

Therefore, $1-\frac{\left(\oslash_{i=1}^{n}\xi_{t_{ij}}^{FL}\right)+\left(\oslash_{i=1}^{n}\xi_{t_{ij}}^{FU}\right)}{2}\leq 1-\frac{\left(\oslash_{i=1}^{n}\xi_{h_{ij}}^{FL}\right)+\left(\oslash_{i=1}^{n}\xi_{h_{ij}}^{FU}\right)}{2}$.

Hence,

$$\frac{1}{2}\times\left[\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_{ti}^{TL})}\right)^{\vartheta_i}\right)^{2q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_{ti}^{TU})}\right)^{\vartheta_i}\right)^{2q}}{2}\right. \\ \left.\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{ti}^{IL})}\right)^{\vartheta_i}\right)^{q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{ti}^{IU})}\right)^{\vartheta_i}\right)^{q}}{2} +1-\frac{\sqrt[q]{\left(\oslash_{i=1}^{n}(\xi_{tij}^{FL})\right)}+\sqrt[q]{\left(\oslash_{i=1}^{n}(\xi_{tij}^{FU})\right)}}{2}\right]$$

$$\leq\frac{1}{2}\times\left[\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_{hi}^{TL})}\right)^{\vartheta_i}\right)^{2q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_{hi}^{TU})}\right)^{\vartheta_i}\right)^{2q}}{2}\right. \\ \left.\frac{\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{hi}^{IL})}\right)^{\vartheta_i}\right)^{q}+\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_{hi}^{IU})}\right)^{\vartheta_i}\right)^{q}}{2} +1-\frac{\sqrt[q]{\left(\oslash_{i=1}^{n}(\xi_{hij}^{FL})\right)}+\sqrt[q]{\left(\oslash_{i=1}^{n}(\xi_{hij}^{FU})\right)}}{2}\right].$$

Hence, $q-rungSRIVNWA\left(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_n\right)\leq q-rungSRIVNWA\left(\overrightarrow{W}_1,\overrightarrow{W}_2,...,\overrightarrow{W}_n\right)$.

5.2 q-rung SRIV weighted geometric(q-rung SRIVNWG) operator

Definition 5.6. Let $\overrightarrow{\Xi}_i=\left\langle[\xi_i^{TL},\xi_i^{TU}],[\xi_i^{IL},\xi_i^{IU}],[\xi_i^{FL},\xi_i^{FU}]\right\rangle$ be the q-rung SRIVNNs. Then q-rung SRIVNWG $(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_n)=\oslash_{i=1}^{n}\overrightarrow{\Xi}_i^{\vartheta_i}$.

Theorem 5.7. *Let* $\overrightarrow{\Xi}_i=\left\langle[\xi_i^{TL},\xi_i^{TU}],[\xi_i^{IL},\xi_i^{IU}],[\xi_i^{FL},\xi_i^{FU}]\right\rangle$ *be the q-rung SRIVNNs. Prove that*

$$\left[\begin{array}{c}\left[\oslash_{i=1}^{n}(\xi_i^{TL})^{\vartheta_i},\oslash_{i=1}^{n}(\xi_i^{TU})^{\vartheta_i}\right],\\ \left[\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\vartheta_i}\right)^{q},\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\vartheta_i}\right)^{q}\right],\\ \left[\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_i^{FL})}\right)^{\vartheta_i}\right)^{2q},\left(1-\oslash_{i=1}^{n}\left(1-\sqrt[2q]{(\xi_i^{FU})}\right)^{\vartheta_i}\right)^{2q}\right]\end{array}\right].$$

Theorem 5.8. *If all* $\overrightarrow{\Xi}_i=\left\langle[\xi_i^{TL},\xi_i^{TU}],[\xi_i^{IL},\xi_i^{IU}],[\xi_i^{FL},\xi_i^{FU}]\right\rangle$ *are equal, then q-rung SRIVNWG*$(\overrightarrow{\Xi}_1,\overrightarrow{\Xi}_2,...,\overrightarrow{\Xi}_n)=\overrightarrow{\Xi}$.

Remark 5.9. It is possible to achieve boundedness and monotonicity of the system by using the SRIVNWG operator with the q-rung.

5.3 Generalized q-rung SRIVNWA (q-rung GSRIVNWA) operator

Definition 5.10. Let $\overrightarrow{\Xi}_i = \left\langle [\xi_i^{TL}, \xi_i^{TU}], [\xi_i^{IL}, \xi_i^{IU}], [\xi_i^{FL}, \xi_i^{FU}] \right\rangle$ be the q-rung SRIVNN. Then q-rung GSRIVNWA $(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n) = \left(\sqcup_{i=1}^n \vartheta_i \overrightarrow{\Xi}_i^{\Omega} \right)^{1/\Omega}$.

