Logarithmic Pythagorean neutrosophic vague aggregating operators and their real-life applications

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Abstract

This article examines Pythagorean neurosophic vague set (PyNVS) problems relevant to multiple attribute decision-making (MADM). Pythagorean vague set (PyVS) and neutrosophic set (NS) can be generalized into Pythagorean neutrosophic vague set (PyNVS). We discuss log Pythagorean neutrosophic vague weighted averaging (log PyNVWA), logarithmic Pythagorean neutrosophic vague weighted geometric (log PyNVWG), log generalized Pythagorean neurosophic vague weighted averaging (log GPyNVWA) and log generalized Pythagorean neutrosophic vague weighted geometric (log GPyNVWG). In this article, we define the Euclidean distance (ED), Hamming distance (HD), operator laws, and flowchart using an algorithm. By analyzing log PyNVS through algebraic operations, we discuss its properties. They can identify the best option more quickly and understand the practicalities better. An illustrative example of this is the fusion of computer science and machine tool technology in agriculture. Furthermore, there are autonomous robot tractors and soil sterilization robots that can harvest crops, weed, and take photos of seed planting with seedlings. A random selection of five farmers (alternatives) has been made. Climate, water, soil, disease, and flooding are all criteria to consider when choosing a farmer. Our goal is to narrow down the options by comparing expert judgments with the criteria.
Keywords: MADM; PyNVWA; PyNVWG; GPyNVWA; GPyNVWG.

1 Introduction

Many authors have contributed to this field of study by using different techniques. As a result of the uncertainties, fuzzy set (FS),[47] intuitionistic set (IFS),[10] interval-valued fuzzy set (IVFS),[18] vague set (VS),[12] Pythagorean set (PyFS),[44] IVPyFS[30] with aggregation operator (AO) and spherical FS (SFS)[9] and neutrosophic set (NS).[36] Xu et al.[43] discussed the concept of regression prediction for fuzzy time series. The membership value (MV) in a set consists of degrees of belongingness that lie between 0 and 1. As a result, Atanassov introduced the concept of an IFS, which is classified by the total of its MV with a non-membership value (NMV) of less than one. The DM approach sometimes generates a single problem when the total of the MVs and NMVs exceeds one. The concept of PyFS was developed by[44] to generalize IFS by ensuring that the square total of its MV and NMV does not exceed one. According to these hypotheses, the neutral state cannot be demonstrated (neither favour nor disfavour).[14] proposed the notion of a picture FS, and used three-pointers; positive, neutral, and negative, with a total of not more than one value. Moreover, it has more advantages than IFS and PyFS for a few applications[1–3,11,16,17,42] for supporting the use of these sets for DM. According to,[22] the idea

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of generalized PyFS with an aggregation operator (AO) was introduced and its applications were described. The characteristics of PyIVFS with AOs are described in.[30–32,45] Rarely, we present a DM approach challenge where the sum of the truth membership value (TMV), the indeterminacy membership value (IMV), and the false membership value (FMV) is greater than one. Thus,[9] propose an SFS with a square total of TMV, IMV, and FMV less than one. According to,[15] SFS can be conceptualized using the TOPSIS approach.

Smarandache et al.[37] introduced the concept of neutrosophic logic and sets. Neutrosophy refers to the knowledge of neutral thought, and that neutrality is the main difference between FS and IFS. The concept of NSS was introduced by.[37] A value of truth, an indeterminacy value, and a falsity value are assigned to each proposition in this logic. NS in which every element of the universe has a value of truth, indeterminacy, and falsity, respectively, between 0 and 1. NS generalizes a classical set, FS, IVFS etc. A method based on AOs for MCDM under interval NS was presented by.[46] A discussion of IVNS in MCDM problems was presented by.[48] The VS was launched by Gau et al.[12] In a VS, two functions are defined, namely, a TMG t_v and a FMG f_v, with $t_v(x)$ denoting the TMV of x derived from the evidence for x, and $f_v(x)$ denoting the FMV of x derived from the evidence against x, as well as $t_v(x)$ and $f_v(x)$ belonging to $[0, 1]$ and $t_v(x)$, in which the total does not exceed one. There are several recognized applications of IVFS and FS, which are extensions of VS.[13,21,41] Zhang et al. introduced the PyFS should be expanded to include multi-criteria decision-making (MCDM) using TOPSIS.[49] Hwang et al.,[19] MADM practical problems are discussed in their discussion. A novel approach for robust single-valued NS with AOs was developed by.[20] A study of spherical vague normal operators and their applications in farmer selection using MADM by Palanikumar et al.[27] Recently, Uluçay et al.[34,35,38–40] discussed the concept of generalization of neutrosophic soft set and its various applications. Palanikumar et al.[26] discussed Pythagorean neutrosophic normal interval-valued operators based on MADM. Recently, Palanikumar et al. discussed many algebraic structures and aggregation operators with applications.[23–25,28,29,33] Many researchers[4–8] discussed the concept of Pythagorean with its extension based on DM.

An introduction is found in Section 1. The PyNS and VS information is described in Section 2. The definition and some operations of log PyNVNs are provided in Section 3. Also, we discuss the relationship between log PyNVN and NFN. Therefore, $\Lambda_1 = ([\log T_{\Lambda 1}, \log(1 - F_{\Lambda 1})], [\log I_{\Lambda 1}, \log I_{\Lambda 1}], [\log F_{\Lambda 1}, \log(1 - T_{\Lambda 1})]) = ([1, 1], [1, 1], [0, 0])$ and $\Lambda_2 = ([\log T_{\Lambda 2}, \log(1 - F_{\Lambda 2})], [\log I_{\Lambda 2}, \log I_{\Lambda 2}], [\log F_{\Lambda 2}, \log(1 - T_{\Lambda 2})]) = ([1, 1], [1, 1], [0, 0])$ which is known as the distance between log PyNVNs is transformed to the distance between NFN. In Section 4, we discuss MADM with the ED and HD with log PyNVNs. There is an interaction between MADM and AOs for log PyNVN in Section 5. The section 6 discusses an application of log PyNVNS, as well as the insert algorithm, flowchart, and numerical example. The conclusion is provided in Section 7. Accordingly, the paper has the following outcomes:

1. Several algebraic properties of log PyNVSs have been established such as associativity, distributivity and idempotency.
2. A log PyNVS is characterized by HD and ED. The purpose of this method is to calculate the ED distance between two log PyNVNs. In addition, we have discussed the idea of converting log PyNVNs into NFNs as well.
3. The purpose of this study is to demonstrate numerically that MADMs and AOs are applied to real-world problems using the log PyNVSs. There is a need to develop an algorithm for a log PyNVS. Additionally, I have to use a log PyNVS algorithm to determine a normalized decision matrix based on the response matrix.
4. It is imperative that an ideal value be determined for each of the concepts referred to in the log PyNVWA, log PyNVWG, log GPyNVWA, and log GPyNVWG.
5. As a result, the decision-maker can select the ranking results based on the decision-maker's preference by log PyNVWA, log PyNVWG, log GPyNVWA, and log GPyNVWG operators in a flexible manner.
6. It is observed that some examples are examined to assess the validity of the proposed models.

2 Basic concepts

In this section, we will review the concepts of PyFS and VS.

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Definition 2.1. [44] Let U be the universal set. The PyFS $\Lambda = \Big\{\xi, \langle \zeta^T_\Lambda(\xi), \zeta^F_\Lambda(\xi)\rangle \big| \xi \in U\Big\}$, $\zeta^T_\Lambda : U \to [0,1]$ and $\zeta^F_\Lambda : U \to [0,1]$ denotes MG and NMG of $\xi \in U$ to Λ, respectively and $0 \preceq (\zeta^T_\Lambda(\xi))^2 + (\zeta^F_\Lambda(\xi))^2 \preceq 1$. For convenience, $\Lambda = \langle \zeta^T_\Lambda, \zeta^F_\Lambda\rangle$ is called the Pythagorean fuzzy number (PyFN).

Definition 2.2. [30] The Pythagorean IVFS (PyIVFS) $\Lambda = \Big\{\xi, \Big\langle \widetilde{\zeta^T_\Lambda}(\xi), \widetilde{\zeta^F_\Lambda}(\xi)\Big\rangle \Big| \xi \in U\Big\}$, where $\widetilde{\zeta^T_\Lambda} : U \to \mathrm{Int}([0,1])$ and $\widetilde{\zeta^F_\Lambda} : U \to \mathrm{Int}([0,1])$ denotes MG and NMG of $\xi \in U$ to Λ, respectively, and $0 \preceq (\zeta^{T+}_\Lambda(\xi))^2 + (\zeta^{F+}_\Lambda(\xi))^2 \preceq 1$. For convenience, $\Lambda = \Big\langle \Big[\zeta^{T-}_\Lambda, \zeta^{T+}_\Lambda\Big], \Big[\zeta^{F-}_\Lambda, \zeta^{F+}_\Lambda\Big]\Big\rangle$ is called the PyIVFN.

Definition 2.3. [26] The PyN $\Lambda = \Big\{\xi, \langle \zeta^T_\Lambda(\xi), \zeta^I_\Lambda(\xi), \zeta^F_\Lambda(\xi)\rangle \big| \xi \in U\Big\}$, where $\zeta^T_\Lambda : U \to [0,1]$, $\zeta^I_\Lambda : U \to [0,1]$ and $\zeta^F_\Lambda : U \to [0,1]$ denotes TMG, IMG and FMG of $\xi \in U$ to Λ, respectively and $0 \preceq (\zeta^T_\Lambda(\xi))^2 + (\zeta^I_\Lambda(\xi))^2 + (\zeta^F_\Lambda(\xi))^2 \preceq 2$. For convenience, $\Lambda = \langle \zeta^T_\Lambda, \zeta^I_\Lambda, \zeta^F_\Lambda\rangle$ is called the PyNFN.

Definition 2.4. [12] (i) A VS Λ in U is a pair (T_Λ, F_Λ), $T_\Lambda : U \to [0,1], F_\Lambda : U \to [0,1]$ are mappings such that $T_\Lambda(\xi) + F_\Lambda(\xi) \preceq 1, \forall \xi \in U$, T_Λ and F_Λ denotes the truth and false membership function, respectively.
(ii) $\Lambda(\xi) = [T_\Lambda(\xi), 1 - F_\Lambda(\xi)]$ is called the vague value of ξ in Λ.

Definition 2.5. [12] (i) A VS Λ is contained in VS Λ_1, $\Lambda \subseteq \Lambda_1$ if and only if $\Lambda(\xi) \preceq \Lambda_1(\xi)$. That is, $T_\Lambda(\xi) \preceq T_{\Lambda_1}(\xi)$ and $1 - F_\Lambda(\xi) \preceq 1 - F_{\Lambda_1}(\xi), \forall \xi \in U$.
(ii) The union of two VSs Λ and Λ_1, as $X = \Lambda \cup \Lambda_1$, $T_X = \max\{T_\Lambda, T_{\Lambda_1}\}$ and $1 - F_X = \max\{1 - F_\Lambda, 1 - F_{\Lambda_1}\} = 1 - \min\{F_\Lambda, F_{\Lambda_1}\}$.
(iii) The intersection of two VSs Λ and Λ_1 as $X = \Lambda \cap \Lambda_1$, $T_X = \min\{T_\Lambda, T_{\Lambda_1}\}$ and $1 - F_X = \min\{1 - F_\Lambda, 1 - F_{\Lambda_1}\} = 1 - \max\{F_\Lambda, F_{\Lambda_1}\}$.

Definition 2.6. [12] A VS Λ of a set U. Then $\forall \xi \in U$,
(i) $T_\Lambda(\xi) = 0$ and $F_\Lambda(\xi) = 1$ is called a zero VS of U,
(ii) $T_\Lambda(\xi) = 1$ and $F_\Lambda(\xi) = 0$ is called a unit VS of U.

3 log PyNVN and its Operations

Several intriguing fundamental operations are described for the log PyNVN.

