# Innovative Perspective on Neutrosophic Cubic Z-Algebras

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**Abstract**

This study explores an innovative perspective on neutrosophic cubic Z-algebras, delving into the theoretical framework within mathematical structures. Through a comprehensive analysis, we uncover unique insights that contribute to the advancement of algebraic methodologies, particularly in handling uncertainties represented by neutrosophic elements. This work aims to present the idea of neutrosophic cubic sets in Z-algebras, as well as the usage of false membership function, truth, and indeterminacy in Z-algebras. Further, the results on $\mathcal{S}$-union, $\mathcal{S}$-intersection, $\mathcal{K}$-union, and $\mathcal{K}$-intersection of neutrosophic cubic Z-subalgebras are provided. This paper also discusses homomorphisms of Z-algebras and its associated characteristics.

**Keywords:** Z-algebra; Z-subalgebra; Cubic set; Cubic Z-subalgebra; Neutrosophic set; Neutrosophic cubic set; Neutrosophic cubic Z-subalgebra.

## 1. Introduction

Zadeh [24] developed fuzzy set theory, which expands on conventional set theory by permitting varying degrees of membership. It deals with imprecision and uncertainty, using linguistic variables to model and flexible way than traditional binary logic. In this way, cubic sets and cubic subgroups were introduced by Jun et al. [7,8]. Fuzzy β-subalgebras of β-algebras were first discussed by Ansari and Chandramouleeswaran [2]. Smarandache [23] investigated an extension of intuitionistic fuzzy sets. Jun et al. [9,10] investigated neutrosophic cubic sets (NCSs) and the relationships between neutrosophic subalgebras (NSs) of various sorts in BCK/BCI algebras. Additionally, Jun [11] covered cubic interval-valued intuitionistic fuzzy sets and how they relate to BCK/BCI algebras. In [13], the idea of soft sets was explored, some new results were studied using neutrosophic sets (NSs). Iqbal et al. [6] discussed cubic subalgebras and neutrosophic cubic closed ideals of B-algebras. β-subalgebras with interval values were described by Hemavathi et al. in their study [5]. The idea of neutrosophic triplet of BI-algebras was first proposed by Akbar and Smarandache [1]. Bordbar et al. [3] proposed the idea of MBJ-neutrosophic subalgebras in BCK/BCI-algebras and produced some interesting results.

Aboeihamd et al. [4] created the neutrosophic logic theory and its applications. Awoloda's [12] research focused on the idea of neutrosophic set-level sets. The concepts of translation and multiplication of the NCS were explained by Khalid et al. [14,15,16], who also established the concept of the T-MBJ-neutrosophic set under the M-subalgebra. Neggers and Kim [19] defined β-algebras and some results. Some parts of cubic fuzzy β-subalgebras were introduced by Muralikrishna et al. [17] in their study of β-algebras. The MBJ-neutrosophic set was used by [20] to incorporate the notion of the ideal and they examined several fascinating outcomes. Nanthini and Pushpalatha [18] studied interval valued neutrosophic topological spaces. The goal



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

of Borzooei et al. [21] generalised neutrosophic subalgebras in BCK/BCI-algebras. Remy and Shalini [22] discussed neutrosophic vague binary in BCK/BCI-algebras.

This article explains the idea of neutrosophic cubic Z-algebras. In the second section, we go over some basic definitions that the paper depends on neutrosophic cubic Z-algebras. Additionally, neutrosophic cubic Z-subalgebras and homomorphism of neutrosophic cubic Z-algebras are covered in the third and fourth sections.

## 2. Preliminaries

We'll go over certain concepts and examples that are essential to the work in this part.

**Definition 2.1:** For each universal set Y, a fuzzy set is defined as a function $\lambda$: Y → [0,1]. The membership value of an element y in Y is denoted by $\lambda$(y) for each y in Y.

**Definition 2.2:** Consider two fuzzy sets $\lambda_1$ and $\lambda_2$ are in Y, if the union of $\lambda_1$ and $\lambda_2$ denoted by $\lambda_1 \cup \lambda_2$ and is characterized by $(\lambda_1 \cup \lambda_2)(y) = \max\{\lambda_1(y), \lambda_2(y)\}$, ∀ y ∈ Y.

**Definition 2.3:** Consider two fuzzy sets $\lambda_1$ and $\lambda_2$ are in Y, if the intersection of $\lambda_1$ and $\lambda_2$ denoted by $\lambda_1 \cap \lambda_2$ and is characterized by $(\lambda_1 \cap \lambda_2)(y) = \min\{\lambda_1(y), \lambda_2(y)\}$, ∀ y ∈ Y.