Theorem 5.11. *Let* $\overrightarrow{\Xi}_i = \left\langle [\xi_i^{TL}, \xi_i^{TU}], [\xi_i^{IL}, \xi_i^{IU}], [\xi_i^{FL}, \xi_i^{FU}] \right\rangle$ *be the q-rung SRIVNNs. Then q-rung GSRIVNWA* $(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n) =$

$$\left[\begin{array}{c} \left[\left(\left(1-\oslash_{i=1}^n\left(1-\left(\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{2q}\right)^{\Omega}, \left(\left(1-\oslash_{i=1}^n\left(1-\left(\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{2q}\right)^{\Omega}\right], \\ \left[\left(\left(1-\oslash_{i=1}^n\left(1-\left(\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{q}\right)^{\Omega}, \left(\left(1-\oslash_{i=1}^n\left(1-\left(\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{q}\right)^{\Omega}\right], \\ \left[\begin{array}{c} \left(1-\left(1-\sqrt[q]{\oslash_{i=1}^n\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{q}, \\ \left(1-\left(1-\sqrt[q]{\oslash_{i=1}^n\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{q} \end{array}\right] \end{array}\right].$$

Proof. It is essential that the following is demonstrated:
$\sqcup_{i=1}^n \vartheta_i \overrightarrow{\Xi}_i^{\Omega} =$

$$\left[\begin{array}{c} \left[\left(1-\oslash_{i=1}^n\left(1-\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)^{\vartheta_i}\right)^{2q} \left(1-\oslash_{i=1}^n\left(1-\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)^{\vartheta_i}\right)^{2q}\right], \\ \left[\left(1-\oslash_{i=1}^n\left(1-\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)^{\vartheta_i}\right)^{q} \left(1-\oslash_{i=1}^n\left(1-\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)^{\vartheta_i}\right)^{q}\right], \\ \left[\oslash_{i=1}^n\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}, \oslash_{i=1}^n\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}\right] \end{array}\right].$$

https://doi.org/10.54216/IJNS.230314

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

Put $n = 2$, $\vartheta_1 \Xi_1 \sqcup \vartheta_2 \Xi_2 =$

$$\begin{bmatrix} \begin{bmatrix} \left(\begin{array}{c} \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_1^{TL})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}} + \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_2^{TL})^{\Omega}}\right)^{\vartheta_2}\right)^{2q}} \\ - \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_1^{TL})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}} \cdot \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_2^{TL})^{\Omega}}\right)^{\vartheta_2}\right)^{2q}} \end{array} \right)^{2q} \\ \left(\begin{array}{c} \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_1^{TU})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}} + \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_2^{TU})^{\Omega}}\right)^{\vartheta_2}\right)^{2q}} \\ - \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_1^{TU})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}} \cdot \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_2^{TU})^{\Omega}}\right)^{\vartheta_2}\right)^{2q}} \end{array} \right)^{2q} \end{bmatrix}, \\ \begin{bmatrix} \left(\begin{array}{c} \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_1^{IL})^{\Omega}}\right)^{\vartheta_1}\right)^{q}} + \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_2^{IL})^{\Omega}}\right)^{\vartheta_2}\right)^{q}} \\ - \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_1^{IL})^{\Omega}}\right)^{\vartheta_1}\right)^{q}} \cdot \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_2^{IL})^{\Omega}}\right)^{\vartheta_2}\right)^{q}} \end{array} \right)^{q} \\ \left(\begin{array}{c} \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_1^{IU})^{\Omega}}\right)^{\vartheta_1}\right)^{q}} + \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_2^{IU})^{\Omega}}\right)^{\vartheta_2}\right)^{q}} \\ - \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_1^{IU})^{\Omega}}\right)^{\vartheta_1}\right)^{q}} \cdot \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_2^{IU})^{\Omega}}\right)^{\vartheta_2}\right)^{q}} \end{array} \right)^{q} \end{bmatrix}, \\ \begin{bmatrix} \left(\left(1-\left(1-\sqrt[q]{(\xi_1^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_1} \cdot \left(\left(1-\left(1-\sqrt[q]{(\xi_2^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_1}, \\ \left(\left(1-\left(1-\sqrt[q]{(\xi_1^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_1} \cdot \left(\left(1-\left(1-\sqrt[q]{(\xi_2^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_1} \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \left[\left(1-\oslash_{i=1}^{2}\left(1-\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)^{\vartheta_i}\right)^{2q} \left(1-\oslash_{i=1}^{2}\left(1-\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)^{\vartheta_i}\right)^{2q} \right], \\ \left[\left(1-\oslash_{i=1}^{2}\left(1-\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)^{\vartheta_i}\right)^{q} \left(1-\oslash_{i=1}^{2}\left(1-\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)^{\vartheta_i}\right)^{q} \right], \\ \left[\oslash_{i=1}^{2}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}, \oslash_{i=1}^{2}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i} \right] \end{bmatrix}.$$

