A Study of the 16-Plithogenic and 17-Plithogenic Square Real Matrices and Their Properties

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Abstract

This paper is dedicated to study the algebraic structures that are related to symbolic 16-plithogenic/17-plithogenic with symbolic plithogenic real entries, where symbolic 16-plithogenic/17-plithogenic eigenvectors and values will be discussed and presented in terms of theorems. As well as, the computation of determinants, inverses, and eigenvalues and vectors.

Keywords: symbolic 16-plithogenic matrix; symbolic 16-plithogenic eigenvalue; symbolic 16- plithogenic eigenvector; symbolic 17-plithogenic matrix; symbolic 17-plithogenic eigenvalue; symbolic 17- plithogenic eigenvector.

1. Introduction

The symbolic n-plithogenic algebra began with the work of Smarandache [2], where he defined for the first time the applications of symbolic n-plithogenic sets in building algebraic generalizations of well-known algebraic structures. Plithogenic matrices for various values of n are a vivid example of the applications of plithogenic sets in the study of algebraic structures and their properties related to them.

The main difference between symbolic n-plithogenic algebraic structure and n-refined neutrosophic structure is the definition of the multiplication operation, where the multiplication between the sub-indices is defined as follows:
$P_iP_j = P_{\max(i,j)}$. For more details about similar systems of neutrosophic and refined neutrosophic matrices, see [12-16].

Symbolic 2-plithogenic rings were defined by Merkepci.et.al [1], 2-plithogenic algebraic structures [3], and 3-plithogenic algebraic structures [4-6].

Recently, the symbolic n-plithogenic matrices have been introduced for different values of n, see [7-11, 17-18]. The algebraic properties of these matrices were studied widely, especially those which are related to the diagonalization problem such as eigenvalues, eigenvectors, and inverses.
In general, the symbolic n-plithogenic square real matrix is defined with the following formula:
$F = F_0 + \sum_{i=1}^{n} F_iP_i$, where F_i are s-square classical matrices with real entries.

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

This has motivated us to follow these efforts, where we show the concept of 16 plithogenic/17-plithogenic matrices with some of their algebraic properties.

2. Main Discussion

Symbolic 16-plithogenic matrices

Definition:

The square symbolic 16-plithogenic matrix is defined as follows:

$\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$; $(\mu_i)_{n\times n}$ is square matrix of real entries.

Example.

Consider the symbolic 16-plithogenic matrix:

$$\Delta = \begin{pmatrix} -3 & -9 \\ 1 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 67 & 1 \end{pmatrix} P_1 + \begin{pmatrix} 12 & -1 \\ 7 & 12 \end{pmatrix} P_2 + \begin{pmatrix} 6 & -8 \\ 9 & -6 \end{pmatrix} P_3 + \begin{pmatrix} 8 & 5 \\ 6 & 1 \end{pmatrix} P_4 + \begin{pmatrix} -5 & -5 \\ -5 & -2 \end{pmatrix} P_5 + \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix} P_6 + \begin{pmatrix} -1 & 7 \\ 9 & 8 \end{pmatrix} P_7 + \begin{pmatrix} 12 & 11 \\ 65 & -1 \end{pmatrix} P_8 + \begin{pmatrix} -1 & 19 \\ -1 & 0 \end{pmatrix} P_{10} + \begin{pmatrix} 8 & -1 \\ 7 & 5 \end{pmatrix} P_{11} + \begin{pmatrix} 42 & -41 \\ 2 & -8 \end{pmatrix} P_{12} + \begin{pmatrix} 41 & -1 \\ 21 & -9 \end{pmatrix} P_{13} + \begin{pmatrix} 4 & -11 \\ 21 & -8 \end{pmatrix} P_{14} + \begin{pmatrix} 41 & -1 \\ 2 & -8 \end{pmatrix} P_{15} + \begin{pmatrix} 41 & -31 \\ 32 & -8 \end{pmatrix} P_{16}.$$

Definition.

Let $\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$ be a symbolic 16-plithogenic matrix of size $n \times n$, hence:

$$\begin{aligned}
\det \Delta = \det(\Delta_0) &+ \left[\det\left(\sum_{i=0}^{1} \Delta_i\right) - \det(\Delta_0)\right] P_1 + \left[\det\left(\sum_{i=0}^{2} \Delta_i\right) - \det\left(\sum_{i=0}^{1} \Delta_i\right)\right] P_2 \\
&+ \left[\det\left(\sum_{i=0}^{3} \Delta_i\right) - \det\left(\sum_{i=0}^{2} \Delta_i\right)\right] P_3 + \left[\det\left(\sum_{i=0}^{4} \Delta_i\right) - \det\left(\sum_{i=0}^{3} \Delta_i\right)\right] P_4 \\
&+ \left[\det\left(\sum_{i=0}^{5} \Delta_i\right) - \det\left(\sum_{i=0}^{4} \Delta_i\right)\right] P_5 + \left[\det\left(\sum_{i=0}^{6} \Delta_i\right) - \det\left(\sum_{i=0}^{5} \Delta_i\right)\right] P_6 \\
&+ \left[\det\left(\sum_{i=0}^{7} \Delta_i\right) - \det\left(\sum_{i=0}^{6} \Delta_i\right)\right] P_7 + \left[\det\left(\sum_{i=0}^{8} \Delta_i\right) - \det\left(\sum_{i=0}^{7} \Delta_i\right)\right] P_8 \\
&+ \left[\det\left(\sum_{i=0}^{9} \Delta_i\right) - \det\left(\sum_{i=0}^{8} \Delta_i\right)\right] P_9 + \left[\det\left(\sum_{i=0}^{10} \Delta_i\right) - \det\left(\sum_{i=0}^{9} \Delta_i\right)\right] P_{10} \\
&+ \left[\det\left(\sum_{i=0}^{11} \Delta_i\right) - \det\left(\sum_{i=0}^{10} \Delta_i\right)\right] P_{11} + \left[\det\left(\sum_{i=0}^{12} \Delta_i\right) - \det\left(\sum_{i=0}^{11} \Delta_i\right)\right] P_{12} \\
&+ \left[\det\left(\sum_{i=0}^{13} \Delta_i\right) - \det\left(\sum_{i=0}^{12} \Delta_i\right)\right] P_{13} + \left[\det\left(\sum_{i=0}^{14} \Delta_i\right) - \det\left(\sum_{i=0}^{13} \Delta_i\right)\right] P_{14} \\
&+ \left[\det\left(\sum_{i=0}^{15} \Delta_i\right) - \det\left(\sum_{i=0}^{14} \Delta_i\right)\right] P_{15} + \left[\det\left(\sum_{i=0}^{16} \Delta_i\right) - \det\left(\sum_{i=0}^{15} \Delta_i\right)\right] P_{16}
\end{aligned}$$

Theorem1.

Let $\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$ be a symbolic 16-plithogenic matrix of size $n \times n$, hence:

1. Δ is invertible if and only if $\det \Delta$ is an invertible symbolic 16-plithogenic real number.
2. $\Delta^{-1} = {\Delta_0}^{-1} + \left[(\sum_{i=0}^{1} \Delta_i)^{-1} - {\Delta_0}^{-1}\right] P_1 + [(\sum_{i=0}^{2} \Delta_i)^{-1} - (\sum_{i=0}^{1} \Delta_i)^{-1}] P_2 + [(\sum_{i=1}^{3} \Delta_i)^{-1} - (\sum_{i=0}^{2} \Delta_i)^{-1}] P_3 + [(\sum_{i=1}^{4} \Delta_i)^{-1} - (\sum_{i=0}^{3} \Delta_i)^{-1}] P_4 + \left[(\sum_{i=1}^{5} \Delta_i)^{-1} - (\sum_{i=0}^{4} \Delta_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} \Delta_i)^{-1} - (\sum_{i=0}^{5} \Delta_i)^{-1}\right] P_6 + [(\sum_{i=1}^{7} \Delta_i)^{-1} - (\sum_{i=0}^{6} \Delta_i)^{-1}] P_7 + [(\sum_{i=1}^{8} \Delta_i)^{-1} - (\sum_{i=0}^{7} \Delta_i)^{-1}] P_8 + [(\sum_{i=1}^{9} \Delta_i)^{-1} - (\sum_{i=0}^{8} \Delta_i)^{-1}] P_9 + [(\sum_{i=1}^{10} \Delta_i)^{-1} - (\sum_{i=0}^{9} \Delta_i)^{-1}] P_{10} + [(\sum_{i=1}^{11} \Delta_i)^{-1} - (\sum_{i=0}^{10} \Delta_i)^{-1}] P_{11} + [(\sum_{i=1}^{12} \Delta_i)^{-1} - (\sum_{i=0}^{11} \Delta_i)^{-1}] P_{12} + [(\Delta)^{-1} - (\sum_{i=0}^{12} \Delta_i)^{-1}] P_{13} + [(\sum_{i=1}^{14} \Delta_i)^{-1} - (\sum_{i=0}^{13} \Delta_i)^{-1}] P_{14} + \left[(\sum_{i=1}^{15} \Delta_i)^{-1} - (\sum_{i=0}^{14} \Delta_i)^{-1}\right] P_{15} + \left[(\sum_{i=1}^{16} \Delta_i)^{-1} - (\sum_{i=0}^{15} \Delta_i)^{-1}\right] P_{16}$.

Definition.

Let $r = r_0 + \sum_{i=1}^{16} r_i P_i$ be a symbolic 16-plithogenic real number and $\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$ be a symbolic 16-plithogenic square real matrix, then r is called symbolic 16-plithogenic eigen value if and only if $\Delta X = rX$.

X is called symbolic 16-plithogenic eigenvector.

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

Theorem2.

Let $u = u_0 + \sum_{i=1}^{16} u_i P_i \in 16 - SP_R, X = X_0 + \sum_{i=1}^{16} X_i P_i$ be a symbolic 16-plithogenic real vector, then u is eigen value of $\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$ with X as the corresponding eigen vector if and only if:
$\sum_{i=0}^{j} u_i$ is eigen value of $\sum_{i=0}^{j} \Delta_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \le j \le 16$.