**Definition 2.4:** The sup-property of the fuzzy set $\lambda$ for the subset S in Y is characterized by $\lambda(s_0)= \sup \lambda(s)$ for some $s_0 \in$ S.

**Definition 2.5:** An interval-valued fuzzy set S is defined on Y by S = {(y, $[\lambda_S^L(y), \lambda_S^U(y)]$) : y ∈ Y} (briefly denoted by S = $[\lambda_S^L(y), \lambda_S^U(y)]$), where $\lambda_S^L(y)$, $\lambda_S^U(y)$ are two fuzzy sets in Y such that $\lambda_S^L(y) \leq \lambda_S^U(y)$, ∀ y ∈ Y.

Let $\bar{\lambda}_S(y) = [\lambda_S^L(y), \lambda_S^U(y)]$ for all y ∈ Y. Consider the family of all closed subintervals of [0,1], which is represented by $\mathfrak{D}[0,1]$. For example, $0 \leq \mathcal{C} \leq 1$, where $\lambda_S^L(y) = \lambda_S^U(y) = \mathcal{C}$, then $\bar{\lambda}_S(y) = [\mathcal{C}, \mathcal{C}]$. Thus $\bar{\lambda}_S(y) \in \mathfrak{D}[0,1]$, ∀ y ∈ Y.

Therefore, the interval-valued fuzzy set S follows, S ={(y, $\bar{\lambda}_S(y)$)}, ∀ y ∈ Y, where $\bar{\lambda}_S: Y \rightarrow \mathfrak{D}[0,1]$.

Now let's define the refined minimum (abbreviated rmin) consist of two elements in $\mathfrak{D}[0,1]$. In addition, describe the characters "≥", "≤" and "=" when $\mathfrak{D}[0,1]$ contains two elements.

Consider, two elements $\mathfrak{D}_1 = [\imath_1, \jmath_1]$ and $\mathfrak{D}_2 = [\imath_2, \jmath_2] \in \mathfrak{D}[0,1]$. Now,

$$\text{rmin}(\mathfrak{D}_1, \mathfrak{D}_2) = [\min\{\imath_1, \imath_2\}, \min\{\jmath_1, \jmath_2\}]$$

$$\mathfrak{D}_1 \geq \mathfrak{D}_2 \text{ if } \imath_1 \geq \imath_2, \jmath_1 \geq \jmath_2.$$

Similarly, $\mathfrak{D}_1 \leq \mathfrak{D}_2$ and $\mathfrak{D}_1 = \mathfrak{D}_2$.

**Definition 2.6:** A Z-algebra (Y,⊗,0) is a non-empty set Y with a binary operation ⊗ and a constant 0 if below properties holds:

1. y ⊗0 = 0

2. 0⊗ y = 0

3. y ⊗ y = y

4. y ⊗ z = z ⊗ y, when y ≠ 0 and z ≠ 0, ∀ y, z ∈ Y.

**Example 2.7:** Assume that the Z-algebra Y = {0, 1, 2, 3} with the Cayley table:

| ⊗ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 0 | 1 | 0 | 1 |



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

| | | | | |
|---|---|---|---|---|
| 2 | 0 | 0 | 2 | 2 |
| 3 | 0 | 2 | 2 | 3 |

**Definition 2.8:** Consider a non-empty subset S of a Z-algebra Y. Then S is referred as a Z-subalgebra of Y if $y \otimes z \in S$, $\forall$ y, z $\in$ S.

**Definition 2.9:** Consider Y be a non-empty set. A cubic set (CS) in Y is characterized by C = {(y, $\bar{\lambda}_c$(y), $\mu_c$(y)) : y $\in$ Y}, where $\bar{\lambda}_c$ is an interval-valued fuzzy set in Y and $\mu_c$ is a fuzzy set in Y.

**Definition 2.10:** Consider a CS C = {(y, $\bar{\lambda}_c$(y), $\mu_c$(y)) : y $\in$ Y} in Y. A cubic Z-subalgebra C satisfies the following properties holds:

1. $\bar{\lambda}_c(y \otimes z) \geq \text{rmin}\{\bar{\lambda}_c(y), \bar{\lambda}_c(z)\}$

2. $\mu_c(y \otimes z) \leq \max\{\mu_c(y), \mu_c(z)\}$.

**Definition 2.11:** An NS in Y is characterized by H = {(y, $\zeta_{T_\sigma}$(y), $\zeta_{I_\sigma}$(y), $\zeta_{F_\sigma}$(y)) : y $\in$ Y}, where $\zeta_{T_\sigma}$ : Y $\longrightarrow$[0,1] is a truth membership function, $\zeta_{I_\sigma}$ : Y $\longrightarrow$[0,1] is an indeterminate membership function, and $\zeta_{F_\sigma}$ : Y $\longrightarrow$[0,1] is a false membership function.