In general,

$$\begin{bmatrix} \left[\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)^{\vartheta_i}\right)^{2q} \left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)^{\vartheta_i}\right)^{2q} \right], \\ \left[\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)^{\vartheta_i}\right)^{q} \left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)^{\vartheta_i}\right)^{q} \right], \\ \left[\oslash_{i=1}^{l}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}, \oslash_{i=1}^{l}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i} \right] \end{bmatrix}.$$

If $n = l + 1$, then $\sqcup_{i=1}^{l} \vartheta_i \overrightarrow{\Xi}_i^{\Omega} + \vartheta_{l+1} \overrightarrow{\Xi}_{l+1}^{\Omega} = \sqcup_{i=1}^{l+1} \vartheta_i \overrightarrow{\Xi}_i^{\Omega}$.

Now,$\sqcup_{i=1}^{l} \vartheta_i \overrightarrow{\Xi}_i^{\Omega} + \vartheta_{l+1} \overrightarrow{\Xi}_{l+1}^{\Omega} = \sqcup_{i=1}^{l+1} \vartheta_i \overrightarrow{\Xi}_i^{\Omega} = \vartheta_1 \overrightarrow{\Xi}_1^{\Omega} \sqcup \vartheta_2 \overrightarrow{\Xi}_2^{\Omega} \sqcup ... \sqcup \vartheta_l \overrightarrow{\Xi}_l^{\Omega} \sqcup \vartheta_{l+1} \overrightarrow{\Xi}_{l+1}^{\Omega}$

https://doi.org/10.54216/IJNS.230314
Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

$$
= \left[\begin{array}{c}
\left[\begin{array}{c}
\left(\begin{array}{c}
\sqrt[2q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)^{\vartheta_i}\right)^{2q}} + \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TL})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}} \\
-\sqrt[2q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)^{\vartheta_i}\right)^{2q}} \cdot \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TL})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}}
\end{array} \right)^{2q} \\
\left(\begin{array}{c}
\sqrt[2q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)^{\vartheta_i}\right)^{2q}} + \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TU})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}} \\
-\sqrt[2q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)^{\vartheta_i}\right)^{2q}} \cdot \sqrt[2q]{\left(1-\left(1-\sqrt[2q]{(\xi_{l+1}^{TU})^{\Omega}}\right)^{\vartheta_1}\right)^{2q}}
\end{array} \right)^{2q}
\end{array} \right], \\
\left[\begin{array}{c}
\left(\begin{array}{c}
\sqrt[q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)^{\vartheta_i}\right)^{q}} + \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IL})^{\Omega}}\right)^{\vartheta_1}\right)^{q}} \\
-\sqrt[q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)^{\vartheta_i}\right)^{q}} \cdot \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IL})^{\Omega}}\right)^{\vartheta_1}\right)^{q}}
\end{array} \right)^{q}, \\
\left(\begin{array}{c}
\sqrt[q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)^{\vartheta_i}\right)^{q}} + \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IU})^{\Omega}}\right)^{\vartheta_1}\right)^{q}} \\
-\sqrt[q]{\left(1-\oslash_{i=1}^{l}\left(1-\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)^{\vartheta_i}\right)^{q}} \cdot \sqrt[q]{\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{IU})^{\Omega}}\right)^{\vartheta_1}\right)^{q}}
\end{array} \right)^{q}
\end{array} \right], \\
\left[\begin{array}{c}
\oslash_{i=1}^{l}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i} \cdot \left(\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_1}, \\
\oslash_{i=1}^{l}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i} \cdot \left(\left(1-\left(1-\sqrt[q]{(\xi_{l+1}^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_1}
\end{array} \right]
\end{array} \right],
$$

Hence,

$$
\sqcup_{i=1}^{l+1}\vartheta_i \overrightarrow{\Xi}_i^{\Omega} = \left[\begin{array}{c}
\left[\left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)^{\vartheta_i}\right)^{2q} \left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)^{\vartheta_i}\right)^{2q} \right], \\
\left[\left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)^{\vartheta_i}\right)^{q} \left(1-\oslash_{i=1}^{l+1}\left(1-\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)^{\vartheta_i}\right)^{q} \right], \\
\left[\oslash_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}, \oslash_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i} \right]
\end{array} \right].
$$

