Some Operations on Trapezoidal Single Valued Neutrosophic Fuzzy Numbers

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Abstract

Almost all situations that arise in applied mathematics involve uncertainty, inconsistency, and indeterminacy. This can be simultaneously handled by the use of single valued neutrosophic fuzzy sets and single valued neutrosophic fuzzy numbers. In this paper, we propose the operations of addition, subtraction, and scalar multiplication on trapezoidal single valued neutrosophic fuzzy numbers. We introduce some component-wise interval operations on the union of closed bounded intervals. Then we show how this can be used to perform the proposed operations on trapezoidal single valued neutrosophic fuzzy numbers with the help of finite $\alpha-$ cuts, finite $\beta-$ cuts, finite $\gamma-$ cuts, and finite $\delta-$ cuts, which we define in this paper itself.

Keywords: Single valued neutrosophic fuzzy number; Trapezoidal single valued neutrosophic fuzzy number; Operations on Trapezoidal single valued neutrosophic fuzzy number.

1 Introduction

A thorough investigation of mathematical structures intended to manage imprecision and uncertainty can be found in the literature on fuzzy set theory. The research included in this literature is a broad and rich tapestry that has developed over many years, contributing significantly to a variety of disciplines including mathematics, artificial intelligence, computer science, decision-making, and control systems. Developed by Lotfi Zadeh[1] in the 1965, fuzzy set theory offers a paradigm-shifting framework for managing and modeling the vagueness, imprecision, and uncertainty that are inherent in real-world phenomena. A detailed introduction to fuzzy set theory that covered operations, fuzzy relations, and applications was given by Dubois D., and Prade H.[7] in 1980. Scholars expanded fuzzy set theory to solve a wide range of problems in the years that followed. By extending fuzzy sets to intuitionistic fuzzy sets in 1986, Atanassov's[5] work improved uncertainty modeling by adding a new degree of hesitancy. Neutrosophic sets, capable of dealing with inconsistency and indeterminacy, were first introduced by Smarandache's[6] work in 1998. The introduction of the notion of single-valued neutrosophic sets by Smarandache et al.[3] in 2010 made the framework of neutrosophic sets applicable to problems in science and engineering. Fuzzy sets were extended in 2020 by S. Das et al.[2] in the framework of neutrosophic sets to neutrosophic fuzzy sets to handle uncertain, inconsistent, and indeterminate problems. They defined the fuzzy membership grade using neutrosophic components. In neutrosophic fuzzy sets, the truth, falsity, and indeterminacy membership functions are related to each membership value. For better applications, they have then defined single valued neutrosophic fuzzy sets. In 2023, Ligin P. et al.[4] have defined single valued neutrosophic fuzzy numbers, trapezoidal single valued neutrosophic fuzzy numbers, and triangular single valued neutrosophic fuzzy numbers.

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The remainder of the article is structured as follows. The preliminary concepts needed for our work are included in the second section. In the third section, we define some component-wise interval operations on the union of closed bounded intervals using the usual interval operations. Some operations on trapezoidal single valued neutrosophic fuzzy numbers are defined in the fourth section. In the fifth section, we demonstrate how the component-wise interval operations on the union of closed bounded intervals can be used to perform the proposed operations on trapezoidal single valued neutrosophic fuzzy numbers. Concluding remarks are included at the end of the paper.

2 Preliminaries

Definition 2.1. [2]
Let X be a set of objects and A be a fuzzy set defined on X given by $A = \{(x, \mu_A(x))|x \in X, \mu_A : X \to [0,1]\}$. Then a Single Valued Neutrosophic Fuzzy Set A in X is defined by $A = \{(x, \mu_A(x), T_{\mu_A}(x), I_{\mu_A}(x), F_{\mu_A}(x))|x \in X, \mu_A(x) : X \to [0,1], T_{\mu_A}(x) : X \to [0,1], I_{\mu_A}(x) : X \to [0,1], F_{\mu_A}(x) : X \to [0,1]\}$

Here, each membership value is expressed with the truth($T_{\mu_A}(x)$), indeterminacy($I_{\mu_A}(x)$), and falsity($F_{\mu_A}(x)$) membership functions with their ranges lying in $[0,1]$.

Definition 2.2. [4]
A single valued neutrosophic fuzzy set A defined on $\mathbb{R}$ with fuzzy, truth, indeterminacy, and falsity membership functions $\mu_A, T_{\mu_A}, I_{\mu_A}$ and F_{μ_A} respectively is called a trapezoidal single valued neutrosophic fuzzy number if

$$\mu_A(x) = \begin{cases} \frac{x-a_\mu}{b_\mu - a_\mu} & \text{for} \quad x \in [a_\mu, b_\mu] \\ 1 & \text{for} \quad x \in [b_\mu, c_\mu] \\ \frac{d_\mu - x}{d_\mu - c_\mu} & \text{for} \quad x \in [c_\mu, d_\mu] \\ 0 & \text{o.w} \end{cases} \quad T_{\mu_A}(x) = \begin{cases} \frac{w_T(x-a_T)+b_T-x}{b_T-a_T} & \text{for} \quad x \in [a_T, b_T] \\ \frac{w_T(x-c_T)+b_T-x}{b_T-c_T} & \text{for} \quad x \in [b_T, c_T] \\ 1 & \text{for} \quad x \in [c_T, d_T] \\ \frac{w_T(x-d_T)+e_T-x}{e_T-d_T} & \text{for} \quad x \in [d_T, e_T] \\ \frac{w_T(x-f_T)+e_T-x}{e_T-f_T} & \text{for} \quad x \in [e_T, f_T] \\ 1 & \text{o.w} \end{cases}$$

$$I_{\mu_A}(x) = \begin{cases} \frac{w_I(x-a_I)}{b_I-a_I} & \text{for} \quad x \in [a_I, b_I] \\ \frac{w_I(c_I-x)}{c_I-b_I} & \text{for} \quad x \in [b_I, c_I] \\ 0 & \text{for} \quad x \in [c_I, d_I] \\ \frac{w_I(x-d_I)}{e_I-d_I} & \text{for} \quad x \in [d_I, e_I] \\ \frac{w_I(f_I-x)}{f_I-e_I} & \text{for} \quad x \in [e_I, f_I] \\ 0 & \text{o.w} \end{cases} \quad F_{\mu_A}(x) = \begin{cases} \frac{w_F(x-a_F)}{b_F-a_F} & \text{for} \quad x \in [a_F, b_F] \\ \frac{w_F(c_F-x)}{c_F-b_F} & \text{for} \quad x \in [b_F, c_F] \\ 0 & \text{for} \quad x \in [c_F, d_F] \\ \frac{w_F(x-d_F)}{e_F-d_F} & \text{for} \quad x \in [d_F, e_F] \\ \frac{w_F(f_F-x)}{f_F-e_F} & \text{for} \quad x \in [e_F, f_F] \\ 0 & \text{o.w} \end{cases}$$

, where $a_\mu = b_T = b_I = b_F,\ d_\mu = e_T = e_I = e_F,\ 0 \leq w_T, w_I, w_F \leq 1, [b_\mu, c_\mu] \cap [c_T, d_T] \cap [c_I, d_I] \cap [c_F, d_F] \neq \phi$.

Remark 2.3. For a trapezoidal SVNFN we use the notation
$A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$, where $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$.

Theorem 2.4. [4] *Let A be a trapezoidal single valued neutrosophic fuzzy number given by* $A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$ *, where* $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$ *with fuzzy, truth, indeterminacy, and falsity membership functions* $\mu_A, T_{\mu_A}, I_{\mu_A}$ *and* F_{μ_A} *respectively.*
Let ${}^\alpha A$, ${}^\beta A$, ${}^\gamma A$ *and* ${}^\delta A$ *denote the* $\alpha-$ *cut,* $\beta-$ *cut,* $\gamma-$ *cut and* $\delta-$ *cut of A.*
Then we have

$${}^\alpha A = [{}^\alpha A_1(\alpha),\ {}^\alpha A_2(\alpha)],$$

$${}^\beta A = \begin{cases} (-\infty,\ \infty) & ,\quad \beta \in [0, w_T] \\ (-\infty,\ {}^\beta A_1(\beta)] \cup [{}^\beta A_2(\beta),\ {}^\beta A_3(\beta)] \cup [{}^\beta A_4(\beta),\ \infty) & ,\quad \beta \in (w_T, 1] \end{cases},$$

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$$^{\gamma}A = \begin{cases} (-\infty,\ \infty) & ,\quad \gamma \in [w_I, 1] \\ (-\infty,\ {}^{\gamma}A_1(\gamma)] \cup [{}^{\gamma}A_2(\gamma),\ {}^{\gamma}A_3(\gamma)] \cup [{}^{\gamma}A_4(\gamma),\ \infty) & ,\quad \gamma \in [0, w_I) \end{cases},$$

$$^{\delta}A = \begin{cases} (-\infty,\ \infty) & ,\quad \delta \in [w_F, 1] \\ (-\infty,\ {}^{\delta}A_1(\delta)] \cup [{}^{\delta}A_2(\delta),\ {}^{\delta}A_3(\delta)] \cup [{}^{\delta}A_4(\delta),\ \infty) & ,\quad \delta \in [0, w_F) \end{cases}$$

where ${}^{\alpha}A_1(\alpha),\ {}^{\beta}A_2(\beta),\ {}^{\beta}A_4(\beta),\ {}^{\gamma}A_1(\gamma),\ {}^{\gamma}A_3(\gamma),\ {}^{\delta}A_1(\delta),\ {}^{\delta}A_3(\delta)$ *are monotonic increasing functions and* ${}^{\alpha}A_2(\alpha),\ {}^{\beta}A_1(\beta),\ {}^{\beta}A_3(\beta),\ {}^{\gamma}A_2(\gamma),\ {}^{\gamma}A_4(\gamma),\ {}^{\delta}A_2(\delta),\ {}^{\delta}A_4(\delta)$ *are monotonic decreasing functions.*

3 Some Component-wise Interval Operations on Union of Closed Intervals

This section introduces the definition of component-wise interval operations of addition, subtraction, and scalar multiplication on the union of closed bounded intervals of the form $[\alpha, \beta] \cup [\gamma, \delta] \cup [\zeta, \eta], (\alpha \le \beta < \gamma \le \delta < \zeta \le \eta)$ using the usual interval operations.