**Definition 2.12:** If Θ = {(y, $\bar{\eta}_{T_\sigma}$(y), $\bar{\eta}_{I_\sigma}$(y), $\bar{\eta}_{F_\sigma}$(y)) : y $\in$ Y} defines an interval neutrosophic set (INS) in Y, where $\bar{\eta}_{T_\sigma}$, $\bar{\eta}_{I_\sigma}$, and $\bar{\eta}_{F_\sigma}$ are interval valued fuzzy sets in Y. These are referred to as an interval truth membership function, an interval indeterminate function, and an interval false membership function respectively, taken the values from Y to $\mathfrak{D}$[0,1].

**Definition 2.13:** Let Y be a non-empty set. An NCS is a pair C = (Θ, H), with Θ = {(y, $\bar{\eta}_{T_\sigma}$(y), $\bar{\eta}_{I_\sigma}$(y), $\bar{\eta}_{F_\sigma}$(y)) : y $\in$ Y} is an INS and H = {(y, $\zeta_{T_\sigma}$(y), $\zeta_{I_\sigma}$(y), $\zeta_{F_\sigma}$(y)) : y $\in$ Y} is an NS.

**Definition 2.14:** Let $C_j$ = ($\Theta_j$, $H_j$), where $\Theta_j$ = {(y, $\bar{\eta}_{jT_\sigma}$(y), $\bar{\eta}_{jI_\sigma}$(y), $\bar{\eta}_{jF_\sigma}$(y)) : y $\in$ Y} and $H_j$ = {(y, $\zeta_{jT_\sigma}$(y), $\zeta_{jI_\sigma}$(y), $\zeta_{jF_\sigma}$(y)) : y $\in$ Y}, for $j \in k$. $\mathcal{S}$-union, $\mathcal{S}$-intersection, $\mathcal{K}$-union, and $\mathcal{K}$-intersection are defined by

$\mathcal{S}$-union: $\cup_{j\in k} C_j = (\cup_{j\in k} \Theta_j, \vee_{j\epsilon k} H_j)$

$\mathcal{S}$-intersection: $\cap_{j\epsilon k} C_j = (\cap_{j\epsilon k} \Theta_j , \wedge_{j\epsilon k} H_j)$

$\mathcal{K}$-union: $\cup_{j\epsilon k} C_j = (\cup_{j\epsilon k} \Theta_j , \wedge_{j\epsilon k} H_j)$

$\mathcal{K}$-intersection: $\cap_{j\epsilon k} C_j = (\cap_{j\epsilon k} \Theta_j, \vee_{j\epsilon k} H_j)$,

where $\cup_{j\epsilon k} \Theta_j$ = {(y, $(\cup_{j\epsilon k} \bar{\eta}_{jT_\sigma})$(y), $(\cup_{j\epsilon k} \bar{\eta}_{jI_\sigma})$(y), $(\cup_{j\epsilon k} \bar{\eta}_{jF_\sigma})$(y)) : y $\in$ Y}

$\vee_{j\epsilon k} H_j$ = {(y, $(\vee_{j\epsilon k}\zeta_{jT_\sigma})$(y), $(\vee_{j\epsilon k}\zeta_{jF_\sigma})$(y), $\vee_{j\epsilon k}\zeta_{jF_\sigma}$)(y)) : y $\in$ Y}

$\cap_{j\epsilon k} \Theta_j$ = {(y, $(\cap_{j\epsilon k} \bar{\eta}_{jT_\sigma})$(y), $(\cap_{j\epsilon k} \bar{\eta}_{jI_\sigma})$(y), $(\cap_{j\epsilon k} \bar{\eta}_{jF_\sigma})$(y)) : y $\in$ Y}

$\wedge_{j\epsilon k} H_j$ = {(y, $(\wedge_{j\epsilon k}\zeta_{jT_\sigma})$(y), $(\wedge_{j\epsilon k}\zeta_{jF_\sigma})$(y), $(\wedge_{j\epsilon k}\zeta_{jF_\sigma})$(y)) : y $\in$ Y}.

## 3. Neutrosophic cubic Z-subalgebras

In this part, we examine several intriguing results and introduce the notion of neutrosophic cubic Z-subalgebras.