Theorem3.

$$\Delta^n = {\Delta_0}^n + P_1\left[\left(\sum_{i=0}^{1}\Delta_i\right)^n - {\Delta_0}^n\right] + \left[\left(\sum_{i=0}^{2}\Delta_i\right)^n - \left(\sum_{i=0}^{1}\Delta_i\right)^n\right]P_2 + \left[\left(\sum_{i=1}^{3}\Delta_i\right)^n - \left(\sum_{i=0}^{2}\Delta_i\right)^n\right]P_3$$
$$+ \left[\left(\sum_{i=1}^{4}\Delta_i\right)^n - \left(\sum_{i=0}^{3}\Delta_i\right)^n\right]P_4 + \left[\left(\sum_{i=1}^{5}\Delta_i\right)^n - \left(\sum_{i=0}^{4}\Delta_i\right)^n\right]P_5 + \left[\left(\sum_{i=1}^{6}\Delta_i\right)^n - \left(\sum_{i=0}^{5}\Delta_i\right)^n\right]P_6$$
$$+ \left[\left(\sum_{i=1}^{7}\Delta_i\right)^n - \left(\sum_{i=0}^{6}\Delta_i\right)^n\right]P_7 + \left[\left(\sum_{i=1}^{8}\Delta_i\right)^n - \left(\sum_{i=0}^{7}\Delta_i\right)^n\right]P_8 + \left[\left(\sum_{i=1}^{9}\Delta_i\right)^n - \left(\sum_{i=0}^{8}\Delta_i\right)^n\right]P_9$$
$$+ \left[\left(\sum_{i=1}^{10}\Delta_i\right)^n - \left(\sum_{i=0}^{9}\Delta_i\right)^n\right]P_{10} + \left[\left(\sum_{i=1}^{11}\Delta_i\right)^n - \left(\sum_{i=0}^{10}\Delta_i\right)^n\right]P_{11}$$
$$+ \left[\left(\sum_{i=1}^{12}\Delta_i\right)^n - \left(\sum_{i=0}^{11}\Delta_i\right)^n\right]P_{12} + \left[\left(\sum_{i=1}^{13}\Delta_i\right)^n - \left(\sum_{i=0}^{12}\Delta_i\right)^n\right]P_{13}$$
$$+ \left[\left(\sum_{i=1}^{14}\Delta_i\right)^n - \left(\sum_{i=0}^{13}\Delta_i\right)^n\right]P_{14} + \left[\left(\sum_{i=1}^{15}\Delta_i\right)^n - \left(\sum_{i=0}^{14}\Delta_i\right)^n\right]P_{15}$$
$$+ \left[\left(\sum_{i=1}^{16}\Delta_i\right)^n - \left(\sum_{i=0}^{15}\Delta_i\right)^n\right]P_{16}$$

Theorem4.

Let $\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$ be a square 16-plithogenic invertible real matrix, then:
1). $\det(\Delta^{-1}) = (\det \Delta)^{-1}$
2). $\det \Delta^t = \det \Delta$
3). $\det(\Delta . C) = \det \Delta . \det C \,;\, C = C_0 + \sum_{i=1}^{16} C_i P_i$.

Definition.

Let $\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$ be a symbolic 16-plithogenic real square matrix, then:
Δ is called orthogonal if and only if $\Delta^t = \Delta^{-1}$.

Theorem5.

Δ is orthogonal if and only if $\sum_{i=0}^{j} \Delta_i \,;\ 0 \le j \le 16$ are orthogonal.

Proof of theorem1.

1). Let $\Delta = \Delta_0 + \sum_{i=1}^{16} \Delta_i P_i$, then Δ is invertible if and only if there exists $T = T_0 + \sum_{i=1}^{16} T_i P_i$ such that:
$\Delta \times T = U_{n\times n}$, hence:

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

$$\begin{cases}\Delta_0 T_0 = U_{n\times n}\\ \sum_{i=0}^{1}\Delta_i\sum_{i=0}^{1}T_i - \Delta_0 T_0 = O_{n\times n}\\ \sum_{i=0}^{2}\Delta_i\sum_{i=0}^{2}T_i - \sum_{i=0}^{1}\Delta_i\sum_{i=0}^{1}T_i = O_{n\times n}\\ \sum_{i=0}^{3}\Delta_i\sum_{i=0}^{3}T_i - \sum_{i=0}^{2}\Delta_i\sum_{i=0}^{2}T_i = O_{n\times n}\\ \sum_{i=0}^{4}\Delta_i\sum_{i=0}^{4}T_i - \sum_{i=0}^{3}\Delta_i\sum_{i=0}^{3}T_i = O_{n\times n}\\ \sum_{i=0}^{5}\Delta_i\sum_{i=0}^{5}T_i - \sum_{i=0}^{4}\Delta_i\sum_{i=0}^{4}T_i = O_{n\times n}\\ \sum_{i=0}^{6}\Delta_i\sum_{i=0}^{6}T_i - \sum_{i=0}^{5}\Delta_i\sum_{i=0}^{5}T_i = O_{n\times n}\\ \sum_{i=0}^{7}\Delta_i\sum_{i=0}^{7}T_i - \sum_{i=0}^{6}\Delta_i\sum_{i=0}^{6}T_i = O_{n\times n}\\ \sum_{i=0}^{8}\Delta_i\sum_{i=0}^{8}T_i - \sum_{i=0}^{7}\Delta_i\sum_{i=0}^{7}T_i = O_{n\times n}\\ \sum_{i=0}^{9}\Delta_i\sum_{i=0}^{9}T_i - \sum_{i=0}^{8}\Delta_i\sum_{i=0}^{8}T_i = O_{n\times n}\\ \sum_{i=0}^{10}\Delta_i\sum_{i=0}^{10}T_i - \sum_{i=0}^{9}\Delta_i\sum_{i=0}^{9}T_i = O_{n\times n}\\ \sum_{i=0}^{11}\Delta_i\sum_{i=0}^{11}T_i - \sum_{i=0}^{10}\Delta_i\sum_{i=0}^{10}T_i = O_{n\times n}\\ \sum_{i=0}^{12}\Delta_i\sum_{i=0}^{12}T_i - \sum_{i=0}^{11}\Delta_i\sum_{i=0}^{11}T_i = O_{n\times n}\\ \sum_{i=0}^{13}\Delta_i\sum_{i=0}^{13}T_i - \sum_{i=0}^{12}\Delta_i\sum_{i=0}^{12}T_i = O_{n\times n}\\ \sum_{i=0}^{14}\Delta_i\sum_{i=0}^{14}T_i - \sum_{i=0}^{13}\Delta_i\sum_{i=0}^{13}T_i = O_{n\times n}\\ \sum_{i=0}^{15}\Delta_i\sum_{i=0}^{15}T_i - \sum_{i=0}^{14}\Delta_i\sum_{i=0}^{14}T_i = O_{n\times n}\\ \sum_{i=0}^{16}\Delta_i\sum_{i=0}^{16}T_i - \sum_{i=0}^{15}\Delta_i\sum_{i=0}^{15}T_i = O_{n\times n}\end{cases}$$

This implies that:

$$\begin{cases}\Delta_0 T_0 = U_{n\times n}\\ \sum_{i=0}^{j}\Delta_i\sum_{i=0}^{j}T_i = U_{n\times n}\end{cases};\ 1\le j\le 16$$

Hence $\det\left(\sum_{i=0}^{j}\Delta_i\right)\neq 0$ for all $1\le j\le 16$, so that $\det(\Delta)$ is invertible in $16-SP_R$.

2). It holds directly as follows:

$\sum_{i=0}^{j}T_i = \left(\sum_{i=0}^{j}\Delta_i\right)^{-1}$ for $1\le j\le 16$, hence:

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Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

$\Delta^{-1}= {\Delta_0}^{-1} + [(\sum_{i=0}^{1}\Delta_i)^{-1} - {\Delta_0}^{-1}]P_1 + [(\sum_{i=0}^{2}\Delta_i)^{-1} - (\Delta)^{-1}]P_2 + [(\sum_{i=1}^{3}\Delta_i)^{-1} - (\sum_{i=0}^{2}\Delta_i)^{-1}]P_3 + [(\sum_{i=1}^{4}\Delta_i)^{-1} - (\sum_{i=0}^{3}\Delta_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5}\Delta_i)^{-1} - (\sum_{i=0}^{4}\Delta_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6}\Delta_i)^{-1} - (\sum_{i=0}^{5}\Delta_i)^{-1}\right]P_6 + [(\sum_{i=1}^{7}\Delta_i)^{-1} - (\sum_{i=0}^{6}\Delta_i)^{-1}]P_7 + [(\sum_{i=1}^{8}\Delta_i)^{-1} - (\sum_{i=0}^{7}\Delta_i)^{-1}]P_8 + [(\sum_{i=1}^{9}\Delta_i)^{-1} - (\sum_{i=0}^{8}\Delta_i)^{-1}]P_9 + [(\sum_{i=1}^{10}\Delta_i)^{-1} - (\sum_{i=0}^{9}\Delta_i)^{-1}]P_{10} + [(\sum_{i=1}^{11}\Delta_i)^{-1} - (\sum_{i=0}^{10}\Delta_i)^{-1}]P_{11} + [(\sum_{i=1}^{12}\Delta_i)^{-1} - (\sum_{i=0}^{11}\Delta_i)^{-1}]P_{12} + [(\sum_{i=1}^{13}\Delta_i)^{-1} - (\sum_{i=0}^{12}\Delta_i)^{-1}]P_{13}+ [(\sum_{i=1}^{14}\Delta_i)^{-1} - (\sum_{i=0}^{13}\mu_i)^{-1}]P_{14}+ \left[(\sum_{i=1}^{15}\Delta_i)^{-1} - (\sum_{i=0}^{14}\Delta_i)^{-1}\right]P_{15} + \left[(\sum_{i=1}^{16}\Delta_i)^{-1} - (\sum_{i=0}^{15}\Delta_i)^{-1}\right]P_{16}.$

Proof of theorem2.

It is clear that r is an eigen value of Δ with X as an eigen vector if and only if:
$\Delta.X = r.X$, which is equivalent to:

$$\begin{cases} \Delta_0 X_0 = r_0 X_0 \\ \sum_{i=0}^{j}\Delta_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} r_i \sum_{i=0}^{j} X_i \end{cases};\ 1 \le j \le 15$$

Which is equivalent to:
$\sum_{i=0}^{j} r_i$ is an eigen value of $\sum_{i=0}^{j}\Delta_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 16$.

Proof of theorem4.