Definition 3.1. Let A and B be two unions of closed bounded intervals given by $A = [\alpha_1, \beta_1] \cup [\gamma_1, \delta_1] \cup [\zeta_1, \eta_1]$ and $B = [\alpha_2, \beta_2] \cup [\gamma_2, \delta_2] \cup [\zeta_2, \eta_2]$. Then we define $A + B$, $A - B$, and $k.A (k \in \mathbb{R}/\{0\})$ as given below.
$A + B = [\alpha_1 + \alpha_2, \beta_1 + \beta_2] \cup [\gamma_1 + \gamma_2, \delta_1 + \delta_2] \cup [\zeta_1 + \zeta_2, \eta_1 + \eta_2]$
$A - B = [\alpha_1 - \eta_2, \beta_1 - \zeta_2] \cup [\gamma_1 - \delta_2, \delta_1 - \gamma_2] \cup [\zeta_1 - \beta_2, \eta_1 - \alpha_2]$
$$k.A = \begin{cases} [k\eta_1, k\zeta_1] \cup [k\delta_1, k\gamma_1] \cup [k\beta_1, k\alpha_1] & ,\quad \text{if } k < 0 \\ [k\alpha_1, k\beta_1] \cup [k\gamma_1, k\delta_1] \cup [k\zeta_1, k\eta_1] & ,\quad \text{if } k > 0 \end{cases}$$

4 Some Operations on Trapezoidal Single valued Neutrosophic Fuzzy Numbers

In this section, we define the operations of addition, subtraction, and scalar multiplication on Trapezoidal SVNFNs. But before defining the said operations, we will define compatible trapezoidal SVNFNs.

Definition 4.1. Two Trapezoidal SVNFNs $A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$, where $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$ and $A' =< (a_{\mu'}, b_{\mu'}, c_{\mu'}, d_{\mu'}), (a_{T'}, b_{T'}, c_{T'}, d_{T'}, e_{T'}, f_{T'}; w_{T'}), (a_{I'}, b_{I'}, c_{I'}, d_{I'}, e_{I'}, f_{I'}; w_{I'}), (a_{F'}, b_{F'}, c_{F'}, d_{F'}, e_{F'}, f_{F'}; w_{F'}) >$, where $a_{\mu'} = b_{T'} = b_{I'} = b_{F'}, d_{\mu'} = e_{T'} = e_{I'} = e_{F'}$ are said to be compatible if $w_T = w_{T'}, w_I = w_{I'}, w_F = w_{F'}$.

Definition 4.2. Let A and A' be two compatible trapezoidal SVNFNs given by $A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$, where $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$ and $A' =< (a_{\mu'}, b_{\mu'}, c_{\mu'}, d_{\mu'}), (a_{T'}, b_{T'}, c_{T'}, d_{T'}, e_{T'}, f_{T'}; w_{T'}), (a_{I'}, b_{I'}, c_{I'}, d_{I'}, e_{I'}, f_{I'}; w_{I'}), (a_{F'}, b_{F'}, c_{F'}, d_{F'}, e_{F'}, f_{F'}; w_{F'}) >$, where $a_{\mu'} = b_{T'} = b_{I'} = b_{F'}, d_{\mu'} = e_{T'} = e_{I'} = e_{F'}, w_T = w_{T'}, w_I = w_{I'}, w_F = w_{F'}$. Then we define the trapezoidal SVNFNs $A + A'$ and $A - A'$ by

$A + A' =< (a_\mu + a_{\mu'}, b_\mu + b_{\mu'}, c_\mu + c_{\mu'}, d_\mu + d_{\mu'}), (a_T + a_{T'}, b_T + b_{T'}, c_T + c_{T'}, d_T + d_{T'}, e_T + e_{T'}, f_T + f_{T'}; w_T), (a_I + a_{I'}, b_I + b_{I'}, c_I + c_{I'}, d_I + d_{I'}, e_I + e_{I'}, f_I + f_{I'}; w_I), (a_F + a_{F'}, b_F + b_{F'}, c_F + c_{F'}, d_F + d_{F'}, e_F + e_{F'}, f_F + f_{F'}; w_F) >$ and

$A - A' =< (a_\mu - d_{\mu'}, b_\mu - c_{\mu'}, c_\mu - b_{\mu'}, d_\mu - a_{\mu'}), (a_T - f_{T'}, b_T - e_{T'}, c_T - d_{T'}, d_T - c_{T'}, e_T - b_{T'}, f_T - a_{T'}; w_T), (a_I - f_{I'}, b_I - e_{I'}, c_I - d_{I'}, d_I - c_{I'}, e_I - b_{I'}, f_I - a_{I'}; w_I), (a_F - f_{F'}, b_F - e_{F'}, c_F - d_{F'}, d_F - c_{F'}, e_F - b_{F'}, f_F - a_{F'}; w_F) >$

Remark 4.3. It is easy to see that the two trapezoidal SVNFNs $A + A'$ and $A - A'$ defined above are indeed trapezoidal SVNFNs.
For, we have $a_\mu < b_\mu < c_\mu < d_\mu$ and $a_{\mu'} < b_{\mu'} < c_{\mu'} < d_{\mu'}$. Therefore, $a_\mu + a_{\mu'} < b_\mu + b_{\mu'} < c_\mu + c_{\mu'} < d_\mu + d_{\mu'}$. Also, we have $a_T < b_T < c_T < d_T < e_T < f_T$ and $a_{T'} < b_{T'} < c_{T'} < d_{T'} < e_{T'} < f_{T'}$. Therefore, $a_T + a_{T'} < b_T + b_{T'} < c_T + c_{T'} < d_T + d_{T'} < e_T + e_{T'} < f_T + f_{T'}$.
Similarly, $a_I + a_{I'} < b_I + b_{I'} < c_I + c_{I'} < d_I + d_{I'} < e_I + e_{I'} < f_I + f_{I'}$ and $a_F + a_{F'} < b_F + b_{F'} < c_F + c_{F'} < d_F + d_{F'} < e_F + e_{F'} < f_F + f_{F'}$.

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Also, we have $a_\mu = b_T = b_I = b_F,\ d_\mu = e_T = e_I = e_F$ and $a_{\mu'} = b_{T'} = b_{I'} = b_{F'},\ d_{\mu'} = e_{T'} = e_{I'} = e_{F'}$. Therefore, $a_\mu + a_{\mu'} = b_T + b_{T'} = b_I + b_{I'} = b_F + b_{F'}$ and $d_\mu + d_{\mu'} = e_T + e_{T'} = e_I + e_{I'} = e_F + e_{F'}$.

Similarly, $[b_\mu, c_\mu] \cap [c_T, d_T] \cap [c_I, d_I] \cap [c_F, d_F] \neq \phi$ and $[b_{\mu'}, c_{\mu'}] \cap [c_{T'}, d_{T'}] \cap [c_{I'}, d_{I'}] \cap [c_{F'}, d_{F'}] \neq \phi$. Therefore, $[b_\mu + b_{\mu'}, c_\mu + c_{\mu'}] \cap [c_T + c_{T'}, d_T + d_{T'}] \cap [c_I + c_{I'}, d_I + d_{I'}] \cap [c_F + c_{F'}, d_F + d_{F'}] \neq \phi$.

Thus, we conclude that $A + A'$ is a trapezoidal SVNFN.

On the other hand, we have $a_\mu < b_\mu < c_\mu < d_\mu$ and $-d_{\mu'} < -c_{\mu'} < -b_{\mu'} < -a_{\mu'}$. Hence, $a_\mu - d_{\mu'} < b_\mu - c_{\mu'} < c_\mu - b_{\mu'} < d_\mu - a_{\mu'}$.
Also, we have $a_T < b_T < c_T < d_T < e_T < f_T$ and $-f_{T'} < -e_{T'} < -d_{T'} < -c_{T'} < -b_{T'} < -a_{T'}$. Therefore, $a_T - f_{T'} < b_T - e_{T'} < c_T - d_{T'} < d_T - c_{T'} < e_T - b_{T'} < f_T - a_{T'}$.
Similarly, $a_I - f_{I'} < b_I - e_{I'} < c_I - d_{I'} < d_I - c_{I'} < e_I - b_{I'} < f_I - a_{I'}$ and $a_F - f_{F'} < b_F - e_{F'} < c_F - d_{F'} < d_F - c_{F'} < e_F - b_{F'} < f_F - a_{F'}$.

Also, $a_\mu = b_T = b_I = b_F,\ d_\mu = e_T = e_I = e_F$ and $a_{\mu'} = b_{T'} = b_{I'} = b_{F'},\ d_{\mu'} = e_{T'} = e_{I'} = e_{F'}$. Therefore, $a_\mu - d_{\mu'} = b_T - e_{T'} = b_I - e_{I'} = b_F - e_{F'}$ and $d_\mu - a_{\mu'} = e_T - b_{T'} = e_I - b_{I'} = e_F - b_{F'}$.

Similarly, $[b_\mu, c_\mu] \cap [c_T, d_T] \cap [c_I, d_I] \cap [c_F, d_F] \neq \phi$ and $[b_{\mu'}, c_{\mu'}] \cap [c_{T'}, d_{T'}] \cap [c_{I'}, d_{I'}] \cap [c_{F'}, d_{F'}] \neq \phi$. Hence $[b_\mu - c_{\mu'}, c_\mu - b_{\mu'}] \cap [c_T - d_{T'}, d_T - c_{T'}] \cap [c_I - d_{I'}, d_I - c_{I'}] \cap [c_F - d_{F'}, d_F - c_{F'}] \neq \phi$.

Thus, we conclude that $A - A'$ is also a trapezoidal SVNFN.