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

**Definition 3.1:** An NCS in Y is characterized by C = {(y, Θ(y), H(y)) : y ∈ Y}. Then the set C is a neutrosophic cubic Z-subalgebra of Y if it holds: for all $y, z \in Y$,
NS1:

$$\bar{\eta}_{T_\sigma}(y \otimes z) \geq \text{rmin}\{\bar{\eta}_{T_\sigma}(y), \bar{\eta}_{T_\sigma}(z)\}$$

$$\bar{\eta}_{I_\sigma}(y \otimes z) \geq \text{rmin}\{\bar{\eta}_{I_\sigma}(y), \bar{\eta}_{I_\sigma}(z)\}$$

$$\bar{\eta}_{F_\sigma}(y \otimes z) \geq \text{rmin}\{\bar{\eta}_{F_\sigma}(y), \bar{\eta}_{F_\sigma}(z)\}$$

NS2:

$$\zeta_{T_\sigma}(y \otimes z) \leq \max\{\zeta_{T_\sigma}(y), \zeta_{T_\sigma}(z)\}$$

$$\zeta_{I_\sigma}(y \otimes z) \leq \max\{\zeta_{I_\sigma}(y), \zeta_{I_\sigma}(z)\}$$

$$\zeta_{F_\sigma}(y \otimes z) \leq \max\{\zeta_{F_\sigma}(y), \zeta_{F_\sigma}(z)\}.$$

The NCS shall be referred to as for convenience as $C = (\bar{\eta}_{T_\sigma, I_\sigma, F_\sigma}, \zeta_{T_\sigma, I_\sigma, F_\sigma}) = \{(y, \bar{\eta}_{T_\sigma, I_\sigma, F_\sigma}(y), \zeta_{T_\sigma, I_\sigma, F_\sigma}(y)) : y \in Y\}$ with conditions

$$\bar{\eta}_{T_\sigma, I_\sigma, F_\sigma}(y \otimes z) \geq \text{rmin}\{\bar{\eta}_{T_\sigma, I_\sigma, F_\sigma}(y), \bar{\eta}_{T_\sigma, I_\sigma, F_\sigma}(z)\}$$

$$\zeta_{T_\sigma, I_\sigma, F_\sigma}(y \otimes z) \leq \max\{\zeta_{T_\sigma, I_\sigma, F_\sigma}(y), \zeta_{T_\sigma, I_\sigma, F_\sigma}(z)\}.$$

**Example 3.2:** A cubic set C = {(y, Θ(y), H(y)) : y ∈ Y} on a Z-algebra Y in the below Example 2.7.

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\zeta_{T_\sigma}$ | 0.3 | 0.5 | 0.2 | 0.1 |
| $\zeta_{I_\sigma}$ | 0.1 | 0.3 | 0.6 | 0.5 |
| $\zeta_{F_\sigma}$ | 0.2 | 0.4 | 0.3 | 0.7 |

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\bar{\eta}_{T_\sigma}$ | [0.2,0.5] | [0.2,0.3] | [0.1,0.6] | [0.3,0.4] |
| $\bar{\eta}_{I_\sigma}$ | [0.3,0.7] | [0.3,0.5] | [0.2,0.6] | [0.2,0.7] |
| $\bar{\eta}_{F_\sigma}$ | [0.4,0.6] | [0.1,0.7] | [0.3,0.6] | [0.1,0.8] |

**Theorem 3.3:** Assume that $C_j = \{(y, \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(y), \zeta_{jT_\sigma, I_\sigma, F_\sigma}(y)) : y \in Y\}$ be a set of neutrosophic cubic Z-subalgebras of Y. Then $\mathcal{K}$-intersection of $C_j$ is also a neutrosophic cubic Z-subalgebra of Y.

**Proof:** Let $y, z \in Y$. Then

$$\cap\, \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(y \otimes z)$$

$$= \text{rinf}\, \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(y \otimes z)$$

$$\geq \text{rinf}\{\text{rmin}\{\bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(y), \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(z)\}\}$$

$$= \text{rmin}\{\text{rinf}\, \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(y), \text{rinf}\, \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(z)\}$$

$$= \text{rmin}\{\cap\, \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(y), \cap\, \bar{\eta}_{jT_\sigma, I_\sigma, F_\sigma}(z)\},$$

$$\vee \zeta_{jT_\sigma, I_\sigma, F_\sigma}(y \otimes z)$$

$$= \sup \zeta_{jT_\sigma, I_\sigma, F_\sigma}(y \otimes z)$$



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

$\leq \sup\{\max\{\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\}$

$= \max\{\sup \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$

$= \max\{\vee\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \vee\zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}.$

Hence, the proof is complete.

**Theorem 3.4:** Assume that $C_j = \{(y, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y)) : y \in Y\}$ is a set of neutrosophic cubic Z-subalgebras of Y. If $\inf\{\max\{\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\} = \max\{\inf \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \inf \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$, $\forall$ y, z $\in$ Y. Then $\mathcal{S}$-intersection of $C_j$ is also a neutrosophic cubic Z-subalgebra of Y.