Definition 3.1. The log PyNVS $\Lambda = \Big\{\xi, \Big\langle \Big[\log T_\Lambda(\xi), \log(1 - F_\Lambda(\xi))\Big], \Big[\log I_\Lambda(\xi), \log I_\Lambda(\xi)\Big], \Big[\log F_\Lambda(\xi), \log(1 - T_\Lambda(\xi))\Big]\Big\rangle \Big| \xi \in U\Big\}$, $\widetilde{\zeta^T_\Lambda} : U \to \mathrm{Int}([0,1])$, $\widetilde{\zeta^I_\Lambda} : U \to \mathrm{Int}([0,1])$ and $\widetilde{\zeta^F_\Lambda} : U \to \mathrm{Int}([0,1])$ denotes TMG, IMG and FMG of $\xi \in U$ to Λ, respectively and $0 \preceq (\log_{\Psi_i} 1 - F_\Lambda(\xi))^2 + (\log_{\Psi_i} I_\Lambda(\xi))^2 + (\log_{\Psi_i} 1 - T_\Lambda(\xi))^2 \preceq 2$, where $\Psi = \prod [T_\Lambda, 1 - F_\Lambda], [I_\Lambda, I_\Lambda], [F_\Lambda, 1 - T_\Lambda]$. For convenience, $\Lambda = \Big\langle \Big[\log T_\Lambda, \log(1 - F_\Lambda)\Big], \Big[\log I_\Lambda, \log I_\Lambda\Big], \Big[\log F_\Lambda, \log(1 - T_\Lambda)\Big]\Big\rangle$ is called the log PyNVN.

Definition 3.2. Let $\Lambda = \Big\langle \Big[\log T_\Lambda, \log(1 - F_\Lambda)\Big], \Big[\log I_\Lambda, \log I_\Lambda\Big], \Big[\log F_\Lambda, \log(1 - T_\Lambda)\Big]\Big\rangle$ be the log PyNVN, the score function of Λ is defined as $\mathbb{S}(\Lambda) = \frac{\mathbb{S}_1(\Lambda) + \mathbb{S}_2(\Lambda)}{2}$, $-1 \preceq \mathbb{S}(\Lambda) \preceq 1$, where

$$\mathbb{S}_1(\Lambda) = \left(\frac{\mathbb{X}}{2} + 1 - \frac{\mathbb{Y}}{2} + 1 - \frac{\mathbb{Z}}{2}\right), \mathbb{S}_2(\Lambda) = \left(\frac{\mathbb{X}}{2} + 1 - \frac{\mathbb{Y}}{2} + 1 - \frac{\mathbb{Z}}{2}\right).$$

The accuracy function of Λ is $\mathbb{A}(\Lambda) = \frac{\mathbb{A}_1(\Lambda) + \mathbb{A}_2(\Lambda)}{2}$, $0 \preceq \mathbb{A}(\Lambda) \preceq 1$,

$$\mathbb{A}_1(\Lambda) = \left(\frac{\mathbb{X}}{2} + 1 + \frac{\mathbb{Y}}{2} + 1 + \frac{\mathbb{Z}}{2}\right), \mathbb{A}_2(\Lambda) = \left(\frac{\mathbb{X}}{2} + 1 + \frac{\mathbb{Y}}{2} + 1 + \frac{\mathbb{Z}}{2}\right),$$

where $\mathbb{X} = (\log_{\Psi_i} T_\Lambda)^2 + (\log_{\Psi_i}(1 - F_\Lambda))^2$, $\mathbb{Y} = (\log_{\Psi_i} I_\Lambda)^2 + (\log_{\Psi_i} I_\Lambda)^2$, and $\mathbb{Z} = (\log_{\Psi_i} F_\Lambda)^2 + (\log_{\Psi_i}(1 - T_\Lambda))^2$.

Definition 3.3. Let $\Lambda = \Big\langle [\log T_\Lambda, \log(1 - F_\Lambda)], [\log I_\Lambda, \log I_\Lambda], [\log F_\Lambda, \log(1 - T_\Lambda)]\Big\rangle$,
$\Lambda_1 = \Big\langle [\log T_{\Lambda 1}, \log(1 - F_{\Lambda 1})], [\log I_{\Lambda 1}, \log I_{\Lambda 1}], [\log F_{\Lambda 1}, \log(1 - T_{\Lambda 1})]\Big\rangle$,
and $\Lambda_2 = \Big\langle [\log T_{\Lambda 2}, \log(1 - F_{\Lambda 2})], [\log I_{\Lambda 2}, \log I_{\Lambda 2}], [\log F_{\Lambda 2}, \log(1 - T_{\Lambda 2})]\Big\rangle$ be any three log PyNVNs and $\Psi = \prod [T_{\Lambda_i}, 1 - F_{\Lambda_i}], [I_{\Lambda_i}, I_{\Lambda_i}], [F_{\Lambda_i}, 1 - T_{\Lambda_i}]$. Their following operations are defined as follows:

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1. $\Lambda_1 \boxplus \Lambda_2 =$
$$\left[\begin{array}{c}\left[\begin{array}{c}\sqrt{(\log_{\Psi_i} T_{\Lambda 1})^2 + (\log_{\Psi_i} T_{\Lambda 2})^2 - (\log_{\Psi_i} T_{\Lambda 1})^2 \cdot (\log_{\Psi_i} T_{\Lambda 2})^2}, \\ \sqrt{(\log_{\Psi_i} (1-F_{\Lambda 1}))^2 + (\log_{\Psi_i} (1-F_{\Lambda 2}))^2 - (\log_{\Psi_i} (1-F_{\Lambda 1}))^2 \cdot (\log_{\Psi_i} (1-F_{\Lambda 2}))^2}\end{array}\right], \\ \left[\begin{array}{c}\sqrt{(\log_{\Psi_i} I_{\Lambda 1})^2 + (\log_{\Psi_i} I_{\Lambda 2})^2 - (\log_{\Psi_i} I_{\Lambda 1})^2 \cdot (\log_{\Psi_i} I_{\Lambda 2})^2}, \\ \sqrt{(\log_{\Psi_i} I_{\Lambda 1})^2 + (\log_{\Psi_i} I_{\Lambda 2})^2 - (\log_{\Psi_i} I_{\Lambda 1})^2 \cdot (\log_{\Psi_i} I_{\Lambda 2})^2}\end{array}\right], \\ \left[\log_{\Psi_i} F_{\Lambda 1} \cdot \log_{\Psi_i} F_{\Lambda 2}, \log_{\Psi_i} (1-T_{\Lambda 1}) \cdot \log_{\Psi_i} (1-T_{\Lambda 2})\right]\end{array}\right],$$

2. $\Lambda_1 \boxtimes \Lambda_2 =$
$$\left[\begin{array}{c}\left[\log_{\Psi_i} T_{\Lambda 1} \cdot \log_{\Psi_i} T_{\Lambda 2}, \log_{\Psi_i} (1-F_{\Lambda 1}) \cdot \log_{\Psi_i} (1-F_{\Lambda 2})\right], \\ \left[\begin{array}{c}\sqrt{(\log_{\Psi_i} I_{\Lambda 1})^2 + (\log_{\Psi_i} I_{\Lambda 2})^2 - (\log_{\Psi_i} I_{\Lambda 1})^2 \cdot (\log_{\Psi_i} I_{\Lambda 2})^2}, \\ \sqrt{(\log_{\Psi_i} I_{\Lambda 1})^2 + (\log_{\Psi_i} I_{\Lambda 2})^2 - (\log_{\Psi_i} I_{\Lambda 1})^2 \cdot (\log_{\Psi_i} I_{\Lambda 2})^2}\end{array}\right], \\ \left[\begin{array}{c}\sqrt{(\log_{\Psi_i} F_{\Lambda 1})^2 + (\log_{\Psi_i} F_{\Lambda 2})^2 - (\log_{\Psi_i} F_{\Lambda 1})^2 \cdot (\log_{\Psi_i} F_{\Lambda 2})^2}, \\ \sqrt{(\log_{\Psi_i} (1-T_{\Lambda 1}))^2 + (\log_{\Psi_i} (1-T_{\Lambda 2}))^2 - (\log_{\Psi_i} (1-T_{\Lambda 1}))^2 \cdot (\log_{\Psi_i} (1-T_{\Lambda 2}))^2}\end{array}\right]\end{array}\right],$$

3. $$\Omega \cdot \Lambda = \left[\begin{array}{c}\left[\sqrt{1-\left(1-(\log_{\Psi_i} T_\Lambda)^2\right)^\Omega}, \sqrt{1-\left(1-(\log_{\Psi_i} (1-F_\Lambda))^2\right)^\Omega}\right], \\ \left[\sqrt{1-\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)^\Omega}, \sqrt{1-\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)^\Omega}\right], \\ \left[(\log_{\Psi_i} F_\Lambda)^\Omega, (\log_{\Psi_i} (1-T_\Lambda))^\Omega\right]\end{array}\right],$$

4. $$\Lambda^\Omega = \left[\begin{array}{c}\left[(\log_{\Psi_i} T_\Lambda)^\Omega, (\log_{\Psi_i} (1-F_\Lambda))^\Omega\right], \\ \left[\sqrt{1-\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)^\Omega}, \sqrt{1-\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)^\Omega}\right], \\ \left[\sqrt{1-\left(1-(\log_{\Psi_i} F_\Lambda)^2\right)^\Omega}, \sqrt{1-\left(1-(\log_{\Psi_i} (1-T_\Lambda))^2\right)^\Omega}\right]\end{array}\right].$$

4 Find the distance between log PyNVNs

Measurements of ED and HD were conducted and some mathematical characteristics of log PyNVNs were examined.

Definition 4.1. Let $\Lambda_1 = \left\langle [\log T_{\Lambda 1}, \log(1-F_{\Lambda_1})], [\log I_{\Lambda 1}, \log I_{\Lambda 1}], [\log F_{\Lambda 1}, \log(1-T_{\Lambda_1})] \right\rangle$ and $\Lambda_2 = \left\langle [\log T_{\Lambda 2}, \log(1-F_{\Lambda_2})], [\log I_{\Lambda 2}, \log I_{\Lambda 2}], [\log F_{\Lambda 2}, \log(1-T_{\Lambda_2})] \right\rangle$ be any two log PyNVNs. Then ED between Λ_1 and Λ_2 is

$$\mathscr{D}_E\Big(\Lambda_1, \Lambda_2\Big) = \frac{1}{2}\sqrt{\begin{array}{c}\left[\begin{array}{c}\frac{1+(\log_{\Psi_i} T_{\Lambda 1})^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} F_{\Lambda 1})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_1}))^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} (1-T_{\Lambda_1}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda_2}))^2}{2}\end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{c}\frac{1+(\log_{\Psi_i} T_{\Lambda 1})^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} F_{\Lambda 1})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_1}))^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} (1-T_{\Lambda_1}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda_2}))^2}{2}\end{array}\right]^2\end{array}}$$

and HD between Λ_1 and Λ_2 is defined as

$$\mathscr{D}_\Lambda\Big(\Lambda_1, \Lambda_2\Big) = \frac{1}{2}\left[\begin{array}{c}\left|\begin{array}{c}\frac{1+(\log_{\Psi_i} T_{\Lambda 1})^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} F_{\Lambda 1})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_1}))^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} (1-T_{\Lambda_1}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda_2}))^2}{2}\end{array}\right| \\ +\frac{1}{2}\left|\begin{array}{c}\frac{1+(\log_{\Psi_i} T_{\Lambda 1})^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} F_{\Lambda 1})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_1}))^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} (1-T_{\Lambda_1}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda_2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda_2}))^2}{2}\end{array}\right|\end{array}\right]$$

Theorem 4.2. *Let* $\Lambda_1 = \left\langle [\log T_{\Lambda 1}, \log(1-F_{\Lambda_1})], [\log I_{\Lambda 1}, \log I_{\Lambda 1}], [\log F_{\Lambda 1}, \log(1-T_{\Lambda_1})] \right\rangle$,
$\Lambda_2 = \left\langle [\log T_{\Lambda 2}, \log(1-F_{\Lambda_2})], [\log I_{\Lambda 2}, \log I_{\Lambda 2}], [\log F_{\Lambda 2}, \log(1-T_{\Lambda_2})] \right\rangle$*, and*
$\Lambda_3 = \left\langle [\log T_{\Lambda 3}, \log(1-F_{\Lambda_3})], [\log I_{\Lambda 3}, \log I_{\Lambda 3}], [\log F_{\Lambda 3}, \log(1-T_{\Lambda_3})] \right\rangle$ *be any three log PyNVNs. Then*

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1. $\mathscr{D}_E(\Lambda_1, \Lambda_2) = 0$ *if and only if* $\Lambda_1 = \Lambda_2$,
2. $\mathscr{D}_E(\Lambda_1, \Lambda_2) = \mathscr{D}_E(\Lambda_2, \Lambda_1)$,
3. $\mathscr{D}_E(\Lambda_1, \Lambda_3) \preceq \mathscr{D}_E(\Lambda_1, \Lambda_2) + \mathscr{D}_E(\Lambda_2, \Lambda_3)$.