Also, $\left(\oslash_{i=1}^{l+1}\vartheta_i \overrightarrow{\Xi}_i^{\Omega}\right)^{1/\Omega} =$

$$
\left[\begin{array}{c}
\left[\left(\left(1-\oslash_{i=1}^{l+1}\left(1-\left(\sqrt[2q]{(\xi_i^{TL})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{2q}\right)^{\Omega}, \left(\left(1-\oslash_{i=1}^{l+1}\left(1-\left(\sqrt[2q]{(\xi_i^{TU})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{2q}\right)^{\Omega} \right], \\
\left[\left(\left(1-\oslash_{i=1}^{l+1}\left(1-\left(\sqrt[q]{(\xi_i^{IL})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{q}\right)^{\Omega}, \left(\left(1-\oslash_{i=1}^{l+1}\left(1-\left(\sqrt[q]{(\xi_i^{IU})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{q}\right)^{\Omega} \right], \\
\left[\begin{array}{c}
\left(1-\left(1-\sqrt[q]{\oslash_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{q}, \\
\left(1-\left(1-\sqrt[q]{\oslash_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{FU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{q}
\end{array} \right]
\end{array} \right].
$$

It valid for any l.

https://doi.org/10.54216/IJNS.230314

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

Remark 5.12. The q-rung GSRIVNWA is switched to the q-rung SRIVNWA if $\Omega = 1$.

Theorem 5.13. *If all* $\overrightarrow{\Xi}_i = \left\langle [\xi_i^{TL}, \xi_i^{TU}], [\xi_i^{IL}, \xi_i^{IU}], [\xi_i^{FL}, \xi_i^{FU}] \right\rangle (i = 1 \ to \ n)$ *are equal, then* *q-rung GSRIVNWA*$(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n) = \overrightarrow{\Xi}$.

Remark 5.14. Using the q-rung GSRIVNWA, it has been demonstrated that monotonicity and boundedness can both be achieved.

5.4 Generalized q-rung SRIVNWG (q-rung GSRIVNWG) operator

Definition 5.15. Let $\overrightarrow{\Xi}_i = \left\langle [\xi_i^{TL}, \xi_i^{TU}], [\xi_i^{IL}, \xi_i^{IU}], [\xi_i^{FL}, \xi_i^{FU}] \right\rangle$ be the q-rung SRIVNNs. Then q-rung GSRIVNWG $(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n) = \frac{1}{\Omega}\Big(\oslash_{i=1}^{n} (\Omega \overrightarrow{\Xi}_i)^{\vartheta_i} \Big)$.

Theorem 5.16. *Let* $\overrightarrow{\Xi}_i = \left\langle [\xi_i^{TL}, \xi_i^{TU}], [\xi_i^{IL}, \xi_i^{IU}], [\xi_i^{FL}, \xi_i^{FU}] \right\rangle$ *be the q-rung SRIVNNs. Prove that q-rung GSRIVNWG*$(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n)$=

$$\left[\begin{array}{c} \left[\begin{array}{c} \left(1-\left(1-\sqrt[2q]{\oslash_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2q]{(\xi_i^{TL})}\right)^{\Omega}\right)^{2q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{2q}, \\ \left(1-\left(1-\sqrt[2q]{\oslash_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2q]{(\xi_i^{TU})}\right)^{\Omega}\right)^{2q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{2q} \end{array}\right], \\ \left[\begin{array}{c} \left(1-\left(1-\sqrt[q]{\oslash_{i=1}^{n}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{IL})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{q}, \\ \left(1-\left(1-\sqrt[q]{\oslash_{i=1}^{n}\left(\left(1-\left(1-\sqrt[q]{(\xi_i^{IU})}\right)^{\Omega}\right)^{q}\right)^{\vartheta_i}}\right)^{\Omega}\right)^{q} \end{array}\right], \\ \left[\left(\left(1-\oslash_{i=1}^{n}\left(1-\left(\sqrt[q]{(\xi_i^{FL})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{q}\right)^{\Omega}, \left(\left(1-\oslash_{i=1}^{n}\left(1-\left(\sqrt[q]{(\xi_i^{FU})^{\Omega}}\right)\right)^{\vartheta_i}\right)^{q}\right)^{\Omega}\right] \end{array}\right].$$

Remark 5.17. The q-rung GSRIVNWG becomes the q-rung SRIVNWG operator if $\Omega = 1$.