**Proof:** Let $y, z \in Y$. Then

$\cap \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$= \text{rinf}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$\geq \text{rinf}\{\text{rmin}\{\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\}$

$= \text{rmin}\{\text{rinf}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \text{rinf}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$

$= \text{rmin}\{\cap \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \cap \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\},$

$\wedge\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$= \inf \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$\leq \inf\{\max\{\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{iT_\sigma,I_\sigma,F_\sigma}(z)\}\}$

$= \max\{\inf \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$

$= \max\{\wedge\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \wedge\zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}.$

Hence, the proof is complete.

**Theorem 3.5:** Assume that $C_j = \{(y, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y)) : y \in Y\}$ be a set of neutrosophic cubic Z-subalgebras of Y. If $\text{rsup}\{\text{rmin}\{\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\} = \text{rmin}\{\text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$, $\forall$ y, z $\in$ Y. Then $\mathcal{S}$-union of $C_j$ is also a neutrosophic cubic Z-subalgebra of Y.

**Proof:** Let $y, z \in Y$. Then

$\cup \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$= \text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$\geq \text{rsup}\{\text{rmin}\{\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\}$

$= \text{rmin}\{\text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$

$= \text{rmin}\{\cup \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \cup \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\},$

$\vee\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$= \sup\{\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)\}$

$\leq \sup\{\max\{\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\}$

$= \max\{\sup \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \sup \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$

$= \max\{\vee\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \vee\zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}.$

Hence, the proof is complete.



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

**Theorem 3.6:** Assume that $C_j$ = {(y, $\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}$(y), $\zeta_{jT_\sigma,I_\sigma,F_\sigma}$(y)) : y ∈ Y} be a set of neutrosophic cubic Z-subalgebras of Y. If inf{max{$\zeta_{jT_\sigma,I_\sigma,F_\sigma}$(y), $\zeta_{jT_\sigma,I_\sigma,F_\sigma}$(z)}} = max{inf $\zeta_{jT_\sigma,I_\sigma,F_\sigma}$(y), $\zeta_{jT_\sigma,I_\sigma,F_\sigma}$(z)} and rsup {rmin{$\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}$(y), $\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}$(z)}} = rmin{rsup $\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}$(y), rsup $\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}$(z)}, ∀ y, z ∈ Y. Then $\mathcal{K}$-union of $C_j$ is also a neutrosophic cubic Z-subalgebra of Y.

**Proof:** Let $y, z \in Y$. Then

$$\cup\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$
$$= \text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$
$$\geq \text{rsup}\{\text{rmin}\{\bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\}$$
$$= \text{rmin}\{\text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \text{rsup}\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$$
$$= \text{rmin}\{\cup\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(y), \cup\, \bar{\eta}_{jT_\sigma,I_\sigma,F_\sigma}(z)\},$$
$$\wedge \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$
$$= \inf \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$
$$\leq \inf\{\max\{\zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}\}$$
$$= \max\{\inf \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \inf \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}$$
$$= \max\{\wedge \zeta_{jT_\sigma,I_\sigma,F_\sigma}(y), \wedge \zeta_{jT_\sigma,I_\sigma,F_\sigma}(z)\}.$$

Hence, the proof is complete.

**Theorem 3.7:** NCS $C_j$ = ($\Theta_{T_\sigma,I_\sigma,F_\sigma}$, $H_{T_\sigma,I_\sigma,F_\sigma}$) of Y is a neutrosophic cubic Z-subalgebra of Y if and only if $\eta^L_{T_\sigma,I_\sigma,F_\sigma}$, $\eta^U_{T_\sigma,I_\sigma,F_\sigma}$, and $\zeta_{T_\sigma,I_\sigma,F_\sigma}$ are fuzzy Z-subalgebras of Y.

**Proof:** Assume that $\eta^L_{T_\sigma,I_\sigma,F_\sigma}$, $\eta^U_{T_\sigma,I_\sigma,F_\sigma}$, and $\zeta_{T_\sigma,I_\sigma,F_\sigma}$ are fuzzy Z-subalgebras of Y. Let y, z ∈ Y. Then

$$\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) \geq \min\{\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^L_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$
$$\eta^U_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) \geq \min\{\eta^U_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$
$$\zeta_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) \leq \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y), \zeta_{T_\sigma,I_\sigma,F_\sigma}(z)\}.$$

Now,

$$\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$
$$= [\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)]$$
$$\geq [\min\{\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^L_{T_\sigma,I_\sigma,F_\sigma}(z)\}, \min\{\eta^U_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(z)\}]$$
$$\geq \text{rmin}\{[\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(z)], [\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(z)]\}$$
$$= \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(z)\}.$$

Hence, $C_j$ is a neutrosophic cubic Z-subalgebra of Y.