Proof. There are clear and concise conditions (1) and (2). It remains to be proved (3). Now,
$\Big(\mathscr{D}_E(\Lambda_1, \Lambda_2) + \mathscr{D}_E(\Lambda_2, \Lambda_3)\Big)^2$

$$= \left[\begin{array}{l} \frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\log_{\Psi_i} T_{\Lambda 1})^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} F_{\Lambda 1})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 1}))^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} (1-T_{\Lambda 1}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda 2}))^2}{2}\end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\log_{\Psi_i} T_{\Lambda 1})^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} F_{\Lambda 1})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 1}))^2-(\log_{\Psi_i} I_{\Lambda 1})^2-(\log_{\Psi_i} (1-T_{\Lambda 1}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda 2}))^2}{2}\end{array}\right]^2\end{array}} \\ +\frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda 2}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 3})^2-(\log_{\Psi_i} I_{\Lambda 3})^2-(\log_{\Psi_i} F_{\Lambda 3})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 3}))^2-(\log_{\Psi_i} I_{\Lambda 3})^2-(\log_{\Psi_i} (1-T_{\Lambda 3}))^2}{2}\Psi_3\end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\log_{\Psi_i} T_{\Lambda 2})^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} F_{\Lambda 2})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 2}))^2-(\log_{\Psi_i} I_{\Lambda 2})^2-(\log_{\Psi_i} (1-T_{\Lambda 2}))^2}{2} \\ -\frac{1+(\log_{\Psi_i} T_{\Lambda 3})^2-(\log_{\Psi_i} I_{\Lambda 3})^2-(\log_{\Psi_i} F_{\Lambda 3})^2+1+(\log_{\Psi_i} (1-F_{\Lambda 3}))^2-(\log_{\Psi_i} I_{\Lambda 3})^2-(\log_{\Psi_i} (1-T_{\Lambda 3}))^2}{2}\end{array}\right]^2\end{array}}\end{array}\right]^2$$

$$\begin{aligned}
&= \frac{1}{4}\Big((\gamma_a - \gamma_b)^2 + \frac{1}{2}(\gamma_a - \gamma_b)^2\Big) + \frac{1}{4}\Big((\gamma_b - \gamma_c)^2 + \frac{1}{2}(\gamma_b - \gamma_c)^2\Big) \\
&\quad + \frac{1}{2}\left(\sqrt{(\gamma_a - \gamma_b)^2 + \frac{1}{2}(\gamma_a - \gamma_b)^2} \times \sqrt{(\gamma_b - \gamma_c)^2 + \frac{1}{2}(\gamma_b - \gamma_c)^2}\right) \\
&\succeq \frac{1}{4}\Big((\gamma_a - \gamma_b)^2 + \frac{1}{2}(\gamma_a - \gamma_b)^2\Big) + \frac{1}{4}\Big((\gamma_b - \gamma_c)^2 + \frac{1}{2}(\gamma_b - \gamma_c)^2\Big) \\
&\quad + \frac{1}{2}\Big((\gamma_a - \gamma_b) \times (\gamma_b - \gamma_c) + \frac{1}{2}(\gamma_a - \gamma_b) \times (\gamma_b - \gamma_c)\Big) \\
&= \frac{1}{4}(\gamma_a - \gamma_b + \gamma_b - \gamma_c)^2 + \frac{1}{8}(\gamma_a - \gamma_b + \gamma_b - \gamma_c)^2 \\
&= \frac{1}{4}(\gamma_a - \gamma_c)^2 + \frac{1}{8}(\gamma_a - \gamma_c)^2 \\
&= \mathscr{D}_E(\Lambda_1, \Lambda_3)^2,
\end{aligned}$$

where

$$\gamma_a = \frac{1 + (\log_{\Psi_i} T_{\Lambda 1})^2 - (\log_{\Psi_i} I_{\Lambda 1})^2 - (\log_{\Psi_i} F_{\Lambda 1})^2 + 1 + (\log_{\Psi_i} (1 - F_{\Lambda 1}))^2 - (\log_{\Psi_i} I_{\Lambda 1})^2 - (\log_{\Psi_i} (1 - T_{\Lambda 1}))^2}{2},$$

$$\gamma_b = \frac{1 + (\log_{\Psi_i} T_{\Lambda 2})^2 - (\log_{\Psi_i} I_{\Lambda 2})^2 - (\log_{\Psi_i} F_{\Lambda 2})^2 + 1 + (\log_{\Psi_i} (1 - F_{\Lambda 2}))^2 - (\log_{\Psi_i} I_{\Lambda 2})^2 - (\log_{\Psi_i} (1 - T_{\Lambda 2}))^2}{2},$$

$$\gamma_c = \frac{1 + (\log_{\Psi_i} T_{\Lambda 3})^2 - (\log_{\Psi_i} I_{\Lambda 3})^2 - (\log_{\Psi_i} F_{\Lambda 3})^2 + 1 + (\log_{\Psi_i} (1 - F_{\Lambda 3}))^2 - (\log_{\Psi_i} I_{\Lambda 3})^2 - (\log_{\Psi_i} (1 - T_{\Lambda 3}))^2}{2}.$$

□

Corollary 4.3. *Let* $\Lambda_1 = \Big\langle(\kappa_1, \delta_1); [\log T_{\Lambda 1}, \log{(1 - F_{\Lambda_1})}], [\log I_{\Lambda 1}, \log I_{\Lambda 1}], [\log F_{\Lambda 1}, \log{(1 - T_{\Lambda_1})}]\Big\rangle$,
$\Lambda_2 = \Big\langle(\kappa_2, \delta_2); [\log T_{\Lambda 2}, \log{(1 - F_{\Lambda_2})}], [\log I_{\Lambda 2}, \log I_{\Lambda 2}], [\log F_{\Lambda 2}, \log{(1 - T_{\Lambda_2})}]\Big\rangle$, *and*
$\Lambda_3 = \Big\langle(\kappa_3, \delta_3); [\log T_{\Lambda 3}, \log{(1 - F_{\Lambda_3})}], [\log I_{\Lambda 3}, \log I_{\Lambda 3}], [\log F_{\Lambda 3}, \log{(1 - T_{\Lambda_3})}]\Big\rangle$ *be any three log PyN-VNs. Then*

1. $\mathscr{D}_\Lambda(\Lambda_1, \Lambda_2) = 0$ *if and only if* $\Lambda_1 = \Lambda_2$,
2. $\mathscr{D}_\Lambda(\Lambda_1, \Lambda_2) = \mathscr{D}_\Lambda(\Lambda_2, \Lambda_1)$,

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3. $\mathscr{D}_\Lambda(\Lambda_1, \Lambda_3) \preceq \mathscr{D}_\Lambda(\Lambda_1, \Lambda_2) + \mathscr{D}_\Lambda(\Lambda_2, \Lambda_3)$.

Proof. This proof is based on Theorem 4.2. □

Remark 4.4. Since $\Lambda_1 = ([\log T_{\Lambda 1}, \log(1 - F_{\Lambda_1})], [\log I_{\Lambda 1}, \log I_{\Lambda 1}], [\log F_{\Lambda 1}, \log(1 - T_{\Lambda_1})])$ $= ([1,1],[1,1],[0,0])$ and $\Lambda_2 = ([\log T_{\Lambda 2}, \log(1 - F_{\Lambda_2})], [\log I_{\Lambda 2}, \log I_{\Lambda 2}], [\log F_{\Lambda 2}, \log(1 - T_{\Lambda_2})]) = ([1,1],[1,1],[0,0])$ which is known as the distance between log PyNVNs is transformed to the distance between NFN.

5 AOs based on log PyNVS approach

A novel concept has been presented in this section including log PyNVWA, log PyNVWG, log GPyNVWA, and log GPyNVWG operators using log PyNVS. Based on the operational rules of log PyNVNs, the weighted averaging operators for log PyNVN are presented.

5.1 log PyNV weighted averaging (log PyNVWA) operator

Definition 5.1. Let $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1 - F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1 - T_{\Lambda_i})] \right\rangle$ be the family of log PyNVNs, $\Upsilon = (\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$ be the weight of Λ_i, $\Upsilon_i \succeq 0$ and $\oslash_{i=1}^n \Upsilon_i = 1$ and $\Psi = \prod [T_{\Lambda_i}, 1 - F_{\Lambda_i}], [I_{\Lambda_i}, I_{\Lambda_i}], [F_{\Lambda_i}, 1 - T_{\Lambda_i}]$. Then log PyNVWA operator is log PyNVWA $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \oslash_{i=1}^n \Upsilon_i \Lambda_i$ for $i = 1, 2, ..., n$.

Theorem 5.2 illustrates the aggregated log PyNVWA result based on the above definition.

Theorem 5.2. *(associativity property) Let* $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1 - F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], \right.$ $\left. [\log F_{\Lambda i}, \log(1 - T_{\Lambda_i})] \right\rangle$ *be the family of log PyNVNs. Then log PyNVWA* $(\Lambda_1, \Lambda_2, ..., \Lambda_n)$ *satisfies the associativity property.*

$$\left[\begin{array}{c} \left[\sqrt{1 - \bigcirc_{i=1}^n \left(1 - (\log_{\Psi_i} T_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1 - \bigcirc_{i=1}^n \left(1 - (\log_{\Psi_i} (1 - F_{\Lambda_i}))^2\right)^{\Upsilon_i}}\right], \\ \left[\sqrt{1 - \bigcirc_{i=1}^n \left(1 - (\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1 - \bigcirc_{i=1}^n \left(1 - (\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}\right], \\ \left[\bigcirc_{i=1}^n (\log_{\Psi_i} F_{\Lambda i})^{\Upsilon_i}, \bigcirc_{i=1}^n (\log_{\Psi_i} (1 - T_{\Lambda_i}))^{\Upsilon_i}\right] \end{array}\right].$$

Proof. Using the induction method, the proof can be made.
If $n = 2$, then log PyNVWA $(\Lambda_1, \Lambda_2) = \Upsilon_1 \Lambda_1 \boxplus \Upsilon_2 \Lambda_2$, where

$$\Upsilon_1 \Lambda_1 = \left[\begin{array}{c} \left[\sqrt{1 - \left(1 - (\log_{\Psi_i} T_{\Lambda 1})^2\right)^{\Upsilon_1}}, \sqrt{1 - \left(1 - (\log_{\Psi_i} (1 - F_{\Lambda_1}))^2\right)^{\Upsilon_1}}\right], \\ \left[\sqrt{1 - \left(1 - (\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}}, \sqrt{1 - \left(1 - (\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}}\right], \\ \left[(\log_{\Psi_i} F_{\Lambda 1})^{\Upsilon_1}, (\log_{\Psi_i} (1 - T_{\Lambda_1}))^{\Upsilon_1}\right] \end{array}\right]$$

and

$$\Upsilon_2 \Lambda_2 = \left[\begin{array}{c} \left[\sqrt{1 - \left(1 - (\log_{\Psi_i} T_{\Lambda 2})^2\right)^{\Upsilon_2}}, \sqrt{1 - \left(1 - (\log_{\Psi_i} (1 - F_{\Lambda_2}))^2\right)^{\Upsilon_2}}\right], \\ \left[\sqrt{1 - \left(1 - (\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}}, \sqrt{1 - \left(1 - (\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}}\right], \\ \left[(\log_{\Psi_i} F_{\Lambda 2})^{\Upsilon_2}, (\log_{\Psi_i} (1 - T_{\Lambda_2}))^{\Upsilon_2}\right] \end{array}\right].$$