1). $det\Delta^{-1}= det({\Delta_0}^{-1}) + P_1[det(\sum_{i=0}^{1}\Delta_i)^{-1} - det({\Delta_0}^{-1})] + [det(\sum_{i=0}^{2}\Delta_i)^{-1} - det(\sum_{i=0}^{1}\Delta_i)^{-1}]P_2 + [det(\sum_{i=1}^{3}\Delta_i)^{-1} - det(\sum_{i=0}^{2}\Delta_i)^{-1}]P_3 + [det(\sum_{i=1}^{4}\Delta_i)^{-1} - det(\sum_{i=0}^{3}\Delta_i)^{-1}]P_4 + \left[det(\sum_{i=1}^{5}\Delta_i)^{-1} - det(\sum_{i=0}^{4}\Delta_i)^{-1}\right]P_5 + \left[det(\sum_{i=1}^{6}\Delta_i)^{-1} - det(\sum_{i=0}^{5}\Delta_i)^{-1}\right]P_6 + [det(\sum_{i=1}^{7}\Delta_i)^{-1} - det(\sum_{i=0}^{6}\Delta_i)^{-1}]P_7 + [det(\sum_{i=1}^{8}\Delta_i)^{-1} - det(\sum_{i=0}^{7}\Delta_i)^{-1}]P_8 + [det(\sum_{i=1}^{9}\Delta_i)^{-1} - det(\sum_{i=0}^{8}\Delta_i)^{-1}]P_9 + [det(\sum_{i=1}^{10}\Delta_i)^{-1} - det(\sum_{i=0}^{9}\Delta_i)^{-1}]P_{10} + [det(\sum_{i=1}^{11}\Delta_i)^{-1} - det(\sum_{i=0}^{10}\Delta_i)^{-1}]P_{11} + [det(\sum_{i=1}^{12}\Delta_i)^{-1} - det(\sum_{i=0}^{11}\Delta_i)^{-1}]P_{12} + [det(\sum_{i=1}^{13}\Delta_i)^{-1} - det(\sum_{i=0}^{12}\Delta_i)^{-1}]P_{13} + [det(\sum_{i=1}^{14}\Delta_i)^{-1} - det(\sum_{i=0}^{13}\Delta_i)^{-1}]P_{14} + \left[det(\sum_{i=1}^{15}\Delta_i)^{-1} - det(\sum_{i=0}^{14}\Delta_i)^{-1}\right]P_{15} + \left[det(\sum_{i=1}^{16}\Delta_i)^{-1} - det(\sum_{i=0}^{15}\Delta_i)^{-1}\right]P_{16} = (det\Delta)^{-1}.$

2). $\Delta^t= {\Delta_0}^t + {\Delta_1}^t P_1 + {\Delta_2}^t P_2 + {\Delta_3}^t P_3 + {\Delta_4}^t P_4 + {\Delta_5}^t P_5 + {\Delta_6}^t P_6 + {\Delta_7}^t P_7 + {\Delta_8}^t P_8 + {\Delta_9}^t P_9 + {\Delta_{10}}^t P_{10} + {\Delta_{11}}^t P_{11} + {\Delta_{12}}^t P_{12} + {\Delta_{13}}^t P_{13} + {\Delta_{14}}^t P_{14} + {\Delta_{15}}^t P_{15} + {\Delta_{16}}^t P_{16}.$

$det\Delta^t= det({\Delta_0}^t) + [det(\sum_{i=0}^{1}{\Delta_i}^t) - det({\Delta_0}^t)]P_1 + [det(\sum_{i=0}^{2}{\Delta_i}^t) - det(\sum_{i=0}^{1}{\Delta_i}^t)]P_2 + [det(\sum_{i=0}^{3}{\Delta_i}^t) - det(\sum_{i=0}^{2}{\Delta_i}^t)]P_3 + [det(\sum_{i=0}^{4}{\Delta_i}^t) - det(\sum_{i=0}^{3}{\Delta_i}^t)]P_4 + [det(\sum_{i=0}^{5}{\Delta_i}^t) - det\ (\sum_{i=0}^{4}{\Delta_i}^t)]P_5 + [det(\sum_{i=0}^{6}{\Delta_i}^t) - det\ (\sum_{i=0}^{5}{\Delta_i}^t)]P_6 + [det(\sum_{i=0}^{7}{\Delta_i}^t) - det\ (\sum_{i=0}^{6}{\Delta_i}^t)]P_7 + [det(\sum_{i=0}^{8}{\Delta_i}^t) - det\ (\sum_{i=0}^{7}{\Delta_i}^t)]P_8 + [det(\sum_{i=0}^{9}{\Delta_i}^t) - det\ (\sum_{i=0}^{8}{\Delta_i}^t)]P_9 + [det(\sum_{i=0}^{10}{\Delta_i}^t) - det\ (\sum_{i=0}^{9}{\Delta_i}^t)]P_{10} + [det(\sum_{i=0}^{11}{\Delta_i}^t) - det\ (\sum_{i=0}^{10}{\Delta_i}^t)]P_{11} + [det(\sum_{i=0}^{12}{\Delta_i}^t) - det\ (\sum_{i=0}^{11}{\Delta_i}^t)]P_{12} + [det(\sum_{i=0}^{13}{\Delta_i}^t) - det\ (\sum_{i=0}^{12}{\Delta_i}^t)]P_{13} + [det(\sum_{i=0}^{14}{\Delta_i}^t) - det\ (\sum_{i=0}^{13}{\Delta_i}^t)]P_{14} + [det(\sum_{i=0}^{15}{\Delta_i}^t) - det\ (\sum_{i=0}^{14}{\Delta_i}^t)]P_{15} + [det(\sum_{i=0}^{16}{\Delta_i}^t) - det\ (\sum_{i=0}^{15}{\Delta_i}^t)]P_{16} = \det(\Delta_0) + [\det(\sum_{i=0}^{1}\Delta_i) - \det(\Delta_0)]P_1 + [\det(\sum_{i=0}^{2}\Delta_i) - \det(\sum_{i=0}^{1}\Delta_i)]P_2 + [\det(\sum_{i=0}^{3}\Delta_i) - \det(\sum_{i=0}^{2}\Delta_i)]P_3 + [\det(\sum_{i=0}^{4}\Delta_i) - \det(\sum_{i=0}^{3}\Delta_i)]P_4 + [\det(\sum_{i=0}^{5}\Delta_i) - \det(\sum_{i=0}^{4}\Delta_i)]P_5 + [\det(\sum_{i=0}^{6}\Delta_i) - \det(\sum_{i=0}^{5}\Delta_i)]P_6 + [\det(\sum_{i=0}^{7}\Delta_i) - \det(\sum_{i=0}^{6}\Delta_i)]P_7 + [\det(\sum_{i=0}^{8}\Delta_i) - \det(\sum_{i=0}^{7}\Delta_i)]P_8 + [\det(\sum_{i=0}^{9}\Delta_i) - \det(\sum_{i=0}^{8}\Delta_i)]P_9 + [\det(\sum_{i=0}^{10}\Delta_i) - \det(\sum_{i=0}^{9}\Delta_i)]P_{10} + [\det(\sum_{i=0}^{11}\Delta_i) - \det(\sum_{i=0}^{10}\Delta_i)]P_{11} + [\det(\sum_{i=0}^{12}\Delta_i) - \det(\sum_{i=0}^{11}\Delta_i)]P_{12} + [\det(\sum_{i=0}^{13}\Delta_i) - \det(\sum_{i=0}^{12}\Delta_i)]P_{13} + [\det(\sum_{i=0}^{14}\Delta_i) - \det(\sum_{i=0}^{13}\Delta_i)]P_{14} + [\det(\sum_{i=0}^{15}\Delta_i) - \det(\sum_{i=0}^{14}\Delta_i)]P_{15} + [\det(\sum_{i=0}^{16}\Delta_i) - \det(\sum_{i=0}^{15}\Delta_i)]P_{16} = det\Delta.$

3). we have:
$\Delta.C = \Delta_0 C_0 + [\sum_{i=0}^{1}\Delta_i \sum_{i=0}^{1} C_i - \Delta_0 C_0]P_1 + [\sum_{i=0}^{2}\Delta_i \sum_{i=0}^{2} C_i - \sum_{i=0}^{1}\Delta_i \sum_{i=0}^{1} C_i]P_2 + [\sum_{i=0}^{3}\Delta_i \sum_{i=0}^{3} C_i - \sum_{i=0}^{2}\mu_i \sum_{i=0}^{2} C_i]P_3 + [\sum_{i=0}^{4}\Delta_i \sum_{i=0}^{4} C_i - \sum_{i=0}^{3}\Delta_i \sum_{i=0}^{3} B_i]P_4 + [\sum_{i=0}^{5}\Delta_i \sum_{i=0}^{5} C_i - \sum_{i=0}^{4}\Delta_i \sum_{i=0}^{4} C_i]P_5 + [\sum_{i=0}^{6}\Delta_i \sum_{i=0}^{6} C_i - \sum_{i=0}^{5}\Delta_i \sum_{i=0}^{5} C_i]P_6 + [\sum_{i=0}^{7}\Delta_i \sum_{i=0}^{7} C_i - \sum_{i=0}^{6}\Delta_i \sum_{i=0}^{6} C_i]P_7 + [\sum_{i=0}^{8}\Delta_i \sum_{i=0}^{8} C_i - \sum_{i=0}^{7}\Delta_i \sum_{i=0}^{7} C_i]P_8 + [\sum_{i=0}^{9}\Delta_i \sum_{i=0}^{9} C_i - \sum_{i=0}^{8}\mu_i \sum_{i=0}^{8} C_i]P_9 + [\sum_{i=0}^{10}\Delta_i \sum_{i=0}^{10} C_i - \sum_{i=0}^{9}\Delta_i \sum_{i=0}^{9} C_i]P_{10} + [\sum_{i=0}^{11}\Delta_i \sum_{i=0}^{11} C_i - \sum_{i=0}^{10}\Delta_i \sum_{i=0}^{10} C_i]P_{11} + [\sum_{i=0}^{12}\Delta_i \sum_{i=0}^{12} C_i - \sum_{i=0}^{11}\Delta_i \sum_{i=0}^{11} C_i]P_{12} + [\sum_{i=0}^{13}\Delta_i \sum_{i=0}^{13} C_i - \sum_{i=0}^{12}\Delta_i \sum_{i=0}^{12} C_i]P_{13} + [\sum_{i=0}^{14}\Delta_i \sum_{i=0}^{14} C_i - \sum_{i=0}^{13}\Delta_i \sum_{i=0}^{13} C_i]P_{14} + [\sum_{i=0}^{15}\Delta_i \sum_{i=0}^{15} C_i - \sum_{i=0}^{14}\Delta_i \sum_{i=0}^{14} C_i]P_{15} + [\sum_{i=0}^{16}\Delta_i \sum_{i=0}^{16} C_i - \sum_{i=0}^{15}\Delta_i \sum_{i=0}^{15} C_i]P_{16}.$