On the other hand, assume that $C_j$ is a neutrosophic cubic Z-subalgebra of Y. Let y, z ∈ Y. Then

$$[\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)]$$
$$= \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$
$$\geq \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$
$$\geq \text{rmin}\{[\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(z)], [\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(z)]\}.$$



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

Thus

$$\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) \geq \min\{\eta^L_{T_\sigma,I_\sigma,F_\sigma}(y),\ \eta^L_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$

$$\eta^U_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) \geq \min\{\eta^U_{T_\sigma,I_\sigma,F_\sigma}(y), \eta^U_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$

$$\zeta_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) \leq \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y), \zeta_{T_\sigma,I_\sigma,F_\sigma}(z)\}.$$

Hence, $\eta^L_{T_\sigma,I_\sigma,F_\sigma}$, $\eta^U_{T_\sigma,I_\sigma,F_\sigma}$, and $\zeta_{T_\sigma,I_\sigma,F_\sigma}$ are fuzzy Z-subalgebras of Y.

**Remark 3.8:** The sets represented by $I_{\eta_{T_\sigma,I_\sigma,F_\sigma}}$ and $I_{\zeta_{T_\sigma,I_\sigma,F_\sigma}}$ are also Z-subalgebras of Y which are defined as $I_{\eta_{T_\sigma,I_\sigma,F_\sigma}} = \{y \in Y : \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y) = \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(0)\}$ and $I_{\zeta_{T_\sigma,I_\sigma,F_\sigma}} = \{y \in Y : \zeta_{T_\sigma,I_\sigma,F_\sigma}(y) = \zeta_{T_\sigma,I_\sigma,F_\sigma}(0)\}$.

**Theorem 3.9:** Let C = $(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}, \zeta_{T_\sigma,I_\sigma,F_\sigma})$ be a neutrosophic cubic Z-subalgebra of Y. Then the sets $I_{\eta_{T_\sigma,I_\sigma,F_\sigma}}$ and $I_{\zeta_{T_\sigma,I_\sigma,F_\sigma}}$ are also Z-subalgebras of Y.

**Proof:** Let $y, z \in I_{\eta_{T_\sigma,I_\sigma,F_\sigma}}$. Then $\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y) = \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(0) = \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(z)$. Thus

$$\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$

$$\geq \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$

$$\geq \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(0), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(0)\}$$

$$= \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(0).$$

Then $\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) = \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(0)$ or $y \otimes z \in I_{\eta_{T_\sigma,I_\sigma,F_\sigma}}$.

Now, let $y, z \in I_{\zeta_{T_\sigma,I_\sigma,F_\sigma}}$. Then $\zeta_{T_\sigma,I_\sigma,F_\sigma}(y) = \zeta_{T_\sigma,I_\sigma,F_\sigma}(0) = \zeta_{T_\sigma,I_\sigma,F_\sigma}(z)$. Thus

$$\zeta_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$

$$\leq \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y), \zeta_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$

$$= \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(0), \zeta_{T_\sigma,I_\sigma,F_\sigma}(0)\}$$

$$= \zeta_{T_\sigma,I_\sigma,F_\sigma}(0).$$

Then $\zeta_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) = \zeta_{T_\sigma,I_\sigma,F_\sigma}(0)$ or $y \otimes z \in I_{\zeta_{T_\sigma,I_\sigma,F_\sigma}}$.

Hence, the sets $I_{\eta_{T_\sigma,I_\sigma,F_\sigma}}$ and $I_{\zeta_{T_\sigma,I_\sigma,F_\sigma}}$ are Z-subalgebras of Y.

**Theorem 3.10:** Let P be a non-empty subset of Y and C = $(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}, \zeta_{T_\sigma,I_\sigma,F_\sigma})$ be a neutrosophic cubic Z-subalgebra of Y by

$$\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y) = \begin{cases} [\alpha_{T_\sigma,I_\sigma,F_\sigma}, \alpha_{T_\sigma,I_\sigma,F_\sigma}]; & \text{if } y \in P \\ [\beta_{T_\sigma,I_\sigma,F_\sigma}, \beta_{T_\sigma,I_\sigma,F_\sigma}]; & \text{otherwise} \end{cases} \quad \text{and} \quad \zeta_{T_\sigma,I_\sigma,F_\sigma}(y) = \begin{cases} \gamma_{T_\sigma,I_\sigma,F_\sigma}; & \text{if } y \in P \\ \delta_{T_\sigma,I_\sigma,F_\sigma}; & \text{otherwise} \end{cases}$$