Example 4.4. Consider two compatible trapezoidal SVNFNs A and A' given by

$A =< (55, 67, 79, 88), (45, 55, 65, 74, 88, 92; 0.6), (51, 55, 59, 71, 88, 92; 0.4), (47, 55, 69, 75, 88, 95; 0.45) >$ and $A' =< (11, 16, 22, 25), (9, 11, 17, 23, 25, 33; 0.6), (7, 11, 14, 20, 25, 29; 0.4), (5, 11, 17, 21, 25, 34; 0.45) >$. Then $A + A' =< (66, 83, 101, 113), (54, 66, 82, 97, 113, 125; 0.6), (58, 66, 73, 91, 113, 121; 0.4), (52, 66, 86, 96, 113, 129; 0.45) >$ and
$A - A' =< (30, 45, 63, 77), (12, 30, 42, 57, 77, 83; 0.6), (22, 30, 39, 57, 77, 85; 0.4), (13, 30, 48, 58, 77, 90; 0.45) >$.

Definition 4.5. Let A be any trapezoidal SVNFN given by $A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$, where $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$ and K be any non-zero scalar. Then we define the trapezoidal SVNFN $K.A$ by

$$K.A = \begin{cases} < (Ka_\mu, Kb_\mu, Kc_\mu, Kd_\mu), (Ka_T, Kb_T, Kc_T, Kd_T, Ke_T, Kf_T; w_T), \\ (Ka_I, Kb_I, Kc_I, Kd_I, Ke_I, Kf_I; w_I), (Ka_F, Kb_F, Kc_F, Kd_F, Ke_F, Kf_F; w_F) > & \text{, if } K > 0 \\ < (Kd_\mu, Kc_\mu, Kb_\mu, Ka_\mu), (Kf_T, Ke_T, Kd_T, Kc_T, Kb_T, Ka_T; w_T), \\ (Kf_I, Ke_I, Kd_I, Kc_I, Kb_I, Ka_I; w_I), (Kf_F, Ke_F, Kd_F, Kc_F, Kb_F, Ka_F; w_F) > & \text{, if } K < 0 \end{cases}$$

Remark 4.6. It is evident that the trapezoidal SVNFN $K.A$ specified above is, in fact, a trapezoidal SVNFN. For, if we take $K > 0$, then, since $a_\mu < b_\mu < c_\mu < d_\mu$, we obtain $Ka_\mu < Kb_\mu < Kc_\mu < Kd_\mu$. Similarly, we have $a_T < b_T < c_T < d_T < e_T < f_T$, and hence $Ka_T < Kb_T < Kc_T < Kd_T < Ke_T < Kf_T$. Similarly, we obtain $Ka_I < Kb_I < Kc_I < Kd_I < Ke_I < Kf_I$ and $Ka_F < Kb_F < Kc_F < Kd_F < Ke_F < Kf_F$.
Also, we have $a_\mu = b_T = b_I = b_F,\ d_\mu = e_T = e_I = e_F$ and hence $Ka_\mu = Kb_T = Kb_I = Kb_F,\ Kd_\mu = Ke_T = Ke_I = Ke_F$.
Similarly, since $[b_\mu, c_\mu] \cap [c_T, d_T] \cap [c_I, d_I] \cap [c_F, d_F] \neq \phi$, we obtain $[Kb_\mu, Kc_\mu] \cap [Kc_T, Kd_T] \cap [Kc_I, Kd_I] \cap [Kc_F, Kd_F] \neq \phi$.
Therefore, we conclude that $< (Ka_\mu, Kb_\mu, Kc_\mu, Kd_\mu), (Ka_T, Kb_T, Kc_T, Kd_T, Ke_T, Kf_T; w_T), (Ka_I, Kb_I, Kc_I, Kd_I, Ke_I, Kf_I; w_I), (Ka_F, Kb_F, Kc_F, Kd_F, Ke_F, Kf_F; w_F) >$ is indeed a trapezoidal SVNFN in this case.

If we take $K < 0$, then, since $a_\mu < b_\mu < c_\mu < d_\mu$, we obtain $Kd_\mu < Kc_\mu < Kb_\mu < Ka_\mu$. Similarly, we have $a_T < b_T < c_T < d_T < e_T < f_T$, and hence $Kf_T < Ke_T < Kd_T < Kc_T < Kb_T < Ka_T$. Similarly, we obtain $Kf_I < Ke_I < Kd_I < Kc_I < Kb_I < Ka_I$ and $Kf_F < Ke_F < Kd_F < Kc_F < Kb_F < Ka_F$.
Also, we have $a_\mu = b_T = b_I = b_F,\ d_\mu = e_T = e_I = e_F$ and hence $Ka_\mu = Kb_T = Kb_I = Kb_F,\ Kd_\mu = Ke_T = Ke_I = Ke_F$.
Similarly, since $[b_\mu, c_\mu] \cap [c_T, d_T] \cap [c_I, d_I] \cap [c_F, d_F] \neq \phi$, we obtain $[Kc_\mu, Kb_\mu] \cap [Kd_T, Kc_T] \cap [Kd_I, Kc_I] \cap$

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$[Kd_F, Kc_F] \neq \phi$.
Therefore, in this case also, we conclude that $< (Kd_\mu, Kc_\mu, Kb_\mu, Ka_\mu), (Kf_T, Ke_T, Kd_T, Kc_T, Kb_T, Ka_T; w_T), (Kf_I, Ke_I, Kd_I, Kc_I, Kb_I, Ka_I; w_I), (Kf_F, Ke_F, Kd_F, Kc_F, Kb_F, Ka_F; w_F) >$ is indeed a trapezoidal SVNFN.

Example 4.7. Consider a trapezoidal SVNFN A given by
$A =< (55, 67, 79, 88), (45, 55, 65, 74, 88, 92; 0.6), (51, 55, 59, 71, 88, 92; 0.4), (47, 55, 69, 75, 88, 95; 0.45) >$.
Let $K = 3$. Then, $K.A =< (165, 201, 237, 264), (135, 165, 195, 222, 264, 276; 0.6), (153, 165, 177, 213, 264, 276; 0.4), (141, 165, 207, 225, 264, 285; 0.45) >$.
If $K = -3$, then $K.A =< (-264, -237, -201, -165), (-276, -264, -222, -195, -165, -135; 0.6), (-276, -264, -213, -177, -165, -153; 0.4), (-285, -264, -225, -207, -165, -141; 0.45) >$.

5 Operations on Trapezoidal Single valued Neutrosophic Fuzzy Numbers using Component-wise Interval Operations on Union of Closed Bounded Intervals

In this section, we will define the arithmetic operations on trapezoidal SVNFNs using component-wise interval operations on the union of closed bounded intervals applied to its $\alpha-$ cut, $\beta-$ cut, $\gamma-$ cut, and $\delta-$ cut. For the application of component-wise interval operations on the union of closed bounded intervals, as mentioned, we will first introduce the concept of finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut of trapezoidal SVNFNs as follows.

Definition 5.1. Let A be a trapezoidal SVNFN given by $A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$, where $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$. Then the finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut of A denoted by ${}^\alpha\tilde{A}$, ${}^\beta\tilde{A}$, ${}^\gamma\tilde{A}$ and ${}^\delta\tilde{A}$ are defined by
${}^\alpha\tilde{A} = {}^\alpha A$, ${}^\beta\tilde{A} = {}^\beta A \cap [a_T, f_T]$, ${}^\gamma\tilde{A} = {}^\gamma A \cap [a_I, f_I]$ and ${}^\delta\tilde{A} = {}^\delta A \cap [a_F, f_F]$.

Remark 5.2. We note from the above definition that
${}^\alpha\tilde{A} = [a_\mu + \alpha(b_\mu - a_\mu), d_\mu - \alpha(d_\mu - c_\mu)]$,

$$
{}^\beta\tilde{A} = \begin{cases} [a_T, f_T] & ,\beta \leq w_T \\ [a_T, \frac{\beta(a_T-b_T)-w_T a_T+b_T}{1th-w_T}] \cup [\frac{\beta(c_T-b_T)-w_T c_T+b_T}{1-w_T}, \frac{\beta(d_T-e_T)-w_T d_T+e_T}{1-w_T}] \cup [\frac{\beta(f_T-e_T)-w_T f_T+e_T}{1-w_T}, f_T] & ,\beta > w_T \end{cases}
$$

$$
{}^\gamma\tilde{A} = \begin{cases} [a_I, f_I] & ,\gamma \geq w_I \\ [a_I, \frac{a_I w_I+\gamma(b_I-a_I)}{w_I}] \cup [\frac{c_I w_I-\gamma(c_I-b_I)}{w_I}, \frac{d_I w_I+\gamma(e_I-d_I)}{w_I}] \cup [\frac{f_I w_I-\gamma(f_I-e_I)}{w_I}, f_I] & ,\gamma < w_I \end{cases}
$$

$$
{}^\delta\tilde{A} = \begin{cases} [a_F, f_F] & ,\delta \geq w_F \\ [a_F, \frac{a_F w_F+\delta(b_F-a_F)}{w_F}] \cup [\frac{c_F w_F-\delta(c_F-b_F)}{w_F}, \frac{d_F w_F+\delta(e_F-d_F)}{w_F}] \cup [\frac{f_F w_F-\delta(f_F-e_F)}{w_F}, f_F] & ,\delta < w_F \end{cases}
$$

Theorem 5.3. *Let A and A' be two compatible trapezoidal SVNFNs with their finite $\alpha-$ cuts, finite $\beta-$ cuts, finite $\gamma-$ cuts, and finite $\delta-$ cuts given by ${}^\alpha\tilde{A}$, ${}^\beta\tilde{A}$, ${}^\gamma\tilde{A}$, ${}^\delta\tilde{A}$ and ${}^\alpha\tilde{A}'$, ${}^\beta\tilde{A}'$, ${}^\gamma\tilde{A}'$, ${}^\delta\tilde{A}'$ respectively. Let $*$ denote any of the two operations $(+)$ and $(-)$. Then the trapezoidal SVNFN $A * A'$ can be obtained by defining its finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut as ${}^\alpha\widetilde{A * A'} = {}^\alpha\tilde{A} * {}^\alpha\tilde{A}'$, ${}^\beta\widetilde{A * A'} = {}^\beta\tilde{A} * {}^\beta\tilde{A}'$, ${}^\gamma\widetilde{A * A'} = {}^\gamma\tilde{A} * {}^\gamma\tilde{A}'$ and ${}^\delta\widetilde{A * A'} = {}^\delta\tilde{A} * {}^\delta\tilde{A}'$. Here, the $*$ on the right-hand side denotes the corresponding component-wise interval operation on the union of closed bounded intervals.*

Proof. Let A and A' be two compatible trapezoidal SVNFNs given by $A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$, where $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$ and $A' =< (a_{\mu'}, b_{\mu'}, c_{\mu'}, d_{\mu'}), (a_{T'}, b_{T'}, c_{T'}, d_{T'}, e_{T'}, f_{T'}; w_{T'}), (a_{I'}, b_{I'}, c_{I'}, d_{I'}, e_{I'}, f_{I'}; w_{I'}), (a_{F'}, b_{F'}, c_{F'}, d_{F'}, e_{F'}, f_{F'}; w_{F'}) >$, where $a_{\mu'} = b_{T'} = b_{I'} = b_{F'}, d_{\mu'} = e_{T'} = e_{I'} = e_{F'}, w_T = w_{T'}, w_I = w_{I'}, w_F = w_{F'}$.