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

$det(\Delta.C) = det(\Delta_0 C_0) + [det(\sum_{i=0}^{1}\Delta_i \sum_{i=0}^{1} C_i) - det(\Delta_0 C_0)]P_1 + [det(\sum_{i=0}^{2}\Delta_i \sum_{i=0}^{2} C_i) - det(\sum_{i=0}^{1}\Delta_i \sum_{i=0}^{1} C_i)]P_2 + [det(\sum_{i=0}^{3}\Delta_i \sum_{i=0}^{3} C_i) - det(\sum_{i=0}^{2}\Delta_i \sum_{i=0}^{2} C_i)]P_3 + [det(\sum_{i=0}^{4}\Delta_i \sum_{i=0}^{4} C_i) - det(\sum_{i=0}^{3}\Delta_i \sum_{i=0}^{3} C_i)]P_4 + [det(\sum_{i=0}^{5}\Delta_i \sum_{i=0}^{5} C_i) - det\ (\sum_{i=0}^{4}\Delta_i \sum_{i=0}^{4} C_i)]P_5 + [det(\sum_{i=0}^{6}\Delta_i \sum_{i=0}^{6} C_i) - det\ (\sum_{i=0}^{5}\Delta_i \sum_{i=0}^{5} C_i)]P_6 + [det(\sum_{i=0}^{7}\Delta_i \sum_{i=0}^{7} C_i) - det\ (\sum_{i=0}^{6}\Delta_i \sum_{i=0}^{6} C_i)]P_7 + [det(\sum_{i=0}^{8}\Delta_i \sum_{i=0}^{8} C_i) - det\ (\sum_{i=0}^{7}\Delta_i \sum_{i=0}^{7} C_i)]P_8 + [det(\sum_{i=0}^{9}\Delta_i \sum_{i=0}^{9} C_i) - det\ (\sum_{i=0}^{8}\Delta_i \sum_{i=0}^{8} C_i)]P_9 + [det(\sum_{i=0}^{10}\Delta_i \sum_{i=0}^{10} C_i) - det\ (\sum_{i=0}^{9}\Delta_i \sum_{i=0}^{9} C_i)]P_{10} + [det(\sum_{i=0}^{11}\Delta_i \sum_{i=0}^{11} C_i) - det\ (\sum_{i=0}^{10}\Delta_i \sum_{i=0}^{10} C_i)]P_{11} + [det(\sum_{i=0}^{12}\Delta_i \sum_{i=0}^{12} C_i) - det\ (\sum_{i=0}^{11}\Delta_i \sum_{i=0}^{11} C_i)]P_{12} + [det(\sum_{i=0}^{13}\Delta_i \sum_{i=0}^{13} C_i) - det\ (\sum_{i=0}^{12}\Delta_i \sum_{i=0}^{12} C_i)]P_{13} + [det(\sum_{i=0}^{14}\Delta_i \sum_{i=0}^{14} C_i) - det\ (\sum_{i=0}^{13}\Delta_i \sum_{i=0}^{13} C_i)]P_{14} + [det(\sum_{i=0}^{15}\Delta_i \sum_{i=0}^{15} C_i) - det\ (\sum_{i=0}^{14}\Delta_i \sum_{i=0}^{14} C_i)]P_{15} + [det(\sum_{i=0}^{16}\Delta_i \sum_{i=0}^{16} C_i) - det\ (\sum_{i=0}^{15}\Delta_i \sum_{i=0}^{15} C_i)]P_{16} = det(\Delta_0)det(C_0) + [det(\sum_{i=0}^{j}\Delta_i).det(\sum_{i=0}^{j} C_i) - det(\Delta).det(\sum_{i=1}^{j-1} C_{i-1})]P_i = det(\Delta)det(C); 1 \le j \le 16.$

Proof of theorem5.

Δ is orthogonal if and only if $\Delta^t = \Delta^{-1}$, hence:

${\Delta_0}^t + \sum_{i=1}^{15}{\Delta_i}^t P_i = {\Delta_0}^{-1} + [(\Delta)^{-1} - {\Delta_0}^{-1}]P_1 + \left[(\sum_{i=0}^{2}\Delta_i)^{-1} - (\sum_{i=0}^{1}\Delta_i)^{-1}\right]P_2 + \left[(\sum_{i=1}^{3}`\Delta_i)^{-1} - (\sum_{i=0}^{2}\Delta_i)^{-1}\right]P_3 + \left[(\sum_{i=1}^{4}\Delta_i)^{-1} - (\sum_{i=0}^{3}\Delta_i)^{-1}\right]P_4 + \left[(\Delta)^{-1} - (\sum_{i=0}^{4}\Delta_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6}\Delta_i)^{-1} - (\sum_{i=0}^{5}\Delta_i)^{-1}\right]P_6 + \left[(\sum_{i=1}^{7}\Delta_i)^{-1} - (\sum_{i=0}^{6}\Delta_i)^{-1}\right]P_7 + \left[(\sum_{i=1}^{8}\Delta_i)^{-1} - (\sum_{i=0}^{7}\Delta_i)^{-1}\right]P_8 + \left[(\sum_{i=1}^{9}\Delta_i)^{-1} - (\sum_{i=0}^{8}\Delta_i)^{-1}\right]P_9 + \left[(\sum_{i=1}^{10}\Delta_i)^{-1} - (\sum_{i=0}^{9}\Delta_i)^{-1}\right]P_{10} + \left[(\sum_{i=1}^{11}\Delta_i)^{-1} - (\sum_{i=0}^{10}\Delta_i)^{-1}\right]P_{11} + \left[(\sum_{i=1}^{12}\Delta_i)^{-1} - (\sum_{i=0}^{11}\Delta_i)^{-1}\right]P_{12} + \left[(\sum_{i=1}^{13}\mu_i)^{-1} - (\sum_{i=0}^{12}\Delta_i)^{-1}\right]P_{13} + \left[(\sum_{i=1}^{14}\Delta_i)^{-1} - (\sum_{i=0}^{13}\Delta_i)^{-1}\right]P_{14} + \left[(\sum_{i=1}^{15}\Delta_i)^{-1} - (\sum_{i=0}^{14}\Delta_i)^{-1}\right]P_{15} + \left[(\sum_{i=1}^{16}\Delta_i)^{-1} - (\sum_{i=0}^{15}\Delta_i)^{-1}\right]P_{16}$, thus:

$$\begin{cases} {\Delta_0}^t = {\Delta_0}^{-1} \\ {\Delta_1}^t = \left(\sum_{i=0}^{1}\Delta_i\right)^{-1} - {\Delta_0}^{-1} \\ {\Delta_2}^t = \left(\sum_{i=0}^{2}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{1}\Delta_i\right)^{-1} \\ {\Delta_3}^t = \left(\sum_{i=0}^{3}\mu_i\right)^{-1} - \left(\sum_{i=0}^{2}\Delta_i\right)^{-1} \\ {\Delta_4}^t = \left(\sum_{i=0}^{4}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{3}\Delta_i\right)^{-1} \\ {\Delta_5}^t = \left(\sum_{i=0}^{5}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{4}\Delta_i\right)^{-1} \\ {\Delta_6}^t = \left(\sum_{i=0}^{6}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{5}\Delta_i\right)^{-1} \\ {\Delta_7}^t = \left(\sum_{i=0}^{7}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{6}\Delta_i\right)^{-1} \\ {\Delta_8}^t = \left(\sum_{i=0}^{8}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{7}\Delta_i\right)^{-1} \\ {\Delta_9}^t = \left(\sum_{i=0}^{9}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{8}\Delta_i\right)^{-1} \\ {\Delta_{10}}^t = \left(\sum_{i=0}^{10}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{9}\Delta_i\right)^{-1} \\ {\Delta_{11}}^t = \left(\sum_{i=0}^{11}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{10}\Delta_i\right)^{-1} \\ {\Delta_{12}}^t = \left(\sum_{i=0}^{12}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{11}\Delta_i\right)^{-1} \\ {\Delta_{13}}^t = \left(\sum_{i=0}^{13}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{12}\Delta_i\right)^{-1} \\ {\Delta_{14}}^t = \left(\sum_{i=0}^{14}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{13}\Delta_i\right)^{-1} \\ {\Delta_{15}}^t = \left(\sum_{i=0}^{15}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{14}\Delta_i\right)^{-1} \\ {\Delta_{16}}^t = \left(\sum_{i=0}^{16}\Delta_i\right)^{-1} - \left(\sum_{i=0}^{15}\Delta_i\right)^{-1} \end{cases}$$

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

This implies that:

$$\begin{cases} {\Delta_0}^t = {\Delta_0}^{-1} \\ \sum_{i=0}^{1} {\Delta_i}^t = (\sum_{i=0}^{1} \Delta_i)^{-1} \\ \sum_{i=0}^{2} {\Delta_i}^t = (\sum_{i=0}^{2} \Delta_i)^{-1} \\ \sum_{i=0}^{3} {\Delta_i}^t = (\sum_{i=0}^{3} \Delta_i)^{-1} \\ \sum_{i=0}^{4} {\Delta_i}^t = (\sum_{i=0}^{4} \Delta_i)^{-1} \\ \sum_{i=0}^{5} {\Delta_i}^t = (\sum_{i=0}^{5} \Delta_i)^{-1} \\ \sum_{i=0}^{6} {\Delta_i}^t = (\sum_{i=0}^{6} \Delta_i)^{-1} \\ \sum_{i=0}^{7} {\Delta_i}^t = (\sum_{i=0}^{7} \Delta_i)^{-1} \\ \sum_{i=0}^{8} {\Delta_i}^t = (\sum_{i=0}^{8} \Delta_i)^{-1}, \\ \sum_{i=0}^{9} {\Delta_i}^t = (\sum_{i=0}^{9} \Delta_i)^{-1} \\ \sum_{i=0}^{10} {\Delta_i}^t = (\sum_{i=0}^{10} \Delta_i)^{-1} \\ \sum_{i=0}^{11} {\Delta_i}^t = (\sum_{i=0}^{11} \Delta)^{-1} \\ \sum_{i=0}^{12} {\Delta_i}^t = (\sum_{i=0}^{12} \Delta_i)^{-1} \\ \sum_{i=0}^{13} {\Delta_i}^t = (\sum_{i=0}^{13} \Delta_i)^{-1} \\ \sum_{i=0}^{14} {\Delta_i}^t = (\sum_{i=0}^{14} \Delta_i)^{-1} \\ \sum_{i=0}^{15} {\Delta_i}^t = (\sum_{i=0}^{15} \Delta_i)^{-1} \\ \sum_{i=0}^{16} {\Delta_i}^t = (\sum_{i=0}^{16} \Delta_i)^{-1} \end{cases}$$

Symbolic 17-plithogenic matrices:

Definition:

The square symbolic 17-plithogenic matrix is defined as follows:

$\Delta = \Delta_0 + \sum_{i=1}^{17} \Delta_i P_i$; $(\mu_i)_{n \times n}$ is square matrix of real entries.