Remark 5.18. A q-rung GSRIVNWG is used in order to meet the properties of boundedness and monotonicity.

Theorem 5.19. *If all* $\overrightarrow{\Xi}_i = \left\langle [\xi_i^{TL}, \xi_i^{TU}], [\xi_i^{IL}, \xi_i^{IU}], [\xi_i^{FL}, \xi_i^{FU}] \right\rangle (i = 1 \ to \ n)$ *are equal, then* *q-rung GSRIVNWG*$(\overrightarrow{\Xi}_1, \overrightarrow{\Xi}_2, ..., \overrightarrow{\Xi}_n) = \overrightarrow{\Xi}$.

6 MADM using q-rung SRIV data

Let $\overrightarrow{\Upsilon} = \{\overrightarrow{\Upsilon}_a, \overrightarrow{\Upsilon}_b, ..., \overrightarrow{\Upsilon}_n\}$ be the alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the attributes, $w = \{\vartheta_1, \vartheta_2, ..., \vartheta_m\}$ be the weights of attributes, $\overrightarrow{\overrightarrow{\Upsilon}}_{ij} = \left\langle [\xi_{ij}^{TL}, \xi_{ij}^{TU}], [\xi_{ij}^{IL}, \xi_{ij}^{IU}], [\xi_{ij}^{FL}, \xi_{ij}^{FU}] \right\rangle$ is denote q-rung SRIVNN of $\overrightarrow{\Upsilon}_i$ in C_j. Here, $\left[\xi_{ij}^{TL}, \xi_{ij}^{TU}\right]$, $\left[\xi_{ij}^{FL}, \xi_{ij}^{FU}\right] \in [0, 1]$ and $0 \leq (\xi_{ij}^{TU}(\vartheta))^q + \sqrt[q]{(\xi_{ij}^{FU}(\vartheta))} \leq 1$.

6.1 Algorithm

Step-1: As the q-rung SRIVNS is one of the most important values, it should have a choice value.
Step-2: A choice is made regarding which values to use for the normalization process. Let $\mathscr{D} = (\overrightarrow{\Upsilon}_{ij})_{n \times m}$ is normalized into $\overrightarrow{\mathscr{D}} = (\overrightarrow{\Upsilon}_{ij})_{n \times m}$; put

$$\overrightarrow{\Upsilon}_{ij} = \left\langle [\overrightarrow{\xi_{ij}^{TL}}, \overrightarrow{\xi_{ij}^{TU}}], [\overrightarrow{\xi_{ij}^{IL}}, \overrightarrow{\xi_{ij}^{IU}}], [\overrightarrow{\xi_{ij}^{FL}}, \overrightarrow{\xi_{ij}^{FU}}] \right\rangle$$

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Step-3: Calculate the positive and negative ideal values:

$$\overrightarrow{\Upsilon}^P = \Big\langle [1,1],[1,1],[0,0] \Big\rangle,$$

$$\overrightarrow{\Upsilon}^N = \Big\langle [0,0],[0,0],[1,1] \Big\rangle.$$

Step-4: It is important to find the difference between the ideal values of each option in order to find the ED:

$$\mathscr{D}_i^P = \mathscr{D}_E\Big(\overrightarrow{\Upsilon}_i, \overrightarrow{\Upsilon}^P\Big); \ \mathscr{D}_i^N = \mathscr{D}_E\Big(\overrightarrow{\Upsilon}_i, \overrightarrow{\Upsilon}^N\Big).$$

Step-5: Using the following formula, one can calculate how close two points are to each other:

$$\mathscr{D}_i^\star = \frac{\mathscr{D}_i^N}{\mathscr{D}_i^P + \mathscr{D}_i^N}.$$

Step-6: If you are looking for the best output in this case, you will need to use $\sup \mathscr{D}_i^\star$. A decision is the process of choosing the right option in order to solve a given problem. The Figure-1 shows that algorithm for the proposed methods.