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Hence,

$$\Upsilon_1\Lambda_1 \boxplus \Upsilon_2\Lambda_2 = \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{\begin{array}{c}\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda 1})^2\right)^{\Upsilon_1}\right)+\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda 2})^2\right)^{\Upsilon_2}\right) \\ -\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda 1})^2\right)^{\Upsilon_1}\right)\cdot\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda 2})^2\right)^{\Upsilon_2}\right),\end{array}} \\ \sqrt{\begin{array}{c}\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_1}))^2\right)^{\Upsilon_1}\right)+\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_2}))^2\right)^{\Upsilon_2}\right) \\ -\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_1}))^2\right)^{\Upsilon_1}\right)\cdot\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_2}))^2\right)^{\Upsilon_2}\right)\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt{\begin{array}{c}\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}\right)+\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}\right) \\ -\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}\right)\cdot\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}\right),\end{array}} \\ \sqrt{\begin{array}{c}\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}\right)+\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}\right) \\ -\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}\right)\cdot\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}\right)\end{array}} \end{array}\right], \\ \left[(\log_{\Psi_i} F_{\Lambda 1})^{\Upsilon_1}\cdot(\log_{\Psi_i} F_{\Lambda 2})^{\Upsilon_2}, (\log_{\Psi_i}(1-T_{\Lambda_1}))^{\Upsilon_1}\cdot(\log_{\Psi_i}(1-T_{\Lambda_2}))^{\Upsilon_2}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{1-\left(1-(\log_{\Psi_i} T_{\Lambda 1})^2\right)^{\Upsilon_1}\cdot\left(1-(\log_{\Psi_i} T_{\Lambda 2})^2\right)^{\Upsilon_2}}, \\ \sqrt{1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_1}))^2\right)^{\Upsilon_1}\cdot\left(1-(\log_{\Psi_i}(1-F_{\Lambda_2}))^2\right)^{\Upsilon_2}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt{1-\left(1-(\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}\cdot\left(1-(\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}}, \\ \sqrt{1-\left(1-(\log_{\Psi_i} I_{\Lambda 1})^2\right)^{\Upsilon_1}\cdot\left(1-(\log_{\Psi_i} I_{\Lambda 2})^2\right)^{\Upsilon_2}} \end{array}\right], \\ \left[(\log_{\Psi_i} F_{\Lambda 1})^{\Upsilon_1}\cdot(\log_{\Psi_i} F_{\Lambda 2})^{\Upsilon_2}, (\log_{\Psi_i}(1-T_{\Lambda_1}))^{\Upsilon_1}\cdot(\log_{\Psi_i}(1-T_{\Lambda_2}))^{\Upsilon_2}\right] \end{array}\right].$$

Thus, log PyNVWA (Λ_1, Λ_2)

$$= \left[\begin{array}{c} \left[\sqrt{1-\bigcirc_{i=1}^{2}\left(1-(\log_{\Psi_i} T_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{2}\left(1-(\log_{\Psi_i}(1-F_{\Lambda_i}))^2\right)^{\Upsilon_i}}\right], \\ \left[\sqrt{1-\bigcirc_{i=1}^{2}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{2}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}\right], \\ \left[\bigcirc_{i=1}^{2}(\log_{\Psi_i} F_{\Lambda i})^{\Upsilon_i}, \bigcirc_{i=1}^{2}(\log_{\Psi_i}(1-T_{\Lambda_i}))^{\Upsilon_i}\right] \end{array}\right].$$

It is valid for $n = l$ and $l \succeq 3$.
Hence, log PyNVWA $(\Lambda_1, \Lambda_2, ..., \Lambda_l)$

$$= \left[\begin{array}{c} \left[\sqrt{1-\bigcirc_{i=1}^{l}\left(1-(\log_{\Psi_i} T_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{l}\left(1-(\log_{\Psi_i}(1-F_{\Lambda_i}))^2\right)^{\Upsilon_i}}\right], \\ \left[\sqrt{1-\bigcirc_{i=1}^{l}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{l}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}\right], \\ \left[\bigcirc_{i=1}^{l}(\log_{\Psi_i} F_{\Lambda i})^{\Upsilon_i}, \bigcirc_{i=1}^{l}(\log_{\Psi_i}(1-T_{\Lambda_i}))^{\Upsilon_i}\right] \end{array}\right].$$

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If $n = l + 1$ and we apply, log PyNVWA $(\Lambda_1, \Lambda_2, ..., \Lambda_l, \Lambda_{l+1})$

$$= \begin{bmatrix} \left[\begin{array}{c} \sqrt{\begin{array}{c} \oslash_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda i})^2\right)^{\Upsilon_i}\right)+\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda l+1})^2\right)^{\Upsilon_{l+1}}\right) \\ -\bigcirc_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda i})^2\right)^{\Upsilon_i}\right)\cdot\left(1-\left(1-(\log_{\Psi_i} T_{\Lambda l+1})^2\right)^{\Upsilon_{l+1}}\right), \end{array}} \\ \sqrt{\begin{array}{c} \oslash_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_i}))^2\right)^{\Upsilon_i}\right)+\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_{l+1}}))^2\right)^{\Upsilon_{l+1}}\right) \\ -\bigcirc_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_i}))^2\right)^{\Upsilon_i}\right)\cdot\left(1-\left(1-(\log_{\Psi_i}(1-F_{\Lambda_{l+1}}))^2\right)^{\Upsilon_{l+1}}\right) \end{array}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt{\begin{array}{c} \oslash_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}\right)+\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda l+1})^2\right)^{\Upsilon_{l+1}}\right) \\ -\bigcirc_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}\right)\cdot\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda l+1})^2\right)^{\Upsilon_{l+1}}\right), \end{array}} \\ \sqrt{\begin{array}{c} \oslash_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}\right)+\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda l+1})^2\right)^{\Upsilon_{l+1}}\right) \\ -\bigcirc_{i=1}^{l}\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}\right)\cdot\left(1-\left(1-(\log_{\Psi_i} I_{\Lambda l+1})^2\right)^{\Upsilon_{l+1}}\right) \end{array}} \end{array} \right], \\ \left[\bigcirc_{i=1}^{l}(\log_{\Psi_i} F_{\Lambda i})^{\Upsilon_i}\cdot(\log_{\Psi_i} F_{\Lambda l+1})^{\Upsilon_{l+1}}, \bigcirc_{i=1}^{l}(\log_{\Psi_i}(1-T_{\Lambda_i}))^{\Upsilon_i}\cdot(\log_{\Psi_i}(1-T_{\Lambda_{l+1}}))^{\Upsilon_{l+1}}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left[\sqrt{1-\bigcirc_{i=1}^{l+1}\left(1-(\log_{\Psi_i} T_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{l+1}\left(1-(\log_{\Psi_i}(1-F_{\Lambda_i}))^2\right)^{\Upsilon_i}}\right], \\ \left[\sqrt{1-\bigcirc_{i=1}^{l+1}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{l+1}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}\right], \\ \left[\bigcirc_{i=1}^{l+1}(\log_{\Psi_i} F_{\Lambda i})^{\Upsilon_i}, \bigcirc_{i=1}^{l+1}(\log_{\Psi_i}(1-T_{\Lambda_i}))^{\Upsilon_i}\right] \end{bmatrix}.$$

□

Theorem 5.3. *(idempotency property) If all* $\Lambda_i = \Big\langle [\log T_{\Lambda i}, \log(1-F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}],$ $[\log F_{\Lambda i}, \log(1-T_{\Lambda_i})]\Big\rangle$ $(i = 1, 2, ..., n)$ *are equal and* $\Lambda_i = \Lambda$*, then log PyNVWA* $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \Lambda$.

Proof. Given that $[\log T_{\Lambda i}, \log(1-F_{\Lambda_i})] = [\log T_\Lambda, \log(1-F_\Lambda)]$, $[\log I_{\Lambda i}, \log I_{\Lambda i}] = [\log I_\Lambda, \log I_\Lambda]$ and $[\log F_{\Lambda i}, \log(1-T_{\Lambda_i})] = [\log F_\Lambda, \log(1-T_\Lambda)]$, for $i = 1, 2, ..., n$ and $\oslash_{i=1}^{n}\Upsilon_i = 1$. Now, log PyNVWA $(\Lambda_1, \Lambda_2, ..., \Lambda_n)$

$$\begin{aligned} &= \begin{bmatrix} \left[\sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i} T_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i}(1-F_{\Lambda_i}))^2\right)^{\Upsilon_i}}\right], \\ \left[\sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}\right], \\ \left[\bigcirc_{i=1}^{n}(\log_{\Psi_i} F_{\Lambda i})^{\Upsilon_i}, \bigcirc_{i=1}^{n}(\log_{\Psi_i}(1-T_{\Lambda_i}))^{\Upsilon_i}\right] \end{bmatrix} \\ &= \begin{bmatrix} \left[\sqrt{1-\left(1-(\log_{\Psi_i} T_\Lambda)^2\right)^{\oslash_{i=1}^{n}\Upsilon_i}}, \sqrt{1\left(1-(\log_{\Psi_i}(1-F_\Lambda))^2\right)^{\oslash_{i=1}^{n}\Upsilon_i}}\right], \\ \left[\sqrt{1-\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)^{\oslash_{i=1}^{n}\Upsilon_i}}, \sqrt{1\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)^{\oslash_{i=1}^{n}\Upsilon_i}}\right], \\ \left[(\log_{\Psi_i} F_\Lambda)^{\oslash_{i=1}^{n}\Upsilon_i}, (\log_{\Psi_i}(1-T_\Lambda))^{\oslash_{i=1}^{n}\Upsilon_i}\right] \end{bmatrix} \\ &= \begin{bmatrix} \left[\sqrt{1-\left(1-(\log_{\Psi_i} T_\Lambda)^2\right)}, \sqrt{1-\left(1-(\log_{\Psi_i}(1-F_\Lambda))^2\right)}\right], \\ \left[\sqrt{1-\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)}, \sqrt{1-\left(1-(\log_{\Psi_i} I_\Lambda)^2\right)}\right], \\ \left[(\log_{\Psi_i} F_\Lambda), (\log_{\Psi_i}(1-T_\Lambda))\right] \end{bmatrix} \\ &= \Lambda. \end{aligned}$$

□

Theorem 5.4. *(boundedness property) Let* $\Lambda_i = \Big\langle [\log T_{\Lambda ij}, \log(1-F_{\Lambda_{ij}})], [\log I_{\Lambda ij}, \log I_{\Lambda ij}], [\log F_{\Lambda ij},$ $\log(1-T_{\Lambda_{ij}})]\Big\rangle$ $(i = 1, 2, ..., n; j = 1, 2, ..., i_j)$ *be the collection of log PyNVWA, where* $\underbrace{\log_{\Psi_i} T_\Lambda} =$