Definition.

Let $\Delta = \Delta_0 + \sum_{i=1}^{17} \Delta_i P_i$ be a symbolic 17-plithogenic matrix of size $n \times n$, hence:

$$\begin{aligned} \det \Delta = \det(\Delta_0) &+ \left[\det\left(\sum_{i=0}^{1} \Delta_i\right) - \det(\Delta_0)\right] P_1 + \left[\det\left(\sum_{i=0}^{2} \Delta_i\right) - \det\left(\sum_{i=0}^{1} \Delta_i\right)\right] P_2 \\ &+ \left[\det\left(\sum_{i=0}^{3} \Delta_i\right) - \det\left(\sum_{i=0}^{2} \Delta_i\right)\right] P_3 + \left[\det\left(\sum_{i=0}^{4} \Delta_i\right) - \det\left(\sum_{i=0}^{3} \Delta_i\right)\right] P_4 \\ &+ \left[\det\left(\sum_{i=0}^{5} \Delta_i\right) - \det\left(\sum_{i=0}^{4} \Delta_i\right)\right] P_5 + \left[\det\left(\sum_{i=0}^{6} \Delta_i\right) - \det\left(\sum_{i=0}^{5} \Delta_i\right)\right] P_6 \\ &+ \left[\det\left(\sum_{i=0}^{7} \Delta_i\right) - \det\left(\sum_{i=0}^{6} \Delta_i\right)\right] P_7 + \left[\det\left(\sum_{i=0}^{8} \Delta_i\right) - \det\left(\sum_{i=0}^{7} \Delta_i\right)\right] P_8 \\ &+ \left[\det\left(\sum_{i=0}^{9} \Delta_i\right) - \det\left(\sum_{i=0}^{8} \Delta_i\right)\right] P_9 + \left[\det\left(\sum_{i=0}^{10} \Delta_i\right) - \det\left(\sum_{i=0}^{9} \Delta_i\right)\right] P_{10} \\ &+ \left[\det\left(\sum_{i=0}^{11} \Delta_i\right) - \det\left(\sum_{i=0}^{10} \Delta_i\right)\right] P_{11} + \left[\det\left(\sum_{i=0}^{12} \Delta_i\right) - \det\left(\sum_{i=0}^{11} \Delta_i\right)\right] P_{12} \\ &+ \left[\det\left(\sum_{i=0}^{13} \Delta_i\right) - \det\left(\sum_{i=0}^{12} \Delta_i\right)\right] P_{13} + \left[\det\left(\sum_{i=0}^{14} \Delta_i\right) - \det\left(\sum_{i=0}^{13} \Delta_i\right)\right] P_{14} \\ &+ \left[\det\left(\sum_{i=0}^{15} \Delta_i\right) - \det\left(\sum_{i=0}^{14} \Delta_i\right)\right] P_{15} + \left[\det\left(\sum_{i=0}^{16} \Delta_i\right) - \det\left(\sum_{i=0}^{15} \Delta_i\right)\right] P_{16} \\ &+ \left[\det\left(\sum_{i=0}^{17} \Delta_i\right) - \det\left(\sum_{i=0}^{16} \Delta_i\right)\right] P_{17} \end{aligned}$$

Theorem1.

Let $\Delta = \Delta_0 + \sum_{i=1}^{17} \Delta_i P_i$ be a symbolic 17-plithogenic matrix of size $n \times n$, hence:

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

1. Δ is invertible if and only if $\det\Delta$ is an invertible symbolic 17-plithogenic real number.
2. $\Delta^{-1}= {\Delta_0}^{-1} + [(\sum_{i=0}^{1}\Delta_i)^{-1} - {\Delta_0}^{-1}]P_1 + [(\sum_{i=0}^{2}\Delta_i)^{-1} - (\sum_{i=0}^{1}\Delta_i)^{-1}]P_2 + [(\sum_{i=1}^{3}\Delta_i)^{-1} - (\sum_{i=0}^{2}\Delta_i)^{-1}]P_3 + [(\sum_{i=1}^{4}\Delta_i)^{-1} - (\sum_{i=0}^{3}\Delta_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5}\Delta_i\right)^{-1} - (\sum_{i=0}^{4}\Delta_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6}\Delta_i)^{-1} - \left(\sum_{i=0}^{5}\Delta_i\right)^{-1}\right]P_6 + [(\sum_{i=1}^{7}\Delta_i)^{-1} - (\sum_{i=0}^{6}\Delta_i)^{-1}]P_7 + [(\sum_{i=1}^{8}\Delta_i)^{-1} - (\sum_{i=0}^{7}\Delta_i)^{-1}]P_8 + [(\sum_{i=1}^{9}\Delta_i)^{-1} - (\sum_{i=0}^{8}\Delta_i)^{-1}]P_9 + [(\sum_{i=1}^{10}\Delta_i)^{-1} - (\sum_{i=0}^{9}\Delta_i)^{-1}]P_{10} + [(\sum_{i=1}^{11}\Delta_i)^{-1} - (\sum_{i=0}^{10}\Delta_i)^{-1}]P_{11} + [(\sum_{i=1}^{12}\Delta_i)^{-1} - (\sum_{i=0}^{11}\Delta_i)^{-1}]P_{12} + [(\Delta)^{-1} - (\sum_{i=0}^{12}\Delta_i)^{-1}]P_{13} + [(\sum_{i=1}^{14}\Delta_i)^{-1} - (\sum_{i=0}^{13}\Delta_i)^{-1}]P_{14} + \left[\left(\sum_{i=1}^{15}\Delta_i\right)^{-1} - (\sum_{i=0}^{14}\Delta_i)^{-1}\right]P_{15} + \left[(\sum_{i=1}^{16}\Delta_i)^{-1} - \left(\sum_{i=0}^{15}\Delta_i\right)^{-1}\right]P_{16} + [(\sum_{i=1}^{17}\Delta_i)^{-1} - (\sum_{i=0}^{16}\Delta_i)^{-1}]P_{17}$.

Definition.

Let $r = r_0 + \sum_{i=1}^{17} r_i P_i$ be a symbolic 17-plithogenic real number and $\Delta= \Delta_0 + \sum_{i=1}^{17}\Delta_i P_i$ be a symbolic 17-plithogenic square real matrix, then r is called symbolic 17-plithogenic eigen value if and only if $\Delta X = rX$.
X is called symbolic 17-plithogenic eigenvector.

Theorem2.

Let $u = u_0 + \sum_{i=1}^{17} u_i P_i \in 17 - SP_R, X = X_0 + \sum_{i=1}^{17} X_i P_i$ be a symbolic 17-plithogenic real vector, then u is eigen value of $\Delta= \Delta_0 + \sum_{i=1}^{17}\Delta_i P_i$ with X as the corresponding eigen vector if and only if:
$\sum_{i=0}^{j} u_i$ is eigen value of $\sum_{i=0}^{j}\Delta_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \le j \le 17$.

Theorem3.

$$\Delta^n= {\Delta_0}^n + P_1\left[\left(\sum_{i=0}^{1}\Delta_i\right)^n - {\Delta_0}^n\right] + \left[\left(\sum_{i=0}^{2}\Delta_i\right)^n - \left(\sum_{i=0}^{1}\Delta_i\right)^n\right]P_2 + \left[\left(\sum_{i=1}^{3}\Delta_i\right)^n - \left(\sum_{i=0}^{2}\Delta_i\right)^n\right]P_3$$
$$+ \left[\left(\sum_{i=1}^{4}\Delta_i\right)^n - \left(\sum_{i=0}^{3}\Delta_i\right)^n\right]P_4 + \left[\left(\sum_{i=1}^{5}\Delta_i\right)^n - \left(\sum_{i=0}^{4}\Delta_i\right)^n\right]P_5 + \left[\left(\sum_{i=1}^{6}\Delta_i\right)^n - \left(\sum_{i=0}^{5}\Delta_i\right)^n\right]P_6$$
$$+ \left[\left(\sum_{i=1}^{7}\Delta_i\right)^n - \left(\sum_{i=0}^{6}\Delta_i\right)^n\right]P_7 + \left[\left(\sum_{i=1}^{8}\Delta_i\right)^n - \left(\sum_{i=0}^{7}\Delta_i\right)^n\right]P_8 + \left[\left(\sum_{i=1}^{9}\Delta_i\right)^n - \left(\sum_{i=0}^{8}\Delta_i\right)^n\right]P_9$$
$$+ \left[\left(\sum_{i=1}^{10}\Delta_i\right)^n - \left(\sum_{i=0}^{9}\Delta_i\right)^n\right]P_{10} + \left[\left(\sum_{i=1}^{11}\Delta_i\right)^n - \left(\sum_{i=0}^{10}\Delta_i\right)^n\right]P_{11}$$
$$+ \left[\left(\sum_{i=1}^{12}\Delta_i\right)^n - \left(\sum_{i=0}^{11}\Delta_i\right)^n\right]P_{12} + \left[\left(\sum_{i=1}^{13}\Delta_i\right)^n - \left(\sum_{i=0}^{12}\Delta_i\right)^n\right]P_{13}$$
$$+ \left[\left(\sum_{i=1}^{14}\Delta_i\right)^n - \left(\sum_{i=0}^{13}\Delta_i\right)^n\right]P_{14} + \left[\left(\sum_{i=1}^{15}\Delta_i\right)^n - \left(\sum_{i=0}^{14}\Delta_i\right)^n\right]P_{15}$$
$$+ \left[\left(\sum_{i=1}^{16}\Delta_i\right)^n - \left(\sum_{i=0}^{15}\Delta_i\right)^n\right]P_{16} + \left[\left(\sum_{i=1}^{17}\Delta_i\right)^n - \left(\sum_{i=0}^{16}\Delta_i\right)^n\right]P_{17}$$

Theorem4.

Let $\Delta= \Delta_0 + \sum_{i=1}^{17}\Delta_i P_i$ be a square 16-plithogenic invertible real matrix, then:
1). $\det(\Delta^{-1}) = (\det\Delta)^{-1}$
2). $\det\Delta^t = \det\Delta$
3). $\det(\Delta.C) = \det\Delta.\det C\,; C = C_0 + \sum_{i=1}^{17} C_i P_i$.

Definition.

Let $\Delta= \Delta_0 + \sum_{i=1}^{17}\Delta_i P_i$ be a symbolic 17-plithogenic real square matrix, then:
Δ is called orthogonal if and only if $\Delta^t= \Delta^{-1}$.

Theorem5.

Δ is orthogonal if and only if $\sum_{i=0}^{j}\Delta_i\,;\ 0 \le j \le 17$ are orthogonal.

Proof of theorem1.