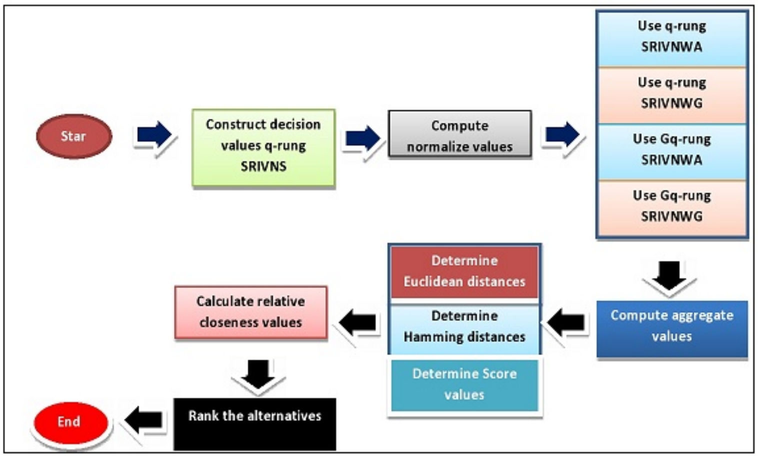

Figure-1. algorithm for proposed models.

6.2 Selection of Courier Service in India

Every business today relies on delivery services. The small and medium enterprises have now joined this sector after the e-commerce industries in their need for road transportation. In addition to that, there is fierce competition between the courier service companies in India, all claiming to be the best. Therefore, you should be careful when choosing a courier partner for your business. When there are so many courier companies available on the market, how can one choose a reliable one? When selecting the best courier service in India, there are many important factors to consider, and here are the most important ones so that you don't waste your time or money. Currently, I have selected five courier companies at random such as company-1, company-2, company-3, company-4 and company-5. What are the important factors that courier company?

1. Reliability C_1: Reliability is one of the most important factors to consider when choosing a courier service. Whether your shipments arrive late or damaged, you may need to consider another supplier. In many cases, courier services advertise extremely low prices, but they end up costing you a lot more than what they advertise. If they promise delivery by a certain time, then they will deliver. The best courier service will offer reliable service at an affordable price. In the long run, it's better to spend more on a reliable courier first; that will save you money on shipping costs in the future.

https://doi.org/10.54216/IJNS.230314

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

2. Turnaround Time C_2: You probably have the turnaround time in mind when choosing a courier service. The speed of delivery is determined by a number of factors, including delivery methods (ground or air), origin and destination locations, and even seasons (that last one is obvious). Identifying your business's priorities is the key. Does it matter if your product arrives three days past its expiration date if it requires two days to ship? Before choosing a courier company, you'll want to research its estimated speeds for a particular shipment if speed is vital.

3. Payment Options C_3: All business transactions involve money, as you know. Transactions are typically processed and facilitated by third parties. The most common party to fulfill such a role is a bank, but other parties can also play the same role. In the case of payments between businesses or individuals, courier services play a similar role. How does one courier service differ from another? Options for making payments. Various services accept in-person payments, as well as credit cards and electronic payments (like PayPal). Your shipping volume may be too large for an all-in-one option if you work with customers who prefer different payment methods.

4. Tracking Capability C_4: In India, you must be informed as soon as your package has been picked up by your courier service. While your shipment is in transit, you will be able to monitor its progress at any time. You will also be able to discover whether or not your shipment has been delayed with an effective tracking system. Your package cannot be tracked without it, causing unnecessary anxiety. If an order is missed due to late delivery, it could also be costly for a business. Make sure you choose your courier wisely.

Table-1 shows that decision making informations are

Table 1: Decision making informations

	C_1	C_2	C_3	C_4
$\overrightarrow{\Upsilon}_a$	$\langle[0.8,0.85],[0.65,0.7],[0.6,0.74]\rangle$	$\langle[0.85,0.9],[0.75,0.8],[0.65,0.75]\rangle$	$\langle[0.65,0.85],[0.65,0.85],[0.6,0.7]\rangle$	$\langle[0.65,0.75],[0.8,0.85],[0.65,0.7]\rangle$
$\overrightarrow{\Upsilon}_b$	$\langle[0.65,0.75],[0.7,0.75],[0.6,0.8]\rangle$	$\langle[0.7,0.8],[0.75,0.85],[0.6,0.85]\rangle$	$\langle[0.75,0.85],[0.7,0.8],[0.6,0.65]\rangle$	$\langle[0.6,0.85],[0.65,0.75],[0.7,0.75]\rangle$
$\overrightarrow{\Upsilon}_c$	$\langle[0.8,0.85],[0.55,0.7],[0.55,0.75]\rangle$	$\langle[0.85,0.95],[0.65,0.7],[0.55,0.8]\rangle$	$\langle[0.8,0.9],[0.75,0.8],[0.45,0.5]\rangle$	$\langle[0.55,0.75],[0.8,0.85],[0.65,0.75]\rangle$
$\overrightarrow{\Upsilon}_d$	$\langle[0.75,0.8],[0.6,0.75],[0.75,0.85]\rangle$	$\langle[0.7,0.75],[0.65,0.8],[0.7,0.85]\rangle$	$\langle[0.85,0.9],[0.65,0.7],[0.7,0.75]\rangle$	$\langle[0.75,0.85],[0.75,0.8],[0.7,0.8]\rangle$
$\overrightarrow{\Upsilon}_e$	$\langle[0.75,0.9],[0.75,0.8],[0.8,0.85]\rangle$	$\langle[0.65,0.8],[0.7,0.85],[0.8,0.9]\rangle$	$\langle[0.75,0.85],[0.75,0.8],[0.8,0.9]\rangle$	$\langle[0.7,0.8],[0.75,0.9],[0.75,0.85]\rangle$