First we take $*$ as the operation $(+)$. Then we have
${}^\alpha\widetilde{A + A'} = {}^\alpha\tilde{A} + {}^\alpha\tilde{A}'$, ${}^\beta\widetilde{A + A'} = {}^\beta\tilde{A} + {}^\beta\tilde{A}'$, ${}^\gamma\widetilde{A + A'} = {}^\gamma\tilde{A} + {}^\gamma\tilde{A}'$ and ${}^\delta\widetilde{A + A'} = {}^\delta\tilde{A} + {}^\delta\tilde{A}'$.

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That is, ${}^{\alpha}\widetilde{A+A'} = [a_\mu + a_{\mu'} + \alpha(b_\mu + b_{\mu'} - a_\mu - a_{\mu'}), d_\mu + d_{\mu'} - \alpha(d_\mu + d_{\mu'} - c_\mu - c_{\mu'})]$,

${}^{\beta}\widetilde{A+A'} = [a_T + a_{T'}, f_T + f_{T'}]$, when $\beta \le w_T$ and

${}^{\beta}\widetilde{A+A'} = [a_T + a_{T'}, \frac{\beta(a_T-b_T+a_{T'}-b_{T'})-w_T(a_T+a_{T'})+b_T+b_{T'}}{1-w_T}] \cup [\frac{\beta(c_T-b_T+c_{T'}-b_{T'})-w_T(c_T+c_{T'})+b_T+b_{T'}}{1-w_T}, \frac{\beta(d_T-e_T+d_{T'}-e_{T'})-w_T(d_T+d_{T'})+e_T+e_{T'}}{1-w_T}] \cup [\frac{\beta(f_T-e_T+f_{T'}-e_{T'})-w_T(f_T+f_{T'})+e_T+e_{T'}}{1-w_T}, f_T + f_{T'}]$,when $\beta > w_T$,

${}^{\gamma}\widetilde{A+A'} = [a_I + a_{I'}, f_I + f_{I'}]$, when $\gamma \ge w_I$ and
${}^{\gamma}\widetilde{A+A'} = [a_I + a_{I'}, \frac{(a_I+a_{I'})w_I+\gamma(b_I-a_I+b_{I'}-a_{I'})}{w_I}] \cup [\frac{(c_I+c_{I'})w_I-\gamma(c_I-b_I+c_{I'}-b_{I'})}{w_I}, \frac{(d_I+d_{I'})w_I+\gamma(e_I-d_I+e_{I'}-d_{I'})}{w_I}] \cup [\frac{(f_I+f_{I'})w_I-\gamma(f_I-e_I+f_{I'}-e_{I'})}{w_I}, f_I + f_{I'}]$,when $\gamma < w_I$,

${}^{\delta}\widetilde{A+A'} = [a_F + a_{F'}, f_F + f_{F'}]$, when $\delta \ge w_F$ and
${}^{\delta}\widetilde{A+A'} = [a_F + a_{F'}, \frac{(a_F+a_{F'})w_F+\delta(b_F-a_F+b_{F'}-a_{F'})}{w_F}] \cup [\frac{(c_F+c_{F'})w_F-\delta(c_F-b_F+c_{F'}-b_{F'})}{w_F}, \frac{(d_F+d_{F'})w_F+\delta(e_F-d_F+e_{F'}-d_{F'})}{w_F}] \cup [\frac{(f_F+f_{F'})w_F-\delta(f_F-e_F+f_{F'}-e_{F'})}{w_F}, f_F + f_{F'}]$,when $\delta < w_F$.

The corresponding fuzzy, truth, falsity, and indeterminacy membership functions are obtained as

$$\mu_{A+A'}(x) = \begin{cases} \frac{x-(a_\mu+a_{\mu'})}{(b_\mu+b_{\mu'})-(a_\mu+a_{\mu'})} & \text{for} \quad x \in [a_\mu + a_{\mu'}, b_\mu + b_{\mu'}] \\ 1 & \text{for} \quad x \in [b_\mu + b_{\mu'}, c_\mu + c_{\mu'}] \\ \frac{(d_\mu+d_{\mu'})-x}{(d_\mu+d_{\mu'})-(c_\mu+c_{\mu'})} & \text{for} \quad x \in [c_\mu + c_{\mu'}, d_\mu + d_{\mu'}] \\ 0 & \text{o.w} \end{cases}$$

$$T_{\mu_{A+A'}}(x) = \begin{cases} \frac{w_T(x-(a_T+a_{T'}))+b_T+b_{T'}-x}{(b_T+b_{T'})-(a_T+a_{T'})} & \text{for} \quad x \in [a_T + a_{T'}, b_T + b_{T'}] \\ \frac{w_T(x-(c_T+c_{T'}))+b_T+b_{T'}-x}{(b_T+b_{T'})-(c_T+c_{T'})} & \text{for} \quad x \in [b_T + b_{T'}, c_T + c_{T'}] \\ 1 & \text{for} \quad x \in [c_T + c_{T'}, d_T + d_{T'}] \\ \frac{w_T(x-(d_T+d_{T'}))+e_T+e_{T'}-x}{(e_T+e_{T'})-(d_T+d_{T'})} & \text{for} \quad x \in [d_T + d_{T'}, e_T + e_{T'}] \\ \frac{w_T(x-(f_T+f_{T'}))+e_T+e_{T'}-x}{(e_T+e_{T'})-(f_T+f_{T'})} & \text{for} \quad x \in [e_T + e_{T'}, f_T + f_{T'}] \\ 1 & \text{o.w} \end{cases}$$

$$I_{\mu_{A+A'}}(x) = \begin{cases} \frac{w_I(x-(a_I+a_{I'}))}{(b_I+b_{I'})-(a_I+a_{I'})} & \text{for} \quad x \in [a_I + a_{I'}, b_I + b_{I'}] \\ \frac{w_I((c_I+c_{I'})-x)}{(c_I+c_{I'})-(b_I+b_{I'})} & \text{for} \quad x \in [b_I + b_{I'}, c_I + c_{I'}] \\ 0 & \text{for} \quad x \in [c_I + c_{I'}, d_I + d_{I'}] \\ \frac{w_I(x-(d_I+d_{I'}))}{(e_I+e_{I'})-(d_I+d_{I'})} & \text{for} \quad x \in [d_I + d_{I'}, e_I + e_{I'}] \\ \frac{w_I((f_I+f_{I'})-x)}{(f_I+f_{I'})-(e_I+e_{I'})} & \text{for} \quad x \in [e_I + e_{I'}, f_I + f_{I'}] \\ 0 & \text{o.w} \end{cases}$$

$$F_{\mu_{A+A'}}(x) = \begin{cases} \frac{w_F(x-(a_F+a_{F'}))}{(b_F+b_{F'})-(a_F+a_{F'})} & \text{for} \quad x \in [a_F + a_{F'}, b_F + b_{F'}] \\ \frac{w_F((c_F+c_{F'})-x)}{(c_F+c_{F'})-(b_F+b_{F'})} & \text{for} \quad x \in [b_F + b_{F'}, c_F + c_{F'}] \\ 0 & \text{for} \quad x \in [c_F + c_{F'}, d_F + d_{F'}] \\ \frac{w_F(x-(d_F+d_{F'}))}{(e_F+e_{F'})-(d_F+d_{F'})} & \text{for} \quad x \in [d_F + d_{F'}, e_F + e_{F'}] \\ \frac{w_F((f_F+f_{F'})-x)}{(f_F+f_{F'})-(e_F+e_{F'})} & \text{for} \quad x \in [e_F + e_{F'}, f_F + f_{F'}] \\ 0 & \text{o.w} \end{cases}$$

Hence we obtain, as in its definition, the trapezoidal SVNFN $A + A'$ as
$A + A' =< (a_\mu + a_{\mu'}, b_\mu + b_{\mu'}, c_\mu + c_{\mu'}, d_\mu + d_{\mu'}), (a_T + a_{T'}, b_T + b_{T'}, c_T + c_{T'}, d_T + d_{T'}, e_T + e_{T'}, f_T + f_{T'}; w_T), (a_I + a_{I'}, b_I + b_{I'}, c_I + c_{I'}, d_I + d_{I'}, e_I + e_{I'}, f_I + f_{I'}; w_I), (a_F + a_{F'}, b_F + b_{F'}, c_F + c_{F'}, d_F + d_{F'}, e_F + e_{F'}, f_F + f_{F'}; w_F) >$

Next, we take $*$ as the operation $(-)$. Then we have
${}^{\alpha}\widetilde{A-A'} = {}^{\alpha}\tilde{A} - {}^{\alpha}\tilde{A}'$, ${}^{\beta}\widetilde{A-A'} = {}^{\beta}\tilde{A} - {}^{\beta}\tilde{A}'$, ${}^{\gamma}\widetilde{A-A'} = {}^{\gamma}\tilde{A} - {}^{\gamma}\tilde{A}'$ and ${}^{\delta}\widetilde{A-A'} = {}^{\delta}\tilde{A} - {}^{\delta}\tilde{A}'$.