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$\min \log_{\Psi_i} T_{\Lambda ij}$, $\overbrace{\log_{\Psi_i} T_{\Lambda}} = \max \log_{\Psi_i} T_{\Lambda ij}$, $\underbrace{\log_{\Psi_i} (1-F_{\Lambda})} = \min \log_{\Psi_i} (1-F_{\Lambda ij})$, $\overbrace{\log_{\Psi_i} (1-F_{\Lambda})} = \max \log_{\Psi_i} (1-F_{\Lambda ij})$, $\underbrace{\log_{\Psi_i} I_{\Lambda}} = \min \log_{\Psi_i} I_{\Lambda ij}$, $\overbrace{\log_{\Psi_i} I_{\Lambda}} = \max \log_{\Psi_i} I_{\Lambda ij}$, $\underbrace{\log_{\Psi_i} I_{\Lambda}} = \min \log_{\Psi_i} I_{\Lambda ij}$, $\overbrace{\log_{\Psi_i} I_{\Lambda}} = \max \log_{\Psi_i} I_{\Lambda ij}$, $\underbrace{\log_{\Psi_i} F_{\Lambda}} = \min \log_{\Psi_i} F_{\Lambda ij}$, $\overbrace{\log_{\Psi_i} F_{\Lambda}} = \max \log_{\Psi_i} F_{\Lambda ij}$, $\underbrace{\log_{\Psi_i} (1-T_{\Lambda})} = \min \log_{\Psi_i} (1-T_{\Lambda ij})$, $\overbrace{\log_{\Psi_i} (1-T_{\Lambda})} = \max \log_{\Psi_i} (1-T_{\Lambda ij})$. *Then*

$$\begin{aligned}
&\Big\langle [\underbrace{\log_{\Psi_i} T_{\Lambda}}, \underbrace{\log_{\Psi_i} (1-F_{\Lambda})}], [\underbrace{\log_{\Psi_i} I_{\Lambda}}, \underbrace{\log_{\Psi_i} I_{\Lambda}}], [\overbrace{\log_{\Psi_i} F_{\Lambda}}, \overbrace{\log_{\Psi_i} (1-T_{\Lambda})}] \Big\rangle \\
&\qquad \preceq \textit{q-Rung} \ \log \ \textit{PyNVNWA}(\Lambda_1, \Lambda_2, ..., \Lambda_n) \\
&\qquad \preceq \Big\langle [\overbrace{\log_{\Psi_i} T_{\Lambda}}, \overbrace{\log_{\Psi_i} (1-F_{\Lambda})}], [\overbrace{\log_{\Psi_i} I_{\Lambda}}, \overbrace{\log_{\Psi_i} I_{\Lambda}}], [\underbrace{\log_{\Psi_i} F_{\Lambda}}, \underbrace{\log_{\Psi_i} (1-T_{\Lambda})}] \Big\rangle,
\end{aligned}$$

where $1 \preceq i \preceq n, \ j = 1, 2, ..., i_j$.

Proof. Since $\underbrace{\log_{\Psi_i} T_{\Lambda}} = \min \log_{\Psi_i} T_{\Lambda ij}, \overbrace{\log_{\Psi_i} T_{\Lambda}} = \max \log_{\Psi_i} T_{\Lambda ij}, \underbrace{\log_{\Psi_i} (1-F_{\Lambda})} = \min \log_{\Psi_i} (1-F_{\Lambda ij})$, $\overbrace{\log_{\Psi_i} (1-F_{\Lambda})} = \max \log_{\Psi_i} (1-F_{\Lambda ij})$ and $\underbrace{\log_{\Psi_i} T_{\Lambda}} \preceq \log_{\Psi_i} T_{\Lambda ij} \preceq \overbrace{\log_{\Psi_i} T_{\Lambda}}$ and $\underbrace{\log_{\Psi_i} (1-F_{\Lambda})} \preceq \log_{\Psi_i} (1-F_{\Lambda})_{ij} \preceq \overbrace{\log_{\Psi_i} (1-F_{\Lambda})}$, we have

$$\begin{aligned}
& \underbrace{\log_{\Psi_i} T_{\Lambda}} + \underbrace{\log_{\Psi_i} (1-F_{\Lambda})} \\
= \ & \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\underbrace{\log_{\Psi_i} T_{\Lambda}})^2\Big)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\underbrace{\log_{\Psi_i} (1-F_{\Lambda})})^2\Big)^{\Upsilon_i}} \\
\preceq \ & \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\log_{\Psi_i} T_{\Lambda ij})^2\Big)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\log_{\Psi_i} (1-F_{\Lambda ij}))^2\Big)^{\Upsilon_i}} \\
\preceq \ & \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\overbrace{\log_{\Psi_i} T_{\Lambda}})^2\Big)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\overbrace{\log_{\Psi_i} (1-F_{\Lambda})})^2\Big)^{\Upsilon_i}} \\
= \ & \overbrace{\log_{\Psi_i} T_{\Lambda}} + \overbrace{\log_{\Psi_i} (1-F_{\Lambda})}.
\end{aligned}$$

Since $\underbrace{\log_{\Psi_i} I_{\Lambda}} = \min \log_{\Psi_i} I_{\Lambda ij}$, $\overbrace{\log_{\Psi_i} I_{\Lambda}} = \max \log_{\Psi_i} I_{\Lambda ij}$, $\underbrace{\log_{\Psi_i} I_{\Lambda}} = \min \log_{\Psi_i} I_{\Lambda ij}$, $\overbrace{\log_{\Psi_i} I_{\Lambda}} = \max \log_{\Psi_i} I_{\Lambda ij}$ and $\underbrace{\log_{\Psi_i} I_{\Lambda}} \preceq \log_{\Psi_i} I_{\Lambda ij} \preceq \overbrace{\log_{\Psi_i} I_{\Lambda}}$ and $\underbrace{\log_{\Psi_i} I_{\Lambda}} \preceq \log_{\Psi_i} I_{\Lambda ij} \preceq \overbrace{\log_{\Psi_i} I_{\Lambda}}$, we have

$$\begin{aligned}
\underbrace{\log_{\Psi_i} I_{\Lambda}} + \underbrace{\log_{\Psi_i} I_{\Lambda}} &= \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\underbrace{\log_{\Psi_i} I_{\Lambda}})^2\Big)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\underbrace{\log_{\Psi_i} I_{\Lambda}})^2\Big)^{\Upsilon_i}} \\
&\preceq \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\log_{\Psi_i} I_{\Lambda ij})^2\Big)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\log_{\Psi_i} I_{\Lambda ij})^2\Big)^{\Upsilon_i}} \\
&\preceq \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\overbrace{\log_{\Psi_i} I_{\Lambda}})^2\Big)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n} \Big(1 - (\overbrace{\log_{\Psi_i} I_{\Lambda}})^2\Big)^{\Upsilon_i}} \\
&= \overbrace{\log_{\Psi_i} I_{\Lambda}} + \overbrace{\log_{\Psi_i} I_{\Lambda}}.
\end{aligned}$$

Since $\underbrace{\log_{\Psi_i} F_{\Lambda}} = \min \log_{\Psi_i} F_{\Lambda ij}$, $\overbrace{\log_{\Psi_i} F_{\Lambda}} = \max \log_{\Psi_i} F_{\Lambda ij}$ $\underbrace{\log_{\Psi_i} (1-T_{\Lambda})} = \min \log_{\Psi_i} (1-T_{\Lambda ij})$, $\overbrace{\log_{\Psi_i} (1-T_{\Lambda})} = \max \log_{\Psi_i} (1-T_{\Lambda ij})$ and $\underbrace{\log_{\Psi_i} F_{\Lambda}} \preceq \log_{\Psi_i} F_{\Lambda ij} \preceq \overbrace{\log_{\Psi_i} F_{\Lambda}}$ and $\underbrace{\log_{\Psi_i} (1-T_{\Lambda})} \preceq$

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$\log_{\Psi_i}(1-T_{\Lambda_{ij}}) \preceq \overbrace{\log_{\Psi_i}(1-T_\Lambda)}$, we have

$$\begin{aligned}
\underbrace{\log_{\Psi_i} F_\Lambda}+\underbrace{\log_{\Psi_i}(1-T_\Lambda)} &= \bigcirc_{i=1}^n(\underbrace{\log_{\Psi_i} F_\Lambda})^{\Upsilon_i}+\bigcirc_{i=1}^n(\underbrace{\log_{\Psi_i}(1-T_\Lambda)})^{\Upsilon_i}\\
&\preceq \bigcirc_{i=1}^n(\log_{\Psi_i} F_{\Lambda ij})^{\Upsilon_i}+\bigcirc_{i=1}^n(\log_{\Psi_i}(1-T_{\Lambda_{ij}}))^{\Upsilon_i}\\
&\preceq \bigcirc_{i=1}^n(\overbrace{\log_{\Psi_i} F_\Lambda})^{\Upsilon_i}+\bigcirc_{i=1}^n(\overbrace{\log_{\Psi_i}(1-T_\Lambda)})^{\Upsilon_i}\\
&= \overbrace{\log_{\Psi_i} F_\Lambda}+\overbrace{\log_{\Psi_i}(1-T_\Lambda)}.
\end{aligned}$$

Therefore,

$$\left[\cfrac{\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\underbrace{\log_{\Psi_i} T_\Lambda})^2\right)^{\Upsilon_i}}\right)^2+\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\underbrace{\log_{\Psi_i}(1-F_\Lambda)})^2\right)^{\Upsilon_i}}\right)^2}{2+1-\cfrac{\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\underbrace{\log_{\Psi_i} I_\Lambda})^2\right)^{\Upsilon_i}}\right)^2+\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\underbrace{\log_{\Psi_i} I_\Lambda})^2\right)^{\Upsilon_i}}\right)^2}{2+1-\cfrac{\left(\bigcirc_{i=1}^n(\overbrace{\log_{\Psi_i} F_\Lambda})^{\Upsilon_i}\right)^2+\left(\bigcirc_{i=1}^n(\overbrace{\log_{\Psi_i}(1-T_\Lambda)})^{\Upsilon_i}\right)^2}{2}}}\right]$$

$$=\left[\cfrac{\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\log_{\Psi_i} T_{\Lambda ij})^2\right)^{\Upsilon_i}}\right)^2+\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\log_{\Psi_i}(1-F_{\Lambda ij}))^2\right)^{\Upsilon_i}}\right)^2}{2+1-\cfrac{\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\log_{\Psi_i} I_{\Lambda ij})^2\right)^{\Upsilon_i}}\right)^2+\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\log_{\Psi_i} I_{\Lambda ij})^2\right)^{\Upsilon_i}}\right)^2}{2+1-\frac{\left(\bigcirc_{i=1}^n(\log_{\Psi_i} F_{\Lambda ij})^{\Upsilon_i}\right)^2+\left(\bigcirc_{i=1}^n(\log_{\Psi_i}(1-T_{\Lambda_{ij}}))^{\Upsilon_i}\right)^2}{2}}}\right]$$

$$=\left[\cfrac{\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\overbrace{\log_{\Psi_i} T_\Lambda})^2\right)^{\Upsilon_i}}\right)^2+\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\overbrace{\log_{\Psi_i}(1-F_\Lambda)})^2\right)^{\Upsilon_i}}\right)^2}{2+1-\cfrac{\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\overbrace{\log_{\Psi_i} I_\Lambda})^2\right)^{\Upsilon_i}}\right)^2+\left(\sqrt{1-\bigcirc_{i=1}^n\left(1-(\overbrace{\log_{\Psi_i} I_\Lambda})^2\right)^{\Upsilon_i}}\right)^2}{2+1-\cfrac{\left(\bigcirc_{i=1}^n(\underbrace{\log_{\Psi_i} F_\Lambda})^{\Upsilon_i}\right)^2+\left(\bigcirc_{i=1}^n(\underbrace{\log_{\Psi_i}(1-T_\Lambda)})^{\Upsilon_i}\right)^2}{2}}}\right].$$

Hence, $\left\langle[\underbrace{\log T_\Lambda},\underbrace{\log(1-F_\Lambda)}],[\underbrace{\log I_\Lambda},\underbrace{\log I_\Lambda}],[\overbrace{\log F_\Lambda},\overbrace{\log(1-T_\Lambda)}]\right\rangle$

$$\begin{aligned}
&\preceq \log\ \text{PyNVWA}(\Lambda_1,\Lambda_2,...,\Lambda_n)\\
&\preceq \left\langle[\overbrace{\log T_\Lambda},\overbrace{\log(1-F_\Lambda)}],[\overbrace{\log I_\Lambda},\overbrace{\log I_\Lambda}],[\underbrace{\log F_\Lambda},\underbrace{\log(1-T_\Lambda)}]\right\rangle.
\end{aligned}$$