The following aggregate data can be obtained using the q-rung SRIVNWA operator. In table-2 it is shown that q-rung SRIVNWG operates.

Table 2: q-rung SRIVNWG operator

	$q-rungSRIVNWA\ operator\ (q=1)$
$\overrightarrow{\Upsilon}_a$	$\langle[0.782,0.86],[0.7008,0.7842],[0.6195,0.7307]\rangle$
$\overrightarrow{\Upsilon}_b$	$\langle[0.6829,0.7991],[0.7116,0.7949],[0.6093,0.7765]\rangle$
$\overrightarrow{\Upsilon}_c$	$\langle[0.8002,0.8948],[0.6579,0.7419],[0.5373,0.7051]\rangle$
$\overrightarrow{\Upsilon}_d$	$\langle[0.7611,0.8186],[0.643,0.7629],[0.7196,0.824]\rangle$
$\overrightarrow{\Upsilon}_e$	$\langle[0.718,0.8566],[0.7359,0.8288],[0.7949,0.8746]\rangle$

Analyze the positive and negative values of the following alternatives and determine the optimum value $\overrightarrow{\Upsilon}^P = \langle[1,1],[1,1],[0,0]\rangle$ and $\overrightarrow{\Upsilon}^N = \langle[0,0],[0,0],[1,1]\rangle$.
ED between each alternatives with both ideal values are $\mathscr{D}_1^P$=0.4875, $\mathscr{D}_2^P$=0.5234, $\mathscr{D}_3^P$=0.482, $\mathscr{D}_4^P$=0.5376, $\mathscr{D}_5^P$=0.5127 and $\mathscr{D}_1^N$=0.431, $\mathscr{D}_2^N$=0.3952, $\mathscr{D}_3^N$=0.4366, $\mathscr{D}_4^N$=0.381, $\mathscr{D}_5^N$=0.4059.
The following formula is used to calculate relative closeness: $\mathscr{D}_1^\star$= 0.4692, $\mathscr{D}_2^\star$= 0.4302, $\mathscr{D}_3^\star$= 0.4753, $\mathscr{D}_4^\star$= 0.4148, $\mathscr{D}_5^\star$= 0.4418.
The following are the alternatives ranked in order of preference: $\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$. As a result, company-3 is the best option.

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6.3 Compared to existing models and proposed models

An interaction MADM approach for Pythagorean neutrosophic normal interval-valued aggregation operators was recently introduced by Palanikumar et al.[16] Several distance techniques have recently been applied to MADM challenges for ranking vague sets by Palanikumar.[17] We proposed ED and HD is based on q-rung SRIVNWA, q-rung SRIVNWG, q-rung GSRIVNWA and q-rung GSRIVNWG, respectively. HD and ED were used in both cases. There are several proposed methods shown in Table-3.

Table 3: Proposed methods

$q=1$	$q-rung$ $SRIVNWA$	$q-rung$ $SRIVNWG$	$q-rung$ $GSRIVNWA$	$q-rung$ $GSRIVNWG$
$TOPSIS-\ Hamming$ $distance\,(proposed)$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$Score\,(proposed)$ $distance\,(proposed)$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$

Table-4 shows that existing methods.

Table 4: Existing methods

$q=1$	$PNIVNWA$	$PNIVNWG$	$GPNIVNWA$	$GPNIVNWG$
$TOPSIS-\ Hamming$ $distance$[16]	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$
$Score\,value$ [16]	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_b$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_d$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_e$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_b$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_d$ $\overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_e$

Table-5 shows that existing methods.

Table 5: Existing methods

$q=1$	$logFVNWA$	$logFVNWG$	$GlogFVNWA$	$GlogFVNWG$
$TOPSIS-\ Hamming$ $distance$[17]	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_d$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_e$ $\overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_d$
$Score\,value$ [17]	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_b$ $\overrightarrow{\Upsilon}_d \succ \overrightarrow{\Upsilon}_e$

6.4 Data Analysis

In the Table-6, we show the relative closeness values and their order based on the q-rung SRIVNWG operator, which provides the following aggregate data for each alternative:

In Figure-2 deals the different q values for proposed approaches.