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That is, ${}^{\alpha}\widetilde{A-A'} = [a_\mu - d_{\mu'} + \alpha(b_\mu - a_\mu + d_{\mu'} - c_{\mu'}),\ d_\mu - a_{\mu'} + \alpha(c_\mu - d_\mu + a_{\mu'} - b_{\mu'})]$,

${}^{\beta}\widetilde{A-A'} = [a_T - f_{T'}, f_T - a_{T'}]$, if $\beta \le w_T$,
${}^{\beta}\widetilde{A-A'} = [a_T - f_{T'}, \frac{\beta(a_T-b_T+e_{T'}-f_{T'})+w_T(f_{T'}-a_T)+b_T-e_{T'}}{1-w_T}] \cup [\frac{\beta(c_T-b_T+e_{T'}-d_{T'})+w_T(d_{T'}-c_T)+b_T-e_{T'}}{1-w_T},$
$\frac{\beta(d_T-e_T+b_{T'}-e_{T'})+w_T(c_{T'}-d_T)+e_T-b_{T'}}{1-w_T}] \cup [\frac{\beta(f_T-e_T+b_{T'}-a_{T'})+w_T(a_{T'}-f_T)+e_T-b_{T'}}{1-w_T}, f_T - a_{T'}]$, if $\beta > w_T$,

${}^{\gamma}\widetilde{A-A'} = [a_I - f_{I'}, f_I - a_{I'}]$, if $\gamma \le w_I$,
${}^{\gamma}\widetilde{A-A'} = [a_I - f_{I'}, \frac{w_I(a_I-f_{I'})+\gamma(b_I-a_I+f_{I'}-e_{I'})}{w_I}] \cup$
$[\frac{w_I(c_I-d_{I'})-\gamma(c_I-b_I+e_{I'}-d_{I'})}{w_I}, \frac{w_I(d_I-c_{I'})+\gamma(e_I-d_I+c_{I'}-b_{I'})}{w_I}]$
$\cup [\frac{w_I(f_I-a_{I'})-\gamma(f_I-e_I+b_{I'}-a_{I'})}{w_I}, f_I - a_{I'}]$, if $\gamma > w_I$,

${}^{\delta}\widetilde{A-A'} = [a_F - f_{F'}, f_F - a_{F'}]$, if $\delta \le w_F$,
${}^{\delta}\widetilde{A-A'} = [a_F - f_{F'}, \frac{w_F(a_F-f_{F'})+\delta(b_F-a_F+f_{F'}-e_{F'})}{w_F}] \cup [\frac{w_F(c_F-d_{F'})-\delta(c_F-b_F+e_{F'}-d_{F'})}{w_F},$
$\frac{w_F(d_F-c_{F'})+\delta(e_F-d_F+c_{F'}-b_{F'})}{w_F}] \cup [\frac{w_F(f_F-a_{F'})-\delta(f_F-e_F+b_{F'}-a_{F'})}{w_F}, f_F - a_{F'}]$, if $\delta > w_F$

The corresponding fuzzy, truth, falsity and indeterminacy membership functions are obtained as

$$\mu_{A-A'}(x) = \begin{cases} \frac{x-(a_\mu-d_{\mu'})}{(b_\mu-c_{\mu'})-(a_\mu-d_{\mu'})} & \text{for} \quad x \in [a_\mu - d_{\mu'}, b_\mu - c_{\mu'}] \\ 1 & \text{for} \quad x \in [b_\mu - c_{\mu'}, c_\mu - b_{\mu'}] \\ \frac{(d_\mu-a_{\mu'})-x}{(d_\mu-a_{\mu'})-(c_\mu-b_{\mu'})} & \text{for} \quad x \in [c_\mu - b_{\mu'}, d_\mu - a_{\mu'}] \\ 0 & \text{o.w} \end{cases}$$

$$T_{\mu_{A-A'}}(x) = \begin{cases} \frac{w_T(x-(a_T-f_{T'}))+b_T-e_{T'}-x}{(b_T-e_{T'})-(a_T-f_{T'})} & \text{for} \quad x \in [a_T - f_{T'}, b_T - e_{T'}] \\ \frac{w_T(x-(c_T-d_{T'}))+b_T-e_{T'}-x}{(b_T-e_{T'})-(c_T-d_{T'})} & \text{for} \quad x \in [b_T - e_{T'}, c_T - d_{T'}] \\ 1 & \text{for} \quad x \in [c_T - d_{T'}, d_T - c_{T'}] \\ \frac{w_T(x-(d_T-c_{T'}))+e_T-b_{T'}-x}{(e_T-b_{T'})-(d_T-c_{T'})} & \text{for} \quad x \in [d_T - c_{T'}, e_T - b_{T'}] \\ \frac{w_T(x-(f_T-a_{T'}))+e_T-b_{T'}-x}{(e_T-b_{T'})-(f_T-a_{T'})} & \text{for} \quad x \in [e_T - b_{T'}, f_T - a_{T'}] \\ 1 & \text{o.w} \end{cases}$$

$$I_{\mu_{A-A'}}(x) = \begin{cases} \frac{w_I(x-(a_I-f_{I'}))}{(b_I-e_{I'})-(a_I-f_{I'})} & \text{for} \quad x \in [a_I - f_{I'}, b_I - e_{I'}] \\ \frac{w_I((c_I-d_{I'})-x)}{(c_I-d_{I'})-(b_I-e_{I'})} & \text{for} \quad x \in [b_I - e_{I'}, c_I - d_{I'}] \\ 0 & \text{for} \quad x \in [c_I - d_{I'}, d_I - c_{I'}] \\ \frac{w_I(x-(d_I-c_{I'}))}{(e_I-b_{I'})-(d_I-c_{I'})} & \text{for} \quad x \in [d_I - c_{I'}, e_I - b_{I'}] \\ \frac{w_I((f_I-a_{I'})-x)}{(f_I-a_{I'})-(e_I-b_{I'})} & \text{for} \quad x \in [e_I - b_{I'}, f_I - a_{I'}] \\ 0 & \text{o.w} \end{cases}$$

$$F_{\mu_{A-A'}}(x) = \begin{cases} \frac{w_F(x-(a_F-f_{F'}))}{(b_F-e_{F'})-(a_F-f_{F'})} & \text{for} \quad x \in [a_F - f_{F'}, b_F - e_{F'}] \\ \frac{w_F((c_F-d_{F'})-x)}{(c_F-d_{F'})-(b_F-e_{F'})} & \text{for} \quad x \in [b_F - e_{F'}, c_F - d_{F'}] \\ 0 & \text{for} \quad x \in [c_F - d_{F'}, d_F - c_{F'}] \\ \frac{w_F(x-(d_F-c_{F'}))}{(e_F-b_{F'})-(d_F-c_{F'})} & \text{for} \quad x \in [d_F - c_{F'}, e_F - b_{F'}] \\ \frac{w_F((f_F-a_{F'})-x)}{(f_F-a_{F'})-(e_F-b_{F'})} & \text{for} \quad x \in [e_F - b_{F'}, f_F - a_{F'}] \\ 0 & \text{o.w} \end{cases}$$

Hence we obtain, as in its definition, the trapezoidal SVNFN $A - A'$ as
$A - A' =< (a_\mu - d_{\mu'}, b_\mu - c_{\mu'}, c_\mu - b_{\mu'}, d_\mu - a_{\mu'}), (a_T - f_{T'}, b_T - e_{T'}, c_T - d_{T'}, d_T - c_{T'}, e_T - b_{T'}, f_T - a_{T'}; w_T), (a_I - f_{I'}, b_I - e_{I'}, c_I - d_{I'}, d_I - c_{I'}, e_I - b_{I'}, f_I - a_{I'}; w_I), (a_F - f_{F'}, b_F - e_{F'}, c_F - d_{F'}, d_F - c_{F'}, e_F - b_{F'}, f_F - a_{F'}; w_F) >$
Hence, the proof. □

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Example 5.4. Consider two trapezoidal SVNFNs A and A' with their fuzzy, truth, indeterminacy, and falsity membership functions given by

$$\mu_A(x)=\begin{cases}0.083x-4.583 & ,x\in[55,67]\\ 1 & ,x\in[67,79]\\ -0.111x+9.778 & ,x\in[79,88]\\ 0 & ,\text{o.w}\end{cases}\quad T_{\mu_A}(x)=\begin{cases}-0.04x+2.8 & ,x\in[45,55]\\ 0.04x-1.6 & ,x\in[55,65]\\ 1 & ,x\in[65,74]\\ -0.029x+3.114 & ,x\in[74,88]\\ 0.1x-8.2 & ,x\in[88,92]\\ 1 & ,\text{o.w}\end{cases}$$

$$I_{\mu_A}(x)=\begin{cases}0.1x-5.1 & ,x\in[51,55]\\ -0.1x+5.9 & ,x\in[55,59]\\ 0 & ,x\in[59,71]\\ 0.024x-1.671 & ,x\in[71,88]\\ -0.1x+9.2 & ,x\in[88,92]\\ 0 & ,\text{o.w}\end{cases}\quad F_{\mu_A}(x)=\begin{cases}0.056x-2.644 & ,x\in[47,55]\\ -0.032x+2.218 & ,x\in[55,69]\\ 0 & ,x\in[69,75]\\ 0.035x-2.596 & ,x\in[75,88]\\ -0.064x+6.107 & ,x\in[88,95]\\ 0 & ,\text{o.w}\end{cases}$$

and

$$\mu_{A'}(x)=\begin{cases}0.2x-2.2 & ,x\in[11,16]\\ 1 & ,x\in[16,22]\\ -0.33x+8.33 & ,x\in[22,25]\\ 0 & ,\text{o.w}\end{cases}\quad T_{\mu_{A'}}(x)=\begin{cases}-0.2x+2.8 & ,x\in[9,11]\\ 0.067x-0.133 & ,x\in[11,17]\\ 1 & ,x\in[17,23]\\ -0.2x+5.6 & ,x\in[23,25]\\ 0.05x-0.65 & ,x\in[25,33]\\ 1 & ,\text{o.w}\end{cases}$$

$$I_{\mu_{A'}}(x)=\begin{cases}0.1x-0.7 & ,x\in[7,11]\\ -0.133x+1.867 & ,x\in[11,14]\\ 0 & ,x\in[14,20]\\ 0.08x-1.6 & ,x\in[20,25]\\ -0.1x+2.9 & ,x\in[25,29]\\ 0 & ,\text{o.w}\end{cases}\quad F_{\mu_{A'}}(x)=\begin{cases}0.075x-0.375 & ,x\in[5,11]\\ -0.075x+1.275 & ,x\in[11,17]\\ 0 & ,x\in[17,21]\\ 0.113x-2.363 & ,x\in[21,25]\\ -0.05x+1.7 & ,x\in[25,34]\\ 0 & ,\text{o.w}\end{cases}$$