$\square$

Theorem 5.5. *(monotonicity property) Let* $\Lambda_i=\Big\langle[\log T_{\Lambda t_{ij}},\log(1-F_{\Lambda t_{ij}})],[\log I_{\Lambda t_{ij}},\log I_{\Lambda t_{ij}}],[\log F_{\Lambda t_{ij}},\log(1-T_{\Lambda t_{ij}})]\Big\rangle$ *and* $\Upsilon_i=\Big\langle[\log T_{\Lambda h_{ij}},\log(1-F_{\Lambda h_{ij}})],[\log I_{\Lambda h_{ij}},\log I_{\Lambda h_{ij}}],[\log F_{\Lambda h_{ij}},\log(1-T_{\Lambda h_{ij}})]\Big\rangle$ $(i=1,2,...,n;j=1,2,...,i_j)$ *be the families of log PyNVWAs. For any i, if there is* $\left(\log_{\Psi_i} T_{\Lambda t_{ij}}\right)^2+\left(\log_{\Psi_i}(1-F_{\Lambda t_{ij}})\right)^2\preceq\left(\log_{\Psi_i} T_{\Lambda h_{ij}}\right)^2+\left(\log_{\Psi_i}(1-F_{\Lambda h_{ij}})\right)^2$ *and* $\left(\log_{\Psi_i} I_{\Lambda t_{ij}}\right)^2+\left(\log_{\Psi_i} I_{\Lambda t_{ij}}\right)^2\succeq\left(\log_{\Psi_i} I_{\Lambda h_{ij}}\right)^2+\left(\log_{\Psi_i} I_{\Lambda h_{ij}}\right)^2$ *and* $\left(\log_{\Psi_i} F_{\Lambda t_{ij}}\right)^2+\left(\log_{\Psi_i}(1-T_\Lambda)_{t_{ij}}\right)^2\succeq\left(\log_{\Psi_i} F_{\Lambda h_{ij}}\right)^2+\left(\log_{\Psi_i}(1-T_{\Lambda h_{ij}})\right)^2$ *or* $\Lambda_i\preceq\Upsilon_i$*. Then log PyNVWA* $(\Lambda_1,\Lambda_2,...,\Lambda_n)\preceq$ *log PyNVWA* $(\Upsilon_1,\Upsilon_2,...,\Upsilon_n)$.

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Proof. For any i, $\left(\log_{\Psi_i} T_{\Lambda_{t_{ij}}}\right)^2 + \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{t_{ij}}}\right)\right)^2 \preceq \left(\log_{\Psi_i} T_{\Lambda_{h_{ij}}}\right)^2 + \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{h_{ij}}}\right)\right)^2$.
Therefore, $1 - \left(\log_{\Psi_i} T_{\Lambda_{t_i}}\right)^2 + 1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{t_i}}\right)\right)^2 \succeq 1 - \left(\log_{\Psi_i} T_{\Lambda_{h_i}}\right)^2 + 1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{h_i}}\right)\right)^2$.
Hence, $\bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} T_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i} + \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{t_i}}\right)\right)^2\right)^{\Upsilon_i} \succeq$
$\bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} T_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i} + \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{h_i}}\right)\right)^2\right)^{\Upsilon_i}$
and $\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} T_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{t_i}}\right)\right)^2\right)^{\Upsilon_i}}$
$\preceq \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} T_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda_{h_i}}\right)\right)^2\right)^{\Upsilon_i}}$.
For any i, $\left(\log_{\Psi_i} I_{\Lambda_{t_{ij}}}\right)^2 + \left(\log_{\Psi_i} I_{\Lambda_{t_{ij}}}\right)^2 \succeq \left(\log_{\Psi_i} I_{\Lambda_{h_{ij}}}\right)^2 + \left(\log_{\Psi_i} I_{\Lambda_{h_{ij}}}\right)^2$.
Therefore, $1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2 + 1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2 \preceq 1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2 + 1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2$.
Hence, $\bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i} + \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i} \preceq \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i} +$
$\bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i}$
implies that $\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i}}$
$\succeq \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i}}$.
Hence, $1 - \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{t_i}}\right)^2\right)^{\Upsilon_i}}$
$\preceq 1 - \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i}} + \sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda_{h_i}}\right)^2\right)^{\Upsilon_i}}$.
For any i, $\left(\log_{\Psi_i} F_{\Lambda_{t_{ij}}}\right)^2 + \left(\log_{\Psi_i}\left(1 - T_{\Lambda_{t_{ij}}}\right)\right)^2 \succeq \left(\log_{\Psi_i} F_{\Lambda_{h_{ij}}}\right)^2 + \left(\log_{\Psi_i}\left(1 - T_{\Lambda_{h_{ij}}}\right)\right)^2$.
Therefore, $1 - \frac{\left(\bigcirc_{i=1}^{n}\log_{\Psi_i} F_{\Lambda_{t_{ij}}}\right)^2 + \left(\bigcirc_{i=1}^{n}\log_{\Psi_i}\left(1 - T_{\Lambda_{t_{ij}}}\right)\right)^2}{2} \preceq 1 - \frac{\left(\bigcirc_{i=1}^{n}\log_{\Psi_i} F_{\Lambda_{h_{ij}}}\right)^2 + \left(\bigcirc_{i=1}^{n}\log_{\Psi_i}\left(1 - T_{\Lambda_{h_{ij}}}\right)\right)^2}{2}$.

$$= \left[\frac{\left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} T_{\Lambda ti}\right)^2\right)^{\Upsilon_i}}\right)^2 + \left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda ti}\right)\right)^2\right)^{\Upsilon_i}}\right)^2}{2} \atop +1 - \frac{\left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda ti}\right)^2\right)^{\Upsilon_i}}\right)^2 + \left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda ti}\right)^2\right)^{\Upsilon_i}}\right)^2}{2} \atop +1 - \frac{\left(\bigcirc_{i=1}^{n}\left(\log_{\Psi_i} F_{\Lambda tij}\right)\right)^2 + \left(\bigcirc_{i=1}^{n}\left(\log_{\Psi_i}\left(1 - T_{\Lambda tij}\right)\right)\right)^2}{2}\right]$$

$$= \left[\frac{\left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} T_{\Lambda hi}\right)^2\right)^{\Upsilon_i}}\right)^2 + \left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i}\left(1 - F_{\Lambda hi}\right)\right)^2\right)^{\Upsilon_i}}\right)^2}{2} \atop +1 - \frac{\left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda hi}\right)^2\right)^{\Upsilon_i}}\right)^2 + \left(\sqrt{1 - \bigcirc_{i=1}^{n}\left(1 - \left(\log_{\Psi_i} I_{\Lambda hi}\right)^2\right)^{\Upsilon_i}}\right)^2}{2} \atop +1 - \frac{\left(\bigcirc_{i=1}^{n}\left(\log_{\Psi_i} F_{\Lambda hij}\right)\right)^2 + \left(\bigcirc_{i=1}^{n}\left(\log_{\Psi_i}\left(1 - T_{\Lambda hij}\right)\right)\right)^2}{2}\right].$$

Hence, log PyNVWA $(\Lambda_1, \Lambda_2, ..., \Lambda_n) \preceq$ log PyNVWA $(\Upsilon_1, \Upsilon_2, ..., \Upsilon_n)$. □

5.2 log PyNV weighted geometric (log PyNVWG) operator

Definition 5.6. Let $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1 - F_{\Lambda i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1 - T_{\Lambda i})] \right\rangle$ be the family of log PyNVNs. Then log PyNVWG operator is the log PyNVWG $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \bigcirc_{i=1}^{n} \Lambda_i^{\Upsilon_i}$ $(i = 1, 2, ..., n)$.

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Theorem 5.7. *Let* $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1-F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1-T_{\Lambda_i})] \right\rangle$ *be the family of log PyNVNs. Then log PyNVWG* $(\Lambda_1, \Lambda_2, ..., \Lambda_n)$

$$= \left[\begin{array}{c} \left[\bigcirc_{i=1}^{n} (\log_{\Psi_i} T_{\Lambda i})^{\Upsilon_i}, \bigcirc_{i=1}^{n} (\log_{\Psi_i} (1-F_{\Lambda_i}))^{\Upsilon_i} \right], \\ \left[\sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i} I_{\Lambda i})^2\right)^{\Upsilon_i}} \right], \\ \left[\sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i} F_{\Lambda i})^2\right)^{\Upsilon_i}}, \sqrt{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Psi_i} (1-T_{\Lambda_i}))^2\right)^{\Upsilon_i}} \right] \end{array} \right].$$

Proof. This proof is based on Theorem 5.2. ☐

Theorem 5.8. *If all* $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1-F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1-T_{\Lambda_i})] \right\rangle$ *are equal and* $\Lambda_i = \Lambda$, *for* $i = 1, 2, ..., n$. *Then log PyNVWG* $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \Lambda$.

Proof. This proof is based on Theorem 5.3. ☐

Corollary 5.9. *The log PyNVWG operator is used to satisfy the boundedness and monotonicity properties.*

Proof. This proof is based on Theorems 5.4 and 5.5. ☐

5.3 Generalized log PyNVWA (log GPyNVWA) operator

As generalizations of the log PyNVWA operators, some generalized log GPyNVWA operators are developed in Section 5.3.

Definition 5.10. Let $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1-F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1-T_{\Lambda_i})] \right\rangle$ be the family of log PyNVNs. Then the log GPyNVWA $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \left(\oslash_{i=1}^{n} \Upsilon_i \Lambda_i^{\Omega} \right)^{1/\Omega}$ is called the log GPyNVWA operator.

Theorem 5.11 illustrates the log GPyNVWA result based on the above definition.

Theorem 5.11. *Let* $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1-F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1-T_{\Lambda_i})] \right\rangle$ *be the family of log PyNVNs. Then log GPyNVWA* $(\Lambda_1, \Lambda_2, ..., \Lambda_n)$

$$= \left[\begin{array}{c} \left[\left(\sqrt{1-\bigcirc_{i=1}^{n}\left(1-\left((\log_{\Psi_i} T_{\Lambda i})^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega}, \left(\sqrt{1-\bigcirc_{i=1}^{n}\left(1-\left((\log_{\Psi_i} (1-F_{\Lambda_i}))^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega} \right], \\ \left[\left(\sqrt{1-\bigcirc_{i=1}^{n}\left(1-\left((\log_{\Psi_i} I_{\Lambda i})^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega}, \left(\sqrt{1-\bigcirc_{i=1}^{n}\left(1-\left((\log_{\Psi_i} I_{\Lambda i})^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega} \right], \\ \left[\begin{array}{c} \sqrt{1-\left(1-\left(\bigcirc_{i=1}^{n}\left(\sqrt{1-\left(1-(\log_{\Psi_i} F_{\Lambda i})^2\right)^2}\right)^{\Upsilon_i}\right)^2\right)^{1/\Omega}}, \\ \sqrt{1-\left(1-\left(\bigcirc_{i=1}^{n}\left(\sqrt{1-\left(1-(\log_{\Psi_i} (1-T_{\Lambda_i}))^2\right)^2}\right)^{\Upsilon_i}\right)^2\right)^{1/\Omega}} \end{array} \right] \end{array} \right].$$

Proof. This proof is based on the Theorem 5.2. ☐

Remark 5.12. If $\Omega = 1$, then the log GPyNVWA operator is modified to the log PyNVWA operator.

Theorem 5.13. *If all* $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1-F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1-T_{\Lambda_i})] \right\rangle$ $(i = 1, 2, ..., n)$ *are equal and* $\Lambda_i = \Lambda$. *Then log GPyNVWA* $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \Lambda$.

Proof. This proof is based on Theorem 5.3. ☐

Remark 5.14. To satisfy the boundedness and monotonicity conditions, we use the log GPyNVWA operator.

Proof. This proof is based on Theorems 5.4 and 5.5. ☐

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5.4 Generalized log PyNVWG (log GPyNVWG) operator

As generalizations of the log PyNVWG operators, some generalized log GPyNVWG operators are developed in Section 5.4.