According to the above data, if $q = 1$, then the order is $\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$. If $2 \leq q \leq 9$, then new alternative order is $\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$. If $q = 10$, then new alternative order is $\overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$. As a result, $\overrightarrow{\Upsilon}_a$ instead of $\overrightarrow{\Upsilon}_c$. Similarly, we can apply to q-rung SRIVNWG, q-rung GSRIVNWA, and q-rung GSRIVNWG operators.

7 Conclusion:

It is presented that the ED and HD measures for q-rung SRIVNN have been computed. In order to demonstrate the validity of ED and HD measures, numerical examples will be used to illustrate the point. A set of AO rules

Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

Table 6: Relative closeness values

$q-values$	$\mathscr{D}_1^\star$	$\mathscr{D}_2^\star$	$\mathscr{D}_3^\star$	$\mathscr{D}_4^\star$	$\mathscr{D}_5^\star$	$Order$
$q=1$	0.6945	0.6396	0.6948	0.6141	0.6574	$\overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=2$	0.6941	0.6395	0.6941	0.6139	0.6573	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=3$	0.6939	0.6395	0.6939	0.6139	0.6573	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=4$	0.6938	0.6394	0.6938	0.6138	0.6572	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=5$	0.6938	0.6394	0.6938	0.6138	0.6572	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=6$	0.6937	0.6394	0.6937	0.6138	0.6572	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=7$	0.6937	0.6394	0.6937	0.6138	0.6572	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=8$	0.6937	0.6394	0.6937	0.6138	0.6572	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=9$	0.6937	0.6394	0.6937	0.6138	0.6572	$\overrightarrow{\Upsilon}_c = \overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$
$q=10$	0.6937	0.6394	0.6936	0.6138	0.6572	$\overrightarrow{\Upsilon}_a \succ \overrightarrow{\Upsilon}_c \succ \overrightarrow{\Upsilon}_e \succ \overrightarrow{\Upsilon}_b \succ \overrightarrow{\Upsilon}_d$

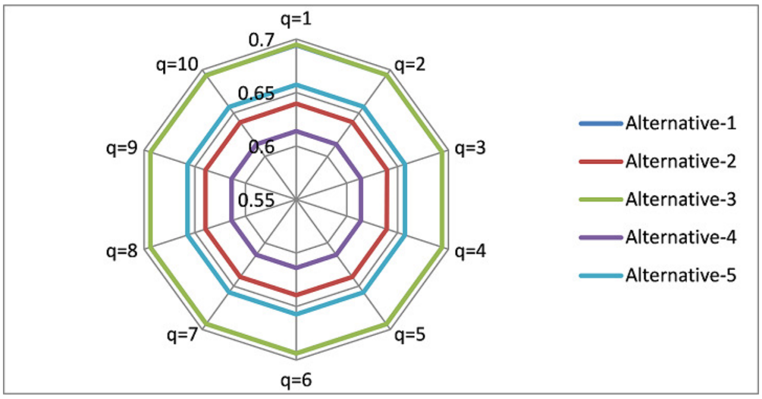

Figure-2. Different q values.

has been proposed for q-rung SRIVNWA, q-rung SRIVNWG, q-rung GSRIVNWA and q-rung GSRIVNWG. The research provided some examples and some of the AO. The application of q-rung SRIVFNN MADM will give people the opportunity to choose the best option under uncertain or inconsistent circumstances in order to make the best choice. In this research, it has been demonstrated through the use of q-rung SRIVNWA, q-rung SRIVNWG, q-rung GSRIVNWA, and q-rung GSRIVNWG it depending on q. Based on the q parameter, a distinct ranking of alternative options can be presented using the the above four operators. As shown in the study, the q have a significant impact on how alternative options are ranked. If a DM is faced with real life problems when making the decision, he or she may select q values accordingly. Many practical applications can be derived from ED and HD measures for data analysis. In this paper, we hope to provide useful information for future academics interested in this area. At the meeting, we will discuss the following topics in greater detail:

(1) It is explored using q-rung SRIVNN in terms of soft sets and expert sets.

(2) Our study is based on the q-rung SRIVNS used to cubic q-rung IVNS.

(3) To solve the problem of MADM, we can use the cubic interval valued NS as well as the complex interval valued NS.

Conflicts of Interest: The authors declare no conflict of interest.

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Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024

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Received: August 28, 2023 Revised: November 17, 2023 Accepted: January 29, 2024