$\forall\ [\alpha_{T_\sigma,I_\sigma,F_\sigma}, \alpha_{T_\sigma,I_\sigma,F_\sigma}], [\beta_{T_\sigma,I_\sigma,F_\sigma}, \beta_{T_\sigma,I_\sigma,F_\sigma}] \in \mathfrak{D}[0,1]$ and $\gamma_{T_\sigma,I_\sigma,F_\sigma}$, $\delta_{T_\sigma,I_\sigma,F_\sigma} \in [0,1]$ with $[\alpha_{T_\sigma,I_\sigma,F_\sigma}, \alpha_{T_\sigma,I_\sigma,F_\sigma}] \geq [\beta_{T_\sigma,I_\sigma,F_\sigma}, \beta_{T_\sigma,I_\sigma,F_\sigma}]$ and $\gamma_{T_\sigma,I_\sigma,F_\sigma} \leq \delta_{T_\sigma,I_\sigma,F_\sigma}$. Then C is a neutrosophic cubic Z-subalgebra of Y if and only if P is a Z-subalgebra of Y.

**Proof:** Assume that C is a neutrosophic cubic Z-subalgebra of Y. Let $y, z \in P$. Then

$$\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)$$

$$\geq \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(z)\}$$

$$\geq \text{rmin}\{[\alpha_{T_\sigma,I_\sigma,F_\sigma}, \alpha_{T_\sigma,I_\sigma,F_\sigma}], [\alpha_{T_\sigma,I_\sigma,F_\sigma}, \alpha_{T_\sigma,I_\sigma,F_\sigma}]\}$$

$$= [\alpha_{T_\sigma,I_\sigma,F_\sigma}, \alpha_{T_\sigma,I_\sigma,F_\sigma}],$$



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

$\zeta_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z)$

$\leq \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y), \zeta_{T_\sigma,I_\sigma,F_\sigma}(z)\}$

$\leq \max\{\gamma_{T_\sigma,I_\sigma,F_\sigma}, \gamma_{T_\sigma,I_\sigma,F_\sigma}\}$

$= \gamma_{T_\sigma,I_\sigma,F_\sigma}$.

Hence, P is a Z-subalgebra of Y.

Conversely, assume that P is a Z-subalgebra of Y. Let $y, z \in Y$.

Case 1: If $y, z \in P$, then $y \otimes z \in P$.

Thus

$$\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) = [\alpha_{T_\sigma,I_\sigma,F_\sigma}, \alpha_{T_\sigma,I_\sigma,F_\sigma}] = \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(z)\},$$

$$\zeta_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) = \gamma_{T_\sigma,I_\sigma,F_\sigma} = \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y), \zeta_{T_\sigma,I_\sigma,F_\sigma}(z)\}.$$

Case 2: If $y, z \notin P$, then

$$\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) = [\beta_{T_\sigma,I_\sigma,F_\sigma}, \beta_{T_\sigma,I_\sigma,F_\sigma}] = \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(z)\},$$

$$\zeta_{T_\sigma,I_\sigma,F_\sigma}(y \otimes z) = \delta_{T_\sigma,I_\sigma,F_\sigma} = \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y), \zeta_{T_\sigma,I_\sigma,F_\sigma}(z)\}.$$

Hence, C is a neutrosophic cubic Z-subalgebra of Y.

## 4. Homomorphisms of neutrosophic cubic Z-subalgebras

This section covers several results about the homomorphism of neutrosophic cubic Z-subalgebras.

**Theorem 4.1:** Assume that $C = (\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}, \zeta_{T_\sigma,I_\sigma,F_\sigma})$ is a neutrosophic cubic Z-subalgebra of Z. Let $f: Y \rightarrow Z$ be a homomorphism of Z-algebras from Y to Z. Then the image $f(C) = \{(y, f_{\text{rsup}}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(y), f_{\inf}(\zeta_{T_\sigma,I_\sigma,F_\sigma})(y) : y \in Y\}$ of C under f is a neutrosophic cubic Z-subalgebra of Z.

**Proof:** Let $y_1, y_2 \in Z$. Then $\{y_1 \otimes y_2 : y_1 \in f^{-1}(z_2), y_2 \in f^{-1}(z_2)\} \subseteq \{y \in Y : y \in f^{-1}(z_1 \otimes z_2)\}$. Now,

$f_{\text{rsup}}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(z_1 \otimes z_2)$

$= \text{rsup}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y) : y \in f^{-1}(z_1 \otimes z_2)\}$

$= \text{rsup}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y_1 \otimes y_2) : y_1 \in f^{-1}(z_1), y_2 \in f^{-1}(z_2)\}$

$\geq \text{rsup}\{\text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y_1), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y_2): y_1 \in f^{-1}(z_1), y_2 \in f^{-1}(z_2)\}$