1). Let $\Delta= \Delta_0 + \sum_{i=1}^{17}\Delta_i P_i$, then Δ is invertible if and only if there exists $T = T_0 + \sum_{i=1}^{17} T_i P_i$ such that:
$\Delta \times T = U_{n\times n}$, hence:

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

$$\begin{cases} \Delta_0 T_0 = U_{n\times n} \\ \sum_{i=0}^{1}\Delta_i \sum_{i=0}^{1} T_i - \Delta_0 T_0 = O_{n\times n} \\ \sum_{i=0}^{2}\Delta_i \sum_{i=0}^{2} T_i - \sum_{i=0}^{1}\Delta_i \sum_{i=0}^{1} T_i = O_{n\times n} \\ \sum_{i=0}^{3}\Delta_i \sum_{i=0}^{3} T_i - \sum_{i=0}^{2}\Delta_i \sum_{i=0}^{2} T_i = O_{n\times n} \\ \sum_{i=0}^{4}\Delta_i \sum_{i=0}^{4} T_i - \sum_{i=0}^{3}\Delta_i \sum_{i=0}^{3} T_i = O_{n\times n} \\ \sum_{i=0}^{5}\Delta_i \sum_{i=0}^{5} T_i - \sum_{i=0}^{4}\Delta_i \sum_{i=0}^{4} T_i = O_{n\times n} \\ \sum_{i=0}^{6}\Delta_i \sum_{i=0}^{6} T_i - \sum_{i=0}^{5}\Delta_i \sum_{i=0}^{5} T_i = O_{n\times n} \\ \sum_{i=0}^{7}\Delta_i \sum_{i=0}^{7} T_i - \sum_{i=0}^{6}\Delta_i \sum_{i=0}^{6} T_i = O_{n\times n} \\ \sum_{i=0}^{8}\Delta_i \sum_{i=0}^{8} T_i - \sum_{i=0}^{7}\Delta_i \sum_{i=0}^{7} T_i = O_{n\times n} \\ \sum_{i=0}^{9}\Delta_i \sum_{i=0}^{9} T_i - \sum_{i=0}^{8}\Delta_i \sum_{i=0}^{8} T_i = O_{n\times n} \\ \sum_{i=0}^{10}\Delta_i \sum_{i=0}^{10} T_i - \sum_{i=0}^{9}\Delta_i \sum_{i=0}^{9} T_i = O_{n\times n} \\ \sum_{i=0}^{11}\Delta_i \sum_{i=0}^{11} T_i - \sum_{i=0}^{10}\Delta_i \sum_{i=0}^{10} T_i = O_{n\times n} \\ \sum_{i=0}^{12}\Delta_i \sum_{i=0}^{12} T_i - \sum_{i=0}^{11}\Delta_i \sum_{i=0}^{11} T_i = O_{n\times n} \\ \sum_{i=0}^{13}\Delta_i \sum_{i=0}^{13} T_i - \sum_{i=0}^{12}\Delta_i \sum_{i=0}^{12} T_i = O_{n\times n} \\ \sum_{i=0}^{14}\Delta_i \sum_{i=0}^{14} T_i - \sum_{i=0}^{13}\Delta_i \sum_{i=0}^{13} T_i = O_{n\times n} \\ \sum_{i=0}^{15}\Delta_i \sum_{i=0}^{15} T_i - \sum_{i=0}^{14}\Delta_i \sum_{i=0}^{14} T_i = O_{n\times n} \\ \sum_{i=0}^{16}\Delta_i \sum_{i=0}^{16} T_i - \sum_{i=0}^{15}\Delta_i \sum_{i=0}^{15} T_i = O_{n\times n} \\ \sum_{i=0}^{17}\Delta_i \sum_{i=0}^{17} T_i - \sum_{i=0}^{16}\Delta_i \sum_{i=0}^{16} T_i = O_{n\times n} \end{cases}$$

This implies that:

$$\begin{cases} \Delta_0 T_0 = U_{n\times n} \\ \sum_{i=0}^{j}\Delta_i \sum_{i=0}^{j} T_i = U_{n\times n} \end{cases} ;\ 1 \le j \le 17$$

Hence $\det\left(\sum_{i=0}^{j}\Delta_i\right) \neq 0$ for all $1 \le j \le 17$, so that $\det(\Delta)$ is invertible in $17 - SP_R$.

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

2). It holds directly as follows:

$\sum_{i=0}^{j} T_i = \left(\sum_{i=0}^{j} \Delta_i\right)^{-1}$ for $1 \le j \le 17$, hence:

$\Delta^{-1} = \Delta_0^{-1} + [(\sum_{i=0}^{1} \Delta_i)^{-1} - \Delta_0^{-1}]P_1 + [(\sum_{i=0}^{2} \Delta_i)^{-1} - (\Delta)^{-1}]P_2 + [(\sum_{i=1}^{3} \Delta_i)^{-1} - (\sum_{i=0}^{2} \Delta_i)^{-1}]P_3 + [(\sum_{i=1}^{4} \Delta_i)^{-1} - (\sum_{i=0}^{3} \Delta_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} \Delta_i)^{-1} - (\sum_{i=0}^{4} \Delta_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6} \Delta_i)^{-1} - (\sum_{i=0}^{5} \Delta_i)^{-1}\right]P_6 + [(\sum_{i=1}^{7} \Delta_i)^{-1} - (\sum_{i=0}^{6} \Delta_i)^{-1}]P_7 + [(\sum_{i=1}^{8} \Delta_i)^{-1} - (\sum_{i=0}^{7} \Delta_i)^{-1}]P_8 + [(\sum_{i=1}^{9} \Delta_i)^{-1} - (\sum_{i=0}^{8} \Delta_i)^{-1}]P_9 + [(\sum_{i=1}^{10} \Delta_i)^{-1} - (\sum_{i=0}^{9} \Delta_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} \Delta_i)^{-1} - (\sum_{i=0}^{10} \Delta_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} \Delta_i)^{-1} - (\sum_{i=0}^{11} \Delta_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} \Delta_i)^{-1} - (\sum_{i=0}^{12} \Delta_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} \Delta_i)^{-1} - (\sum_{i=0}^{13} \mu_i)^{-1}]P_{14} + \left[(\sum_{i=1}^{15} \Delta_i)^{-1} - (\sum_{i=0}^{14} \Delta_i)^{-1}\right]P_{15} + \left[(\sum_{i=1}^{16} \Delta_i)^{-1} - (\sum_{i=0}^{15} \Delta_i)^{-1}\right]P_{16} + [(\sum_{i=1}^{17} \Delta_i)^{-1} - (\sum_{i=0}^{16} \Delta_i)^{-1}]P_{17}.$

Proof of theorem2.

It is clear that r is an eigen value of Δ with X as an eigen vector if and only if:

$\Delta.X = r.X$, which is equivalent to:

$$\begin{cases} \Delta_0 X_0 = r_0 X_0 \\ \sum_{i=0}^{j} \Delta_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} r_i \sum_{i=0}^{j} X_i \end{cases} ;\ 1 \le j \le 17$$

Which is equivalent to:

$\sum_{i=0}^{j} r_i$ is an eigen value of $\sum_{i=0}^{j} \Delta_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 17$.

Proof of theorem4.

1). $det\Delta^{-1} = det(\Delta_0^{-1}) + P_1[det(\sum_{i=0}^{1} \Delta_i)^{-1} - det(\Delta_0^{-1})] + [det(\sum_{i=0}^{2} \Delta_i)^{-1} - det(\sum_{i=0}^{1} \Delta_i)^{-1}]P_2 + [det(\sum_{i=1}^{3} \Delta_i)^{-1} - det(\sum_{i=0}^{2} \Delta_i)^{-1}]P_3 + [det(\sum_{i=1}^{4} \Delta_i)^{-1} - det(\sum_{i=0}^{3} \Delta_i)^{-1}]P_4 + \left[det(\sum_{i=1}^{5} \Delta_i)^{-1} - det(\sum_{i=0}^{4} \Delta_i)^{-1}\right]P_5 + \left[det(\sum_{i=1}^{6} \Delta_i)^{-1} - det(\sum_{i=0}^{5} \Delta_i)^{-1}\right]P_6 + [det(\sum_{i=1}^{7} \Delta_i)^{-1} - det(\sum_{i=0}^{6} \Delta_i)^{-1}]P_7 + [det(\sum_{i=1}^{8} \Delta_i)^{-1} - det(\sum_{i=0}^{7} \Delta_i)^{-1}]P_8 + [det(\sum_{i=1}^{9} \Delta_i)^{-1} - det(\sum_{i=0}^{8} \Delta_i)^{-1}]P_9 + [det(\sum_{i=1}^{10} \Delta_i)^{-1} - det(\sum_{i=0}^{9} \Delta_i)^{-1}]P_{10} + [det(\sum_{i=1}^{11} \Delta_i)^{-1} - det(\sum_{i=0}^{10} \Delta_i)^{-1}]P_{11} + [det(\sum_{i=1}^{12} \Delta_i)^{-1} - det(\sum_{i=0}^{11} \Delta_i)^{-1}]P_{12} + [det(\sum_{i=1}^{13} \Delta_i)^{-1} - det(\sum_{i=0}^{12} \Delta_i)^{-1}]P_{13} + [det(\sum_{i=1}^{14} \Delta_i)^{-1} - det(\sum_{i=0}^{13} \Delta_i)^{-1}]P_{14} + \left[det(\sum_{i=1}^{15} \Delta_i)^{-1} - det(\sum_{i=0}^{14} \Delta_i)^{-1}\right]P_{15} + \left[det(\sum_{i=1}^{16} \Delta_i)^{-1} - det(\sum_{i=0}^{15} \Delta_i)^{-1}\right]P_{16} + [det(\sum_{i=1}^{17} \Delta_i)^{-1} - det(\sum_{i=0}^{16} \Delta_i)^{-1}]P_{17} = (det\Delta)^{-1}.$

2). $\Delta^t = \Delta_0^t + \Delta_1^t P_1 + \Delta_2^t P_2 + \Delta_3^t P_3 + \Delta_4^t P_4 + \Delta_5^t P_5 + \Delta_6^t P_6 + \Delta_7^t P_7 + \Delta_8^t P_8 + \Delta_9^t P_9 + \Delta_{10}^t P_{10} + \Delta_{11}^t P_{11} + \Delta_{12}^t P_{12} + \Delta_{13}^t P_{13} + \Delta_{14}^t P_{14} + \Delta_{15}^t P_{15} + \Delta_{16}^t P_{16} + \Delta_{17}^t P_{17}.$