Then the finite $\alpha-$ cuts, finite $\beta-$ cuts, finite $\gamma-$ cuts and finite $\delta-$ cuts of A and A' are given by

$${}^{\alpha}\tilde{A}=\ [55+12\alpha,88-9\alpha]$$

$${}^{\beta}\tilde{A}=\begin{cases}[45,92] & ,\beta\leq 0.6\\ [45,70-25\beta]\cup[40+25\beta,109-35\beta]\cup[82+10\beta,92] & ,\beta>0.6\end{cases}$$

$${}^{\gamma}\tilde{A}=\begin{cases}[51,92] & ,\gamma\geq 0.4\\ [51,51+10\gamma]\cup[59-10\gamma,71+42.5\gamma]\cup[92-10\gamma,92] & ,\gamma<0.4\end{cases}$$

$${}^{\delta}\tilde{A}=\begin{cases}[47,95] & ,\delta\geq 0.45\\ [47,47+17.778\delta]\cup[69-31.11\delta,75+28.89\delta]\cup[95-15.56\delta,95] & ,\delta<0.45\end{cases}$$

and

$${}^{\alpha}\tilde{A}'=\ [11+5\alpha,25-3\alpha]$$

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$$^{\beta}\tilde{A}' = \begin{cases} [9,33] & ,\beta \leq 0.6 \\ [9,14-5\beta] \cup [2+15\beta, 28-5\beta] \cup [13+20\beta, 33] & ,\beta > 0.6 \end{cases}$$

$$^{\gamma}\tilde{A}' = \begin{cases} [7,29] & ,\gamma \geq 0.4 \\ [7,7+10\gamma] \cup [14-7.5\gamma, 20+12.5\gamma] \cup [29-10\gamma, 29] & ,\gamma < 0.4 \end{cases}$$

$$^{\delta}\tilde{A}' = \begin{cases} [5,34] & ,\delta \geq 0.45 \\ [5,5+13.33\delta] \cup [17-13.33\delta, 21+8.89\delta] \cup [34-20\delta, 34] & ,\delta < 0.45 \end{cases}$$

Hence, by our previous theorem and by the component-wise interval operations on the union of closed bounded intervals, we get the finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut of $A + A'$ as

$$^{\alpha}\widetilde{A+A'} = {}^{\alpha}\tilde{A} + {}^{\alpha}\tilde{A}' = [66+17\alpha, 113-12\alpha]$$

$$^{\beta}\widetilde{A+A'} = {}^{\beta}\tilde{A} + {}^{\beta}\tilde{A}' = \begin{cases} [54,125] & ,\beta \leq 0.6 \\ [54,84-30\beta] \cup [42+40\beta, 137-40\beta] \cup [95+30\beta, 125] & ,\beta > 0.6 \end{cases}$$

$$^{\gamma}\widetilde{A+A'} = {}^{\gamma}\tilde{A} + {}^{\gamma}\tilde{A}' = \begin{cases} [58,121] & ,\gamma \geq 0.4 \\ [58,58+20\gamma] \cup [73-17.5\gamma, 91+55\gamma] \cup [121-20\gamma, 121] & ,\gamma < 0.4 \end{cases}$$

$$^{\delta}\widetilde{A+A'} = {}^{\delta}\tilde{A} + {}^{\delta}\tilde{A}' = \begin{cases} [52,129] & ,\delta \geq 0.45 \\ \begin{array}{l} [52,52+31.108\delta] \cup [86-44.448\delta, 96+37.78\delta] \cup \\ [129-35.56\delta, 129] \end{array} & ,\delta < 0.45 \end{cases}$$

The corresponding fuzzy, truth, indeterminacy, and falsity membership functions of $A + A'$ are obtained as follows.

$$\mu_{A+A'}(x) = \begin{cases} 0.059x - 3.882 & ,x \in [66,83] \\ 1 & ,x \in [83,101] \\ -0.083x + 9.417 & ,x \in [101,113] \\ 0 & ,\text{o.w} \end{cases}$$

$$T_{\mu_{A+A'}}(x) = \begin{cases} -0.033x + 2.8 & ,x \in [54,66] \\ 0.025x - 1.05 & ,x \in [66,82] \\ 1 & ,x \in [82,97] \\ -0.025x + 3.425 & ,x \in [97,113] \\ 0.033x - 3.167 & ,x \in [113,125] \\ 1 & ,\text{o.w} \end{cases}$$

$$I_{\mu_{A+A'}}(x) = \begin{cases} 0.05x - 2.9 & ,x \in [58,66] \\ -0.057x + 4.171 & ,x \in [66,73] \\ 0 & ,x \in [73,91] \\ 0.018x - 1.655 & ,x \in [91,113] \\ -0.05x + 6.05 & ,x \in [113,121] \\ 0 & ,\text{o.w} \end{cases}$$

$$F_{\mu_{A+A'}}(x) = \begin{cases} 0.032x - 1.672 & ,x \in [52,66] \\ -0.022x + 1.935 & ,x \in [66,86] \\ 0 & ,x \in [86,96] \\ 0.026x - 2.541 & ,x \in [96,113] \\ -0.028x + 3.628 & ,x \in [113,129] \\ 0 & ,\text{o.w} \end{cases}$$

Similarly, we find the finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut and finite $\delta-$ cut of $A - A'$ as

$$^{\alpha}\widetilde{A-A'} = {}^{\alpha}\tilde{A} - {}^{\alpha}\tilde{A}' = [30+15\alpha, 77-14\alpha]$$

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$$^{\beta}\widetilde{A-A'} = {}^{\beta}\tilde{A} - {}^{\beta}\tilde{A}' = \begin{cases} [12,83] & ,\beta \leq 0.6 \\ [12,57-45\beta] \cup [12+30\beta, 107-50\beta] \cup [68+15\beta, 83] & ,\beta > 0.6 \end{cases}$$

$$^{\gamma}\widetilde{A-A'} = {}^{\gamma}\tilde{A} - {}^{\gamma}\tilde{A}' = \begin{cases} [22,85] & ,\gamma \geq 0.4 \\ [22,22+20\gamma] \cup [39-22.5\gamma, 57+50\gamma] \cup [85-20\gamma, 85] & ,\gamma < 0.4 \end{cases}$$

$$^{\delta}\widetilde{A-A'} = {}^{\delta}\tilde{A} - {}^{\delta}\tilde{A}' = \begin{cases} [13,90] & ,\delta \geq 0.45 \\ [13,13+37.778\delta] \cup [48-40\delta, 58+42.22\delta] \cup & \\ [90-28.89\delta, 90] & ,\delta < 0.45 \end{cases}$$

The corresponding fuzzy, truth, indeterminacy, and falsity membership functions of $A - A'$ are obtained as follows.

$$\mu_{A-A'}(x) = \begin{cases} 0.067x - 2 & ,x \in [30,45] \\ 1 & ,x \in [45,63] \\ -0.071x + 5.5 & ,x \in [63,77] \\ 0 & ,\text{o.w} \end{cases} \quad T_{\mu_{A-A'}}(x) = \begin{cases} -0.022x + 1.267 & ,x \in [12,30] \\ 0.033x - 0.4 & ,x \in [30,42] \\ 1 & ,x \in [42,57] \\ -0.02x + 2.14 & ,x \in [57,77] \\ 0.067x - 4.533 & ,x \in [77,83] \\ 1 & ,\text{o.w} \end{cases}$$

$$I_{\mu_{A-A'}}(x) = \begin{cases} 0.05x - 1.1 & ,x \in [22,30] \\ -0.044x + 1.733 & ,x \in [30,39] \\ 0 & ,x \in [39,57] \\ 0.02x - 1.14 & ,x \in [57,77] \\ -0.05x + 4.25 & ,x \in [77,85] \\ 0 & ,\text{o.w} \end{cases} \quad F_{\mu_{A-A'}}(x) = \begin{cases} 0.026x - 0.344 & ,x \in [13,30] \\ -0.025x + 1.2 & ,x \in [30,48] \\ 0 & ,x \in [48,58] \\ 0.024x - 1.374 & ,x \in [58,77] \\ -0.035x + 3.115 & ,x \in [77,90] \\ 0 & ,\text{o.w} \end{cases}$$

Theorem 5.5. *Let A be any trapezoidal SVNFN with its finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut given by ${}^{\alpha}\tilde{A}$, ${}^{\beta}\tilde{A}$, ${}^{\gamma}\tilde{A}$, and ${}^{\delta}\tilde{A}$ respectively. Let K be any non-zero scalar. Then the trapezoidal SVNFN $K.A$ can be obtained by defining its finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut as ${}^{\alpha}\widetilde{K.A} = K.{}^{\alpha}\tilde{A}$, ${}^{\beta}\widetilde{K.A} = K.{}^{\beta}\tilde{A}$, ${}^{\gamma}\widetilde{K.A} = K.{}^{\gamma}\tilde{A}$ and ${}^{\delta}\widetilde{K.A} = K.{}^{\delta}\tilde{A}$. Here, the '.' on the right-hand side of the equations denotes the component-wise scalar multiplication on the union of closed bounded intervals.*