Definition 5.15. Let $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1 - F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1 - T_{\Lambda_i})] \right\rangle$ be the family of log PyNVNs. Then the log GPyNVWG $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \frac{1}{\Omega}\left(\bigcirc_{i=1}^{n} (\Omega\Lambda_i)^{\Upsilon_i} \right)$ $(i = 1, 2, ..., n)$ is called the log GPyNVWG operator.

Theorem 5.16. *Let $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1 - F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1 - T_{\Lambda_i})] \right\rangle$ be the family of log PyNVNs. Then log GPyNVWG $(\Lambda_1, \Lambda_2, ..., \Lambda_n)$*

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt{1 - \left(1 - \left(\bigcirc_{i=1}^{n} \left(\sqrt{1 - \left(1 - (\log_{\Psi_i} T_{\Lambda i})^2\right)^2} \right)^{\Upsilon_i} \right)^2 \right)^{1/\Omega}}, \\ \sqrt{1 - \left(1 - \left(\bigcirc_{i=1}^{n} \left(\sqrt{1 - \left(1 - (\log_{\Psi_i} (1 - F_{\Lambda_i}))^2\right)^2} \right)^{\Upsilon_i} \right)^2 \right)^{1/\Omega}} \end{array} \right], \\ \left[\left(\sqrt{1 - \bigcirc_{i=1}^{n} \left(1 - \left((\log_{\Psi_i} I_{\Lambda i})^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega}, \left(\sqrt{1 - \bigcirc_{i=1}^{n} \left(1 - \left((\log_{\Psi_i} I_{\Lambda i})^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega} \right], \\ \left[\left(\sqrt{1 - \bigcirc_{i=1}^{n} \left(1 - \left((\log_{\Psi_i} F_{\Lambda i})^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega}, \left(\sqrt{1 - \bigcirc_{i=1}^{n} \left(1 - \left((\log_{\Psi_i} (1 - T_{\Lambda_i}))^2\right)^2\right)^{\Upsilon_i}} \right)^{1/\Omega} \right] \end{array} \right].$$

Proof. This proof is based on the Theorem 5.2. □

Remark 5.17. If $\Omega = 1$, then the log GPyNVWG operator is converted to the log PyNVWG operator.

Remark 5.18. Boundedness and monotonicity properties can be met using the log GPyNVWG operator.

Proof. This proof is based on Theorems 5.4 and 5.5. □

Corollary 5.19. *If all $\Lambda_i = \left\langle [\log T_{\Lambda i}, \log(1 - F_{\Lambda_i})], [\log I_{\Lambda i}, \log I_{\Lambda i}], [\log F_{\Lambda i}, \log(1 - T_{\Lambda_i})] \right\rangle$ $(i = 1, 2, ..., n)$ are equal and $\Lambda_i = \Lambda$, then log GPyNVWG $(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \Lambda$.*

Proof. This proof is based on Theorem 5.3. □

6 MADM method based on log PyNVN

The complexity of real-world systems increases daily, making it difficult for decision-makers to choose the right option. It is difficult to summarize the objective of achieving a single goal, but it is not impossible. Motivating employees, setting goals, and addressing opinions and complications were challenges for many organizations. The DM process, whether by an individual or a committee, must be transparent and consider multiple objectives simultaneously. According to this reflection, each decision-maker is prevented from achieving an ideal solution under each of the criteria involved in practical problems. Therefore, decision-makers can develop more effective and reliable methods to identify the most appropriate option. DM problems involving ambiguity and uncertainty do not always respond well to classical or crisp methods. In this paper, we propose a MADM approach based on log PyNVN with AOs. An algorithm for selecting the most suitable option from a set of options in the MADM problem, using log PyNVN with AOs. The purpose of this paper is to propose a multi-attribute neutrosophic approach to DM problems with weights.

Let $\Lambda = \{\Lambda_1, \Lambda_2, ..., \Lambda_n\}$ be a set of n-alternatives, $\mathcal{W} = \{\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_m\}$ be a set of m-attributes and weights $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_m\}$, where $\Upsilon_i \in [0, 1]$ and $\sum_i^m \Upsilon_i = 1$.

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Let $\Lambda_{ij} = \Big\langle [\log_{\Psi_i} T_{\Lambda ij}, \log_{\Psi_i}(1-F_{\Lambda_{ij}})], [\log_{\Psi_i} I_{\Lambda ij}, \log_{\Psi_i} I_{\Lambda ij}], [\log_{\Psi_i} F_{\Lambda ij}, \log_{\Psi_i}(1-T_{\Lambda_{ij}})] \Big\rangle$ denote the log PyNVN of alternative Λ_i in attribute $\mathcal{W}_j$, $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$.
Since $\Big[\log_{\Psi_i} T_{\Lambda ij}, \log_{\Psi_i}(1-F_{\Lambda_{ij}})\Big], \Big[\log_{\Psi_i} I_{\Lambda ij}, \log_{\Psi_i} I_{\Lambda ij}\Big], \Big[\log_{\Psi_i} F_{\Lambda ij}, \log_{\Psi_i}(1-T_{\Lambda_{ij}})\Big] \in \mathrm{Int}([0,1])$ and $0 \preceq (\log_{\Psi_i} 1 - F_{\Lambda ij}(\xi))^2 + (\log_{\Psi_i} I_{\Lambda ij}(\xi))^2 + (\log_{\Psi_i} 1 - T_{\Lambda ij}(\xi))^2 \preceq 2$. The MADM process is represented by the following flowchart based on the log PyNVN.

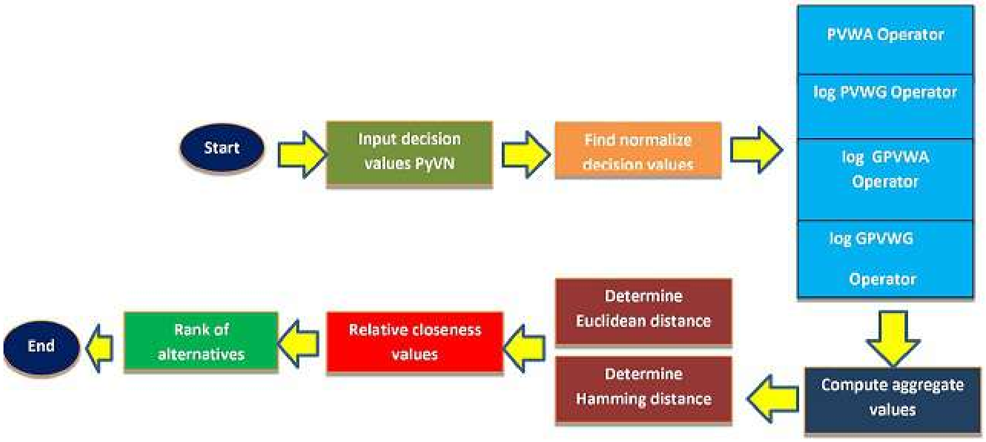

Figure 1: Flowchart of the MADM algorithm.

6.1 Algorithm

Using the proposed method, we can summarize the process depicted in Figure 1 as follows:
Step-1: Decision values for the log PyNVN.
Step-2: Compute the normalised decision values. The decision matrix $\mathscr{D} = (\widetilde{\mathscr{C}}_{ij})_{n\times m}$ is normalized into $\widehat{\mathscr{D}} = (\breve{\mathscr{C}}_{ij})_{n\times m}$, where
$\breve{\mathscr{C}}_{ij} = \Big\langle [\widehat{\log_{\Psi_i} T_{\Lambda ij}}, \widehat{\log_{\Psi_i}(1-F_{\Lambda_{ij}})}], [\widehat{\log_{\Psi_i} I_{\Lambda ij}}, \widehat{\log_{\Psi_i} I_{\Lambda ij}}], [\widehat{\log_{\Psi_i} F_{\Lambda ij}}, \widehat{\log_{\Psi_i}(1-T_{\Lambda_{ij}})}] \Big\rangle$ and
$\widehat{\log_{\Psi_i} T_{\Lambda ij}} = \log_{\Psi_i} T_{\Lambda ij}$, $\widehat{\log_{\Psi_i}(1-F_{\Lambda_{ij}})} = \log_{\Psi_i}(1-F_{\Lambda_{ij}})$, where
$\Psi_i = \prod[T_{\Lambda_i}, 1-F_{\Lambda_i}], [I_{\Lambda_i}, I_{\Lambda_i}], [F_{\Lambda_i}, 1-T_{\Lambda_i}]$.
Step-3: Find the aggregate values for every alternative. Based on the log PyNVN AOs, attribute $\mathcal{W}_j$ in $\widetilde{\mathscr{C}}_i, \breve{\mathscr{C}}_{ij} = \Big\langle [\widehat{\log_{\Psi_i} T_{\Lambda ij}}, \widehat{\log_{\Psi_i}(1-F_{\Lambda_{ij}})}], [\widehat{\log_{\Psi_i} I_{\Lambda ij}}, \widehat{\log_{\Psi_i} I_{\Lambda ij}}], [\widehat{\log_{\Psi_i} F_{\Lambda ij}}, \widehat{\log_{\Psi_i}(1-T_{\Lambda_{ij}})}] \Big\rangle$ is aggregated into $\breve{\mathscr{C}}_i = \Big\langle [\widehat{\log_{\Psi_i} T_{\Lambda i}}, \widehat{\log_{\Psi_i}(1-F_{\Lambda_i})}], [\widehat{\log_{\Psi_i} I_{\Lambda i}}, \widehat{\log_{\Psi_i} I_{\Lambda i}}], [\widehat{\log_{\Psi_i} F_{\Lambda i}}, \widehat{\log_{\Psi_i}(1-T_{\Lambda_i})}] \Big\rangle$.
Step-4: The ideal values for each option can be calculated as follows:
$\breve{\mathscr{C}}^+ = [1,1], [1,1], [0,0]$ and $\breve{\mathscr{C}}^- = [0,0], [0,0], [1,1]$
Step-5: The EDs between each alternative with various ideal values are as follows: $\mathscr{D}_i^+ = \mathscr{D}_E\Big(\breve{\mathscr{C}}_i, \breve{\mathscr{C}}^+\Big)$ and $\mathscr{D}_i^- = \mathscr{D}_E\Big(\breve{\mathscr{C}}_i, \breve{\mathscr{C}}^-\Big)$.
Step-6: The relative closeness values are calculated as follows: $\mathscr{D}_i^* = \frac{\mathscr{D}_i^-}{\mathscr{D}_i^+ + \mathscr{D}_i^-}$.
Step-7: The greatest value is $\max \mathscr{D}_i^*$.

6.2 Selection process of best farmer

Multiple conflicting criteria can be improved and evaluated by the MADM process in all areas of data mining. In today's competitive market, businesses must respond more effectively and accurately to customer needs. Therefore, MADM is capable of handling multiple contradictory criteria successfully. Using expert analysis, experts make decisions based on an analysis of every characteristic of an alternative. Using the log PyNVN aggregation operators, we will also present the model for MADM and how to construct it. As a practical example, MADM is demonstrated here with log PyNVN AOs. Over the past few years, the agriculture sector

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in India has undergone a significant transformation from conventional to smart farming. A variety of cutting-edge strategies have been developed to boost productivity in rural agriculture thanks to technology. To feed an expanding population, farmers must increase their productivity and produce high yields of crops. Artificial intelligence is combined with technology, such as drones and moisture sensors, to achieve sustainable growth. Using log PyNVNs as an example, this subsection demonstrates how MADM can be utilized. People in developing nations can escape poverty through farming. Environmental healing can be achieved through it. Developing countries rely on it to maintain their economic systems. In agriculture, we gain fresh air from plants that are non-polluting. As the population grows, agriculture provides food for them. In addition to providing food and clothing, it also provides shelter. The following are the descriptions and classifications for farmers:

1. Climate and water:
   Climate plays an important role in agriculture and fisheries. There are some places where temperature increases and carbon dioxide (CO2) can increase crop yields. As a result of climate change, habitat ranges are shifting and planting dates are shifting, and droughts and floods can hinder farming activities. Food production is being threatened by climate change. The use of crop water and drought have led to a decline in crop yields in many countries. Land competition may increase as climate changes make some areas unsuitable for farming. Water is consumed most by irrigated agriculture worldwide. In many countries of the Organization for Economic Cooperation and Development (OECD), agriculture irrigation accounts for more than 40% of water use. Freshwater withdrawals for agriculture account for approximately 70% of all withdrawals. In this sector, irrigation, pesticides, fertilizers, and livestock are the most common uses. In agriculture, rainfall provides most of the water needed. Plant roots can only grow with water, and soil organisms need water for nutrition and growth. In a plant, water plays a vital role in the hydraulic process. By converting starch into sugar, it aids in the digestion process. Agricultural production is therefore highly dependent on water.