$det\Delta^t = det(\Delta_0^t) + [det(\sum_{i=0}^{1} \Delta_i^t) - det(\Delta_0^t)]P_1 + [det(\sum_{i=0}^{2} \Delta_i^t) - det(\sum_{i=0}^{1} \Delta_i^t)]P_2 + [det(\sum_{i=0}^{3} \Delta_i^t) - det(\sum_{i=0}^{2} \Delta_i^t)]P_3 + [det(\sum_{i=0}^{4} \Delta_i^t) - det(\sum_{i=0}^{3} \Delta_i^t)]P_4 + [det(\sum_{i=0}^{5} \Delta_i^t) - det\ (\sum_{i=0}^{4} \Delta_i^t)]P_5 + [det(\sum_{i=0}^{6} \Delta_i^t) - det\ (\sum_{i=0}^{5} \Delta_i^t)]P_6 + [det(\sum_{i=0}^{7} \Delta_i^t) - det\ (\sum_{i=0}^{6} \Delta_i^t)]P_7 + [det(\sum_{i=0}^{8} \Delta_i^t) - det\ (\sum_{i=0}^{7} \Delta_i^t)]P_8 + [det(\sum_{i=0}^{9} \Delta_i^t) - det\ (\sum_{i=0}^{8} \Delta_i^t)]P_9 + [det(\sum_{i=0}^{10} \Delta_i^t) - det\ (\sum_{i=0}^{9} \Delta_i^t)]P_{10} + [det(\sum_{i=0}^{11} \Delta_i^t) - det\ (\sum_{i=0}^{10} \Delta_i^t)]P_{11} + [det(\sum_{i=0}^{12} \Delta_i^t) - det\ (\sum_{i=0}^{11} \Delta_i^t)]P_{12} + [det(\sum_{i=0}^{13} \Delta_i^t) - det\ (\sum_{i=0}^{12} \Delta_i^t)]P_{13} + [det(\sum_{i=0}^{14} \Delta_i^t) - det\ (\sum_{i=0}^{13} \Delta_i^t)]P_{14} + [det(\sum_{i=0}^{15} \Delta_i^t) - det\ (\sum_{i=0}^{14} \Delta_i^t)]P_{15} + [det(\sum_{i=0}^{16} \Delta_i^t) - det\ (\sum_{i=0}^{15} \Delta_i^t)]P_{16} + [det(\sum_{i=0}^{17} \Delta_i^t) - det\ (\sum_{i=0}^{16} \Delta_i^t)]P_{17} = \det(\Delta_0) + [\det(\sum_{i=0}^{1} \Delta_i) - \det(\Delta_0)]P_1 + [\det(\sum_{i=0}^{2} \Delta_i) - \det(\sum_{i=0}^{1} \Delta_i)]P_2 + [\det(\sum_{i=0}^{3} \Delta_i) - \det(\sum_{i=0}^{2} \Delta_i)]P_3 + [\det(\sum_{i=0}^{4} \Delta_i) - \det(\sum_{i=0}^{3} \Delta_i)]P_4 + [\det(\sum_{i=0}^{5} \Delta_i) - \det(\sum_{i=0}^{4} \Delta_i)]P_5 + [\det(\sum_{i=0}^{6} \Delta_i) - \det(\sum_{i=0}^{5} \Delta_i)]P_6 + [\det(\sum_{i=0}^{7} \Delta_i) - \det(\sum_{i=0}^{6} \Delta_i)]P_7 + [\det(\sum_{i=0}^{8} \Delta_i) - \det(\sum_{i=0}^{7} \Delta_i)]P_8 + [\det(\sum_{i=0}^{9} \Delta_i) - \det(\sum_{i=0}^{8} \Delta_i)]P_9 + [\det(\sum_{i=0}^{10} \Delta_i) - \det(\sum_{i=0}^{9} \Delta_i)]P_{10} + [\det(\sum_{i=0}^{11} \Delta_i) - \det(\sum_{i=0}^{10} \Delta_i)]P_{11} + [\det(\sum_{i=0}^{12} \Delta_i) - \det(\sum_{i=0}^{11} \Delta_i)]P_{12} + [\det(\sum_{i=0}^{13} \Delta_i) - \det(\sum_{i=0}^{12} \Delta_i)]P_{13} + [\det(\sum_{i=0}^{14} \Delta_i) - \det(\sum_{i=0}^{13} \Delta_i)]P_{14} + [\det(\sum_{i=0}^{15} \Delta_i) - \det(\sum_{i=0}^{14} \Delta_i)]P_{15} + [\det(\sum_{i=0}^{16} \Delta_i) - \det(\sum_{i=0}^{15} \Delta_i)]P_{16} + [\det(\sum_{i=0}^{17} \Delta_i) - \det(\sum_{i=0}^{16} \Delta_i)]P_{17} = det\Delta.$

3). we have:

$\Delta.C = \Delta_0 C_0 + [\sum_{i=0}^{1} \Delta_i \sum_{i=0}^{1} C_i - \Delta_0 C_0]P_1 + [\sum_{i=0}^{2} \Delta_i \sum_{i=0}^{2} C_i - \sum_{i=0}^{1} \Delta_i \sum_{i=0}^{1} C_i]P_2 + [\sum_{i=0}^{3} \Delta_i \sum_{i=0}^{3} C_i - \sum_{i=0}^{2} \mu_i \sum_{i=0}^{2} C_i]P_3 + [\sum_{i=0}^{4} \Delta_i \sum_{i=0}^{4} C_i - \sum_{i=0}^{3} \Delta_i \sum_{i=0}^{3} B_i]P_4 + [\sum_{i=0}^{5} \Delta_i \sum_{i=0}^{5} C_i - \sum_{i=0}^{4} \Delta_i \sum_{i=0}^{4} C_i]P_5 + [\sum_{i=0}^{6} \Delta_i \sum_{i=0}^{6} C_i - \sum_{i=0}^{5} \Delta_i \sum_{i=0}^{5} C_i]P_6 + [\sum_{i=0}^{7} \Delta_i \sum_{i=0}^{7} C_i - \sum_{i=0}^{6} \Delta_i \sum_{i=0}^{6} C_i]P_7 + [\sum_{i=0}^{8} \Delta_i \sum_{i=0}^{8} C_i - \sum_{i=0}^{7} \Delta_i \sum_{i=0}^{7} C_i]P_8 + [\sum_{i=0}^{9} \Delta_i \sum_{i=0}^{9} C_i - \sum_{i=0}^{8} \mu_i \sum_{i=0}^{8} C_i]P_9 + [\sum_{i=0}^{10} \Delta_i \sum_{i=0}^{10} C_i - \sum_{i=0}^{9} \Delta_i \sum_{i=0}^{9} C_i]P_{10} + [\sum_{i=0}^{11} \Delta_i \sum_{i=0}^{11} C_i - \sum_{i=0}^{10} \Delta_i \sum_{i=0}^{10} C_i]P_{11} + [\sum_{i=0}^{12} \Delta_i \sum_{i=0}^{12} C_i - \sum_{i=0}^{11} \Delta_i \sum_{i=0}^{11} C_i]P_{12} + [\sum_{i=0}^{13} \Delta_i \sum_{i=0}^{13} C_i - \sum_{i=0}^{12} \Delta_i \sum_{i=0}^{12} C_i]P_{13} + [\sum_{i=0}^{14} \Delta_i \sum_{i=0}^{14} C_i - \sum_{i=0}^{13} \Delta_i \sum_{i=0}^{13} C_i]P_{14} + [\sum_{i=0}^{15} \Delta_i \sum_{i=0}^{15} C_i - \sum_{i=0}^{14} \Delta_i \sum_{i=0}^{14} C_i]P_{15} + [\sum_{i=0}^{16} \Delta_i \sum_{i=0}^{16} C_i - \sum_{i=0}^{15} \Delta_i \sum_{i=0}^{15} C_i]P_{16} + [\sum_{i=0}^{17} \Delta_i \sum_{i=0}^{17} C_i - \sum_{i=0}^{16} \Delta_i \sum_{i=0}^{16} C_i]P_{17}.$

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

$det(\Delta.C) = det(\Delta_0 C_0) + [det(\sum_{i=0}^{1} \Delta_i \sum_{i=0}^{1} C_i) - det(\Delta_0 C_0)]P_1 + [det(\sum_{i=0}^{2} \Delta_i \sum_{i=0}^{2} C_i) - det(\sum_{i=0}^{1} \Delta_i \sum_{i=0}^{1} C_i)]P_2 + [det(\sum_{i=0}^{3} \Delta_i \sum_{i=0}^{3} C_i) - det(\sum_{i=0}^{2} \Delta_i \sum_{i=0}^{2} C_i)]P_3 + [det(\sum_{i=0}^{4} \Delta_i \sum_{i=0}^{4} C_i) - det(\sum_{i=0}^{3} \Delta_i \sum_{i=0}^{3} C_i)]P_4 + \left[det\left(\sum_{i=0}^{5} \Delta_i \sum_{i=0}^{5} C_i\right) - det\ (\sum_{i=0}^{4} \Delta_i \sum_{i=0}^{4} C_i)\right]P_5 + \left[det(\sum_{i=0}^{6} \Delta_i \sum_{i=0}^{6} C_i) - det\ \left(\sum_{i=0}^{5} \Delta_i \sum_{i=0}^{5} C_i\right)\right]P_6 + [det(\sum_{i=0}^{7} \Delta_i \sum_{i=0}^{7} C_i) - det\ (\sum_{i=0}^{6} \Delta_i \sum_{i=0}^{6} C_i)]P_7 + [det(\sum_{i=0}^{8} \Delta_i \sum_{i=0}^{8} C_i) - det\ (\sum_{i=0}^{7} \Delta_i \sum_{i=0}^{7} C_i)]P_8 + [det(\sum_{i=0}^{9} \Delta_i \sum_{i=0}^{9} C_i) - det\ (\sum_{i=0}^{8} \Delta_i \sum_{i=0}^{8} C_i)]P_9 + [det(\sum_{i=0}^{10} \Delta_i \sum_{i=0}^{10} C_i) - det\ (\sum_{i=0}^{9} \Delta_i \sum_{i=0}^{9} C_i)]P_{10} + [det(\sum_{i=0}^{11} \Delta_i \sum_{i=0}^{11} C_i) - det\ (\sum_{i=0}^{10} \Delta_i \sum_{i=0}^{10} C_i)]P_{11} + [det(\sum_{i=0}^{12} \Delta_i \sum_{i=0}^{12} C_i) - det\ (\sum_{i=0}^{11} \Delta_i \sum_{i=0}^{11} C_i)]P_{12} + [det(\sum_{i=0}^{13} \Delta_i \sum_{i=0}^{13} C_i) - det\ (\sum_{i=0}^{12} \Delta_i \sum_{i=0}^{12} C_i)]P_{13} + [det(\sum_{i=0}^{14} \Delta_i \sum_{i=0}^{14} C_i) - det\ (\sum_{i=0}^{13} \Delta_i \sum_{i=0}^{13} C_i)]P_{14} + \left[det\left(\sum_{i=0}^{15} \Delta_i \sum_{i=0}^{15} C_i\right) - det\ (\sum_{i=0}^{14} \Delta_i \sum_{i=0}^{14} C_i)\right]P_{15} + \left[det(\sum_{i=0}^{16} \Delta_i \sum_{i=0}^{16} C_i) - det\ \left(\sum_{i=0}^{15} \Delta_i \sum_{i=0}^{15} C_i\right)\right]P_{16} + [det(\sum_{i=0}^{17} \Delta_i \sum_{i=0}^{17} C_i) - det\ (\sum_{i=0}^{16} \Delta_i \sum_{i=0}^{16} C_i)]P_{17} = det(\Delta_0)det(C_0) + \left[det\left(\sum_{i=0}^{j} \Delta_i\right).det\left(\sum_{i=0}^{j} C_i\right) - det(\Delta).det\left(\sum_{i=1}^{j-1} C_{i-1}\right)\right]P_i = det(\Delta)det(C); 1 \leq j \leq 17.$

Proof of theorem5.