Proof. Let A be any trapezoidal SVNFN given by $A =< (a_\mu, b_\mu, c_\mu, d_\mu), (a_T, b_T, c_T, d_T, e_T, f_T; w_T), (a_I, b_I, c_I, d_I, e_I, f_I; w_I), (a_F, b_F, c_F, d_F, e_F, f_F; w_F) >$, where $a_\mu = b_T = b_I = b_F, d_\mu = e_T = e_I = e_F$ and K be any non-zero scalar.
First, we assume that $K > 0$. Then,

$${}^{\alpha}\widetilde{K.A} = [Ka_\mu + \alpha(Kb_\mu - Ka_\mu), Kd_\mu - \alpha(Kd_\mu - Kc_\mu)],$$

$${}^{\beta}\widetilde{K.A} = \begin{cases} [Ka_T, Kf_T] & ,\beta \leq w_T \\ [Ka_T, \frac{\beta(Ka_T - Kb_T) - w_T Ka_T + Kb_T}{1-w_T}] \cup & \\ [\frac{\beta(Kc_T - Kb_T) - w_T Kc_T + Kb_T}{1-w_T}, \frac{\beta(Kd_T - Ke_T) - w_T Kd_T + Ke_T}{1-w_T}] \cup & ,\beta > w_T \\ [\frac{\beta(Kf_T - Ke_T) - w_T Kf_T + Ke_T}{1-w_T}, Kf_T] & \end{cases}$$

$${}^{\gamma}\widetilde{K.A} = \begin{cases} [Ka_I, Kf_I] & ,\gamma \geq w_I \\ [Ka_I, \frac{Ka_I w_I + \gamma(Kb_I - Ka_I)}{w_I}] \cup [\frac{Kc_I w_I - \gamma(Kc_I - Kb_I)}{w_I}, \frac{Kd_I w_I + \gamma(Ke_I - Kd_I)}{w_I}] \cup & \\ [\frac{Kf_I w_I - \gamma(Kf_I - Ke_I)}{w_I}, Kf_I] & ,\gamma < w_I \end{cases}$$

$${}^{\delta}\widetilde{K.A} = \begin{cases} [Ka_F, Kf_F] & ,\delta \geq w_F \\ [Ka_F, \frac{Ka_F w_F + \delta(Kb_F - Ka_F)}{w_F}] \cup [\frac{Kc_F w_F - \delta(Kc_F - Kb_F)}{w_F}, \frac{Kd_F w_F + \delta(Ke_F - Kd_F)}{w_F}] \cup & \\ [\frac{Kf_F w_F - \delta(Kf_F - Ke_F)}{w_F}, Kf_F] & ,\delta < w_F \end{cases}$$

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The corresponding fuzzy, truth, falsity, and indeterminacy membership functions are obtained as

$$\mu_{K.A}(x)=\begin{cases}\frac{x-Ka_\mu}{K(b_\mu-a_\mu)} & \text{for} \quad x\in[Ka_\mu,Kb_\mu]\\ 1 & \text{for} \quad x\in[Kb_\mu,Kc_\mu]\\ \frac{Kd_\mu-x}{K(d_\mu-c_\mu)} & \text{for} \quad x\in[Kc_\mu,Kd_\mu]\\ 0 & \text{o.w}\end{cases}$$

$$T_{\mu_{K.A}}(x)=\begin{cases}\frac{x(1-w_T)+K(w_Ta_T-b_T)}{K(a_T-b_T)} & \text{for} \quad x\in[Ka_T,Kb_T]\\ \frac{x(1-w_T)+K(w_Tc_T-b_T)}{K(c_T-b_T)} & \text{for} \quad x\in[Kb_T,Kc_T]\\ 1 & \text{for} \quad x\in[Kc_T,Kd_T]\\ \frac{x(1-w_T)+K(w_Td_T-e_T)}{K(d_T-e_T)} & \text{for} \quad x\in[Kd_T,Ke_T]\\ \frac{x(1-w_T)+K(w_Tf_T-e_T)}{K(f_T-e_T)} & \text{for} \quad x\in[Ke_T,Kf_T]\\ 1 & \text{o.w}\end{cases}$$

$$I_{\mu_{K.A}}(x)=\begin{cases}\frac{(x-Ka_I)w_I}{K(b_I-a_I)} & \text{for} \quad x\in[Ka_I,Kb_I]\\ \frac{(Kc_I-x)w_I}{K(c_I-b_I)} & \text{for} \quad x\in[Kb_I,Kc_I]\\ 0 & \text{for} \quad x\in[Kc_I,Kd_I]\\ \frac{(x-Kd_I)w_I}{K(e_I-d_I)} & \text{for} \quad x\in[Kd_I,Ke_I]\\ \frac{(Kf_I-x)w_I}{K(f_I-e_I)} & \text{for} \quad x\in[Ke_I,Kf_I]\\ 0 & \text{o.w}\end{cases}$$

$$F_{\mu_{K.A}}(x)=\begin{cases}\frac{(x-Ka_F)w_F}{K(b_F-a_F)} & \text{for} \quad x\in[Ka_F,Kb_F]\\ \frac{(Kc_F-x)w_F}{K(c_F-b_F)} & \text{for} \quad x\in[Kb_F,Kc_F]\\ 0 & \text{for} \quad x\in[Kc_F,Kd_F]\\ \frac{(x-Kd_F)w_F}{K(e_F-d_F)} & \text{for} \quad x\in[Kd_F,Ke_F]\\ \frac{(Kf_F-x)w_F}{K(f_F-e_F)} & \text{for} \quad x\in[Ke_F,Kf_F]\\ 0 & \text{o.w}\end{cases}$$

Hence we obtain, as in its definition, the trapezoidal SVNFN $K.A$ as
$K.A=<(Ka_\mu,Kb_\mu,Kc_\mu,Kd_\mu),(Ka_T,Kb_T,Kc_T,Kd_T,Ke_T,Kf_T;w_T),$
$(Ka_I,Kb_I,Kc_I,Kd_I,Ke_I,Kf_I;w_I),(Ka_F,Kb_F,Kc_F,Kd_F,Ke_F,Kf_F;w_F)>$ in this case.

Now, we assume that $K<0$. Then,

$$^{\alpha}\widetilde{K.A}=\ [Kd_\mu-\alpha(Kd_\mu-Kc_\mu),Ka_\mu+\alpha(Kb_\mu-Ka_\mu)],$$

$$^{\beta}\widetilde{K.A}=\begin{cases}[Kf_T,Ka_T] & ,\beta\le w_T\\ [Kf_T,\frac{\beta(Kf_T-Ke_T)-w_TKf_T+Ke_T}{1-w_T}]\cup & \\ [\frac{\beta(Kd_T-Ke_T)-w_TKd_T+Ke_T}{1-w_T},\frac{\beta(Kc_T-Kb_T)-w_TKc_T+Kb_T}{1-w_T}]\cup & ,\beta>w_T\\ [\frac{\beta(Ka_T-Kb_T)-w_TKa_T+Kb_T}{1-w_T},Ka_T] & \end{cases}$$

$$^{\gamma}\widetilde{K.A}=\begin{cases}[Kf_I,Ka_I] & ,\gamma\ge w_I\\ [Kf_I,\frac{Kf_Iw_I-\gamma(Kf_I-Ke_I)}{w_I}]\cup[\frac{Kd_Iw_I+\gamma(Ke_I-Kd_I)}{w_I},\frac{Kc_Iw_I-\gamma(Kc_I-Kb_I)}{w_I}]\cup & \\ [\frac{Ka_Iw_I+\gamma(Kb_I-Ka_I)}{w_I},Ka_I] & ,\gamma<w_I\end{cases}$$

$$^{\delta}\widetilde{K.A}=\begin{cases}[Kf_F,Ka_F] & ,\delta\ge w_F\\ [Kf_F,\frac{Kf_Fw_F-\delta(Kf_F-Ke_F)}{w_F}]\cup[\frac{Kd_Fw_F+\delta(Ke_F-Kd_F)}{w_F},\frac{Kc_Fw_F-\delta(Kc_F-Kb_F)}{w_F}]\cup & \\ [\frac{Ka_Fw_F+\delta(Kb_F-Ka_F)}{w_F},Ka_F] & ,\delta<w_F\end{cases}$$

The corresponding fuzzy, truth, falsity, and indeterminacy membership functions are obtained as

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$$\mu_{K.A}(x) = \begin{cases} \frac{Kd_\mu - x}{K(d_\mu - c_\mu)} & \text{for} \quad x \in [Kd_\mu, Kc_\mu] \\ 1 & \text{for} \quad x \in [Kc_\mu, Kb_\mu] \\ \frac{x - Ka_\mu}{K(b_\mu - a_\mu)} & \text{for} \quad x \in [Kb_\mu, Ka_\mu] \\ 0 & \text{o.w} \end{cases}$$

$$T_{\mu_{K.A}}(x) = \begin{cases} \frac{x(1-w_T)+K(w_T f_T - e_T)}{K(f_T - e_T)} & \text{for} \quad x \in [Kf_T, Ke_T] \\ \frac{x(1-w_T)+K(w_T d_T - e_T)}{K(d_T - e_T)} & \text{for} \quad x \in [Ke_T, Kd_T] \\ 1 & \text{for} \quad x \in [Kd_T, Kc_T] \\ \frac{x(1-w_T)+K(w_T c_T - b_T)}{K(c_T - b_T)} & \text{for} \quad x \in [Kc_T, Kb_T] \\ \frac{x(1-w_T)+K(w_T a_T - b_T)}{K(a_T - b_T)} & \text{for} \quad x \in [Kb_T, Ka_T] \\ 1 & \text{o.w} \end{cases}$$

$$I_{\mu_{K.A}}(x) = \begin{cases} \frac{(Kf_I - x)w_I}{K(f_I - e_I)} & \text{for} \quad x \in [Kf_I, Ke_I] \\ \frac{(x - Kd_I)w_I}{K(e_I - d_I)} & \text{for} \quad x \in [Ke_I, Kd_I] \\ 0 & \text{for} \quad x \in [Kd_I, Kc_I] \\ \frac{(Kc_I - x)w_I}{K(c_I - b_I)} & \text{for} \quad x \in [Kc_I, Kb_I] \\ \frac{(x - Ka_I)w_I}{K(b_I - a_I)} & \text{for} \quad x \in [Kb_I, Ka_I] \\ 0 & \text{o.w} \end{cases}$$

$$F_{\mu_{K.A}}(x) = \begin{cases} \frac{(Kf_F - x)w_F}{K(f_F - e_F)} & \text{for} \quad x \in [Kf_F, Ke_F] \\ \frac{(x - Kd_F)w_F}{K(e_F - d_F)} & \text{for} \quad x \in [Ke_F, Kd_F] \\ 0 & \text{for} \quad x \in [Kd_F, Kc_F] \\ \frac{(Kc_F - x)w_F}{K(c_F - b_F)} & \text{for} \quad x \in [Kc_F, Kb_F] \\ \frac{(x - Ka_F)w_F}{K(b_F - a_F)} & \text{for} \quad x \in [Kb_F, Ka_F] \\ 0 & \text{o.w} \end{cases}$$