2. Soil:
   There are several different kinds of particles in soil, such as organic matter, slit sand, and clay, which combine to form aggregates. In addition to alluvial soil, desert soil, black soil, forest soil, mountain soil, laterite soil, and arid soil, there are many other types of soil. Due to its loanable texture and humus content, alluvial soil is the most fertile. As a result, it is capable of absorbing and retaining a large amount of water.

3. Disease:
   A disease management strategy in agriculture has the goal of minimizing the spread of diseases so crops can produce a greater quantity or higher quality of harvest. Crop diseases range from fungus, oomycetes, bacteria, viruses, viroids, and virus-like organisms, to phytoplasmas, protozoa, and nematodes. Dormancy, reproduction, dispersal, and pathogenesis are all interrelated stages of pathogen biology. Inhibiting disease-causing pathogen growth or killing them reduces a wide range of diseases.

4. Flood:
   The floods are eroding, sedimentating, and flooding a large area of agricultural land, causing crops to disappear. Floods have the most impact on agriculture. Floods primarily damage agriculture by causing water logging in the cropping areas. There are benefits and disasters to flooding. Sand, silt, and debris are deposited on surrounding lands when rivers overflow their banks. The organic matter and minerals in the flood water make the soil fertile after it recedes. Biogeochemical cycling and greenhouse gas production are important in flooded soils.

Suppose that five farmers (alternatives) such as $\mathscr{F} = \{\mathscr{F}_1, \mathscr{F}_2, \mathscr{F}_3, \mathscr{F}_4, \mathscr{F}_5\}$. Four attributes are considered, climate and water $(\mathcal{W}_1)$, soil $(\mathcal{W}_2)$, disease $(\mathcal{W}_3)$, flood$(\mathcal{W}_4)$ and their corresponding weights are $\gamma = \{0.4, 0.3, 0.2, 0.1\}$. Based on expert assessments against the criteria, we select the best from many alternatives.
Step:1 DM information is

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	$\mathcal{W}_1$	$\mathcal{W}_2$	$\mathcal{W}_3$	$\mathcal{W}_4$
$\mathscr{F}_a$	$\begin{bmatrix}[0.5,0.65],\\ [0.7,0.8],\\ [0.35,0.5]\end{bmatrix}$	$\begin{bmatrix}[0.55,0.6],\\ [0.75,0.8],\\ [0.4,0.45]\end{bmatrix}$	$\begin{bmatrix}[0.75,0.8],\\ [0.5,0.55],\\ [0.2,0.25]\end{bmatrix}$	$\begin{bmatrix}[0.45,0.5],\\ [0.55,0.6],\\ [0.5,0.55]\end{bmatrix}$
$\mathscr{F}_b$	$\begin{bmatrix}[0.55,0.6],\\ [0.75,0.8],\\ [0.4,0.45]\end{bmatrix}$	$\begin{bmatrix}[0.4,0.5],\\ [0.7,0.75],\\ [0.5,0.6]\end{bmatrix}$	$\begin{bmatrix}[0.65,0.7],\\ [0.55,0.65],\\ [0.3,0.35]\end{bmatrix}$	$\begin{bmatrix}[0.5,0.6],\\ [0.35,0.4],\\ [0.4,0.5]\end{bmatrix}$

	$\mathcal{W}_1$	$\mathcal{W}_2$	$\mathcal{W}_3$	$\mathcal{W}_4$
$\mathscr{F}_c$	$\begin{bmatrix}[0.5,0.55],\\ [0.45,0.5],\\ [0.45,0.5]\end{bmatrix}$	$\begin{bmatrix}[0.7,0.75],\\ [0.5,0.55],\\ [0.25,0.3]\end{bmatrix}$	$\begin{bmatrix}[0.5,0.6],\\ [0.65,0.7],\\ [0.4,0.5]\end{bmatrix}$	$\begin{bmatrix}[0.65,0.7],\\ [0.4,0.45],\\ [0.3,0.35]\end{bmatrix}$
$\mathscr{F}_d$	$\begin{bmatrix}[0.65,0.7],\\ [0.6,0.65],\\ [0.3,0.35]\end{bmatrix}$	$\begin{bmatrix}[0.5,0.65],\\ [0.55,0.65],\\ [0.35,0.5]\end{bmatrix}$	$\begin{bmatrix}[0.75,0.8],\\ [0.4,0.5],\\ [0.2,0.25]\end{bmatrix}$	$\begin{bmatrix}[0.5,0.55],\\ [0.45,0.5],\\ [0.45,0.5]\end{bmatrix}$
$\mathscr{F}_e$	$\begin{bmatrix}[0.3,0.35],\\ [0.7,0.75],\\ [0.65,0.7]\end{bmatrix}$	$\begin{bmatrix}[0.7,0.75],\\ [0.65,0.75],\\ [0.25,0.3]\end{bmatrix}$	$\begin{bmatrix}[0.8,0.85],\\ [0.45,0.5],\\ [0.15,0.2]\end{bmatrix}$	$\begin{bmatrix}[0.7,0.75],\\ [0.6,0.65],\\ [0.25,0.3]\end{bmatrix}$

Step:2 Based on log PyNVWA operator, aggregated information regarding alternatives.

$\breve{\mathscr{F}}_a$	$\breve{\mathscr{F}}_b$	$\breve{\mathscr{F}}_c$	$\breve{\mathscr{F}}_d$	$\breve{\mathscr{F}}_e$
$\begin{bmatrix}[0.2601,0.2482],\\ [0.2374,0.2373],\\ [0.2467,0.2355]\end{bmatrix}$	$\begin{bmatrix}[0.264,0.2654],\\ [0.2237,0.2251],\\ [0.2386,0.2383]\end{bmatrix}$	$\begin{bmatrix}[0.2714,0.2766],\\ [0.2547,0.2555],\\ [0.2344,0.2343]\end{bmatrix}$	$\begin{bmatrix}[0.2542,0.2438],\\ [0.2367,0.2337],\\ [0.2521,0.2459]\end{bmatrix}$	$\begin{bmatrix}[0.3907,0.4072],\\ [0.2455,0.2478],\\ [0.1813,0.1793]\end{bmatrix}$

Step:3 Among the various ideal values of every alternative are
$\breve{\mathscr{F}}^+ = \big[[1,1],[1,1],[0,0]\big]$ and $\breve{\mathscr{F}}^- = \big[[0,0],[0,0],[1,1]\big]$.
Step:4 The EDs between every alternative with various ideal values are as follows: $\mathscr{D}_1^+ = 0.0379, \mathscr{D}_2^+ = 0.0426, \mathscr{D}_3^+ = 0.0416, \mathscr{D}_4^+ = 0.0359, \mathscr{D}_5^+ = 0.0763$, and $\mathscr{D}_1^- = 0.2683, \mathscr{D}_2^- = 0.2636, \mathscr{D}_3^- = 0.2646, \mathscr{D}_4^- = 0.2703, \mathscr{D}_5^- = 0.2299$.
Step:5 Relative closeness values are $\mathscr{D}_1^* = 0.8762, \mathscr{D}_2^* = 0.8609, \mathscr{D}_3^* = 0.8642, \mathscr{D}_4^* = 0.8829, \mathscr{D}_5^* = 0.7509$.
Step:7 Ranking of alternatives are $\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_c > \mathscr{F}_b > \mathscr{F}_e$.
Thus the farmer $\mathscr{F}_d$ is also the most desirable alternative based on the log PyNVWG operator.

6.3 Comparative study between the suggested and the existing approach

In the MADM process, EDs and HDs are extremely important. The use of ED and HD criterion values to compare alternatives, including neutrosophic sets, interval neutrosophic sets, and vague neutrosophic sets, has been scarcely studied. ED and HD methods will be compared here to validate the feasibility of our proposed DM method. Palanikumar et al.[26] discussed Pythagorean neutrosophic normal interval-valued operators based on MADM. This example illustrates its usefulness and benefits. The approach uses log PyNVWA, log PyNVWG, log GPyNVWA and log GPyNVWG using ED and HD respectively. The following table presents existing and proposed methods for the comparative study:

	log PyNVWA	log PyNVWG	log GPyNVWA	log GPyNVWG
TOPSIS-ED (**Proposed**)	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_c > \mathscr{F}_b > \mathscr{F}_e$	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_e > \mathscr{F}_c > \mathscr{F}_b$	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_c > \mathscr{F}_b > \mathscr{F}_e$	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_e > \mathscr{F}_c > \mathscr{F}_b$
TOPSIS-HD (**Proposed**)	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_c > \mathscr{F}_b > \mathscr{F}_e$	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_e > \mathscr{F}_c > \mathscr{F}_b$	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_c > \mathscr{F}_b > \mathscr{F}_e$	$\mathscr{F}_d > \mathscr{F}_a > \mathscr{F}_e > \mathscr{F}_c > \mathscr{F}_b$
TOPSIS-ED[26]	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_e > \mathscr{F}_a > \mathscr{F}_b$	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_a > \mathscr{F}_b > \mathscr{F}_e$	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_e > \mathscr{F}_a > \mathscr{F}_b$	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_a > \mathscr{F}_b > \mathscr{F}_e$
TOPSIS-HD[26]	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_e > \mathscr{F}_a > \mathscr{F}_b$	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_a > \mathscr{F}_b > \mathscr{F}_e$	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_e > \mathscr{F}_a > \mathscr{F}_b$	$\mathscr{F}_d > \mathscr{F}_c > \mathscr{F}_a > \mathscr{F}_b > \mathscr{F}_e$

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Consider the reliability of the circumstances when analyzing the MADM approach and alternatives. Rankings and values of closeness are as follows: The developed methods demonstrate superiority and greater capability over existing methods, based on this analysis. An algorithm called log PyNVWA is used to estimate it.

7 Conclusion

Multiple conflicting criteria can be improved and evaluated in all areas of data mining using MADM. Experts analyze each factor of an alternative when making an intelligent decision. The best decision can only be reached by carefully preparing and analyzing all aspects of an alternative. They can make a good decision if they have all the necessary data and information. Several methods have been developed for dealing with uncertain information, such as FSs, IFSs, PyFSs, NSSs, and VSs. DM problems arising from log PyNVN are the subject of this research. Several conclusions were drawn in our discussion of some AOs for log PyNVNs. AO rules have been proposed for log PyNVWA, log PyNVWG, log GPyNVWA, and log GPyNVWG. Different rankings can be computed using log PyNVWA, log PyNVWG, log GPyNVWA, and log GPyNVWG. Therefore, we also examined factors that affect alternative rankings the most. Given the examples, the neutrosophic DM method is better suited for real science and engineering applications than the classical DM method. A decision-maker determines the optimal ranking based on a real-life scenario. It is proposed that by utilizing the proposed approaches, MCDM and stepwise decision-making can be facilitated. To demonstrate that the proposed model is more effective than existing methods, a numerical illustration of the developed method is provided, as well as a comparison between some existing methods and the proposed method. Future discussions will cover the following topics:

1. An investigation of the log PyNVS of soft sets and expert sets.
2. Investigating Pythagorean cubic FSs and spherical cubic FSs.
3. The problem of MADM can be solved using other decision-making methodologies based on square root Fermatean cubic fuzzy sets.

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