$= \text{rmin}\{\text{rsup}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y_1) : y_1 \in f^{-1}(z_1)\}, \text{rsup}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(y_2) : y_2 \in f^{-1}(z_2)\}\}$

$= \text{rmin}\{f_{\text{rsup}}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(z_1), f_{\text{rsup}}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(z_2)\},$

$f_{\inf}(\bar{\zeta}_{T_\sigma,I_\sigma,F_\sigma})(z_1 \otimes z_2)$

$= \inf\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y) : y \in f^{-1}(z_1 \otimes z_2)\}$

$= \inf\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y_1 \otimes y_2) : y_1 \in f^{-1}(z_1), y_2 \in f^{-1}(z_2)\}$

$\leq \inf\{\max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y_1), \zeta_{T_\sigma,I_\sigma,F_\sigma}(y_2) : y_1 \in f^{-1}(z_1), y_2 \in f^{-1}(z_2)\}$

$= \max\{\inf\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y_1) : y_1 \in f^{-1}(z_1)\}, \inf\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(y_2) : y_2 \in f^{-1}(z_2)\}\}$

$= \max\{f_{\inf}(\bar{\zeta}_{T_\sigma,I_\sigma,F_\sigma})(z_1), f_{\inf}(\bar{\zeta}_{T_\sigma,I_\sigma,F_\sigma})(z_2)\}.$

Hence, f(C) is a neutrosophic cubic Z-subalgebra of Y.



DOI: https://doi.org/10.54216/IJNS.230307
Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024

**Theorem 4.2:** Assume that C = $(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}, \zeta_{T_\sigma,I_\sigma,F_\sigma})$ is a neutrosophic cubic Z-subalgebra of Z. Let $f: Y \to Z$ be a homomorphism of Z-algebras. Then the inverse-image $f^{-1}(C) = \{(y, f^{-1}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(y), f^{-1}(\zeta_{T_\sigma,I_\sigma,F_\sigma})(y)) : y \in Y\}$ of C under f is a neutrosophic cubic Z-subalgebra of Y.

**Proof:** Let $y, z \in Y$. Then

$$f^{-1}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(y \otimes z)$$
$$= \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(f(y \otimes z))$$
$$= \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(f(y) \otimes f(z))$$
$$\geq \text{rmin}\{\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(f(y)), \bar{\eta}_{T_\sigma,I_\sigma,F_\sigma}(f(z))\}$$
$$= \text{rmin}\{f^{-1}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(y), f^{-1}(\bar{\eta}_{T_\sigma,I_\sigma,F_\sigma})(z)\},$$
$$f^{-1}(\zeta_{T_\sigma,I_\sigma,F_\sigma})(y \otimes z)$$
$$= \zeta_{T_\sigma,I_\sigma,F_\sigma}(f(y \otimes z))$$
$$= \zeta_{T_\sigma,I_\sigma,F_\sigma}(f(y) \otimes f(z))$$
$$\leq \max\{\zeta_{T_\sigma,I_\sigma,F_\sigma}(f(y)), \zeta_{T_\sigma,I_\sigma,F_\sigma}(f(z))\}$$
$$= \max\{f^{-1}(\zeta_{T_\sigma,I_\sigma,F_\sigma})(y), f^{-1}(\zeta_{T_\sigma,I_\sigma,F_\sigma})(z)\}.$$

Hence, $f^{-1}(C)$ is a neutrosophic cubic Z-subalgebra of Y.

## 5. Conclusion

In this study, neutrosophic cubic Z-subalgebras of Z-algebras are defined. Further we given the new kinds of various conditions for neutrosophic cubic $\mathcal{S}$-union, $\mathcal{S}$-intersection, $\mathcal{K}$-union, and $\mathcal{K}$-intersection and demonstrated them with examples. In addition, we investigated a lot of results that used the Z-subalgebra notation for a better understanding. The exploration of novel methodologies in this work, for addressing uncertainties within neutrosophic systems provides a solid foundation for practical applications. As we move forward, this research not only enhances our theoretical understanding but also establishes a framework for implementation in diverse fields. The findings of this study make a substantial contribution to the development of neutrosophic cubic Z-algebras, providing valuable insights for emerging researchers seeking advanced mathematical solutions. Moreover, this can be employed in diverse cubic algebraic scenarios in our forthcoming research endeavors.

**Funding:** This research project was supported by the Thailand Science Research and Innovation Fund and the University of Phayao (Grant No. FF67-UoE-Aiyared-Iampan).

**Conflicts of Interest:** The authors declare no conflict of interest.

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Received: August 25, 2023 Revised: November 28, 2023 Accepted: January 24, 2024