Hence we obtain, in this case also, as in its definition, the trapezoidal SVNFN $K.A$ as
$K.A =< (Kd_\mu, Kc_\mu, Kb_\mu, Ka_\mu), (Kf_T, Ke_T, Kd_T, Kc_T, Kb_T, Ka_T; w_T),$
$(Kf_I, Ke_I, Kd_I, Kc_I, Kb_I, Ka_I; w_I), (Kf_F, Ke_F, Kd_F, Kc_F, Kb_F, Ka_F; w_F) >.$ □

Example 5.6. Consider a trapezoidal SVNFN A with its fuzzy, truth, indeterminacy, and falsity membership functions given by

$$\mu_A(x) = \begin{cases} 0.08x - 4.58 & , x \in [55, 67] \\ 1 & , x \in [67, 79] \\ -0.11x + 9.78 & , x \in [79, 88] \\ 0 & ,\text{o.w} \end{cases} \quad T_{\mu_A}(x) = \begin{cases} -0.04x + 2.8 & , x \in [45, 55] \\ 0.04x - 1.6 & , x \in [55, 65] \\ 1 & , x \in [65, 74] \\ -0.03x + 3.11 & , x \in [74, 88] \\ 0.1x - 8.2 & , x \in [88, 92] \\ 1 & ,\text{o.w} \end{cases}$$

$$I_{\mu_A}(x) = \begin{cases} 0.1x - 5.1 & , x \in [51, 55] \\ -0.1x + 5.9 & , x \in [55, 59] \\ 0 & , x \in [59, 71] \\ 0.02x - 1.67 & , x \in [71, 88] \\ -0.1x + 9.2 & , x \in [88, 92] \\ 0 & ,\text{o.w} \end{cases} \quad F_{\mu_A}(x) = \begin{cases} 0.06x - 2.64 & , x \in [47, 55] \\ -0.03x + 2.22 & , x \in [55, 69] \\ 0 & , x \in [69, 75] \\ 0.04x - 2.6 & , x \in [75, 88] \\ -0.06x + 6.11 & , x \in [88, 95] \\ 0 & ,\text{o.w} \end{cases}$$

Then the finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut and finite $\delta-$ cut of A is given by

$^{\alpha}\tilde{A} = \ [55 + 12\alpha, 88 - 9\alpha]$

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$$^{\beta}\tilde{A} = \begin{cases} [45,92] & ,\beta \leq 0.6 \\ [45, 70-25\beta] \cup [40+25\beta, 109-35\beta] \cup [82+10\beta, 92] & ,\beta > 0.6 \end{cases}$$

$$^{\gamma}\tilde{A} = \begin{cases} [51,92] & ,\gamma \geq 0.4 \\ [51, 51+10\gamma] \cup [59-10\gamma, 71+42.5\gamma] \cup [92-10\gamma, 92] & ,\gamma < 0.4 \end{cases}$$

$$^{\delta}\tilde{A} = \begin{cases} [47,95] & ,\delta \geq 0.45 \\ [47, 47+17.78\delta] \cup [69-31.12\delta, 75+28.89\delta] \cup [95-15.56\delta, 95] & ,\delta < 0.45 \end{cases}$$

Let $K = 3$. Then, by our previous theorem and by component-wise interval operations on the union of closed bounded intervals, we get the finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut of $K.A$ as

$$^{\alpha}\widetilde{K.A} = K.^{\alpha}\tilde{A} = [165+36\alpha, 264-27\alpha]$$

$$^{\beta}\widetilde{K.A} = K.^{\beta}\tilde{A} = \begin{cases} [135,276] & ,\beta \leq 0.6 \\ [135, 210-75\beta] \cup [120+75\beta, 327-105\beta] \cup [246+30\beta, 276] & ,\beta > 0.6 \end{cases}$$

$$^{\gamma}\widetilde{K.A} = K.^{\gamma}\tilde{A} = \begin{cases} [153,276] & ,\gamma \geq 0.4 \\ [153, 153+30\gamma] \cup [177-30\gamma, 213+127.5\gamma] \cup [276-30\gamma, 276] & ,\gamma < 0.4 \end{cases}$$

$$^{\delta}\widetilde{K.A} = K.^{\delta}\tilde{A} = \begin{cases} [141,285] & ,\delta \geq 0.45 \\ [141, 141+53.34\delta] \cup [207-93.36\delta, 225+86.67\delta] \cup [285-46.68\delta, 285] & ,\delta < 0.45 \end{cases}$$

The corresponding fuzzy, truth, indeterminacy, and falsity membership functions of $K.A$ are obtained as follows.

$$\mu_{K.A}(x) = \begin{cases} 0.03x-4.62 & ,x \in [165,201] \\ 1 & ,x \in [201,237] \\ -0.04x+9.78 & ,x \in [237,264] \\ 0 & ,\text{o.w} \end{cases} \quad T_{\mu_{K.A}}(x) = \begin{cases} -0.01x+2.8 & ,x \in [135,165] \\ 0.01x-1.6 & ,x \in [165,195] \\ 1 & ,x \in [195,222] \\ -0.01x+3.11 & ,x \in [222,264] \\ 0.03x-8.2 & ,x \in [264,276] \\ 1 & ,\text{o.w} \end{cases}$$

$$I_{\mu_{K.A}}(x) = \begin{cases} 0.03x-5.1 & ,x \in [153,165] \\ -0.03x+5.9 & ,x \in [165,177] \\ 0 & ,x \in [177,213] \\ 0.01x-1.67 & ,x \in [213,264] \\ -0.03x+9.2 & ,x \in [264,276] \\ 0 & ,\text{o.w} \end{cases} \quad F_{\mu_{K.A}}(x) = \begin{cases} 0.02x-2.64 & ,x \in [141,165] \\ -0.01x+2.22 & ,x \in [165,207] \\ 0 & ,x \in [207,225] \\ 0.01x-2.6 & ,x \in [225,264] \\ -0.02x+6.11 & ,x \in [264,285] \\ 0 & ,\text{o.w} \end{cases}$$

If $K = -3$, we find the finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut and finite $\delta-$ cut of $K.A$ as

$$^{\alpha}\widetilde{K.A} = K.^{\alpha}\tilde{A} = [-264+27\alpha, -165-36\alpha]$$

$$^{\beta}\widetilde{K.A} = K.^{\beta}\tilde{A} = \begin{cases} [-276,-135] & ,\beta \leq 0.6 \\ [-276, -246-30\beta] \cup [-327+105\beta, -120-75\beta] \cup [-210+75\beta, -135] & ,\beta > 0.6 \end{cases}$$

$$^{\gamma}\widetilde{K.A} = K.^{\gamma}\tilde{A} = \begin{cases} [-276,-153] & ,\gamma \geq 0.4 \\ [-276, -276+30\gamma] \cup [-213-127.5\gamma, -177+30\gamma] \cup [-153-30\gamma, -153] & ,\gamma < 0.4 \end{cases}$$

$$^{\delta}\widetilde{K.A} = K.^{\delta}\tilde{A} = \begin{cases} [-285,-141] & ,\delta \geq 0.45 \\ [-285, -285+46.68\delta] \cup [-225-86.67\delta, -207+93.36\delta] \cup \\ [-141-53.34\delta, -141] & ,\delta < 0.45 \end{cases}$$

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The corresponding fuzzy, truth, indeterminacy, and falsity membership functions of $K.A$ are obtained as follows.

$$\mu_{K.A}(x) = \begin{cases} 0.04x + 9.78 & , x \in [-264, -237] \\ 1 & , x \in [-237, -201] \\ -0.03x - 4.58 & , x \in [-201, -165] \\ 0 & ,\text{o.w} \end{cases} \quad T_{\mu_{K.A}}(x) = \begin{cases} -0.03x - 8.2 & , x \in [-276, -264] \\ 0.01x + 3.11 & , x \in [-264, -222] \\ 1 & , x \in [-222, -195] \\ -0.01x - 1.6 & , x \in [-195, -165] \\ 0.01x + 2.8 & , x \in [-165, -135] \\ 1 & ,\text{o.w} \end{cases}$$

$$I_{\mu_{K.A}}(x) = \begin{cases} 0.03x + 9.2 & , x \in [-276, -264] \\ -0.01x - 1.67 & , x \in [-264, -213] \\ 0 & , x \in [-213, -177] \\ 0.03x + 5.9 & , x \in [-177, -165] \\ -0.03x - 5.1 & , x \in [-165, -153] \\ 0 & ,\text{o.w} \end{cases}$$

$$F_{\mu_{K.A}}(x) = \begin{cases} 0.02x + 6.11 & , x \in [-285, -264] \\ -0.01x - 2.6 & , x \in [-264, -225] \\ 0 & , x \in [-225, -207] \\ 0.01x + 2.22 & , x \in [-207, -165] \\ -0.02x - 2.64 & , x \in [-165, -141] \\ 0 & ,\text{o.w} \end{cases}$$

6 Conclusion

In this paper, we have defined addition, subtraction, and scalar multiplication on trapezoidal SVNFNs, offering useful tools for practitioners in various fields. We have presented some component-wise interval operations on the union of closed bounded intervals. Further, we have demonstrated how this can be used to carry out the proposed operations on trapezoidal SVNFNs with the help of the finite $\alpha-$ cut, finite $\beta-$ cut, finite $\gamma-$ cut, and finite $\delta-$ cut which were defined in this paper.

The results of this research open up new avenues for development and make it easier to incorporate trapezoidal SVNFNs into practical uses. This study adds to the theoretical knowledge about trapezoidal SVNFNs and lays the foundation for researchers and practitioners to use these ideas to solve intricate issues that involve uncertainty, inconsistency, and indeterminacy. Essentially, this work adds to the current discussion on trapezoidal SVNFNs by providing insightful information primarily about its computational aspects, which can serve as a basis for future research in this fascinating and demanding area.

Declarations

Author contribution: Ligin P. Mathew studied and wrote the manuscript, and Lovelymol and Baiju Thankachan reviewed the article.

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