Δ is orthogonal if and only if $\Delta^t = \Delta^{-1}$, hence:

${\Delta_0}^t + \sum_{i=1}^{15} {\Delta_i}^t P_i = {\Delta_0}^{-1} + \left[(\Delta)^{-1} - {\Delta_0}^{-1}\right]P_1 + [(\sum_{i=0}^{2} \Delta_i)^{-1} - (\sum_{i=0}^{1} \Delta_i)^{-1}]P_2 + [(\sum_{i=1}^{3} `\Delta_i)^{-1} - (\sum_{i=0}^{2} \Delta_i)^{-1}]P_3 + [(\sum_{i=1}^{4} \Delta_i)^{-1} - (\sum_{i=0}^{3} \Delta_i)^{-1}]P_4 + [(\Delta)^{-1} - (\sum_{i=0}^{4} \Delta_i)^{-1}]P_5 + [(\sum_{i=1}^{6} \Delta_i)^{-1} - (\sum_{i=0}^{5} \Delta_i)^{-1}]P_6 + [(\sum_{i=1}^{7} \Delta_i)^{-1} - (\sum_{i=0}^{6} \Delta_i)^{-1}]P_7 + [(\sum_{i=1}^{8} \Delta_i)^{-1} - (\sum_{i=0}^{7} \Delta_i)^{-1}]P_8 + [(\sum_{i=1}^{9} \Delta_i)^{-1} - (\sum_{i=0}^{8} \Delta_i)^{-1}]P_9 + [(\sum_{i=1}^{10} \Delta_i)^{-1} - (\sum_{i=0}^{9} \Delta_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} \Delta_i)^{-1} - (\sum_{i=0}^{10} \Delta_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} \Delta_i)^{-1} - (\sum_{i=0}^{11} \Delta_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} \mu_i)^{-1} - (\sum_{i=0}^{12} \Delta_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} \Delta_i)^{-1} - (\sum_{i=0}^{13} \Delta_i)^{-1}]P_{14} + [(\sum_{i=1}^{15} \Delta_i)^{-1} - (\sum_{i=0}^{14} \Delta_i)^{-1}]P_{15} + [(\sum_{i=1}^{16} \Delta_i)^{-1} - (\sum_{i=0}^{15} \Delta_i)^{-1}]P_{16} + [(\sum_{i=1}^{17} \Delta_i)^{-1} - (\sum_{i=0}^{16} \Delta_i)^{-1}]P_{17}$, thus:

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

$$\begin{cases} {\Delta_0}^t = {\Delta_0}^{-1} \\ {\Delta_1}^t = \left(\sum_{i=0}^{1} \Delta_i\right)^{-1} - {\Delta_0}^{-1} \\ {\Delta_2}^t = \left(\sum_{i=0}^{2} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{1} \Delta_i\right)^{-1} \\ {\Delta_3}^t = \left(\sum_{i=0}^{3} \mu_i\right)^{-1} - \left(\sum_{i=0}^{2} \Delta_i\right)^{-1} \\ {\Delta_4}^t = \left(\sum_{i=0}^{4} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{3} \Delta_i\right)^{-1} \\ {\Delta_5}^t = \left(\sum_{i=0}^{5} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{4} \Delta_i\right)^{-1} \\ {\Delta_6}^t = \left(\sum_{i=0}^{6} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{5} \Delta_i\right)^{-1} \\ {\Delta_7}^t = \left(\sum_{i=0}^{7} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{6} \Delta_i\right)^{-1} \\ {\Delta_8}^t = \left(\sum_{i=0}^{8} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{7} \Delta_i\right)^{-1} \\ {\Delta_9}^t = \left(\sum_{i=0}^{9} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{8} \Delta_i\right)^{-1} \\ {\Delta_{10}}^t = \left(\sum_{i=0}^{10} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{9} \Delta_i\right)^{-1} \\ {\Delta_{11}}^t = \left(\sum_{i=0}^{11} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{10} \Delta_i\right)^{-1} \\ {\Delta_{12}}^t = \left(\sum_{i=0}^{12} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{11} \Delta_i\right)^{-1} \\ {\Delta_{13}}^t = \left(\sum_{i=0}^{13} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{12} \Delta_i\right)^{-1} \\ {\Delta_{14}}^t = \left(\sum_{i=0}^{14} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{13} \Delta_i\right)^{-1} \\ {\Delta_{15}}^t = \left(\sum_{i=0}^{15} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{14} \Delta_i\right)^{-1} \\ {\Delta_{16}}^t = \left(\sum_{i=0}^{16} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{15} \Delta_i\right)^{-1} \\ {\Delta_{17}}^t = \left(\sum_{i=0}^{17} \Delta_i\right)^{-1} - \left(\sum_{i=0}^{16} \Delta_i\right)^{-1} \end{cases}$$

This implies that:

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$$\begin{cases} \Delta_0{}^t = \Delta_0{}^{-1} \\ \sum_{i=0}^{1} \Delta_i{}^t = (\sum_{i=0}^{1} \Delta_i)^{-1} \\ \sum_{i=0}^{2} \Delta_i{}^t = (\sum_{i=0}^{2} \Delta_i)^{-1} \\ \sum_{i=0}^{3} \Delta_i{}^t = (\sum_{i=0}^{3} \Delta_i)^{-1} \\ \sum_{i=0}^{4} \Delta_i{}^t = (\sum_{i=0}^{4} \Delta_i)^{-1} \\ \sum_{i=0}^{5} \Delta_i{}^t = (\sum_{i=0}^{5} \Delta_i)^{-1} \\ \sum_{i=0}^{6} \Delta_i{}^t = (\sum_{i=0}^{6} \Delta_i)^{-1} \\ \sum_{i=0}^{7} \Delta_i{}^t = (\sum_{i=0}^{7} \Delta_i)^{-1} \\ \sum_{i=0}^{8} \Delta_i{}^t = (\sum_{i=0}^{8} \Delta_i)^{-1} \\ \sum_{i=0}^{9} \Delta_i{}^t = (\sum_{i=0}^{9} \Delta_i)^{-1}, \\ \sum_{i=0}^{10} \Delta_i{}^t = (\sum_{i=0}^{10} \Delta_i)^{-1} \\ \sum_{i=0}^{11} \Delta_i{}^t = (\sum_{i=0}^{11} \Delta)^{-1} \\ \sum_{i=0}^{12} \Delta_i{}^t = (\sum_{i=0}^{12} \Delta_i)^{-1} \\ \sum_{i=0}^{13} \Delta_i{}^t = (\sum_{i=0}^{13} \Delta_i)^{-1} \\ \sum_{i=0}^{14} \Delta_i{}^t = (\sum_{i=0}^{14} \Delta_i)^{-1} \\ \sum_{i=0}^{15} \Delta_i{}^t = (\sum_{i=0}^{15} \Delta_i)^{-1} \\ \sum_{i=0}^{16} \Delta_i{}^t = (\sum_{i=0}^{16} \Delta_i)^{-1} \\ \sum_{i=0}^{17} \Delta_i{}^t = (\sum_{i=0}^{17} \Delta_i)^{-1} \end{cases}$$

3. Conclusion

In this work, we have found the algebraic properties of the symbolic 16-plithogenic/17-plithogenic matrices, where we have established many theorems that describe the algebraic behavior of these matrices, such as determinants, inverses, and eigenvalues. Also, the relationships between symbolic 16-plithogenic and 17-plithogenic matrices and their classical components are presented.

References

[1] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[2] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.

[3] Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.

[4] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and TheirAlgebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.

[5] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[6] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.

[7] Alfahal, A.; Alhasan, Y.; Abdulfatah, R.; Mehmood, A.; Kadhim, M. On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties. *Int. J. Neutrosophic Sci.* **2023**, *21*.

[8] Merkepci, H., "On Novel Results about the Algebraic Properties of Symbolic 3-Plithogenic and 4-Plithogenic Real Square Matrices", Symmetry, MDPI, 2023.

[9] Soueycatt, M., Charchekhandra, B., Abu Hakmeh, R., " On The Algebraic Properties Of Symbolic 6-Plithogenic Integers", Neutrosophic Sets and Systems, vol. 59, 2023.

[10] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.

[11] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Symbolic 8-Plithogenic Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.

[12] Merkepci, M., and Abobala, M., " On Some Novel Results About Split-Complex Numbers, The Diagonalization Problem And Applications To Public Key Asymmetric Cryptography", Journal of Mathematics, Hindawi, 2023.

[13] Abobala, M., Hatip, A., and Bal, M., " A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol.17, 2021.

DOI: https://doi.org/10.54216/IJNS.230306
Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024

[14] Abobala, M. On Refined Neutrosophic Matrices and Their Applications in Refined Neutrosophic Algebraic Equations. *J. Math.* **2021**, *2021*, 5531093.
[15] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
[16] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of neutrosophic Science, Vol. 16, pp. 72-79, 2021.
[17] Soueycatt, M., Charchekhandra, B., Abu Hakmeh, R., " On The Foundations Of Symbolic 5-Plithogenic Number Theory", Neutrosophic Sets and Systems, vol. 59, 2023.
[18] Merkepci, H., and Rawashdeh, A., " On The Symbolic 2-Plithogenic Number Theory and Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.

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Received: August 17, 2023 Revised: November 26, 2023 Accepted: January 27, 2024